Self-excited oscillations in a bounded beam-plasma system

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A problem of nonlinear self-excited oscillations in a beam-plasma system bounded in the longitudinal direction by planes held at the same potential (Pierce conditions) is formulated and solved. There is no preliminary modulation of the beam. No assumptions of any sort are made regarding standing or traveling waves. The eigenfrequencies of the system are found numerically from the condition that the nonlinear equations derived here have a solution. The amplitudes of the corresponding waves excited by evolving bunches and the axial profiles of these amplitudes are calculated. The critical lengths of the system are found.

There are two standard formulations of the problem of the excitation of electrostatic electron waves in a plasma penetrated by an electron beam. One formulation deals with the time evolution of a perturbation which is specified throughout the space at the initial time ($t=0$). This formulation of the problem, for an unbounded plasma with a monoenergetic beam, has been studied theoretically in considerable detail, to the point that the particular features of the wave process have been determined in the nonlinear stage of the instability, where the beam breaks up into bunches.[1–3]

The other standard approach deals with the steady-state injection into the plasma of a beam which is modulated at a given frequency ω in the injection plane ($z=0$). In the steady state, at nonzero z, the amplitude of the oscillations at each point is independent of the time, and a wave of frequency ω exists in the system. The evolution of this wave as a function of distance is also determined by the bunching of the beam.[4–6]

With regard to the oscillations which are observed when an unmodulated beam is injected into a plasma, the situation is not yet clear, although the oscillations in this case obviously result from a beam-plasma instability. Extensive experiments and numerical simulations indicate that the waves which are excited are highly regular and that there is a steady-state oscillation, which is evidently equivalent to neither of these two standard formulations of the problem of the collective interaction of a beam with a plasma. This steady-state oscillation may be an interesting version of self-excited oscillations.[7]

An important point is that both the numerical simulations[8,9] and the actual laboratory experiments usually deal with a system which is bounded in the longitudinal direction. The importance of this boundedness of the plasma in a beam-plasma interaction was recognized quite early in the research, even in work on the linear stage.[10] The wavelength of the oscillations which grow in the course of the instability was linked directly with the length of the system. In contrast, the steady-state nonlinear self-excited oscillations in the interaction of a beam with a plasma have not been studied theoretically.

Our purpose in the present paper is to analyze self-propagation in a bounded beam-plasma system.

BUNCHING OF AN ELECTRON BEAM IN AN OPPOSITELY DIRECTED WAVE

We know that an electron beam will interact efficiently with a traveling wave when the two are synchronized i.e., when the phase velocity of the wave is approximately equal to the beam velocity. The effect on a beam of waves which are not synchronized with it is ignored. On the other hand, it is simple to show that such waves can cause significant bunching of the beam, providing the feedback required for self-excited oscillation.

Let us assume that a monoenergetic electron beam is injected into the half-space $z>0$ across the $z=0$ plane, in the direction normal to this plane. The beam velocity in the injection plane is v_0. The beam is not modulated here. A longitudinal electric-field wave $E(z,t)=E_0\cos(\omega t+kz)$ is propagating in the opposite direction.

To integrate the equation of motion of the beam electrons,

$$m\ddot{z}=-eE_0\cos(\omega t+kz) \tag{1}$$

we switch to Lagrangian variables z, t_0. The variable t_0 is a parameter representing the time at which some electron enters the system ($z=0$). The time t at which this electron arrives at the point with coordinate z is given by

$$t=t_0+\int_0^z\frac{dz}{v_0+\tilde{v}(z,t_0)}=t_0+\frac{z}{v_0}-\frac{1}{\omega}g(z,t_0), \tag{2}$$

where $\tilde{v}(z,t_0)$ is the increment in the electron velocity caused by the electric field of the wave, and $g(z,t_0)$ is a function—unknown at this point—characterizing the degree of bunching of the beam.[11] Everywhere below, we assume the case $\tilde{v}/v_0\ll 1$. According to (2), we then have

$$g(z,t_0)\approx\frac{\omega}{{v_0}^2}\int_0^z\tilde{v}(z,t_0)\,dz. \tag{3}$$

Substituting (2) into (1), and discarding terms quadratic in $\tilde{v}/v_0$, we find the equation of motion

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$$\frac{d\tilde{v}}{dz}=-\frac{eE_0}{mv_0}\cos\left[\omega t_0+\left(\frac{\omega}{v_0}+k\right)z\right]. \qquad (4)$$

