Coherent oscillations in small tunnel junctions

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A microscopic theory is developed for current-biased tunnel junctions having a small capacitance and a low conductance. The theory gives a natural description of the discreteness of both the one-electron (quasiparticle) and pair (Josephson) components of the tunnel current. It is shown that even in the absence of the pair component (for example, in tunneling between normal metals) coherent oscillations of the electric charge and voltage can arise at the junction, with a frequency proportional to the average current. The characteristics of these "one-electron" oscillations is calculated, and the question of their coexistence with "Bloch" oscillations in Josephson junctions is considered. The feasibility of experimental observation of these effects is discussed briefly.

1. INTRODUCTION

Tunnel junctions of very small area S (fractions of a square micron) have recently become the subject of active theoretical and experimental research. At low temperatures, such junctions can exhibit "secondary" macroscopic quantum effects (see, e.g., the review of Larkin *et al.*[1]). The most qualitatively new of these effects is the predicted transition from the ordinary Josephson oscillations with the frequency

$$\omega_J = (2e/\hbar)\bar{V} \tag{1}$$

to "Bloch" oscillations with a frequency[2,3]

$$\omega_B = (\pi/e)(\bar{I} - G_s\bar{V}), \tag{2}$$

as the size of the Josephson junction is decreased ($\bar{I}$ and $\bar{V}$ are the average values of the current and voltage, and G_s is the quasiparticle conductance shunting the Josephson supercurrent).

The theory developed previously[2,3] for this effect was based on the extremely simple "adiabatic" description of the Josephson supercurrent and on the assumption of thermodynamic equilibrium of the quasiparticle ensemble responsible for the conductance G_s. The latter assumption is valid only in the case when the junction is strongly shunted by an external conductance of a nontunneling nature: $G_s \gg G_T$, where G_T is the tunneling quasiparticle conductance (and at not too large a current). It is also of interest to consider the opposite case ($G_s \lesssim G_T$), in which the discrete character not only of the superconducting component but also of the quasiparticle component of the current can become important. This "secondary" quantization can[4,5] give rise to oscillations at a frequency

$$\omega_s = (2\pi/e)(\bar{I} - G_s\bar{V}) \tag{3}$$

even in the absence of a Josephson coupling between electrons, e.g., in purely one-electron tunneling between normal metals.

The goal of the present study was to create a consistent microscopic theory of the "one-electron" oscillations (3) and to study their influence on the Bloch oscillations (2) in Josephson junctions.

2. STATEMENT OF THE PROBLEM AND DERIVATION OF THE FUNDAMENTAL EQUATION

Let us consider a tunnel junction between metals 1 and 2 which is connected to a fixed external current source $I(t)$ and in the general case shunted by an external conductance G_s of a nontunneling (metallic) nature. The Hamiltonian of such a system is[2,3]

$$H = H_0 + H_T + [I_s - I(t)]\Phi, \qquad H_0 = H_1 + H_2 + H_s + Q^2/2c, \tag{4}$$

where $H_{1,2}$ and H_s describe the internal degrees of freedom of the metals and shunt, respectively, I_s is the operator for the current through the shunt, $Q^2/2c$ is the electrostatic energy of the junction as a capacitor, Φ is the operator for the variable which is canonically conjugate to the electric charge Q,

$$\dot{\Phi} = Q/c, \tag{5}$$

and which therefore satisfies the commutation relation[6]

$$[\Phi, Q] = i\hbar \tag{6}$$

(for a superconducting junction $\Phi = (\hbar/2e)\varphi$, where φ is the Josephson phase difference). Unlike Refs. 2 and 3, we take the tunneling operator H_T in the form of the standard tunneling Hamiltonian[7]

$$H_T = H_+ + H_-, \quad H_+ = \sum_{k_1k_2} T_{k_1k_2} c_{k_1}^+ c_{k_2}, \quad H_- = H_+^+, \tag{7}$$

where c_k^+ and c_k are the electron creation and annihilation operators, and the sum is over all the electronic states of metals 1 and 2. The charge on the capacitance is expressed in terms of the same operators as

$$Q = -\frac{e}{2}\Big(\sum_{k_1} c_{k_1}^+ c_{k_1} - \sum_{k_2} c_{k_2}^+ c_{k_2}\Big) + \text{const}, \quad e = |e|, \tag{8}$$

