

Energy relaxation time in a two-dimensional electron gas at a (001) surface of silicon

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Three different experimental methods were used to measure the temperature dependence of the energy relaxation time in silicon (001) MIS structures. It is shown that $\tau_\epsilon \propto T^{-3}$ regardless of the parameters of the specific sample. A decrease of the energy relaxation time with increasing density of the two-dimensional electrons was observed in the interval $N_s = (2 - 8) \cdot 10^{12} \text{ cm}^{-2}$. The energy relaxation time for energy transfer to bulk and surface phonons is calculated, and it is shown that if $kT \ll 2p_{F,s}$ both energy-transfer mechanisms lead to the relation $\tau_\epsilon \propto T^{-3}$.

A drawing electric field can not only cause a two-dimensional electron gas to drift as a result of transport of the electric current, but also change the energy of the disordered thermal motion. It can be assumed that the electron temperature T_e that is established in the electron system differs from the lattice temperature T_{ph} if the following time hierarchy obtains:

$$\tau \ll \tau_i \ll \tau_\epsilon, \quad (1)$$

where τ is the time of the momentum relaxation, τ_i the time in which the electron loses energy by electron-electron scattering, and τ_ϵ is the energy relaxation time that determines the rate of energy lost by the electron system as a whole.

At helium temperatures, a typical value of τ for silicon (001) MIS structures is 10^{-12} to 10^{-13} s, depending on the mobility of the particular sample.¹ The value of τ_i can be determined by experimental observation of the logarithmic conductivity corrections due to the weak localization, and is of the order of 10^{-11} s in this temperature range.²⁻⁴ The energy lost by the electron system as a whole can be due only to emission of acoustic phonons or of surface acoustic waves, while the characteristic times are of the order of 10^{-8} s. We shall therefore assume hereafter that thermal equilibrium sets in, with a temperature T_e determined by the value of the electric field if the lattice temperature is fixed.

There are two methods of determining τ_ϵ in experiment.⁵ The first is based on the energy balance:

$$C(T_e)(T_e - T_{ph})/\tau_\epsilon = W, \quad T_e - T_{ph} \ll T_e, \quad T_{ph}, \quad (2)$$

where $C(T_e)$ is the heat capacity of the electron gas and W is the power input to the electron system. This method was used⁶⁻⁹ to determine τ_ϵ in silicon MIS structures. The second is based on measuring T_e as a function of time after turning on the heating current. It was used to determine τ_ϵ of electrons in the interior of semiconductors¹⁰ and semimetals.¹¹ Both methods are used in the present paper to determine the energy relaxation time in a two-dimensional electron gas on a (001) surface of silicon.

Principal attention was paid in the preceding studies⁶⁻⁹ to the temperature dependence of τ_ϵ . No temperature dependence whatever was observed for τ_ϵ in Ref. 6. A relation $\tau_\epsilon \propto T^{-3}$ was later found in Ref. 7 and was attributed to electron interaction with the surface (Rayleigh) phonons

propagating along the Si-SiO₂ interface. Still later,⁸ the measured samples had a higher electron mobility and the relation $\tau_\epsilon \propto T^{-2}$ was obtained. A similar relation was ascribed on the basis of the theoretical results of Refs. 7 and 12 to the interaction of electrons with bulk phonons in Si. It was concluded that the electron system loses energy by emission of surface phonons in samples having low electron mobility and by emission of bulk phonons from samples having high mobility. On the other hand, a relation $\tau_{ph,e} \propto T^{-3}$ was obtained in an investigation¹³ of dragging of inversion layer charges of either polarity by phonons in samples of sufficiently high mobility.

The present study was aimed at investigating τ_ϵ as a function of temperature and of the electron density N_s in silicon (001) MIS structures.

SAMPLES AND EXPERIMENTAL TECHNIQUE

The measurements were performed on seven samples, with both *n*- and *p*-type substrates. The sample parameters are listed in Table I. Six samples were prepared for the conductivity measurements by the capacitive-coupling technique.^{14,15} Measurements of this type require that the MIS structure gate have a high-resistance part bordered by low-resistance banks. A gate consisting of three concentric metallic films bounded by circles of radii 0.75, 1.75, and 2.25 mm was therefore placed on the (100) surface of silicon on top of an SiO₂ layer. The high-resistance part of the gate was a ring with radii $r_1 = 0.75 \text{ mm} < r < 1.75 \text{ mm} = r_2$, made for different samples of nichrome film of resistance R from 3 to 12 k Ω squares. The banks with $r < r_1$ and $r > r_2$ were low-resistance aluminum films. All films were deposited by sputtering in 10^{-5} Torr vacuum.