Integrating it under the initial condition $\tilde{v}(0, t_0)=0$, we find

$$\tilde{v}(z,t_0)=\frac{eE_0}{m\omega_1}\sin\omega t_0-\frac{eE_0}{m\omega_1}\sin\left(\omega t_0+\frac{\omega_1 z}{v_0}\right), \qquad (5)$$

where $\omega_1=\omega+v_0k$ is the frequency in the coordinate system moving with the beam. We see that electrons which are injected into the system at different times drift at different velocities. This effect is of the same nature as the drift of electrons in a uniform *rf* field, which is associated with the phase at which the electrons are produced,[12] but in the case of a beam this effect leads to a substantial bunching of the electrons—the formation of bunches. Substituting (5) into (3), we find

$$g=\frac{eE_0\omega z}{mv_0^2\omega_1}\sin\omega t_0-\frac{2eE_0\omega}{mv_0\omega_1^2}\sin\frac{\omega_1 z}{2v_0}\sin\left(\omega t_0+\frac{\omega_1 z}{2v_0}\right). \qquad (6)$$

Denoting the beam current density at the points $z=0$ and z by j_0 and $j_b(z)$, respectively, using the charge conservation condition

$$j_0 dt_0=j_b(z)\,|dt|, \qquad (7)$$

and also using (2), we find

$$j_b(z)=j_0\left(1-\frac{1}{\omega}\frac{dg}{dt_0}\right)^{-1}. \qquad (8)$$

It can be seen from (6) and (8) that the bunching during the *rf* drift increases with increasing z, and at the coordinate $S_{\rm dr}\approx mv_0^2\omega_1/eE_0\omega$ the density of the bunches which have formed approaches infinity. Phase focusing occurs.

In this discussion we have ignored the inverse effect of the beam on the electric field. If the process occurs in vacuum, the space-charge field of the bunches which arises will prevent bunching. In a medium with a negative dielectric constant (a plasma at frequencies $\omega<\omega_p$, where ω_p is the plasma frequency), on the other hand, a field which is synchronized with a bunch arises and intensifies the bunching. In a synchronous field, the bunching can quickly become dominant.

EQUATIONS OF THE SELF-EXCITED OSCILLATIONS

To analyze the self-consistent self-propagation in a one-dimensional beam-plasma system we use the linearized hydrodynamic equations of a cold plasma,

$$\partial v_p/\partial t=eE/m, \quad j_p=\rho_0 v_p, \qquad (9)$$

where v_p is the velocity of the plasma electrons, ρ_0 is their unperturbed charge density, and j_p is the current density of the plasma electrons, along with Maxwell's equation for the current density,

$$j_b+j_p+\frac{1}{4\pi}\frac{\partial E}{\partial t}=j_i, \qquad (10)$$

where j_i is the density of the induced current. This current density does not vanish in the case of system of finite size.

Using (9) and (10) and the known expression for the current induced in the external circuit,

$$j_i=\frac{1}{L}\int_0^L (j_b+j_p)\,dz, \qquad (11)$$

where L is the length of the system, we find an equation which relates the electric field E in the plasma to the convection current of the beam:

$$\frac{\partial^2 E}{\partial t^2}+\omega_p^2 E=-4\pi\frac{\partial j_b}{\partial t}+\frac{4\pi}{L}\int_0^L\frac{\partial j_b}{\partial t}dz. \qquad (12)$$

We assume that the system is bounded at $z=0$ and $z=L$ by conducting planes which are held at a common potential, by analogy with the approach taken in the analysis of the Pierce instability, i.e., $\int_0^L E dz=0$. Because of the integral on the right side of (12), the j_b dependence of E ceases to be a local dependence peculiar to the plasma when the second boundary is absent, i.e., when $L\to\infty$. At small z, the field now depends on j_b at large z. Although this field is not in the form of a wave synchronized with a beam, an initial bunching of the beam may occur, as we showed above, with the consequence that a nontrivial solution arises, describing a steady-state self-excited oscillation with homogeneous boundary conditions at the injection plane:

$$j_b|_{z=0}=0, \quad \tilde{v}|_{z=0}=0. \qquad (13)$$

We thus assume that self-excited oscillations, periodic in time, are established in the system. At this point we do not know the fundamental frequency ω of these oscillation. Without making nay assumptions regarding the phases or velocities of the waves, we can then write the beam current density as the Fourier series

$$j_b(z,t)=j_0+\sum_{n=1}^{\infty}(A_n(z)\sin n\omega t+B_n(z)\cos n\omega t) \qquad (14)$$

with the coefficients

$$A_n(z)=\frac{\omega}{\pi}\int_0^{2\pi/\omega} j_b(z,t)\sin n\omega t\,dt,$$
$$B_n(z)=\frac{\omega}{\pi}\int_0^{2\pi/\omega} j_b(z,t)\cos n\omega t\,dt. \qquad (15)$$

