Absolute determination of the intensity of light from photocurrent statistics

D. N. Klyshko

M. V. Lomonosov State University, Moscow
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It is shown that a certain class of states of an electromagnetic field gives rise to a photocurrent the statistics of which carries information on the average radiation intensity and on the detector efficiency. This can be used to develop absolute (standard-free) photometry. Radiation statistics with the following properties is suitable for absolute calibration of photodetectors: the normalized factorial moments $g^{(k)}$ of the photon numbers n should depend on the average photon number $\bar{n}$ and the nature of this dependence should be known. It is shown that such "calibrating" radiation can be obtained from ordinary radiation by parametric scattering and two-photon absorption effects. Radiation from a laser operating at its threshold can also have the required properties. However, harmonic generation and many-photon detectors are of no interest in such photometry.

§1. INTRODUCTION

A method for absolute (standard-free) determination of the quantum efficiency η and of the photon flux by "two-photon light" consisting of separate pairs of practically simultaneously generated photons has been developed.[1,2] In this method, two-photon light is directed to two photon counters connected by a coincidence circuit. If a pulse appears at the output of one detector and not at the output of the other, it follows that the second detector has "missed" a photon. Consequently, if the counting time is sufficiently long, the ratio of the number of coincidence pulses m_c to the number of pulses in the first channel m_1 is equal to the quantum efficiency of the second detector: $\eta_2 = m_c/m_1$. This method gives also information on the number of photons which reach the detectors in the counting time ($n = m_1 m_2/m_c$) and makes it possible to construct a generator producing a selected number of photons. Two-photon light can also be used in calibration of analog detectors with a continuous output signal, image converters, and—in principle—photographic films. At first sight counting of a number of photons using a detector with an *a priori* unknown efficiency seems impossible and initially this method seems to be mysterious.

The purpose of the present paper is to present the general principle of the method and ways in which it can be modified. It is shown that in addition to two-photon light, it is possible to use other types of radiation for absolute calibration of detectors and also that two detectors are not essential for the purpose (and this is why the essence of the method will be considered dealing mainly with a one-detector system). It is found that a certain class of states of an electromagnetic field creates a photocurrent with the statistics that determine the average number of photons $\bar{n}$ which reach a detector during the sampling time T (here and later we shall consider continuous radiation and assume that all the quantities apply to a time interval T and a photocathode section A, i.e., n is the number of photons in the detection region $V_{\rm det} = cTA$).

It is known that there are certain distributions of the photon number P_n, including the most widely encountered Poisson and geometric distributions, which are not affected by the process of one-photon absorption. The distribution of the number of photocounts P'_m repeats the initial form of P_n on a modified scale: $\bar{m} = \eta\bar{n}$. In this case the photocurrent statistics provides no information on the value of $\bar{n}$. However, there are distributions (in particular, N-photon distributions) which are not invariant under one-photon absorption and in such cases the observed function P'_m carries full information on P_n, including $\bar{n}$ and other parameters of the distribution. Naturally, the form of the initial distribution should be known *a priori* and the degree of the change in the distribution should depend on $\bar{n}$.

Another simple example is two-photon light in a system with one detector. In the case of the one-photon photoeffect when $\bar{m} \ll 1$ the probabilities of single and double pulses are clearly of the following form:

$$P_1' = \eta(1-\eta)\bar{n}, \quad P_2' = \eta^2\bar{n}/2 \tag{1.1}$$

(whereas in the case of the initial radiation we have $P_1 = 0$ and $P_2 = \bar{n}/2$). Hence, we find that

$$\eta = (1 + P_1'/2P_2')^{-1}. \tag{1.2}$$

A more rigorous formulation of the requirements in respect of the statistics of radiation suitable for absolute photometry with one-photon detectors and the general scheme of a possible experimental procedure are given in the next section. Later, in §3 we shall show that the required radiation cannot be generated by doubling the frequency of the laser of the thermal radiation (with the Poisson and geometric distributions, respectively). Two-photon light will be discussed in greater detail in §4 and in the Appendix. In §5 we shall demonstrate that ordinary radiation passing through matter exhibiting two-photon absorption acquires properties essential for photometry. In §6 we shall consider laser radiation in the case when the excess above the threshold is not too large. The statistics of such radiation is, in accordance with the familiar model of Scully and Lamb, also not

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invariant under linear absorption. Finally, in §7 we shall discuss briefly the possibility of using the two-photon photoelectric effect.