The electron temperatures on samples 1-6 were determined from the amplitudes of the Shubnikov-de Haas oscillations. Two different methods, which will be considered below, were used to measure τ_ϵ .

1. Method based on energy balance

To determine τ_ϵ we first measured the temperature dependence of the amplitude A of the Shubnikov-de Haas oscillations of the ac conductivity σ_{xx} in the linear regime. The connection between the current I through the sample and

TABLE I.

Sample No.	Substrate type	μ_{\max} , $\text{cm}^2/\text{V} \cdot \text{s}$	Dopant density, cm^{-3}	d , \AA .	Measurement method
1, 2	<i>p</i>	1 800	$5 \cdot 10^{15}$	2200	1, 2
3, 4	<i>p</i>	2 000	$2 \cdot 10^{16}$	4100	1
5, 6	<i>n</i>	3 300	$5 \cdot 10^{15}$	2200	1, 2
7	<i>p</i>	13 400		1300	3

the voltage V between the low-resistance banks of the gate, measured in a magnetic field and in a Corbino geometry, is given by

$$\frac{I}{U} = \frac{2\pi(\sigma_{xx} + R^{-1})}{\ln r_2/r_1} [1 + (1-i)\delta]^{-1}, \quad (3)$$

where

$$\delta = \frac{R\sigma_{xx}(r_1^{-1} + r_2^{-1})}{2 \ln(r_2/r_1) [1 + (1 + R\sigma_{xx})^{1/2}] \alpha},$$

$$\alpha = \left[\frac{\omega c_0}{2} (R + \sigma_{xx}^{-1}) \right]^{1/2}, \quad \alpha_1 = \left[\frac{\omega c_0}{2} \sigma_{xx}^{-1} \right]^{1/2}$$

provided that $\alpha_1 r_1 \gg 1$. Here c_0 is the capacitor, per unit area, between the gate and the two-dimensional electron layer, and $\delta \ll 1$ (a detailed derivation of (3) will be published elsewhere).

The measurements were performed at a frequency $\omega/2\pi = 30$ MHz. Several measurements were made at 15 MHz for comparison. Assuming that the oscillation amplitude is determined exclusively by the electron temperature, an $A(T_e)$ calibration curve was obtained. An example of such a curve is shown in Fig. 1. It can be seen from the figure that when the amplitude of the oscillations is changed by 10–15% the $A(T)$ plot can be approximated with good accuracy by a straight line.

To determine the energy relaxation time, we measured the oscillation amplitudes at a fixed temperature T_{ph} , while increasing gradually the current through the sample. The calibration curve was used to find the dependence of $(T_e - T_{ph})$ on the average power $\bar{W} = IU/2\pi(r_2^2 - r_1^2)$ per unit sample area. The value of τ_e^* was determined, using Eq.

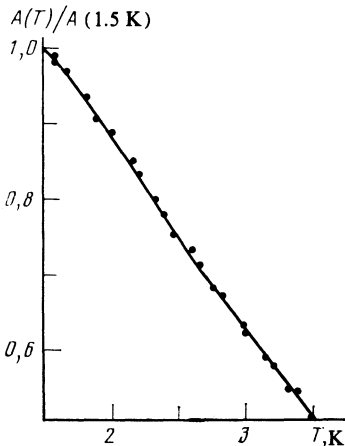


FIG. 1. Calibration curve. Shubnikov-de Haas oscillation amplitude versus temperature. Sample No. 6, $H = 4.7$ T, $N_s = 1.55 \cdot 10^{12} \text{ cm}^{-2}$.

(2), from the initial section of this dependence ($T_e - T_{ph} \ll T_{ph}$).

It should be noted that the quantity obtained in this manner coincides with τ_e only if $r_2 - r_1 \ll r_1$ and $\omega\tau_e \gg 1$. Neither condition was met in the experiment. This means, first, that the electron-system temperature depended on the coordinate r and, second, an electron temperature oscillation of frequency 2ω was superimposed on the average temperature. These two circumstances can be easily taken into account, and a connection can be established between τ_e and the value of τ_e^* obtained by the procedure indicated above. If we can put $C(T) = \text{const}$ and $\partial A/\partial T = \text{const}$ in the employed temperature interval $T_e - T_{ph} \ll T_{ph} = T$ and if the conductivity change due to heating of the electron system is assumed small compared with σ_{xx} , the relation between τ_e and τ_e^* takes the form

$$\frac{\tau_e^*}{\tau_e} = \frac{(r_2/r_1 - r_1/r_2)^2}{4 \ln^2(r_2/r_1)} \left[1 + \frac{1}{2(1 + 4\omega^2\tau_e^2)} \right]. \quad (4)$$

We used (4) to determine τ_e . It was verified that at an oscillation amplitude not higher than 20% of σ_{xx} the values obtained in this manner depended neither on the measurement frequency nor on the constant magnetic field strength.