so that Q and H_T do not commute. Specifically, as is easily verified by substitution, the following relation holds for any operator function $F(Q)$:

$$H_\pm F(Q) = F(Q \pm e) H_\pm. \tag{9}$$

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For any real tunnel junction the number of electronic states N in the metals is very large, so that a charge of moderate size ($|Q| \ll eN$) will not affect their internal properties, and it can be assumed that

$$[H_{1,2}, Q]=0. \tag{10}$$

In the most realistic case, when external current, tunneling current, and shunt current are small and do not disturb the equilibrium of the internal degrees of freedom of the metals and shunt, relations (6), (9), and (10) yield a simple equation in closed form for the density matrix

$$\rho=\mathrm{Sp}_{k_1,k_2,s}\rho_\Sigma, \tag{11}$$

where the trace is taken over the internal states of the electrodes k and shunt s. In the interaction representation this equation is of the form

$$\dot\rho=F_I+F_s+F_T, \tag{12}$$

where F_I, F_s, and F_T are the terms describing the influence of the external current, shunt, and tunneling, respectively, as calculated independently of one another in the first nonvanishing order of standard perturbation theory. The terms F_I and F_s have actually been evaluated previously,[3] and Eqs. (68) and (71) of Ref. 3 can be written in our case as

$$(F_I)_{QQ'}(t)=-I(t)\left(\frac{\partial}{\partial Q}+\frac{\partial}{\partial Q'}\right)\rho_{QQ'}, \tag{13}$$

$$(F_s)_{QQ'}(t)=\int_0^\infty d\tau\exp\{i\omega_{QQ'}(t-\tau)\}\left\{-iA(\tau)\left[\left(\frac{\partial^2}{\partial Q^2}-\frac{\partial^2}{\partial Q'^2}\right)\right.\right.$$
$$-i\tau\omega'_{QQ'}\left(\frac{\partial}{\partial Q}-\frac{\partial}{\partial Q'}\right)+B(\tau)\left[\left(\frac{\partial}{\partial Q}+\frac{\partial}{\partial Q'}\right)^2 \tag{14}$$
$$\left.\left.-i\tau\omega'_{QQ'}\left(\frac{\partial}{\partial Q}+\frac{\partial}{\partial Q'}\right)\right]\right\}\exp\{-i\omega_{QQ'}(t-\tau)\}\rho_{QQ'}(t-\tau),$$

where we have introduced the notation

$$\omega_{QQ'}\equiv(Q^2-Q'^2)/2\hbar c,\quad \omega'_{QQ'}\equiv\left(\frac{\partial}{\partial Q}+\frac{\partial}{\partial Q'}\right)\omega_{QQ'}, \tag{15}$$

and the kernels $A(\tau)$ and $B(\tau)$ can be expressed in terms of the temperature T and the complex conductance $Y_s(\omega)$ of the shunt:

$$A(\tau)=\frac{i}{\pi}\int_{-\infty}^{+\infty}d\omega\,\hbar\omega\,\mathrm{Re}\,Y_s(\omega)e^{i\omega\tau}, \tag{16}$$

$$B(\tau)=\frac{1}{\pi}\int_{-\infty}^{+\infty}d\omega\,\hbar\omega\,\mathrm{cth}\left\{\frac{\hbar\omega}{2T}\right\}\mathrm{Re}\,Y_s(\omega)e^{i\omega\tau}. \tag{17}$$

Merely from the structure of relations (13) and (14) we see[1)] that the electric charge flowing through the nontunneling conductance G_s and the current source $I(t)$ does not exhibit discreteness. This description is correct because in bulk metallic conductors and transported charge is a collective variable, i.e., a function of a large number of coordinates of the current carriers, and is not quantized on the scale of e.

In contrast, the tunneling current is naturally discrete, and this is reflected in the form of the F_T term. In fact, taking the general formula of perturbation theory to second order in H_T in the form

$$F_T(t)=-\hbar^{-2}\int_0^\infty d\tau\,\mathrm{Sp}_{k_1k_2}\{[H_T(t),[H_T(t-\tau),\rho(t-\tau)f]]\}, \tag{18}$$

$$f=Z^{-1}\exp\left\{-\frac{H_1+H_2}{T}\right\},\quad Z=\mathrm{Sp}_{k_1k_2}\left\{\exp\left\{-\frac{H_1+H_2}{T}\right\}\right\} \tag{19}$$