§2. ABSOLUTE DETERMINATION OF THE NUMBER OF PHOTONS

We shall mention briefly the relationship beween the statistics of photons in the radiation incident on a detector and the statistics of photocounts at the detector output (for details see, for example, Refs. 3–6). For the sake of simplicity, we shall assume that the selected detector is of the one-photon and one-mode type, i.e., we shall assume that the volume of the detection region $V_{\rm det}$ is much less than the volume of the field coherence region $V_{\rm coh}$. In this approximation the process of detection involves "binomial" transformation of the distribution:

$$P_m' = \sum_{n=m}^{\infty} \binom{n}{m} \eta^m (1-\eta)^{n-m} P_n = \langle : \hat{m}^m \exp(-\hat{m}) : \rangle / m!, \qquad (2.1)$$

where the angular brackets [like the bar in Eq. (1.1)] denote averaging over the states of the incident field; $\hat{m} = \eta\hat{n}$ is the photocount number operator; $\hat{n}$ is the photon number operator for the detection region; the colon in the above equation denotes the operation of normal ordering. The same transformation describes also the change in the photon statistics as a result of linear attenuation (or amplification if the spontaneous radiation is ignored) of the field in a material with a transmission coefficient η (Ref. 6). The relevant equations are of the form[5–7]

$$dP_n/d\tau = (n+1)P_{n+1} - nP_n, \qquad (2.2)$$

where τ is proportional to the transition probability, to the density of atoms at the lower level (it is assumed that the upper level is empty), and to the layer thickness. Assuming that $\eta = \exp(-\tau)$, we can readily show that Eq. (2.1) is the solution of Eq. (2.2).

The fairly complex transformation of the distribution (2.1) corresponds to an elementary transformation of the generating function:

$$Q'(x) = Q(\eta x), \qquad (2.3)$$

where the generating function for the number of photocounts is defined as follows:

$$Q'(x) \equiv \sum_{m=0}^{\infty} (1+x)^m P_m' = \langle : \exp(x\hat{m}) : \rangle. \qquad (2.4)$$

It follows from Eq. (2.3) that the inverse transformation of Eq. (2.1) is of the same form, but with η replaced with $1/\eta$. Equation (2.1) then describes the change in the photon statistics under ideal amplification conditions when no additional noise is introduced. In particular, the distribution of Eq. (1.1) reduces to the initial two-photon distribution.

According to Eq. (2.4), the derivatives of the generating function at the points $x = 0$ and $x = -1$ determine the factorial moments and the probability, respectively. A simple relationship between the factorial moments for photons and photocounts follows from Eq. (2.3): $G^{(k)\prime} = \eta^k G^{(k)}$, so that the normalized factorial moments $g^{(k)} \equiv G^{(k)}/\bar{n}^k$ are invariant under linear attenuation: $g^{(k)\prime} = g^{(k)}$. Therefore, determination of the photocurrent statistics when $\bar{n}$ and η are not known can give information only on the relative quantities, viz., the normalized factorial moments of the photon distribution. The question now arises: how to obtain the absolute value of the average number of photons $\bar{n}$ from relative measurements?

We shall find the generating function for the normalized factorial moments and distributions: $\tilde{Q}(x) \equiv Q(x/\bar{n})$. According to Eq. (2.3), we have $\tilde{Q}'(x) = \tilde{Q}(x)$. The derivatives of $\tilde{Q}(x)$ at the point 0 and $-\bar{n}$ determine the normalized factorial moments $g^{(k)}$ and the distributions $\tilde{P}_n \equiv P_n/\bar{n}^n$. The function $\tilde{Q}(x)$ carries all the information on the photon distribution which can be obtained using a photon counter with an unknown value of η. Clearly, in absolute photometry we can use only the radiation for which $\tilde{Q}$ depends on $\bar{n}$ or on other parameters of the distribution governing $\bar{n}$: $\tilde{Q} = f(x, a_1, a_2, \ldots)$.

