Nonergodicity and nonequilibrium character of spin glasses

S. L. Ginzburg

B. P. Konstantinov Institute of Nuclear Physics, Academy of Sciences of the USSR, Leningrad
(Submitted 28 August 1985)
Zh. Eksp. Teor. Fiz. **90**, 754–766 (February 1986)

In the theory of the nonergodicity of spin glasses a new approach, corresponding to a continuous spectrum of relaxation times, is proposed. It is shown how to introduce a generalized field conjugate to the order parameter. A phenomenological generalization of the results obtained is performed, and makes it possible to describe nonequilbrium phenomena in spin glasses. Several examples of nonequilibrium phenomena are considered. Qualitative agreement with the experimental data is obtained.

1. INTRODUCTION

At the present time it is clear that the physics of spin glasses is connected with the appearance of highly nontrivial nonergodicity below the phase-transition point (see, e.g., the review in Ref. 1). This nontriviality is due to the fact that in spin glasses there is an infinite number of valleys, forming a hierarchical structure with so-called ultrametric topology.[2,3] In the theory the transition times of transitions from some valleys to others are infinite quantities, and the hierarchical structure of the valleys leads to an infinite hierarchy of infinite transition times.[4] The fact that the transition times are infinite implies that the nonergodicity is absolute, i. e., if the systems falls into some particular state it will never emerge from that state. In this case the existence of the other valleys become unobservable and consequently unimportant.

In experiment, however, one observes an entirely different situation, which we shall call effective nonergodicity. It turns out that in spin glasses there is a continuous spectrum of relaxation times, which starts from paramagnetic times $\tau \sim 10^{-12}$ sec (Ref. 5) and stretches out to astronomical times $t_{\max} \sim 10^{20}$ sec (Ref. 6). Here the logarithm of the relaxation times is distributed almost uniformly over this entire interval of times.[7,8] When effective nonergodicity is compared with absolute nonergodicity, the impression is created that we are dealing, as before, with a hierarchical structure of valleys, but that there is a certain weak mechanism leading to transitions from one valley to another, and that it is this which leads to the appearance of effective nonergodicity instead of absolute nonergodicity.

At the same time, the nonergodicity should lead to the result that spin glasses should be nonequilibrium systems. However, in the case of absolute nonergodicity this nonequilibrium cannot be manifested in any way, and is fundamentally unobservable. In the real situation of effective nonergodicity, however, the nonequilibrium not only is manifested but also should be the principal phenomenon determining the entire physics of spin glasses. Spin glasses should be essentially nonequilibrium systems, since equilibrium can be reached only after a time of the order of $t_{\max}$.

We shall discuss this question in more detail, since until now almost no attention has been paid to this aspect of the problem of nonergodicity. It has been tacitly assumed that we have an equilibrium system, characterizable by various external parameters (e.g., the temperature and external magnetic field), and different phenomena in this equilibrium system have been studied. However, since the experiments show that the maximum relaxation times are very long, we cannot speak of any equilibrium state of the spin glass. The state of the spin glass should depend to a very large degree on the history of the system, the method of cooling, the application of a magnetic field, etc. Since a nonequilibrium state should tend to equilibrium, the most diverse physical quantities, e.g., the magnetic moment or susceptibility, should depend on the relaxation time. Precisely this dependence on the observation time and prior history is the clearest manifestation of the nonequilibrium character of spin glasses.[6–8] Especially interesting are the results of Ref. 6, in which the pattern of the establishment of the magnetic moment was observed to depend on the time for which the system was held at the given temperature before the magnetic field was switched on. Thus, the authors of Ref. 6 established not only the long-time dependence of the magnetic moment on the observation time, but also the dependence of this process on the history of the system. The results of Ref. 6 tell us that in order to obtain reproducible experimental data pertaining to spin glasses it is necessary to monitor and describe all stages of the manipulation with the external parameters after the line of the phase transition to the spin glass has been crossed. All of this is a consequence of the nonequilibrium character of spin glasses.

Although nonequilibrium phenomena are already being studied experimentally, there are not yet any theoretical papers devoted to this question.

In the present paper we shall discuss from a theoretical point of view certain questions associated with the nonequilibrium character of spin glasses. It would be ideal, of course, to learn how to solve completely the problem of describing such a strongly nonequilibrium state as a functional of the entire history of the system. However, in its general form this problem is too complicated. Therefore, we shall confine ourselves to solving a simpler problem, in which, however, all the principal features of the dependence of the state on the history and observation time will be reflected.

We shall assume that we have an equilibrium system,

 0038-5646/86/020439-08$04.00

characterized by a certain temperature, in zero magnetic field. At a certain time t_0 we switch on an external magnetic field or change the temperature of the system, and then study how the susceptibility of the system changes or what happens in the process of relaxation of the magnetic moment. If the field were switched on or the temperature were changed at the time $t_0 = -\infty$, the system would have come to equilibrium and we would have an equilibrium correction to the susceptibility or to the relaxation process. In our case of finite t_0 this correction will be a nonequilibrium quantity and, consequently, will depend on the observation time and history, i.e., on the holding time in the new conditions. It is this nonequilibrium correction that we shall study, using perturbation theory. This simplification of the problem gives the possibility of solving it to completion. At the same time, it is perfectly clear that all the qualitative effects due to the nonequilibrium character will be obtained in this way.