we find that

$$F_T=F_p+F_q, \tag{20a}$$

$$(F_p)_{QQ'}(t)$$
$$=-\hbar^{-2}\int_0^\infty d\tau\sum_\pm[\rho_{Q,Q'\pm2e}\langle H_\mp H_\mp(\tau)\rangle\exp\{-i(\omega_{Q'\pm2e,Q'\pm e}\tau$$
$$-\omega_{Q'\pm2e,Q'}t)\}+\rho_{Q\pm2e,Q'}\langle H_\pm(\tau)H_\pm\rangle\exp\{i(\omega_{Q\pm2e,Q\pm e}\tau-\omega_{Q\pm2e,Q}t)\}$$
$$-\rho_{Q\pm e,Q'\mp e}(\langle H_\pm H_\pm(\tau)\rangle\exp\{-i\omega_{Q'\mp e,Q'}\tau\}+\langle H_\pm(\tau)H_\pm\rangle$$
$$\times\exp\{-i\omega_{Q,Q\pm e}\tau\})\exp\{i(\omega_{Q'\mp e,Q'}+\omega_{Q,Q\pm e})t\}], \tag{20b}$$

$$(F_q)_{QQ'}(t)=-\hbar^{-2}\int_0^\infty d\tau\sum_\pm[\rho_{QQ'}(\langle H_\pm(\tau)H_\mp\rangle\exp\{-i\omega_{Q\pm e,Q}\tau\}$$
$$+\langle H_\pm H_\mp(\tau)\rangle\exp\{i\omega_{Q'\pm e,Q'}\tau\})+\exp\{i(\omega_{Q'\pm e,Q'}+\omega_{Q,Q\pm e})t\}$$
$$\times\rho_{Q\pm e,Q'\pm e}(\langle H_\mp H_\pm(\tau)\rangle\exp\{-i\omega_{Q'\pm e,Q'}\tau\}$$
$$+\langle H_\mp(\tau)H_\pm\rangle\exp\{-i\omega_{Q,Q\pm e}\tau\})], \tag{20c}$$

where

$$\langle\ldots\rangle\equiv\mathrm{Sp}_{k_1k_2}\{\ldots f\},$$

and everywhere under the integral sign the density matrix ρ is taken at time $t-\tau$. The averages of the products of the operators $H_\pm$ in Eqs. (20) can be expressed[7-11] in terms of the functions $I_{p,q}(\omega)$,

$$\langle H_+H_\mp(\tau)\rangle=\langle H_-H_\pm(\tau)\rangle=\frac{\hbar^2}{4\pi e}\int_{-\infty}^{+\infty}d\omega\,\mathrm{Im}\,I_{q,p}(\omega)E_+(\omega,T), \tag{21a}$$

$$\langle H_+(\tau)H_\mp\rangle=\langle H_-(\tau)H_\pm\rangle=\frac{\hbar^2}{4\pi e}\int_{-\infty}^{+\infty}d\omega\,\mathrm{Im}\,I_{q,p}(\omega)E_-(\omega,T), \tag{21b}$$

$$E_\pm(\omega,T)=\mathrm{cth}\left\{\frac{\hbar\omega}{2T}\right\}\cos\omega\tau\pm i\sin\omega\tau, \tag{21c}$$

which determine the average value of the tunneling current under conditions of a fixed static voltage $\bar V$ across the junction:

$$I(t)=\mathrm{Re}\,I_p(e\bar V/\hbar)\sin\varphi+\mathrm{Im}\,I_p(e\bar V/\hbar)\cos\varphi+\mathrm{Im}\,I_q(e\bar V/\hbar), \tag{22a}$$

$$\varphi=(2e/\hbar)\bar Vt+\mathrm{const}. \tag{22b}$$

The functions $I_{p,q}(\omega)$ have well-known[6-8] expressions in terms of the normal conductance of the junction and the internal properties of metals 1 and 2, and they can be regarded as given. Therefore, expressions (12)–(17), (20), and (21) form a closed system of equations for the density matrix ρ; this system of equations describes the dynamics of the charge $Q = cV$ on the junction at a fixed external current $I(t)$.