This is not true of the Poisson distribution, when we have

$$\tilde{Q}(x) = e^x, \quad g^{(k)} = 1, \quad \tilde{P}_n = e^{-\bar{n}}/n!, \qquad (2.5)$$

or in the case of the geometric distribution, which is characterized by

$$\tilde{Q}(x) = 1/(1-x), \quad g^{(k)} = k!, \quad \tilde{P}_n = 1/(1+\bar{n})^{1+n}. \qquad (2.6)$$

The nature (functional form) of these distributions does not change as a result of the transformation described by Eq. (2.1), which alters only the scale of the distribution ($\bar{n} \to \eta\bar{n}$).

The distributions (2.5) and (2.6) are governed by one parameter. A two-parameter distribution describing the sum of a coherent signal and Gaussian noise has the following generating function[4]:

$$Q(x, S, N) = (1-xN)^{-1} \exp[xS/(1-xN)]. \qquad (2.7)$$

In this case the process of linear absorption, i.e., the $x \to \eta x$ transformation of the argument, once again changes only the values of the parameters S and N by the factor η but leaves the form of the generating function unaltered. This applies also to the two-parameter Pascal distribution which describes the statistics of photons in M modes of thermal radiation.

$$Q(x, \bar{n}, M) = (1 - x\bar{n}/M)^{-M}. \qquad (2.8)$$

However, in some cases the form of the distribution is affected by linear absorption and this makes it possible to determine the absolute values of $\bar{n}$ and η. Let us assume that we know *a priori* the functional forms of the photon distribution and the corresponding generating function, both determined by a set of p parameters $a_1, \ldots, a_p$. The factorial moments are then certain known functions of these parameters:

$$\bar{n} = f_1(a_1, \ldots, a_p), \quad g^{(k)} = f_k(a_1, \ldots, a_p), \quad k \geqslant 2. \qquad (2.9)$$

Studies of the statistics of photocounts makes it possible, in principle, to determine (after elimination of the effects of the "dead time," the finite volume of the detection region, etc.) the probabilities P'_m and the moments $\overline{m^k}$. Let us assume that, for example, the moments $\overline{m}$, $\overline{m^2}$, ..., $\overline{m^{p+1}}$ are determined; their combinations give the normalized factorial moments $g^{(k)}$. As a result, we obtain a system of equations for the parameters of the photon distribution a_i:

$$f_k(a_1, \ldots, a_p) = g^{(k)}, \quad k=2, 3, \ldots, p+1. \tag{2.10}$$

If this system is complete, its solution can give complete information on the statistics of photons, including the average number $\bar{n}$ (without the use of calibrated energy meters!). Some examples of application of this procedure will be given later.

Determination of higher moments with $k > p+1$ can be used to check the initial hypothesis on the nature of the statistics of the radiation incident on a detector. The, the formula $\eta = \overline{m}/\bar{n}$ can be used to find the main detector characteristic which is its quantum efficiency. If the incident radiation is monochromatic and if calibrated frequency and time-interval meters are available, it is possible to determine also the average energy $U = \hbar\omega\bar{n}$ and the power U/T.

It should be stessed that the absolute determination of these parameters is possible essentially because of the quantum-optical effects. According to the semiclassical Mandel formula, the normalized factorial moments of photocounts are equal to the normalized ordinary moments for the energy distribution during the sampling time:

$$g^{(k)\prime} = \overline{U^k}/\overline{U}^k. \tag{2.11}$$

The dimensionless numbers $g^{(k)\prime}$ cannot give information on the dimensional quantity $\overline{U}$, irrespective of the density of the distribution $p(U)$.

§3. TRANSFORMATION OF PHOTON STATISTICS AS A RESULT OF FREQUENCY CONVERSION

Light with nontrivial statistics can be obtained from ordinary light with the aid of the nonlinear-optics effects, i.e., with the aid of many-photon processes. The occurrence of such processes in the atoms, molecules, or crystals, alters the statistical properties of the incident radiation[5,7–9] excites new field modes, and results in a correlation between photons belonging to modes of different frequencies.[10,11] It would be of interest to consider the possibility of utilizing photon bunching ($g^{(2)} > 1$) and antibunching ($g^{(2)} < 1$) effects in absolute photometry.