Since, as we have said, the nonequilibrium character is determined by effective nonergodicity, to describe the nonequilibrium phenomena it would be natural to use the existing theory of nonergodicity. At present we have only a theory of absolute nonergodicity. It would be desirable, therefore, to try to apply this theory to the description of effective nonergodicity as well. It turns out, however, that it is not so simple to do this. In the attempt to adapt the existing theories of absolute nonergodicity to the description of effective nonergodicity two problems arise. The first is associated with the absence in the theory of absolute nonergodicity of a continuous spectrum of relaxation times, while the second arises from the fact that in absolute nonergodicity the density of relaxation times is not determined uniquely. Because of this there arises strong degeneracy, which is called gauge invariance. Therefore, first of all it is necessary to construct a theory of absolute nonergodicity with a continuous spectrum of relaxation times and to fix the gauge in some way. After this the theory can then be extended to effective nonergodicty.

In this extension, naturally, we shall have to forgo infinite transition times from one valley to another, i.e., we shall have to assume the existence of some mechanism leading to intervalley transitions. We note that such a mechanism should automatically determine the density of relaxation times, i.e., should determine the gauge uniquely. Of course, we shall not attempt to introduce any specific mechanism, but shall assume that its action reduces entirely to determining t_{max} and fixing the gauge. In this phenomenological approach, however, the general structure of the theory of absolute nonergodicity is preserved. Since this structure reflects the ultrametric topology of the valleys, we hope that our phenomenological theory of effective nonergodicity will also reflect correctly the hierarchical arrangement of the valley, which is the most important element of the physis of spin glasses.

The next two Sections will be devoted to the construction of a theory of absolute nonergodicity with a continous spectrum of relaxation times and to the question of the fixing of the gauge in this theory by means of the introduction of a generalized field, but first we shall discuss briefly the existing approaches in the theory of nonergodicity.

In the theory of absolute nonergodicity of spin glasses there are two approaches. First, there is Parisi's concept of broken symmetry of replicas (see, e.g., Refs. 9–12), and secondly there is the dynamical approach of Sompolinsky.[4,13,14] These two approaches are intimately related, but in the discussion of nonequilibrium phenomena it is natural to attempt to use the dynamical approach. In Sompolinsky's approach, however, there appear an infinite number of infinite relaxation times $\tau_i \to \infty$, and their ratio $\tau_i/\tau_{i+1} \to \infty$. For the description of absolute nonergodicity such an approach is entirely admissible. However, we must have a theory with a continuous specrum of relaxation times.

It turns out that one can construct a theory of absolute nonergodicity by replacing the Sompolinsky condition by the condition $\tau_i \to \infty$, $\tau_i/\tau_{i+1} \to 1$. It is easy to see that this condition corresponds to a continuous spectrum of infinite relaxation times. This is the approach proposed in the present paper. It is interesting that the equations that are obtained with this assumption are, as before, the Sompolinsky equations, but the quantitites appearing in these equations have another meaning and turn out to be related directly to the physical time.

Next, we show how to introduce a generalized field conjugate to the order parameter. We discuss questions associated with the degeneracy due to the gauge invariance and questions associated with the lifting of this degeneracy by the generalized field.

Then, with the aid of a phenomenological hypothesis, the results obtained in the description of absolute nonergodicity are used to describe effective nonergodicity and the consequent nonequilibrium character of spin glasses.

2. BASIC EQUATIONS

In this Section we shall show how one can modify the dynamical approach in order to use it to attempt to describe the experimental data. For this we shall consider the so-called soft model of a spin glass.[13,14] The Sompolinsky equations for this model have been obtained in a paper by the author.[15] In the following we shall adhere to the method of this paper.

The Hamiltonian of the soft model has the form

$$H = -\sum_{ik} J_{ik} m_i m_k + \sum_i U(m_i),$$
$$U(m) = \frac{m^2}{2b} + \frac{um^4}{8}, \quad \langle J_{ik}{}^2 \rangle = I_{ik}. \tag{1}$$

In (1) the m_i are classical fields, and the J_{ik} are random exchange integrals with a Gaussian distribution. In the model adopted the dynamical equations have the form of Langevin equations with random forces:

$$\frac{1}{\Gamma T}\frac{\partial m_i}{\partial t} = -\frac{1}{T}\frac{\partial H}{\partial m_i} + \varepsilon_i(t),$$
$$\langle \varepsilon_i(t)\, \varepsilon_j(t') \rangle = \frac{2}{\Gamma T}\delta_{ij}\delta(t-t'). \tag{2}$$

Here T is the temperature, and Γ^{-1} is the bare relaxation time. In Ref. 15 an expression was derived for the complete stochastic functional in this model, together with equations for the correlation function $D(t)$ of the fields and for the advanced Green function $G_+(t)$ and retarded Green function $G_-(t)$. We shall write out these equations in the paramagnetic region, keeping the notation of Ref. 15:

$$G_\pm^{-1}-S_\pm+\frac{4I_0}{T^2}G_\pm=0,\quad D=-BG_+G_-\left(1-\frac{4I_0}{T^2}G_+G_-\right)^{-1},$$

$$G_0=S_0G_+G_-\left(1+\frac{4I_0}{T^2}G_+G_-\right)^{-1},$$

$$G_1=-S_1G_+G_-\left(1-\frac{4I}{T^2}G_+G_-\right)^{-1},$$

$$S_\pm=\frac{1}{T}\left(\frac{1}{b}\pm\frac{i\omega}{\Gamma}\right)-\Sigma_\pm,\quad B=-\frac{2}{\Gamma T}-\sigma,\qquad (3)$$
$$G_\pm=G_0\pm G_1,\quad S_\pm=S_0\pm S_1,$$
$$\Sigma_\pm=\Sigma_0\pm\Sigma_1,\quad I_0=\sum_k I_{ik}.$$