2. Time dependence of the oscillation amplitude

The presence of a high-resistance gate in the samples permitted direct determination of τ_e by measuring the time dependence of the amplitude of the quantum oscillations. To this end, rectangular voltage pulses of duration 50 ns and repetition frequency $\omega_1/2\pi = 2$ MHz were applied between the low-resistance beams of the gate. The amplitude of the

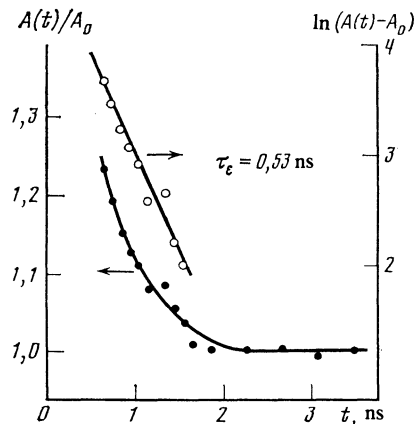


FIG. 2. Time dependence of Shubnikov-de Haas oscillation amplitude. The procedure of determining τ_e is illustrated. Sample No. 1, $H = 7$ T, $T = 4.2$ K, $N_s = 4.3 \cdot 10^{12} \text{ cm}^{-2}$.

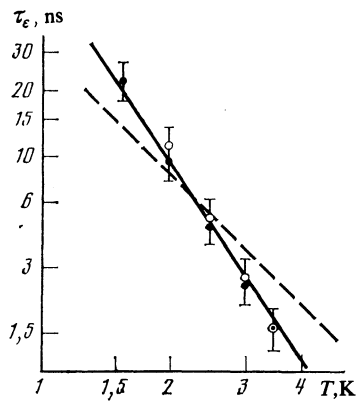


FIG. 3. Temperature dependence of the energy-relaxation time for samples 5 and 6. The solid curve corresponds to $\tau_\epsilon \sim T^{-3}$ and the dashed to $\tau_\epsilon \sim T^{-2}$. The results were obtained from the energy balance equation at $N_s = 1.55 \cdot 10^{12} \text{ cm}^{-2}$.

quantum oscillations at various points of time after tuning on the heating-current pulse were measured.

If the current through the sample remains constant after turning-on the heating pulse, the temperature at any point of the sample varies like

$$T_e = T_{ph} + \frac{W(r) \tau_\epsilon}{C(T)} [1 - \exp(-t/\tau_\epsilon)].$$

On the linear section of the $A(T)$ calibration curve, the time dependence of the oscillation amplitude is given by

$$A(t) = A(0) + [A(\infty) - A(0)] [1 - \exp(-t/\tau_\epsilon)]$$

regardless of how uniform the electron temperature is over the sample. The long-time steady-state amplitude A_∞ can be obtained from experiment. Therefore, recognizing that

$$\ln [A(t) - A(\infty)] = -t/\tau_\epsilon + \text{const} \quad (5)$$

the energy-relaxation time can be determined from the slope of the plot of $\ln[A(t) - A(\infty)]$ vs time.

In real samples, however, even if the pulse voltage is maintained constant, the current through the two-dimensional electron layer decreases with time, with a characteristic constant on the order of 200 ns. The significant times in most measurements of τ_ϵ were shorter than 10 ns, during which the current through the sample could be regarded as approximately constant. With decreasing sample temperature and with increasing τ_ϵ , the significant time intervals became longer (up to 30–40 ns) and it was necessary to introduce corrections for the variation of the current through the sample.

The rise time of the current pulse was ≤ 1 ns, so that the measurements were started approximately 1 ns after the start of the current pulse. The minimum time step was 0.1 ns. The pulse repetition frequency was chosen such as to minimize the average heating of the sample. Typical experimental plots of $A(t)$ and $\ln[A(t) - A(\infty)]$ are shown in Fig. 2.