We shall first consider the effect of frequency doubling $\omega_0 + \omega_0 \to \omega$ in a transparent material. Using the approximation of constant single-mode pumping and the Heisenberg equations, we obtain the relationship $a = i\tau a_0^2$, where a and a_0 are the photon annihilation operators and τ is the amplitude conversion coefficient. Hence, using the equality $\langle a^{+k} a^k \rangle = G^{(k)}$, we obtain

$$G^{(k)} = \tau^{2k} G_0^{(2k)}, \quad g^{(k)} = g_0^{(2k)} / g_0^{(2)k}. \tag{3.1}$$

Consequently, only the Poisson distribution (exhibited by coherence of the field)[3] is invariant under frequency multiplication conditions. In the case of geometric pumping, it follows from Eq. (2.6) that $g^{(k)} = k!(2k-1)!!$, i.e., that strong photon bunching occurs ($g^{(2)} = 6$). However, according to Eq. (3.1), the normalized factorial moments of the second-harmonic field (and the correlation $\overline{nn_0} \sim G_0^{(3)}$) are independent of the photon distribution parameters, i.e., they provide no additional information, so that absolute measurements are impossible. Generation of higher harmonics yields similar results.

Generation of the sum frequency $\omega_1 + \omega_2 \to \omega$ also fails to produce radiation with the required statistics, because in this case we have $a = i\tau a_1 a_2$ and this yields

$$g^{(k)} = g_1^{(k)} g_2^{(k)}. \tag{3.2}$$

Generation of the difference frequency $\omega_0 - \omega_1 \to \omega_2$ is more interesting and in this case the first order treatment gives

$$a_2' = a_2 + i\tau a_0 a_1^+. \tag{3.3}$$

Hence, $\bar{n}_2 = \tau^2 \bar{n}_0(\bar{n}_1 + 1)$, where $n_i = a_i^+ a_i$ are the numbers of photons in the modes. The term independent of $\bar{n}_1$ is the quantum noise of the converter. A comparison of $\overline{m}_2$ in the presence and absence of a field of frequency ω_1 makes it possible to determine $\bar{n}_1$. In this alternative quantum photometry method[12] the comparison standard is in the form of spontaneous radiation emitted by the frequency converter.

We shall be interested in the case when $\bar{n}_1 = 0$ and a crystal is a source of two-photon light. This effect is known as parametric scattering or the frequency splitting effect (see, for example, Refs. 10–13).

§4. PARAMETRIC SCATTERING

Spontaneous parametric scattering can be interpreted as a result of decay of the pump photons into pairs of photons, i.e., this is a process which is opposite to the generation of the sum frequency: $\omega_0 \to \omega_1 + \omega_2$. The photons belonging to the same pair ("biphoton") are created simultaneously (within the limits of 1 psec) at the same point in a crystal[1] or along directions linked by the phase-matching condition ($\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$).

It follows from Eq. (3.3) and from the corresponding expressions for a_1, obtained in the first order with respect to $\bar{n}_0$, that

$$\overline{n_1 n_2} = \bar{n}_1 = \bar{n}_2 = \tau^2 \bar{n}_0, \quad g_{12}^{(2)} = 1/\bar{n}_1. \tag{4.1}$$

Hence, we find that $\eta_1 \bar{n}_1 = \eta_1 \eta_2 \overline{n_1 n_2}$ (Ref. 10). This relationship was confirmed experimentally in Refs. 2 and 14.

In the degenerate case ($\mathbf{k}_1 = \mathbf{k}_2$), it follows from Eq. (3.3) that

$$G^{(2)} = G^{(1)} = \tau^2 \bar{n}_0, \quad g^{(2)} = 1/\bar{n}. \tag{4.2}$$

This last formula is equivalent to Eq. (1.2) because if $\overline{m} \ll 1$, it follows from Eq. (2.1) that

$$P_1' = \eta G^{(1)} - \eta^2 G^{(2)}, \quad P_2' = \eta^2 G^{(2)}/2. \tag{4.3}$$

Therefore, in the first approximation with respect to the pump intensity we have an equality $G^{(2)} = G^{(1)}$ typical of two-photon light and it follows from this equality and from Eq. (4.2) that $P_1 = 0$ and $P_2 = \bar{n}/2$.