The self-energy parts $\Sigma_\pm$ and σ appearing in (3) are expanded in a series in the anharmonicity constant u. We shall write out explicit expressions for them in the t-representation to terms of order u^2:

$$\Sigma_\pm(t)=-\frac{3u}{2T}D(t=0)\delta(t)+\frac{9u^2}{2T^2}G_\pm(t)D^2(t),$$
$$\sigma(t)=(3u^2/2T^2)D^3(t).\qquad (4)$$

The point of the phase transition to the spin glass is determined from the condition that as $\omega\to0$ the function $D(\omega)$ becomes singular. This gives the following equation:

$$(4I_0/T^2)g^2=1,\quad g=G_\pm(\omega=0)=D(t=0).\qquad (5)$$

We turn now to the spin-glass region. We shall assume that in this region

$$D(t)=D_0(t)+D_s(t),\quad G_\pm(t)=G_{\pm0}(t)+G_{\pm s}(t).\qquad (6)$$

Here D_0 and $G_{\pm0}$ are ordinary thermodynamic functions satisfying the fluctuation-dissipation theorem (FDT):

$$D_0(\omega)=2\omega^{-1}\,\mathrm{Im}\,G_{-0}(\omega).\qquad (6a)$$

The equations for these quantities were written out in Ref. 15. They will be of no further interest to us.

The functions $D_s(t)$ and $G_{\pm s}(t)$ describe the nonergodic behavior of spin glasses that is of interest to us. A principal feature in the theory of nonergodicity is the postulating of an explicit form for these functions. Sompolinsky's hypotheses reduces to the following.[4] Let

$$D_s(\omega)=\sum_{j=0}^{k}\frac{2q_j'\Gamma_j}{\omega^2+\Gamma_j^2},\quad G_{-s}(\omega)=-\sum_{j=0}^{k}\frac{i\Delta_j'\Gamma_j}{\omega+i\Gamma_j},$$

$$\Delta_i=-\sum_{j=i}^{k}\Delta_j',\quad q_i=\sum_{j=0}^{i}q_j',\quad \Gamma_i\to0,\quad \Gamma_i/\Gamma_{i+1}\to0.\qquad (7)$$

Next it is necessary to substitute (7) into Eqs. (3) and (4); certain equations for Δ_i and q_i are then obtained. Next it is necessary to let $k\to\infty$; then i/k becomes a continuous variable x, varying on the interval [0, 1], and q_i and Δ_i go over into functions $q(x)$ and $\Delta(x)$, which are the order parameters. For these functions for the soft model the following equations were obtained in Ref. 15:

$$\left\{1-\left[\frac{4I_0}{T^2}+\frac{9u^2}{2T^2}q^2(x)\right][g+\Delta(x)]^2\right\}q'(x)=0,$$
$$\left\{1-\left[\frac{4I_0}{T^2}+\frac{9u^2}{2T^2}q^2(x)\right][g+\Delta(x)]^2\right\}\Delta'(x)=0,\qquad (8)$$

which are the Sompolinsky equations for our problem. As we have already said in the Introduction, this approach is fully satisfactory for the description of absolute nonergodicity. However, one cannot attempt to apply it even phenomenologically to the description of the effective nonergodicity observed in experiment or, consequently, to the description of nonequilibrium processes. This is connected with the fact that, as can be seen from (7), $D_s(\omega)$ and $G_{-s}(\omega)$ are described by a sum of singular functions, while in experiment one observes continuous functions of ω.

Therefore, in order that the theory somehow reflect the experimental situation, it is necessary to advance a hypothesis that is compatible with the continuity of the functions $D_s(\omega)$ and $G_{-s}(\omega)$ and preserves, as before, the condition $\Gamma_i\to0$. It is clear that for this it is necessary that $\Gamma_i/\Gamma_{i+1}\to1$; here Γ_i will be distributed so densely that in (7) it will be possible to go over from a sum to an integral. We set, for example,

$$\Gamma_i=\frac{1}{\tau}\exp\left(-\frac{i}{l\alpha}\right),\qquad (9)$$

where l is an integer and α is some parameter. We identify the time τ with the paramagnetic relaxation time, i.e., τ is a finite time. In (9), and correspondingly in (7), we now take the following limit:

$$k\to\infty,\quad l\to\infty,\quad \alpha\to0,\quad k/l\to\infty,\quad l\alpha\to\infty,$$
$$\left|\frac{\Gamma_i-\Gamma_{i-1}}{\Gamma_i}\right|\to0,\quad \Gamma_i(i\sim l)\to0.\qquad (10)$$

We note that the theory of Sompolinsky corresponds to the case $l\alpha\to0$, $k=l$. Whereas the difference between the conditions $k=l$ and $k\gg l$ is unimportant, the difference between the Sompolinsky condition $l\alpha\to0$ and our condition $l\alpha\to\infty$ is fundamental. Our condition ensures a sufficiently dense set of Γ_i, which gives us the possibility of going over to the continuum limit in (7). As a result we obtain

$$D_s(\omega)=2\int_\infty^0 dy\,q'(y)\frac{\Gamma(y)}{\omega^2+\Gamma^2(y)},$$

$$G_{-s}(\omega)=-i\int_\infty^0 dy\,\Delta'(y)\frac{\Gamma(y)}{\omega+i\Gamma(y)},\qquad (11)$$

$$\Gamma(y)=\frac{1}{\tau}e^{-y/\alpha},\quad y=\frac{i}{l}.$$

In (11) there remains a single parameter — namely, α. All the other parameters have already been used.