Substituting (14) into (12), we find the steady-state alternating electric field

$$E=\sum_{n=1}^{\infty}(\xi_n(z)\sin n\omega t+\eta_n(z)\cos n\omega t),$$

$$\xi_n=\left[-4\pi n\omega B_n+\frac{4\pi n\omega}{L}\int_0^L B_n dz\right](n^2\omega^2-\omega_p^2)^{-1}, \qquad (16)$$

$$\eta_n=\left[4\pi n\omega A_n-\frac{4\pi n\omega}{L}\int_0^L A_n dz\right](n^2\omega^2-\omega_p^2)^{-1}.$$

Changing the notation in (2) to correspond to our case of self-excited oscillations, i. e., changing g to g_s and finding

$$g_s=\frac{e\omega}{mv_0^2}\int_0^z dz'\int_0^{z'} E(z'',t_0)\,dz'' \qquad (17)$$

from the equation of motion, we obtain a closed system of

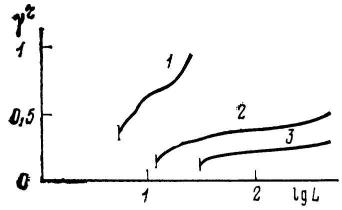

FIG. 1. The eigenvalues γ^2 versus the dimensionless length of the system, L.

equations: (2), (7), and (15)–(17).

We further assume that ω is close to ω_p, so that the second and higher-order harmonics are far from resonance. Under this condition, even though j_b may be very anharmonic, the electric field will be dominated by the fundamental frequency, and we can discard all terms with $n \neq 1$ in (16). The condition that the process be periodic in time allows us to seek the unknown function g_s in the form (see Appendix 1)

$$g_s\ (z,\ t_0) = x(z)\sin\omega t_0 + y(z)\cos\omega t_0. \tag{18}$$

Substituting (18) into (2), and using (7), we can express the coefficients $A_1(z)$ and $B_1(z)$ in terms of the Bessel functions J_0 and J_1 of x and y. We then use (16) and (17) to construct a system of ordinary nonlinear equations for $x(z)$ and $y(z)$, which we write as follows, denoting $\omega z/v_0$ by z_1 and $\omega L/v_0$ by L_1 (for convenience, we omit the indices on z and L everywhere below):

$$\frac{d^2x}{dz^2} - 2\gamma^2 J_0(x)J_1(x)\,[J_0^2(y) - J_1^2(y)] = 2\gamma^2(J_1(x)J_1(y)\sin z$$
$$-J_0(x)J_0(y)\cos z)I_2(L) - 2\gamma^2(J_1(x)J_1(y)\cos z$$
$$+J_0(x)J_0(y)\sin z)I_1(L), \tag{19}$$
$$\frac{d^2y}{dz^2} - 2\gamma^2 J_0(y)J_1(y)\,[J_0^2(x) - J_1^2(x)] = 2\gamma^2(J_1(x)J_1(y)\sin z$$
$$+J_0(x)J_0(y)\cos z)I_1(L) + 2\gamma^2(J_1(x)J_1(y)\cos z$$
$$-J_0(x)J_0(y)\sin z)I_2(L),$$

where

$$I_1(L) = \frac{1}{L}\int_0^L (J_1(x)J_0(y)\sin z - J_1(y)J_0(x)\cos z)\,dz,$$

$$I_2(L) = \frac{1}{L}\int_0^L (J_1(x)J_0(y)\cos z + J_1(y)J_0(x)\sin z)\,dz.$$

Here $\gamma^2 = \omega_b^2/(\omega_p^2 - \omega^2)$ where $\omega_b^2 = 4\pi j_0 e^2/mv_0$ is the eigenfrequency of the beam. Eqs. (19) must be solved under the initial conditions

$$x|_{z=0} = y|_{z=0} = 0,\quad \left.\frac{dx}{dz}\right|_{z=0} = \left.\frac{dy}{dz}\right|_{z=0} = 0. \tag{20}$$

Because of the functionals $I_1(L)$ and $I_2(L)$ in (19), our problem is converted into an eigenvalue problem. The eigenvalues γ for a given L and the corresponding solutions $x(z)$ and $y(z)$ are found numerically (Appendix 2).