3. ONE-ELECTRON OSCILLATIONS

Let us consider the case in which Josephson tunneling is absent, $I_p(\omega) \equiv 0$, i.e., either at least one of the metals is normal or the supercurrent is suppressed by a magnetic field. Then, as we see from expression (21), the F_T term in Eq. (18), like F_I, couples only those elements of the density matrix $\rho_{QQ'}$ which are equidistant from the principal diagonal ($Q - Q' = \text{const}$), and so cannot generate off-diagonal elements. The shunt, in turn, leads to a damping of the off-diagonal elements, with a time constant $\tau \sim \tau_s$, where

$$\tau_s \equiv c/G_s, \tag{23}$$

provided that its conductance is not too large[3,12]:

$$G_s R_Q \leqslant 1, \qquad R_Q = \pi\hbar/2e^2 \approx 6.7\ \text{k}\Omega. \tag{24}$$

In this case the density matrix ρ rapidly becomes diagonal and remains so:

$$\rho_{QQ'}(t) = \sigma(Q,t)\delta(Q-Q'). \tag{25}$$

Let us also suppose that the maximum frequency ($\omega_{\max}$) for variations in σ is much smaller than the characteristic frequencies in the integrand in expressions (20):

$$\hbar\omega_{\max} \ll \max\left\{\frac{e}{c}\min[(Q\pm 2e), (Q\pm e)], T\right\}. \tag{26}$$

In this case σ can be taken outside the integral with respect to τ, and the equation for this quantity assumes the simple form

$$\frac{\partial\sigma}{\partial t} = -I(t)\frac{\partial\sigma}{\partial Q} + \tau_s^{-1}\frac{\partial}{\partial Q}\left(cT\frac{\partial\sigma}{\partial Q} + \sigma Q\right) + F_+ + F_- - F_0, \tag{27a}$$

$$F_\pm(Q) = f_\pm\left(Q \pm \frac{e}{2}\right)\sigma(Q\pm e),$$

$$F_0(Q) = \left[f_+\left(Q - \frac{e}{2}\right) + f_-\left(Q + \frac{e}{2}\right)\right]\sigma(Q), \tag{27b}$$

$$f_\pm(Q) = e^{-1}\,\text{Im}\, I_q\left(\frac{eQ}{\hbar c}\right)\frac{e(\pm)}{e(+) - e(-)},$$

$$e(\pm) = \exp\left\{\pm\frac{eQ}{2cT}\right\} \tag{27c}$$

(this kinetic equation can also be obtained directly from the quantum mechanical "golden rule"). It follows from this equation that the deviation of the behavior of this system from the "classical" behavior, i.e., from the behavior at a fixed voltage, is manifested most clearly at low temperatures

$$T \ll E_Q = e^2/2c. \tag{28}$$

Then

$$f_\pm(Q) \approx e^{-1}\left|\,\text{Im}\, I_q\left(\frac{eQ}{\hbar c}\right)\right|\theta(\pm Q), \tag{29}$$

i.e., if the charge Q is concentrated in the region

$$-e/2 < Q < e/2, \tag{30}$$

then the tunneling is completely suppressed: $F_0 \to 0$. The physical reason for this is that when the voltage V on the contact is not fixed, the tunneling of a single electron changes the Coulomb energy $Q^2/2c$ by an amount

$$\Delta E = [(Q\pm e)^2 - Q^2]/2c. \tag{31}$$

If the charge is localized in region (30), then $\Delta E > 0$, and so this process is suppressed at low temperatures (28). An analogous phenomenon has been observed in tunnel junctions having a metal-impregnated oxide layer[13–15] and has been treated theoretically in Ref. 16.

Let us now consider the case of a constant external current $I(t) = I$. If

$$\bar{I} < I_t \equiv e/2\tau_s, \tag{32}$$

then Eq. (27) has a solution which describes a steady state

$$\sigma(Q) = ((2\pi)^{1/2}\tilde{Q})^{-1}\exp\{-(Q-Q_0)^2/2\tilde{Q}^2\}, \quad Q_0 = I\tau_s, \quad \tilde{Q} = (cT)^{1/2} \ll e. \tag{33}$$