Next, it is necessary that typical functions $\Gamma(y)$ be infinitesimally small as $\alpha\to0$. For this we must assume that the main contribution to (11) is given by $y\sim1$. This assumption is very important in our approach.

Now the integrals in (11), and also the correlator $D_s(t)$ and the Green function $G_{-s}(t)$ in the t-representation, are easily calculated. As a result we obtain

$$D_s(\omega)=-\frac{\pi\alpha}{|\omega|}q'(y),\quad G_{-s}(\omega)=\Delta(y)+i\frac{\pi\alpha}{2}\Delta'(y),$$

$$D_s(t)=q(z),\quad G_{-s}(t)=(\alpha/t)\Delta'(z)\vartheta(t), \tag{12}$$

$$y=-\alpha\ln(|\omega|\tau),\quad z=\alpha\ln(|t|/\tau).$$

Formula (12) is the basis of our whole theory. We note here one important circumstance. From (12) it can be seen that $D_s(\omega)\propto 1/\omega$ for small ω; this differs sharply from the usual behavior of the correlator, which is finite as $\omega\to 0$. In exactly the same way, $\mathrm{Im}G_s(\omega)\to$const in this limit, while, as usual, $\mathrm{Im}G(\omega)\propto\omega$. All of this implies singular behavior of these quantities at low frequencies, and it is these singularities which distinguish G_s and D_s from correlators of ordinary thermodynamic fluctuations. In essence, the $1/\omega$ singularity and the condition $\alpha\to 0$ determine a certain new class of generalized functions, which replaces the sum of δ-functions in the Sompolinsky approach.

Furthermore, the fact that all the quantities in (12) are continuous functions of ω and t make possible a phenomenological relation between theory and experiment, or, in other words, a relation between absolute and effective nonergodicity. In experiment, as we have already said, although the maximum relaxation time $t_{\max}$ is astronomical, it is nevertheless finite. Real experiments give for the quantity $\ln(t_{\max}/\tau)$, which we shall identify with $1/\alpha$, an estimate of the order of 50–100. It can be assumed that α is not an infinitesimal quantity but simply a small parameter, and the proposed theory studies phenomena that arise in the lowest nonvanishing order in α. Having established this phenomenological point of view, we can relate the absolute and effective nonergodicities and attempt to describe specific experiments; this we shall do in the last Section of this paper. This phenomenology implies that the transition from absolute to effective nonergodicity occurs by means of a smearing out of the above-mentioned generalized functions. However, it turns out that one such smearing out is not sufficient to relate the absolute and effective nonergodicities. It is necessary also to fix the gauge, and we shall discuss this in the next Section.

We shall show now that $q(y)$ and $\Delta(y)$ satisfy the Sompolinsky equations. Since, as we have already said, $D_s(\omega)$ and $G_s(\omega)$ are singular functions, by substituting the expressions (12) into (3) and separating the singular parts of the equations from the regular parts we can obtain equations for $q(y)$ and $\Delta(y)$. Here it is necessary to take it into account that, in the region of times and frequencies of interest to us, $G_{\pm 0}(\omega)=g$ and the exact Green function is equal to $g+\Delta(y)$. In this same region, $D_0(t)=0$. Taking all this into account, we obtain from (3), (4), and (12) the following expressions for the singular parts $\Sigma_s(t)$ and $\sigma_s(t)$ in the t- and ω-representations:

$$\sigma_s(t)=\frac{3u^2}{2T^2}q^3(z),\quad \Sigma_s(t)=\frac{9u^2}{2T^2}\frac{\alpha}{t}\Delta'(z)q^2(z)\vartheta(t),$$

$$\sigma_s(\omega)=-\frac{\pi\alpha}{|\omega|}\frac{9u^2}{2T^2}q^2(y)q'(y), \tag{13}$$

$$\Sigma_s(\omega)=\frac{9u^2}{2T^2}\left\{\int_0^y\Delta'(x)q^2(x)\,dx+i\frac{\pi\alpha}{2}\Delta'(y)q^2(y)\right\}.$$

Substituting (13) into (3) and separating the singular parts from the regular parts in the equations for D and G, we obtain the following equations for $q(y)$ and $\Delta(y)$:

$$\left\{1-\left[\frac{4I_0}{T^2}+\frac{9u^2}{2T^2}q^2(y)\right][g+\Delta(y)]^2\right\}q'(y)=0,$$

$$\left\{1-\left[\frac{4I_0}{T^2}+\frac{9u^2}{2T^2}q^2(y)\right][g+\Delta(y)]^2\right\}\Delta'(y)=0, \tag{14}$$

$$\frac{1}{g+\Delta(y)}+\frac{4I_0}{T^2}[g+\Delta(y)]-\frac{1}{bT}+\frac{9u^2}{2T^2}\int_0^y\Delta'(x)q^2(x)\,dx+\alpha(g)=0,$$

$$\alpha(g)=-\frac{3u}{2T}(q+g)+\frac{9u^2}{2T^2}\left(q^2g+qg^2+\frac{1}{3}g^3\right),$$

$$q=q(z=0).$$

The first two equations coincide with the corresponding Sompolinsky equations (8). The third equation is not independent. If we differentiate it with respect to y, we obtain the first equation. Thus, we see that the equations for $q(y)$ and $\Delta(y)$ coincide with the Sompolinsky equations for our model.