CALCULATED RESULTS AND DISCUSSION

Figure 1 shows the functional dependence of γ on L found from the calculations. We see that this is generally a multivalued function. At a fixed L, there can be a self-excited oscillation at various frequencies corresponding to different branches of the function $\gamma(L)$. The oscillation frequencies are given by

$$\omega_s^2 = \omega_p^2 - \omega_b^2/\gamma_s^2, \tag{21}$$

where s in the index of the branch of $\gamma(L)$. On each branch, the frequency of the self-excited oscillation thus increases only slightly with increasing L. At very small values $L < L_{\min\,s}$, there are no eigenvalues γ and thus no nonvanishing solutions of Eqs. (19): no oscillation is possible.

The terminations of the curves at the right were deliberately introduced; they mean that the calculations were pursued no further. For branch 1, the calculation was carried out for only rather small values of L since our approximation that the plasma oscillations are linear is equivalent to the requirement $\gamma^2 \ll 1$, and it is totally unjustified at values $\gamma^2 \gtrsim 1$. Branches 2 and 3, on the other hand, are terminated at the right because of the limitations on computer time. For the same reason, we did not search the branches with index 4 or higher, which lie below branch 3, although we would fully expect such branches to exist on the basis of an extrapolation of the data in Fig. 1.

The theory predicts that the possible oscillation frequencies will form a discrete spectrum, and this conclusion is in agreement with the experimental data. As an example, we show in Fig. 2 an oscillation spectrum which we measured with the help of an S4-5 spectrum analyzer. The oscillations were detected by an rf probe inserted into the beam-plasma interaction zone. A ribbon-shaped beam with cross-sectional dimensions of about 10×0.4 cm was injected without preliminary modulation into the interior of a copper cylinder, along the diameter of the cylinder.[13] The beam-plasma discharge produced a plasma. We see that that the spectrum of the oscillations which are excited is choppy; the frequency interval between the lines is, as follows from (21), far smaller than the oscillation frequencies themselves, which lie near the plasma frequency.

The results of these calculations also furnish, in princi-

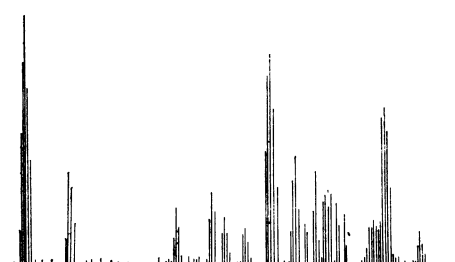

FIG. 2. Measured spectrum of self-excited oscillations in a beam-plasma discharge. The beam current is $I = 200$ mA; the plasma frequency is $f_p = 1700$ MHz; and the beam energy is $eU_0 = 400$ eV.

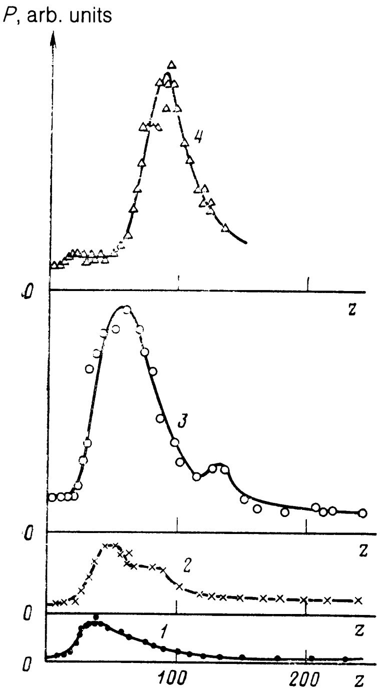

FIG. 3. Measured axial profiles of the oscillation intensity P for $I = 200$ mA and $f_p = 2500$ MHz. 1—$f = 2130$ MHz, $eU_0 = 100$ eV, $L = 450$; 2—1980 MHz, 200 eV, 300; 3—2100 MHz, 400 eV, 220; 4—1860 MHz, 1000 eV, 120.