The above formula for $\overline{n_1n_2}$ does not allow for "accidental" coincidences proportional to $\bar{n}_1\bar{n}_2 = \bar{n}_1^2$. It was pointed out in Ref. 15 that in the case of parametric interactions described by a Hamiltonian of the type $a_0^+ a_1a_2$ + H.c., in addition to the Manley-Rowe relationships, there are also integrals $\langle (n_1 - n_2)^k \rangle$ = const. In particular if $k = 2$ and for the initial vacuum state, we have $\langle n_1n_2\rangle = \langle n_1^2\rangle$, so that

$$G_{12}^{(2)} = G_1^{(2)} + G_1^{(1)} = \bar{n}_1 + g_1^{(2)}\bar{n}_1^2, \quad g_{12}^{(2)} = 1/\bar{n}_1 + g_1^{(2)}. \tag{4.4}$$

This last relationship makes it possible to determine $\bar{n}_1$ from the correlation of photocounts in two channels and from the bunching factor of one of the channels.[1)]

When the pump statistics is known, we can also calculate $g_1^{(2)}$. It is shown in the Appendix that

$$G_1^{(1)} = \tau^2G_0^{(1)} + (^1/_3)\tau^4(G_0^{(2)} - G_0^{(1)}) + \dots,$$
$$G_1^{(2)} = 2\tau^4G_0^{(2)} + \dots, \quad g_1^{(2)} = 2g_0^{(2)} + O(\tau^2) \tag{4.5}$$

(in the degenerate interaction case the pump photon bunching factor $g_0^{(2)}$ is multiplied by 3 instead of 2). The inequality $g_1^{(2)} > g_0^{(2)}$ which is then obtained means that photons are bunched more strongly in each channel than in the pump field (even in the case of spontaneous parametric scattering). This is in conflict with the self-evident concept of random decay of pump photons, for which in accordance with Eq. (2.3) we should have $g_1^{(2)} = g_0^{(2)}$.

Terms of the order of ${}^4G_0^{(1)}$ in Eq. (4.5) appear because of allowance for the noncommutative nature of the operators a_0 and a_0^+, and allowance for their change as a result of the interaction. Let us assume that, as usual, the conditions $\bar{n}_0 \gg \bar{n}_1$ and $G_0^{(k+1)} \gg G_0^{(k)}$ (i.e., $\bar{n}_0 \gg g_0^{(k)}/g_0^{(k+1)}$) are satsified; then, the pump radiation can be regarded as a given classical quantity. The factorial moments then reduce to the ordinary moments and we thus obtain (see the Appendix)

$$G_1^{(1)} = \langle \mathrm{sh}^2 x\rangle = \tau^2\bar{n}_0 + (^1/_3)\tau^4\overline{n_0^2} + \dots,$$
$$G_1^{(2)} = 2\langle \mathrm{sh}^4 x\rangle = 2\tau^4\overline{n_0^2} + (^4/_3)\tau^6\overline{n_0^3} + \dots, \tag{4.6}$$
$$g_1^{(2)} = 2\langle \mathrm{sh}^4 x\rangle/\langle \mathrm{sh}^2 x\rangle^2 = 2g_0^{(2)} + (^4/_3)\tau^2\bar{n}_0(g_0^{(3)} - g_0^{(2)2}) + \dots .$$

Here, $x = \tau n_0^{1/2}$ and the averaging is carried out over the classical distribution of n_0. In the Poisson distribution cases with $\bar{n}_0 \gg 1$, we have $\overline{n_0^k} = \bar{n}_0^k$, so that $\overline{f(x)} = f(\bar{x})$ and $g_1^{(2)} = 2g_0^{(2)}$. Only in this case can we justify the "parametric" approximation for determinate, classical, and given pumping. Results similar to Eq. (4.6) but with the replacement of $2g_0^{(2)}$ by $3g_0^{(2)}$ are obtained for degenerate parametric scattering.

A two-channel coincidence detector can also be used in the case of one-mode calibrating radiation if we employ a semitransparent mirror that separates the probabilities p and $q = 1 - p$ a photon flux into two halves (Brown-Twiss intensity interferometer). The probability of coincidence is then proportional to $\overline{m_1m_2} = pq\eta_1\eta_2G^{(2)}$ so that the normalized output signal from the interferometer is identical with the bunching factor $g^{(2)}$ of the incident radiation. This result can be generalized in an obvious manner to an N-channel coincidence detector. Such a method for determination of the factorial moments has practical advantages over the one-channel method.