3. GAUGE INVARIANCE. THE FIELD CONJUGATE TO THE ORDER PARAMETER

It is well known that the Sompolinsky equations possess degeneracy connected with the presence of gauge invariance in these equations.[4] We shall show this for our example. First of all we note that to Eqs. (14) in our model it is necessary to add two more equations.[15] The essence of these is that they give the possibility of determining the boundary values of Δ and q:

$$q(0)=q,\quad \Delta(0)=0,$$
$$q(\infty)=q_0,\quad \Delta(\infty)=\Delta, \tag{15}$$

and the parameters q, q_0, and Δ are determined uniquely. At the same time, it can be seen from the first two equations (14) that if we assume that $q'\neq 0$ and $\Delta'\neq 0$ in the spin-glass region, then Eqs. (14) determine in the entire interval of variation of y only a relation

$$\left[\frac{4I_0}{T^2}+\frac{9u^2}{2T^2}q^2(y)\right][g+\Delta(y)]^2=1 \tag{16}$$

between the two monotonic functions $q(y)$ and $\Delta(y)$ satisfying the boundary conditions (15). Naturally, Eq. (16) and the monotonicity requirement do not determine $q(y)$ and $\Delta(y)$ uniquely. Enormous arbitrariness remains. Thus, the theory is found to be invariant under a large group of functional transformations that leave the conditions (15) and

(16) invariant and satisfy the monotonicity condition

$$q'(y)\neq 0, \quad \Delta'(y)\neq 0. \tag{17}$$

It is this invariance that we call gauge invariance. The existence of this group leads to strong degeneracy. A degeneracy of this kind can be lifted only with the aid of a field conjugate to the order parameter. At the present time the form of such a field for spin glasses is unknown.

In this paper we shall show how one can introduce such a field. As is well known, the field conjugate to the order parameter is introduced in the likeness of the order parameter itself. In our case the order parameters are the singular parts of the correlator and Green function. In the same way, we introduce a generalized field. For this we make two changes in Eq. (2). First, we add to $\varepsilon_i(t)$ a term with a random field $h_i(t)$ with a long-time correlator for $h_i(t)$, and, secondly, we add to the left-hand side of Eq. (2) a long-time response function.

In place of (2) we set

$$\begin{gathered}\frac{1}{\Gamma T}\frac{\partial m_i}{\partial t}-\frac{1}{T^2}\int dt'\,F(t-t')\,m_i(t')\\ =-\frac{1}{T}\frac{\partial H}{\partial m_i}+\frac{1}{T}h_i(t)+\varepsilon_i(t),\\ \langle h_i(t)\,h_j(t')\rangle=\delta_{ij}K(t-t'),\\ F(t)=\frac{\alpha}{t}\,\xi'(z)\,\vartheta(t),\quad K(t)=\rho(z),\\ F(\omega)=\xi(y)+i\frac{\pi\alpha}{2}\xi'(y),\quad K(\omega)=-\frac{\pi\alpha}{|\omega|}\rho'(y).\end{gathered} \tag{18}$$

The two quantities $\xi(y)$ and $\rho(y)$ are completely equivalent in their properties to the order parameters $\Delta(y)$ and $q(y)$, and are the generalized fields conjugate to the order parameters. It is not difficult to show that the first two equations (14) take the form

$$\begin{gathered}\left\{1-\left[\frac{4I_0}{T^2}+\frac{9u^2}{2T^2}q^2(y)\right][g+\Delta(y)]^2\right\}\Delta'(y)\\ =\frac{1}{T^2}[g+\Delta(y)]^2\xi'(y),\\ \left\{1-\left[\frac{4I_0}{T^2}+\frac{9u^2}{2T^2}q^2(y)\right][g+\Delta(y)]^2\right\}q'(y)\\ =\frac{1}{T^2}[g+\Delta(y)]^2\rho'(y).\end{gathered} \tag{19}$$

The equations (19) determine $\Delta(y)$ and $q(y)$ uniquely, and this is not surprising, since the presence of an external field of finite magnitude always leads to a unique definition of the order parameter. However, it is known that, in the ordered phase, for a unique definition of the order parameter an infinitesimal field that fixes only the direction of the order parameter is sufficient. What plays the role of the direction in our case? To answer this question, we introduce the function

$$w(y)=-\xi'(y)/\rho'(y) \tag{20}$$

and assume that it is monotonic function satisfying the boundary conditions

$$w(0)=w_0, \quad w(\infty)=0. \tag{21}$$

From (19) and (20) it can be seen that

$$\Delta'(y)=-w(y)\,q'(y). \tag{22}$$

Now we can set $\xi'=\rho'=0$ in (19), and we obtain Eqs. (14) and (22), which determine $\Delta(y)$ and $q(y)$ uniquely. Thus, Eq. (22) completes the determination of the order parameter, giving it a "direction" in the functional space. All that we needed from the generalized field was the ratio of ξ' and ρ' in (20), and ξ and ρ themselves can be infinitesimal quantities. This is completely equivalent to, e.g., the ordinary Heisenberg ferromagnet, in which an infinitesimal field fixes the direction of the magnetic moment.