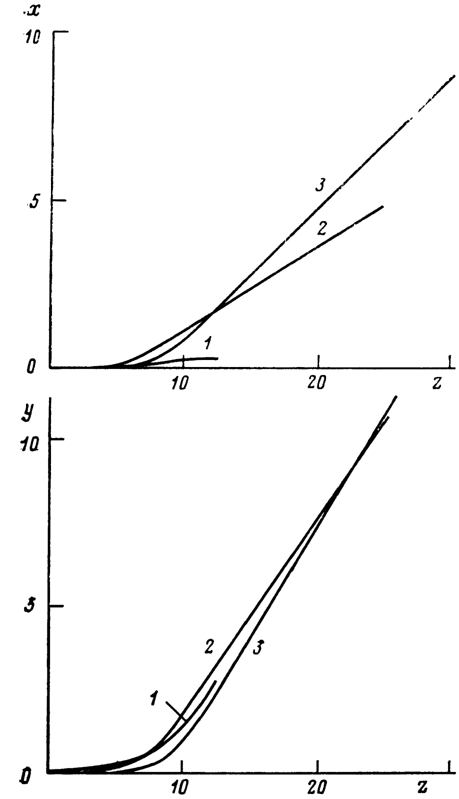

FIG. 5. Solutions of Eqs. (19). 1—$L = 12.57$; 2—25; 3—300.

ple, an explanation for such a well-established experimental fact as the disruption of the self-excited oscillations as the length of the beam-plasma system is reduced to a critical value.[14,15] On the other hand, we must not fail to note that under beam-plasma discharge conditions the physical situation is actually far more complicated, since the plasma density, which is determined to a large extent by the diffusion of particles, also depends on L. Furthermore, the onset or disruption of oscillation is accompanied by corresponding sharp changes in the ionization rate. For this reason, it would hardly make sense to attempt a quantitative comparison of the measured and theoretical values of the critical length for self-excited oscillation.

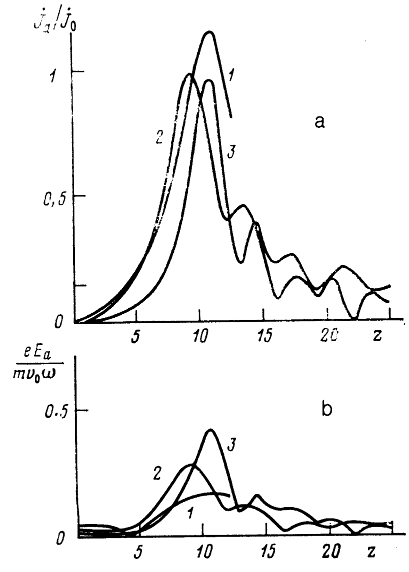

FIG. 4. Calculated axial profiles of the amplitude of the first harmonic of the beam current (a) and of the electric field (b). 1—$L = 12.57$; 2—25; 3—300.

There is also a qualitative agreement between experiment and theory in terms of the axial profiles of the intensity of the oscillations which are excited. Figure 3 shows corresponding experimental curves which we measured in the apparatus mentioned above for various effective lengths of the beam-plasma system. The effective length was varied, without changing the geometry, by varying the velocity of the beam and the frequency which was excited. Theoretical results on the amplitudes $j_a = (A_1^2 + B_1^2)^{1/2}$, $E_a = (\xi_1^2 + \eta_1^2)^{1/2}$ are shown for various values of L in Fig. 4. These curves are plotted for branch 2 on the basis of the solutions of Eqs. (19) shown in Fig. 5. We see that the oscillations are typically localized in a relatively short region, whose position varies only slightly as L is varied. The results for the other branches are similar, but the coordinates of the oscillation zone are different. For branch 1 they are in the region $z \approx 4$–5, while for branch 3 they are in the region $z \approx 16$–17. Comparing these results with Fig. 1, we see that for self-excited oscillation on any branch the corresponding oscillation zone must lie nearly entirely within the length of the system. The distance to the zone increases with increasing index of the branch because of both a decrease in the spatial growth rate and a decrease in the amplitude of the modulating field at the injection plane (Fig. 6).