§5. TWO-PHOTON ABSORPTION

We shall now assume that the investigated radiation crosses a layer of a cold material exhibiting two-photon absorption. A change in the statistics is described by the following equations[5,8,9]:

$$dP_n/d\tau = (n+1)(n+2)P_{n+2} - (n-1)nP_n. \tag{5.1}$$

A general solution of Eq. (5.1) is obtained in Refs. 8 and 9. In view of the low probability of two-photon transitions it is sufficient to consider only a solution which is of the first order in τ. Then, Eq. (5.1) yields the following correction to the generating function:

$$\delta Q(x) = -\tau(2x + x^2)\,d^2Q_0/dx^2. \tag{5.2}$$

The derivatives of this function at $x = 0$ determine the corrections to the factorial moments:

$$\delta G^{(k)} = -k\tau[2G_0^{(k+1)} + (k-1)G_0^{(k)}],$$
$$\delta g^{(k)} = -k\tau g_0^{(k)}[k - 1 + 2\bar{n}_0(g_0^{(k+1)}/g_0^{(k)} - g_0^{(2)})]. \tag{5.3}$$

Consequently, the rate of "unbunching" in the Poisson pumping case is independent of the initial intensity $\bar{n}_0$:

$$g^{(k)} = 1 - k(k-1)\tau, \tag{5.4}$$

whereas in the geometric pumping case it does depend on this intensity:

$$g^{(k)} = k![1 - k(k-1)\tau(1 + 2\bar{n}_0)]. \tag{5.5}$$

Therefore, if we measure $g^{(2)}$ for two values of $\bar{n}_0$, we can then in principle find $\bar{n}_0$ and τ with the aid of Eq. (5.5). However, it is simpler to determine τ using the Poisson pumping and the relationship (5.4), and then apply Eq. (5.5).

On the other hand, determination of three quantities—$g^{(2)}$ and the average numbers of photocounts $\bar{m}_0$ and $\bar{m}$ at the entry and exit from the absorber—also gives full information. In fact, Eq. (5.4) and the relationships

$$\bar{m}_0 = \eta\bar{n}_0, \quad \bar{m} = \bar{m}_0 - 2\tau\eta G_0^{(2)} \tag{5.6}$$

in the Poisson pumping case yield

$$\bar{n}_0 = (1-\varepsilon)/(1-g^{(2)}), \quad 2\tau = 1 - g^{(2)}, \tag{5.7}$$

where $\varepsilon = \bar{m}/\bar{m}_0$. In the geometric pumping case, we find that

$$\bar{n}_0 = (1-\varepsilon)/(2\varepsilon - g^{(2)}), \quad 2\tau = \varepsilon - g^{(2)}/2. \tag{5.8}$$

It should be noted that two-photon absorption can also be used in other forms of absolute measurements. For example, the amplitude of a magnetic field H_1 in the rf range has been determined[16] from the ratio of one- and two-photon absorption under magnetic resonance conditions (the field H_1 is then expressed in units of ω/γ, where γ is the gyromagnetic ratio).

§6. LASER RADIATION

Below the excitation threshold a single-mode laser is known to generate not only ordinary thermal radiation, but also a flux of photons with the geometric distribution. When the threshold is exceeded significantly, the saturation effect limits the field amplitude and its state then approaches coherence and the photon distribution approaches the Poisson form.[3-5]

We shall be interested in the intermediate case when, in accordance with the Scully-Lamb model,[4,5,17] the following two-parameter distribution describes a number of photons in the laser resonator:

$$P_n = P_0 \beta! \alpha^n/(\beta+n)! \approx \alpha^{\beta+n}/(\beta+n)! e^{\alpha}, \tag{6.1}$$

$$P_0 = 1/\Phi(\alpha) \equiv \left[\beta! \sum_{n=0}^{\infty} \alpha^n/(\beta+n)!\right]^{-1} \approx \alpha^{\beta}/\beta!\, e^{\alpha}, \tag{6.2}$$

where $\Phi(\alpha) \equiv \Phi(1, 1+\beta; \alpha)$ is a confluent hypergeometric function. Approximate expressions are valid beginning from a certain excess above the threshold in the range $\alpha/\beta \gtrsim 2$ (Ref. 5); we then have $P_0 \ll 1$ and $\bar{n} \approx \alpha - \beta$.