We return now to Eqs. (14) and (22). Since $w(y)$ is a monotonic function, it is easy to see that we can choose w as the independent variable. With this change of variables the structure of the equations does not change. We then obtain

$$\begin{gathered}\left\{1-\left[\frac{4I_0}{T^2}+\frac{9u^2}{2T^2}q^2(w)\right][g+\Delta(w)]^2\right\}q'(w)=0,\\ \Delta'(w)=-wq'(w).\end{gathered} \tag{23}$$

It is easy to show that Eqs. (23) are simply the Parisi equations for our model. Thus, the introduction of an infinitesimal field conjugate to the order parameter fixes the gauge in the Sompolinsky equations and leads in a natural way to the Parisi equations. From the point of view of the external field the Parisi equations are entirely natural. But from the point of view of the equations themselves another choice of gauge would be much more natural. It is very convenient, e.g., to choose as the independent variable the function $q(y)$. Then all physical quantities would be functions only of the variable q; e.g., $\Delta=\Delta(q)$.

We now note that Eq. (22) with the boundary conditions for (21) contradicts the FDT, which, as is easily shown, has in our case the form

$$\Delta'(y)=-q'(y). \tag{24}$$

Therefore, for the singular correlators $D_s(t)$ and $G_s(t)$ the FDT is not fulfilled. It is this property, together with the singular character of the correlators (which, as we have already said, are generalized functions), that leads to the result that these quantities describe the nonergodicity in spin glasses. Unlike the singular correlators, the regular quantities $D_0(t)$ and $G_0(t)$ satisfy the FDT and describe intravalley transitions, while $D_s(t)$ and $G_s(t)$ describe intervalley transitions.

We shall discuss the question of the gauge invariance in more detail. Since, as can be seen from (11), $\Delta'(y)$ and $q'(y)$ determine the density of relaxation times in the Green function and in the correlator, the gauge invariance implies that only the ratio of these two quantities is determined, and not the two quantitites separately. Fixing the gauge leads to a unique determination of both quantities. The presence of the gauge invariance in the theory of absolute nonergodicity is evidently connected with the absence in this theory of any concrete mechanism leading to transitions from one valley to another.

In the case of effective nonergodicity, some entirely specific mechanism giving rise to intervalley transitions is bound to exist. This mechanism, first, should determine the maximum relaxation time $t_{\max}$, i.e., should fix α, and, secondly, should determine completely the densities of relaxation times, i.e., should fix the gauge. Since the introduction of $w(y)$ into (20) and (22) leads to the same result, this implies that $w(y)$ should be determined by a specific relaxation mechanism in the system. Since the generalized field also fixes the gauge in the theory of absolute nonergodicity, this means that, in essence, it also determines a certain mechanism of intravalley transitions in the case of infinite relaxation times. In the case of effective nonergodicity, however, the introduction of a finite α and a specific gauge is a way of introducing a specific mechanism of intervalley transitions phenomenologically. It is in this way that we go from absolute to effective nonergodicity.

Here it is desirable to note that the most important element of the physics — namely, the hierarchical arrangement of the valleys, i.e., their ultrametric topology, passes over from the absolute to the effective nonergodicity. Thus, the phenomenological approach that we have proposed preserves this most important property inherent to the theory of absolute nonergodicity.

4. NONEQUILIBRIUM PHENOMENA IN SPIN GLASSES

In this Section the theory developed in the preceding Sections will be applied to the description of nonequilibrium phenomena in spin glasses. For this we shall need, first, to assume that the gauge is fixed by some external field, and, secondly, to assume that α is not infinitesimal but is simply a small quantity. We have already discussed these questions in detail in the preceding Sections.

As already mentioned in the Introduction, we shall consider the nonequilibrium correction introduced into the susceptibility by switching on an external magnetic field or changing the temperature. But first we consider the nonequilibrium magnetic moment associated with the switching on of a magnetic field. Suppose that we switched on a magnetic field h_1 at time t_1; then from (6) and (12) we obtain

$$M(t)\equiv M(t-t_1)=h_1\int_{t_1}^{\infty}dt'\,G_-(t-t')=h_1[g+\Delta(z_1)],$$
$$\partial M/\partial z_1=h_1\Delta'(z_1),\quad z_1=z(t-t_1). \tag{25}$$

In (25) g has arisen from the regular part $G_0(t)$, and we have also taken into account that $\Delta(0)=0$. Formula (25) describes the well known long-time relaxation of the magnetic moment. Comparison of (25) with the expression for G_s in (11) shows that the logarithmic derivative of the magnetic moment with respect to the time is proportional to the density of relaxation times. This relationship was indicated empirically in Refs. 6 and 8. We note that the magnetization associated with h_1g is established in a paramagnetic time, whereas the equilibrium magnetization is equal to $h_1(g+\Delta)$ and is established in the time $t_{\max}$.

An analogous situation will also arise for all the other cases, which we now consider. Each expression will consist of two parts—a part relaxing in a paramagnetic time, and a nonequilibrium correction that changes over times of the order of $t_{\max}$.

We now consider the correction introduced into the susceptibility by switching on a constant magnetic field h_0 at time t_0. Obviously, the correction of first order in the anharmonicity constant is equal to

$$\Delta G_-(t,t')=\int dt_1 G_-(t-t_1)G_-(t_1-t')\Sigma(t_1-t_0),$$
$$\Sigma(t)=-\frac{3u}{2T}\mathcal{M}^2(t)=-\frac{3u}{2T}h_0^2[g+\Delta(z)]^2. \tag{26}$$

We note first of all that ΔG depends not on the difference $t-t'$, but on t and t' separately, as is entirely natural in a nonequilibrium situation.