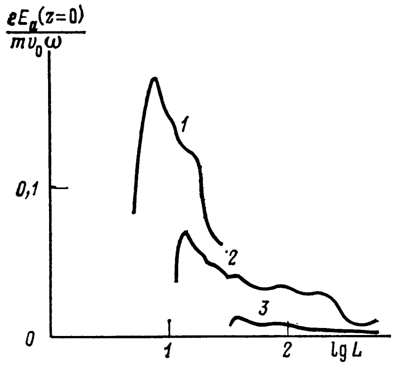

FIG. 6. Amplitude of the varying field at the injection plane ($z = 0$) versus the length L of the system. For clarity, the value of E_a for curve 2 has been increased by a factor of three, and that for curve 3 by a factor of five.

The existence of a localized oscillation region is a consequence of the phase focusing and defocusing of the beam, just as in the case of steady-state injection of a premodulated beam into a plasma. The spatial evolution of the bunches which form during the self-excited oscillation is illustrated in Fig. 7.

CONCLUSION

In summary, the nonlinear process of self-excited oscillation in a beam-plasma system of finite length which is usually observed and which has not previously been explained satisfactorily may proceed as follows: A longitudinal rf electric field consisting of two components is established in the system. One component is a wave, synchronized with the beam, which evolves over distance. The amplitude of this wave increases from zero at $z = 0$ to a maximum at some point and then decreases. This behavior results from the dynamics of the bunches which are formed, as in the injection of premodulated beams into a plasma. The other component of the *rf* field is in phase over the entire space and is small in comparison with the maximum value of the first component. Despite the fact it is not synchronized with the beam, however, it does cause bunching of the beam at small values of z, thereby exciting a growing synchronized wave.

In turn, the oscillation amplitude of the asynchronous field is determined by the variable beam current, in an integrated fashion over the entire volume, so that the process is self-consistent. Self-consistent oscillation phases and amplitudes are possible only for the relation among the parameters which was found in this study and which determines the frequencies of the self-excited oscillations.

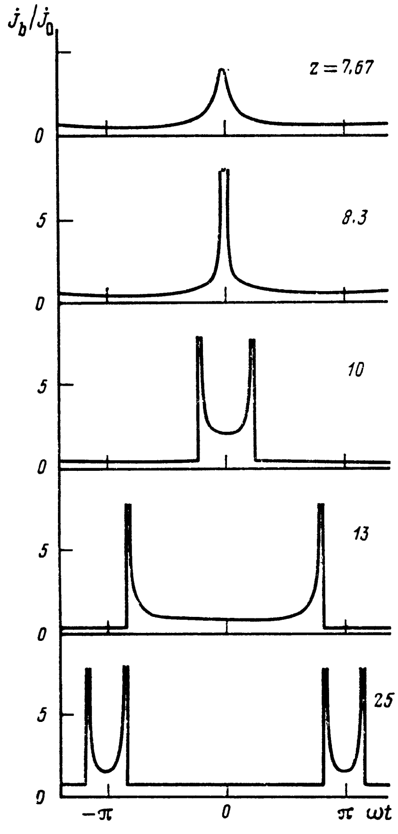

FIG. 7. Temporal profiles of the current-density wave of the beam in the coordinate system moving with the center of a bunch at various values of z for branch 2 ($L = 25$).

APPENDIX 1

Generally speaking, the function g_s should be sought as the series

$$g_s(z,t_0)=\sum_{i=0}^{\infty}[x_i(z)\sin i\omega t_0+y_i(z)\cos i\omega t_0]. \qquad (22)$$

Seeking a solution in the form in (18), on the other hand, is an approximate approach. If we substitute (19) into (17) and use (2), (7), and (16), we find on the right side of (17), along with the first-harmonic terms $x \sin \omega t_0$ and $y \cos \omega t_0$, second, third, and zeroth harmonics, whose amplitudes (x_2, y_2, y_3, and y_0) are given by

$$x_2=\int_0^z dz\int_0^z [\psi(-\cos zJ_1(x)J_0(y)+\sin z\,J_0(x)J_1(y)) +\chi(\cos zJ_0(x)J_1(y) +\sin zJ_1(x)J_0(y))]dz,$$

$$y_2=\int_0^z dz\int_0^z [\psi(-\cos zJ_0(x)J_1(y)-\sin zJ_1(x)J_0(y)) +\chi(-\cos zJ_1(x)J_0(y)+\sin zJ_0(x)J_1(y))]dz,$$