Using the definition (2.4), we can find the generating function for the distribution of Eq. (6.1):

$$Q(x) = P_0 \Phi[(1+x)\alpha]. \tag{6.3}$$

It should be noted that the $x \to \eta x$ transformation does not reduce to a simple change of the scale so that the distribution (6.1) changes its form as a result of linear attenuation of the radiation (for example, as a result of passage through the exit semitransparent mirror of a resonator). In accordance with the definition of Eq. (6.2), the function $\Phi(z)$ satisfies the equation

$$d\Phi/dz = (1-\beta/z)\Phi + \beta/z, \tag{6.4}$$

which yields

$$G^{(1)} = \alpha - \beta + \beta P_0 \approx \alpha - \beta, \quad G^{(2)} = (\alpha-\beta-1)G^{(1)} + \alpha \approx \bar{n}^2 + \beta,$$
$$G^{(3)} = (\alpha-\beta)G^{(2)} + 2(1+\beta)G^{(1)} - 2\alpha \approx \bar{n}^3 + 3\beta\bar{n} - 2\beta. \tag{6.5}$$

Above the threshold the normalized factorial moments are

$$g^{(2)} = 1 + \beta/\bar{n}^2, \quad g^{(3)} = 1 + \beta(3 - 2/\bar{n})/\bar{n}^2. \tag{6.6}$$

Solving this system, we find that

$$\bar{n} = 2(3 - h_3/h_2), \quad \alpha = \bar{n}(1 + h_2\bar{n}), \quad \beta = h_2\bar{n}^2, \tag{6.7}$$

where $h_k \equiv g^{(k)} - 1$. The quantum efficiency is given by $\eta = \bar{m}/\bar{n}$. We recall that η includes also the losses in the optical channel between the laser resonator and the detector, i.e., it includes the exit mirror of the resonator.

§7. TWO-PHOTON DETECTOR

We shall now consider the possibility of using the two-photon photoelectric effect in absolute photometry. A simple generalization of the derivation of Eq. (2.1) shows that the generating function is again given by Eq. (2.4), if we assume that $\hat{m} = \eta{:}\hat{n}^2$. Differentiating the generating function k times, we obtain

$$G^{(k)\prime} = \langle : \hat{m}^k : \rangle = \eta^k G^{(2k)}. \tag{7.1}$$

As expected, this result is analogous to Eq. (3.1) describing transformation of the photon statistics in the course of frequency doubling. Although the detection process alters the statistics of the non-Poisson radiation, such a change does not depend on $\bar{n}$.

Generalizing this discussion, we can draw the conclusion that n-photon detection, like n-fold frequency multiplication, is of no interest in absolute photometry.

CONCLUSIONS

Absolute photometry depends on the quantum nature of light and on the nature of the process of photodetection. This applies also to analog detectors.[1] Although at present two-photon light is the optimal representative of calibrating radiations, it is desirable to seek for variants suitable also at other wavelengths and at other intensities. The saturation effect may be of interest from this point of view.

We have considered here the problem of existence and possibility of generation, in principle, of calibrating radiations so that no numerical estimates were attempted. The practical feasibility of these methods will require separate investigation, but it is already clear that the two-photon absorption effect will most probably be too weak for photometric applications.

APPENDIX

Dependence of the statistics of parametric scattering on the pumping statistics

We shall first consider the degenerate case ($\mathbf{k}_1 = \mathbf{k}_2$). The effective Hamiltonian[12] corresponding to the exact phase matching yields the following Heisenberg equations:

$$\dot{a} \equiv da/d\tau = ca^+, \quad \dot{c} = -a^2/2, \tag{A.1}$$

where $c \equiv ia_0$ and the operators a and a_0 include the factors $\exp(i\omega t)$ and $\exp(i\omega_0 t)$.