We now consider the case when $|t-t'|\ll|t-t_0|$. In this case, since $G_-(t)\propto\theta(t)$, we have $|t-t_1|\sim|t_1-t'|\ll|t-t_0|$. Then, obviously, from (26) we obtain

$$\Delta G_-(t,t')=\Sigma(t-t_0)\int dt_1 G_-(t-t_1)G_-(t_1-t'),$$
$$\Delta G_-(\omega,t)=\int dt' e^{i\omega t'}\Delta G_-\left(t+\frac{t'}{2},t-\frac{t'}{2}\right)$$
$$=\Sigma(t-t_0)\{g+\Delta[y(\omega)]\}^2. \tag{27}$$

The second formula in (27) expresses the Fourier transform of $\Delta G_-(t,t')$ with respect to the difference $t-t'$. It can be seen from this formula that when an external constant magnetic field is switched on the *ac* susceptibility begins to depend on the observation time. This simple effect is very closely related to the relaxation of the magnetic moment in (25). However, for some reason, there has not yet been a single experiment to study it.

We now consider the general case. We shall study the quantity

$$\Delta M(t)=h_1\int_{t_1}^{\infty}\Delta G_-(t,t')\,dt', \tag{28}$$

i.e., at time t_1 we switch on the measurement magnetic field h_1 and consider the term $\Delta M(t)\propto h_0^2h_1$ as a function of the time t in relation to the times t_0 and t_1. This implies that we are studying the dc susceptibility with respect to the measurement field h_1. This kind of problem is, in essence, equivalent to the experiment of Ref. 6. Here we shall consider only the case $t_1>t_0$; i.e., the measurement field is switched on after the external action on the susceptibility. Using the explicit form of the Green function in (12) and the condition $\alpha\ll1$, in this case we can easily obtain from (26) and (28)

$$\Delta M(t)=h_1\Sigma(z_{tot})[g+\Delta(z_s)]^2,$$
$$z_{tot}=z(t-t_0),\quad z_s=z(t-t_1). \tag{29}$$

Here we have introduced the notation of Ref. 6. In the latter paper two times were introduced–the time $t_s=t-t_1$ of action of the measurement field, and the total time $t_{\text{tot}} = t - t_0$ of the external action. The waiting time, i.e., the time for which the external action is held before the switching on of the measurement field, is expressed in terms of these two times: $t_w = t_{\text{tot}} - t_s = t_1 - t_0$. We note now that for $\alpha \ll 1$

$$z_{tot} = \begin{cases} z_s, & z_s > z_w \\ z_w, & z_w > z_s, \end{cases} \qquad z_w = z(t_1 - t_0). \tag{30}$$

Then from (29) we have

$$\begin{aligned} \Delta M(t) &= h_1 \Sigma(z_w) [g + \Delta(z_s)]^2, \quad z_w > z_s, \\ \Delta M(t) &= h_1 \Sigma(z_s) [g + \Delta(z_s)]^2, \quad z_s > z_w. \end{aligned} \tag{31}$$

From this it can be seen that $\Delta M(t)$ has a discontinuity at $z_s = z_w$. We note that the first formula (31) corresponds to the *ac* susceptibility (27). It is interesting to note the following fact. Even in the case when the observation time is much shorter than the waiting time ($z_w > z_s$ in (31), and formula (27) applies), the susceptibility is by no means equal to its equilibrium value. The equilibrium value is obtained only at $z_w = \infty$. This fact is simply a consequence of the nonequilibrium character of spin glasses, discussed in the Introduction.

In Ref. 6 the logarithmic derivative of $\Delta M(t)$ with respect to time was studied as a function of z_{tot} and z_s. We too shall calculate it. From (29) we have

$$\begin{aligned} \partial \Delta M / \partial z_s = h_1 \{ & 2\Delta'(z_s) [g + \Delta(z_s)] \Sigma(z_{tot}) \\ & + \exp\left(-\frac{z_{tot} - z_s}{\alpha} \right) \Sigma'(z_{tot}) [g + \Delta(z_s)]^2 \}. \end{aligned} \tag{32}$$

Formula (32) is quite remarkable. Since $\alpha \ll 1$, for $z_{\text{tot}} - z_s \gg \alpha$ the second term can be discarded and we have a smooth curve. Then, at $z_{\text{tot}} - z_s \sim \alpha$, the second term begins to operate and the whole expression grows rapidly by an amount of the order of itself, while for $z_{\text{tot}} < z_s$ the whole quantity vanishes by purely kinematic considerations, since the total time cannot be shorter than the observation time. Thus, on the graph of the dependence on z_s we obtain a kind of wave, moving forward with increase of z_{tot}. It was this wave that was observed by the authors of Ref. 6. Thus, formula (32) gives a qualitatively correct description of the experimental situation.

We now consider the case of change of the temperature. Unlike the switching on of a magnetic field, a change in temperature cannot be introduced directly into the Hamiltonian. Therefore, we shall proceed in a manner analogous to the way in which a change of temperature is treated in the theory of phase transitions. Near the transition temperature T_c the temperature appears in the effective Hamiltonian in the form $(T - T_c) m^2$, and a change of temperature at the time t_0 can be described by including in the Hamiltonian the term

$$\Delta H = a\vartheta(t - t_0) m^2. \tag{33}$$

It can be shown, however, that the effect of a change in temperature is not confined to the term of the type (33), which determines only the main effect of this change. We shall study the effect on a spin glass of a term of the type (33) and we shall speak of a change of temperature, but in doing this we must always keep in mind the formal character of this identification.