The set of such states gives a linear region, in which there is no tunneling current, on the voltage-ampere characteristic of the junction:

$$\bar{V} = \bar{I}/G_s \quad \text{for } |\bar{I}| < I_t. \tag{34}$$

If $I > I_t$, then Eq. (27) describes coherent oscillations at a frequency (3) which is twice as large as the frequency of the Bloch oscillations. In the simplest case, that of a junction between normal metals under the additional conditions

$$G_s \ll G_T, \tag{35}$$

$$I \ll \frac{e}{\tau_T}, \qquad \tau_T \equiv \frac{c}{G_T}, \qquad G_T \equiv \left.\frac{d\,\text{Im}\, I_q(e\bar{V}/\hbar)}{d\bar{V}}\right|_{\bar{V}=0}, \tag{36}$$

$$\bar{I} \gg I_t, \tag{37}$$

the solution describing these oscillations, i.e., the solution of Eq. (27) with the initial condition $\sigma(Q,0) = \delta(Q)$, has the following form for $t \ll \tau_s$ (for the sake of definiteness we take $I > 0$):

$$\sigma(Q,t) = f(Q)\sum_{n=-\infty}^{+\infty}\delta(Q + en - \bar{I}t), \tag{38a}$$

$$f(Q) = \begin{cases} 0, & Q < -\dfrac{e}{2}, \\ 1 - \exp\left\{-\dfrac{(Q-e/2)^2}{2\bar{I}e\tau_T}\right\}, & Q \in \left[-\dfrac{e}{2}, \dfrac{e}{2}\right], \\ \exp\left\{-\dfrac{(Q-e/2)^2}{2\bar{I}e\tau_T}\right\}, & Q > \dfrac{e}{2}. \end{cases} \tag{38b}$$

Solution (38) describes a periodic process consisting of the motion of charge over region (30) at a rate $\bar{I}$ and the subsequent rapid transfer of probability from the region $Q \approx +e/2$ to the region $Q \approx -e/2$ (this transfer corresponds to the tunneling of a single electron). The physical cause of these "one-electron" oscillations is that the tunneling of a single electron decreases the voltage across the junction by an amount $\Delta V = e/c$, i.e., alters the tunneling conditions for the remaining electrons, thereby correlating the tunneling current.

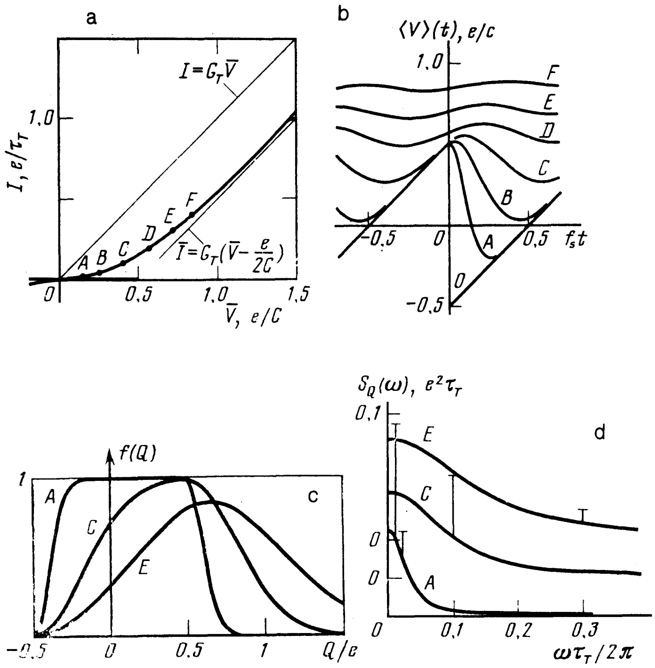

FIG. 1. One-electron oscillations in the absence of Josephson tunneling: a) volt-ampere characteristic of the junction; b) shape of the oscillations; c) steady-state distribution of the charge probability density; d) spectral density of the oscillations of the charge on the junction. The values of the spectral components of the one-electron oscillations would correspond to measurements on a spectrum analyzer with a passband $\Delta\omega = 1/4\tau_T$.

By finding the ensemble average of the voltage across the junction,

$$\langle V\rangle(t)=\frac{e}{c}\sum_n f(\bar It-en), \tag{39}$$

we easily determine that the one-electron oscillations correspond to a region on the volt-ampere characteristic beginning at

$$\bar V=\frac{e}{c}\left(\frac{\pi}{2}\frac{\bar I\tau_T}{e}\right)^{1/4} \quad \text{for} \quad \frac{\bar I\tau_T}{e}\ll 1. \tag{40}$$