We shall seek the factorial moments in the form of a series in τ. The higher derivatives can be obtained with the aid of Eq. (A.1). For example,

$$\ddot{a} = cc^+a - a^2a^+/2, \quad \dddot{a} = c(c^+c - a^+a - {}^1/_2)a^+. \tag{A.2}$$

Here and later we shall drop the terms which do not contribute in the vacuum state of the incident subharmonic field.

Using Eqs. (A.1) and (A.2), we obtain

$$d^2G^{(1)}/d\tau^2 = 2\langle \dot{a}^+\dot{a}\rangle = 2\langle c^+c\rangle\langle aa^+\rangle = 2G_0^{(1)},$$
$$d^4G^{(1)}/d\tau^4 = 8\langle \dot{a}^+\ddot{a}\rangle = 8\langle c^+cc^+c\rangle\langle aa^+\rangle - 4\langle c^+c\rangle(\langle aa^+\rangle + 2\langle aa^+aa^+\rangle). \tag{A.3}$$

The negative term is the result of an allowance for the change in the pump field. It follows from the commutation relationships that

$$\langle c^+cc^+c\rangle = \langle n_0^2\rangle = G_0^{(2)} + G_0^{(1)}, \quad d^4G^{(1)}/d\tau^4 = 8G_0^{(2)} - 4G_0^{(1)}. \tag{A.4}$$

Similarly,

$$
\begin{aligned}
d^2G^{(2)}/d\tau^2 &= 2\langle \dot{a}^+a^+a\dot{a}\rangle = 2G_0^{(1)},\\
d^4G^{(2)}/d\tau^4 &= 8\,\langle \dot{a}^+a^+a\ddot{a} + 3\,(\dot{a}^+)^2\,(\dot{a})^2 + 3\dot{a}^+a^+\ddot{a}\dot{a}\rangle\\
&= 80G_0^{(2)} - 4G_0^{(1)}.
\end{aligned}
\qquad (A.5)
$$

As a result, we obtain

$$G^{(1)} = \tau^2G_0^{(1)} + (1/6)\,\tau^4\,(2G_0^{(2)} - G_0^{(1)}) + \ldots,$$

$$G^{(2)} = \tau^2G_0^{(1)} + (1/6)\,\tau^4\,(20G_0^{(2)} - G_0^{(1)}) + \ldots, \qquad (A.6)$$

$$g^{(2)} = 1/\bar{n} + 3g_0^{(2)} + O(\tau^2).$$

In the case of nondegenerate parametric scattering, we find from Eqs. (A.1) and (A.2) that

$$
\begin{aligned}
&\dot{a} = cb^+, \quad \dot{b} = ca^+, \quad \dot{c} = -ab,\\
&\ddot{a} = (cc^+ - bb^+)\,a, \quad \dddot{a} = c\,(c^+c - b^+b - 1)\,b^+,
\end{aligned}
\qquad (A.7)
$$

where $a \equiv a_1$ and $b \equiv a_2$. Hence, we obtain the derivatives

$$
\begin{aligned}
d^2G_1^{(1)}/d\tau^2 &= 2\,\langle \dot{a}^+\dot{a}\rangle = 2G_0^{(1)},\\
d^4G_1^{(1)}/d\tau^4 &= 8\,\langle \dot{a}^+\ddot{a}\rangle = 8\,(G_0^{(2)} - G_0^{(1)}),\\
d^4G_1^{(2)}/d\tau^4 &= 24\langle (\dot{a}^+)^2\,(\dot{a})^2\rangle = 48G_0^{(2)}.
\end{aligned}
\qquad (A.8)
$$

The result is Eq. (4.5).

Assuming then that $\bar{n}_0 \gg \bar{n}$ and $\bar{n}_0 \gg g_0^{(k)}/g_0^{(k+1)}$, we go over to constant classical pumping. Then, the system (A.7) has the following solutions

$$a(\tau) = ua(0) + ve^{i\varphi}b^+(0), \quad b(\tau) = ub(0) + ve^{i\varphi}a^+(0), \qquad (A.9)$$

where $u = \cosh x$, $v = \sinh x$, $x = \tau n_0^{1/2}$, and $\varphi = \arg(c)$ are random classical quantities. Hence, we can readily find $G^{(k)}$ [see Eq. (4.6)].

[1)] In practice, accidental coincidences are found by moving one of the detectors out of the two-photon coherence region.[2,14]

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