It is easy to show that in zeroth order in the anharmonicity constant the term (33) does not give a nonequilibrium correction to the Green function, although in the correlator such a correction does arise:

$$\begin{aligned} \Delta q(t, t') &= a \int_{t_0}^{\infty} dt_1 \{ q(t' - t_1) G_+(t_1 - t) + q(t - t_1) G_+(t_1 - t') \} \\ &= \Delta q_1(t, t') + \Delta q_1(t', t). \end{aligned} \tag{34}$$

Because of the nonequilibrium character, Δq depends on t and t' separately, rather than on $t - t'$. Using (12) and the condition $\alpha \ll 1$ we can easily calculate (34); we obtain

$$\begin{aligned} \Delta q_1(t, t') &= a \left\{ [g + \Delta(z)] q(z) + \int^{z_0} dx\, \Delta'(x) q(x) \right\}, \\ \Delta q(t, t) &= 2a \left\{ gq + \int_0^{z_0} dx\, \Delta'(x) q(x) \right\}, \\ z_0 &= z(t - t_0), \quad z = z(t - t'), \quad q = q(0). \end{aligned} \tag{35}$$

It is easy to show that a nonequilibrium correction to the Green function arises in the next order in u. This correction coincides exactly with the expression (26), if in the latter we set

$$\Sigma(t - t_0) = -(3u/2T) \Delta q(t, t), \tag{36}$$

where $\Delta q(t,t)$ is given in (35) and depends on the difference $t - t_0$. After this, the entire analysis performed for formula (26) can be extended also to the case of a change of temperature.

We shall consider one last example of nonequilibrium phenomena. We shall suppose that the field h_1 in (28) was switched on infinitely long ago. Then in the spin glass we have the equilibrium magnetic moment, which is determined by formula (25) with $z_1 = \infty$. At time t_0 we switch on the external action, e.g., of the form (36). Then the magnetic moment acquires a nonequilibrium correction associated with this action. To obtain this correction it is necessary to calculate (28) with $t_1 < t_0$ and $z(t_0 - t_1) > z(t - t_0)$. We then obtain

$$\begin{aligned} \Delta M(t) &= h_1 [g + \Delta(z_0)] \Sigma(z_0) [g + \Delta(z_1)], \\ z_0 &= z(t - t_0), \quad z_1 = z(t - t_1). \end{aligned} \tag{37}$$

For $z_0 = z_1 = \infty$ (37) gives the equilibrium correction to the magnetic moment. The nonequilibrium correction of interest to us is obtained if $z_1 = \infty$ and z_0 is finite.

Thus, we have shown, in the framework of perturbation theory, that any change in the external conditions leads to a nonequilibrium state of the spin glass. Relaxation of this nonequilibrium state occurs over times of the order of the maximum relaxation time. In our examples this corresponds to the fact that only for $t \gg t_{\max}$ can we replace $\Sigma(t)$ by $\Sigma(\infty)$ in formulas (25), (26) and the expressions obtained from them. Since $\Sigma(\infty)$ corresponds to the equilibrium correction, this implies that equilibrium is established over precisely such times.

It is perfectly clear that this situation also obtained outside the framework of perturbation theory. This implies that the process by which the equilibrium state of a spin glass is reached from some particular initial state occurs over times of the order of the maximum relaxation time, which is an astronomical quantity. Therefore, an equilibrium spin glass is an unobservable object, and the main problem of the physics of spin glasses is the problem of studying the nonequilibrium situation. One of the consequences of this nonequilibrium character is the dependence of all physical quantities on the oberservation times.

In conclusion the author thanks S. V. Maleev and B. P. Toperverg for discussions about this work.

[1]R. G. Palmer, Adv. Phys. **31**, 669 (1982).
[2]M. Mézard, G. Parisi, N. Sourlas, G. Toulouse, and M. Virasoro, Phys. Rev. Lett. **52**, 1156 (1984).
[3]M. Mézard, G. Parisi, N. Sourlas, G. Toulouse, and M. Virasoro, J. Phys. (Paris) **45**, 843 (1984).
[4]H. Sompolinsky, Phys. Rev. Lett. **47**, 935 (1981).
[5]Y. J. Uemura, D. R. Harshman, M. Senba, E. J. Ansaldo, and A. P. Murani, Phys. Rev. B **30**, 1606 (1984).
[6]L. Lundgren, P. Svedlindh, P. Nordblad, and O. Beckman, Phys. Rev. Lett. **51**, 911 (1983).
[7]L. Lundgren, P. Svedlindh, and O. Beckman, J. Phys. F **12**, 2663 (1982).
[8]L. Lundgren, P. Svedlindh, and O. Beckman, Phys. Rev. B **26**, 3990 (1982).
[9]G. Parisi, Phys. Rev. Lett. **43**, 1754 (1979).
[10]G. Parisi, J. Phys. A **13**, L115 (1980).
[11]G. Parisi, J. Phys. A **13**, 1101 (1980).
[12]G. Parisi, J. Phys. A **13**, 1887 (1980).
[13]H. Sompolinsky and A. Zippelius, Phys. Rev. Lett. **47**, 359 (1981).
[14]H. Sompolinsky and A. Zippelius, Phys. Rev. B **25**, 6860 (1982).
[15]S. L. Ginzburg, Zh. Eksp. Teor. Fiz. **85**, 2171 (1983) [Sov. Phys. JETP **58**, 1260 (1983)].

Translated by P. J. Shepherd