The spectrum of the junction voltage in this approximation consists of monochromatic lines of the harmonics of these oscillations (of frequency $2\pi n\bar I/e$) and a low-frequency noise pedestal characterized by a height

$$S_V(0)\sim e^2/cG_T \tag{41a}$$

and a cutoff frequency

$$\omega_{max}\sim(\bar I/e\tau_T)^{1/2}. \tag{41b}$$

For $\bar I\sim e/\tau_T$ formula (38b) becomes invalid, and the junction characteristic can be calculated by substituting solution (38a) into kinetic equation (27). The results of such a calculation are shown in Fig. 1. It turns out that when the external current is increased to $\bar I\gtrsim 0.1e/\tau_T$ the aforementioned correlations in the motion of the individual electron gradually vanish, the amplitude of the one-electron oscillations falls off rapidly (Fig. 1b), and the noise pedestal increases, going over to the usual shot noise:

$$S_V(\omega)=S_I(\omega)[G_T^2+(\omega c)^2]^{-1}, \quad S_I(\omega)=e\bar I/2\pi. \tag{42}$$

The volt-ampere characteristic of the junction then tends toward the linear asymptotic relation

$$\bar V=G_T^{-1}\bar I+\frac{e}{2c}\operatorname{sign}\bar I. \tag{43}$$

A nonvanishing shunt conductance will introduce a number of changes in the dynamics of the one-electron oscillations. First, a transition region with a nonzero current width arises between the linear region of the volt-ampere characteristic (34) and region (40). For $G_s\ll G_T$ the shape of this region and the oscillation frequency in it are given by formulas analogous to the case of the Bloch oscillations[2]:

$$\bar V=G_s^{-1}\left[\bar I-2I_t\ln^{-1}\left(\frac{\bar I-I_t}{\bar I+I_t}\right)\right], \quad \bar I\gg I_t, \tag{44a}$$

$$\omega_s=\frac{2\pi}{\tau_s}\ln^{-1}\left(\frac{\bar I-I_t}{\bar I+I_t}\right). \tag{44b}$$

At large currents ($\bar I\gg e/\tau_T, I_t$) the volt-ampere characteristic approaches the straight line

$$\bar V=(G_s+G_T)^{-1}\left(\bar I+\frac{e}{2\tau_T}\operatorname{sign}\bar I\right). \tag{45}$$

In addition, because of the nonuniformity of the motion of the "packets" in different regions on the Q axis, the packets broaden, leading to a nonzero width 2Γ of the spectral components of the one-electron oscillations. For a small current (36), for which the probability density is given by expression (38), this width is of the order of

$$\Gamma\sim\frac{1}{\tau_s}\left(\frac{\bar I\tau_T}{e}\right)^{1/2}. \tag{46}$$

Increasing G_s to above G_T completely smears out the lines of the one-electron oscillations.

A nonzero but small [condition (28)] temperature T causes fluctuations of the shunt current, leading to an additional broadening of the spectral lines of the one-electron oscillations. Using general rules[17] to rewrite the corresponding part of Eq. (27) in the form of the equivalent Langevin equation

$$\dot Q_0=I(t)-\tau_s^{-1}Q_0+\tilde I(t), \quad \langle\tilde I(t)\rangle=0,$$
$$\langle\tilde I(t)\tilde I(t+\tau)\rangle=2G_sT\delta(\tau) \tag{47}$$

and proceeding as in the case of the Bloch oscillations,[2] we obtain for the temperature part of the linewidth

$$\Gamma=f(\bar I/I_t)\Gamma_T, \quad \Gamma_T=(2\pi/e)^2G_sT, \tag{48}$$

where f is a function which goes rapidly to unity for $I\gg I_t$. In addition, an increase in temperature leads to a smearing of the corners of the functions $f_\pm(Q)$ in (27c) and, as a result, to a decrease in the amplitude of the one-electron oscillations. On a further increase in the temperature (to a value $T\sim E_Q$), the one-electron oscillations are completely suppressed.