$$x_3=\int_0^z dz\int_0^z (\chi\cos z-\psi\sin z)J_1(x)J_1(y)\,dz,$$

$$y_3=\int_0^z dz\int_0^z (\psi\cos z-\chi\sin z)J_1(x)J_1(y)\,dz,$$

$$y_0=\int_0^z dz\int_0^z [\psi(-\cos zJ_0(x)J_1(y)+\sin zJ_1(x)J_0(y)) +\chi(\cos zJ_1(x)J_0(y)+\sin zJ_0(x)J_1(y))]dz,$$

where

$$\psi=2\gamma^2\left(\cos zJ_1(x)J_0(y)+\sin zJ_1(y)J_0(x)-\frac{I_2}{L}\right),$$

$$\chi=-2\gamma^2\left(\sin zJ_1(x)J_0(y)-\cos zJ_1(y)J_0(x)-\frac{I_1}{L}\right).$$

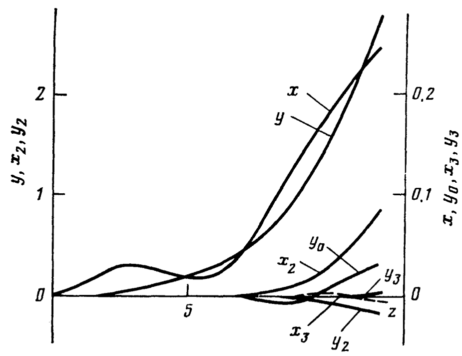

FIG. 8. Axial profiles of the amplitudes of the fundamental (x, y) and additional $(x_2, y_2, x_3, y_3, y_0)$ harmonics for $L = 12.57$.

This simplification is justified if the values of $|x_2|$, $|y_2|$, $|x_3|$, $|y_3|$, $|y_0|$ for the solutions x and y found are far smaller than the larger of $|x|$ and $|y|$ over the entire interval of z.

Figure 8 makes the corresponding comparison for $L = 12.57$. We see that in this case, even at $z = L$, each of the additional harmonics is smaller in magnitude than 30% of the fundamental (the ordinate scales in this figure are different). At $L = 25$, according to the calculations, the situation is worse, especially at large values of z, but again in this case the maximum value of the second (and largest) harmonic does not exceed 50% of the fundamental. Roughly the same relative sizes are found for the harmonics at $L = 300$. Apparently, therefore, the use of approximation (18) for $g(z, t_0)$ correctly conveys the basic features of the self-propagation.

APPENDIX 2

Equations (19) have been solved numerically for a fixed length L of the system by the following method. The initial values $\gamma^{(0)}$, $I_1^{(0)}$, $I_2^{(0)}$ of the parameter γ and of the integrals I_1 and I_2 are generated in an arbitrary way. The corresponding values of $x^{(0)}(z_j)$, $y^{(0)}(z_j)$, $j = \overline{1, N}$ are found by an iterative procedure, where N is the number of partitions of the segment $[0, L]$ along the z axis. Knowing $x^{(0)}$ and $y^{(0)}$, we calculated new values of the integrals, $I_1^{(1)}$ and $I_2^{(1)}$, which we then used to find new values of the functions, $x^{(1)}(z_j)$ and $y^{(1)}(s_j)$. We then constructed a trial function

$$F^{(k)}:=\sum\left\{\left|\frac{x^{(k)}(z_j)-x^{(k-1)}(z_j)}{x^{(k)}(z_j)}\right| + \left|\frac{y^{(k)}(z_j)-y^{(k-1)}(z_j)}{y^{(k)}(z_j)}\right|\right\}.$$

Here k is the index of the iteration; for the first step of the iteration procedure we have $k = 1$. We then use the minimum of the trial function F to alternately optimize the parameter γ at fixed values of $I_1^{(k)}$, $I_2^{(k)}$ and calculate the optimum values of $I_{1\,\text{opt}}^{(k)}$ and $I_{2\,\text{opt}}^{(k)}$, corresponding to the optimum value γ_{opt} which has been found. The process of choosing the best value of γ (in the k-th iteration) at the optimum values of I_1 and I_2 [calculated in the $(k-1)$st iteration] and the calculation of the best values of I_1 and I_2 at the optimum value found for γ are terminated when the condition $|F^{(k)}| \leqslant 10^{-2}$ becomes satisfied.

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