4. COEXISTENCE OF THE ONE-ELECTRON AND BLOCH OSCILLATIONS

In the presence of Josephson tunneling, $I_p(\omega)\not\equiv 0$, Eq. (12) for the density matrix $\rho(Q,Q')$ in the general case cannot be reduced to an equation like (27) for the diagonal part

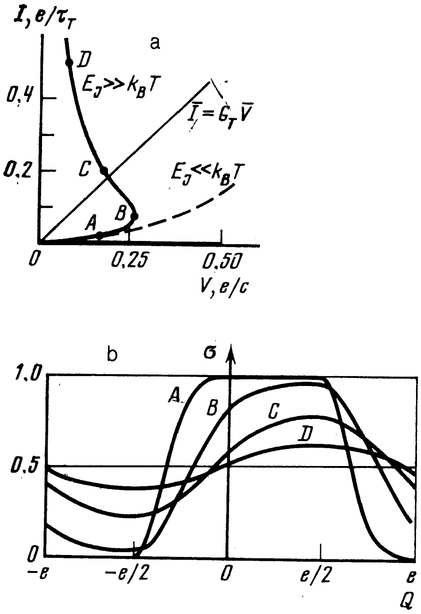

FIG. 2. Coexistence of the one-electron and Bloch oscillations: a) volt-ampere characteristic of the junction; b) steady-state distribution of the charge probability density at various currents.

$\sigma(Q)$. However, in the case of the greatest practical importance, when not only the frequencies (26) for variations in ρ are small but so is the scale of the voltage on the contact during the one-electron (and Bloch) oscillations:

$$e/c \lesssim \Delta_{1,2}(T)/e, \text{ i.e. } E_Q \lesssim \Delta_{1,2}(T), \tag{49}$$

we can assume that[8,9]

$$I_p(eV/\hbar) \approx I_c \equiv 2eE_J/\hbar \tag{50}$$

and, using Eq. (21) and the Kramers-Kronig relations, we can write expression (20b) as

$$(F_p)_{QQ'}(t) = \frac{iE_J}{2\hbar} \sum_{\pm} [\rho_{Q\pm 2e,Q'} \exp\{i\omega_{Q,Q\pm 2e}t\} - \rho_{Q,Q'\pm 2e} \exp\{i\omega_{Q'\pm 2e,Q'}t\}]. \tag{51}$$

Equation (51) agrees with the expression obtained from the "adiabatic" form of the tunneling Hamiltonian[1-3]:

$$H_T = -E_J \cos\varphi, \tag{52}$$

and therefore, for describing the influence of the supercurrent on the dynamics of the charge, we can use the results of Refs. 2 and 3.

This influence is simplest in the case

$$T \ll E_J \ll E_Q, \tag{53}$$

when the effect of the supercurrent is felt only in a small neighborhood $\Delta Q \sim e(E_J/E_Q)$ of the points $Q = \pm e$. Outside these regions the quasicharge[2,3] q is the same as the charge Q, and $F_p \approx 0$, so that the simplest kinetic equation (27) again holds. The influence of the supercurrent is manifested in the tunneling of Cooper pairs when the charge Q reaches one of the boundaries $\pm e$, resulting in the reflection of the system to the opposite point $Q = \mp e$ (strictly speaking, only if one can neglect the thermal and Zener excitation of the higher bands of the energy spectrum[2,3]). This effect can be described by imposing on Eq. (27) the cyclic boundary conditions

$$\sigma(-e,t) = \sigma(+e,t), \tag{54}$$

so that outside the interval $[-e, +e]$ we have $\sigma(Q) \equiv 0$.

Figures 2 and 3 show the solution of Eq. (27) with boundary conditions (54) in the limit $T \to 0$, $G_s \to 0$. We see that when the current is increased to $\sim 0.07\, e/\tau_T$, the shape of the volt-ampere characteristic goes over from the typical shape for one-electron oscillations to the typical shape for Bloch oscillations. The presence of one-electron tunneling, however, introduces substantial changes in the dynamics of the process even at large currents $\bar{I} \gg e/\tau_T$. In fact, the solution of Eqs. (27) and (54) with the initial condition $\sigma(Q,0) = \delta(Q)$ would be of the form of a single probability "packet"

$$\sigma(Q,t) = \delta(Q - Q(t)), \quad Q(t) = (\bar{I}t + e) \bmod (2e) - e \tag{55}$$

moving over the interval $[-e, e]$ and would thus have a

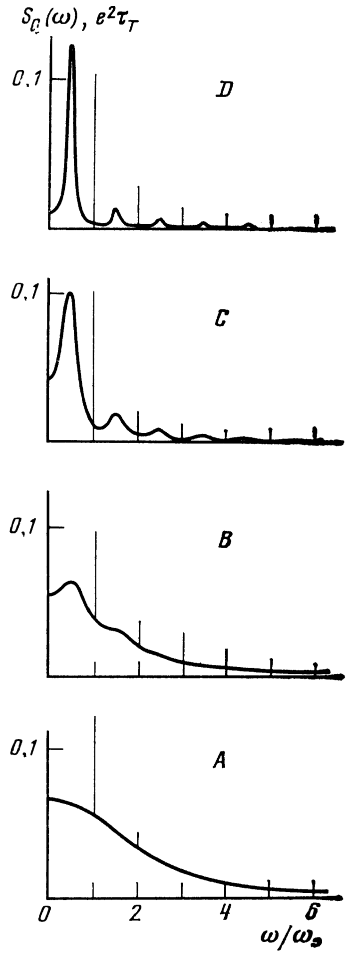

FIG. 3. Spectral density of the oscillations of the charge on the junction; the curves reflect the transition from one-electron oscillations to a superposition of one-electron and Bloch oscillations. The spectral components of the one-electron oscillations are represented as in Fig. 1. The values of the current in A, B, C, and D correspond to Fig. 2.

period of $2e/\bar{I}$. However, even for a small one-electron tunneling the steady-state solution of this equation will be a double "packet"

$$\sigma(Q,t)={}^1/_2\{\delta(Q-Q(t))+\delta(Q-Q(t)-e\,\mathrm{sign}\,Q)\}, \qquad (56)$$

to which solution (55) will go over with a time constant $\tau\sim\tau_T$. At large currents the process can be treated as a kind of superposition of coherent Bloch and one-electron oscillations, with the Bloch component acquiring a nonzero linewidth $\Gamma\sim\tau_T^{-1}$ even for $G_s=0$, $T=0$, whereas the one-electron component remains monochromatic [in the absence of the broadening mechanisms of Eqs. (46) and (48), of course]; see Fig. 3.

Interestingly, in the present case, unlike the case in which there is no Josephson tunneling (see Sec. 3), the presence of a shunt with a large conductance $G_s > G_T$ is still not a sufficient condition for the smearing out of the spectral components of the one-electron oscillations. The physical cause of the destruction of the one-electron oscillations is the nonuniform motion of the probability "packets" (56). At large currents there is a decrease in the relative nonuniformity of this motion, and, as a result, the spectral components are not completely smeared out, although they have a nonzero width.

5. CONCLUSION

We have seen that when a tunnel junction of small capacitance and conductance is connected to an external current source, two types of oscillations, having frequencies in the simple relationship $\omega_s=2\omega_B$, can arise at low temperatures. The one-electron oscillations ω_s can also occur in the absence of Josephson tunneling. In a Josephson junction the two effects form a single oscillatory effect with a gradual transition from the one-electron oscillations to a superposition of the one-electron and Bloch oscillations as the external current is increased.

The conditions for experimental observation of the one-electron oscillations [these conditions are expressed by inequalities (24), (26), (28), (35), and (36)] are basically the same as those for the observation of the Bloch oscillations (except that the metals can be normal). These conditions, and also the possibilities for practical applications of the Bloch (and, consequently, of the one-electron) oscillations, were discussed in detail in Ref. 3. The necessary conditions are satisfied by the following set of parameters, for example:

$$c\approx3\cdot10^{-15}\ \mathrm{F}\ (S\approx1\ \mu\mathrm{m}^2),\ G_T^{-1}\approx30\ \mathrm{k}\Omega,$$
$$G_s^{-1}\approx100\ \mathrm{k}\Omega,\quad T\leqslant0.3\ \mathrm{K}. \qquad (57)$$

With these parameters the scale of the volt-ampere characteristic (Fig. 1a) should be $e/\tau_T\approx1$ nA along the current axis and $e/c\approx50\ \mu$V along the voltage axis, and the typical frequency of the one-electron oscillations should be of the order of 10^9 Hz.

Importantly, the theory considered here remains valid for nonideal tunnel junctions in which the oxide layer is disrupted by metallic microshorts and, generally, for any weak junctions. The only fundamental condition here is that the dimensions of the connectors joining the metals be much smaller than the electron energy free path in them, because then the discreteness of the charge transfer through the connector will be preserved.

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[1)] This conclusion can be reached merely from the form of Hamiltonian (3), which has a nonperiodic dependence on Φ. As we know, the momentum of a quantum mechanical system assumes a discrete set of values only if a condition of spatial periodicity is imposed on its wave function.

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