3. Measurements of τ_ϵ in the absence of a constant magnetic field

Sample No. 7 was a field-effect transistor with long gate and with potential contacts to the inversion layer. The con-

ductivity of a two-dimensional electron gas in a transistor with high enough carrier mobility is known^{16,17} to increase as the temperature is lowered in the liquid-helium region. This effect is due not to freezeout of the electron-phonon scattering,¹⁷ but to a change of the elastic relaxation time.^{18–20} It was used by us to measure the electron-gas temperature. The plot of $\sigma(T)$ in the linear regime served as the calibration curve. We measured the dependence of the conductivity on the power input to the electron system, and τ_ϵ was determined from Eq. (2). If $\tau_\epsilon \propto T^{-n}$, it follows from (2) that

$$W \propto T_e^{n+2} - T_{ph}^{n+2}. \quad (6)$$

We determined T_e by measuring the $\sigma(T_e)$ dependence in a wide range $T_e - T_{ph} \gtrsim T_{ph}$ of electron temperatures, and the results were in the form (6).

EXPERIMENTAL RESULTS

We note first that the results obtained for samples cut from the same washer (samples 1 and 2, 3 and 4, 5 and 6, respectively) agreed within the limits of error (see, e.g., Fig. 3). The energy relaxation time measured by different methods on one sample, at fixed electron density N_s and at fixed lattice temperature T_{ph} , was independent of the method used (Fig. 4). The values of τ_ϵ for different samples at equal temperatures and equal N_s differed more strongly. The values of τ_ϵ for different samples at $T = 3.5$ K are listed in Table II. It can be seen from the table that τ_ϵ can differ by a factor of 30 from sample to sample. The temperature dependence of the energy-relaxation time could be reliably determined for samples 5, 6, and 7. It can be seen from Figs. 3 and 5 that a $\tau_\epsilon \propto T^{-3}$ relation was obtained for samples 5 and 6 both by the method based on the energy balance and by direct measurement of τ_ϵ . The measurement results for samples 1–4 do not contradict such a dependence, although the results obtained for these samples can also be fitted, with experimental accuracy, to the relation $\tau_\epsilon \propto T^{-2}$.

The measurements of sample No. 7 were made at lower densities N_s and in a larger temperature interval (from 0.4 to

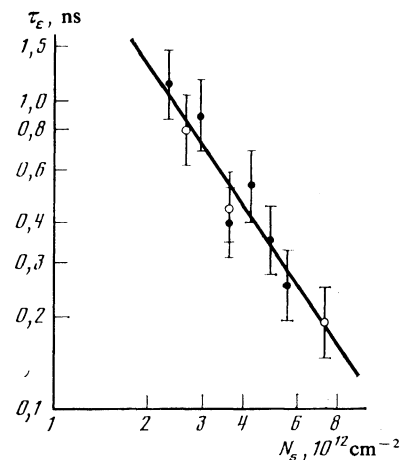


FIG. 4. Dependence of the energy-relaxation time on the electron density in the layer. Sample No. 1, $T = 4.2$ K. Light circles—data obtained from the energy-balance equation, dark—by the pulse technique. The straight line corresponds to $\tau_\epsilon \sim N_s^{-3/2}$.

TABLE II.

Sample No.	T_{ph} (K)	N_s , cm^{-2}	τ_e , ns ($\pm 20\%$)
6	3.5	$1.55 \cdot 10^{12}$	1.6
1	3.5	$1.5 \cdot 10^{12}$	3.4
4	3.5	$1.7 \cdot 10^{12}$	0.11

4.2 K). The results for two densities are shown in Figs. 6–8. It can be seen from Fig. 6 that at an electron density $N_s = 8.5 \cdot 10^{11} \text{ cm}^{-2}$ the dependence of the power input to the electron system on the electron temperature takes in the interval from 0.4 to 3.5 K the form $W \propto T_e^5 - T_{ph}^5$, corresponding to $\tau_e \propto T^{-3}$. With further increase of T_e , the exponents in relation for W tend to decrease (Fig. 7). A similar behavior was obtained also for the density $N_s = 1.34 \cdot 10^{12} \text{ cm}^{-2}$ (Fig. 8). In this case the temperature interval in which the relation $W \propto T_e^5 - T_{ph}^5$ was observed was somewhat larger. The measurement results at $N_s = 5 \cdot 10^{11} \text{ cm}^{-2}$ could be fitted, within the limits of error, to difference between either fourth or fifth powers of the electron and lattice temperatures.

In the density interval $N_s = (2-8) \cdot 10^{12} \text{ cm}^{-2}$, the values of τ_e of all samples decreased with increasing electron density. Detailed measurements of $\tau_e(N_s)$ were made for sample No. 1. The result of these measurements, shown in Fig. 4, shows that the observed dependence can be expressed in the form $\tau_e \propto N_s^{-p}$, where $p = 1.5 \pm 0.3$.

DISCUSSION

Energy transfer from a two-dimensional electron system to a lattice is made possible by emission of bulk acoustic and surface phonons. At helium temperature, in the investigated density range, the characteristic phonon energies $\hbar\omega \sim kT$ are considerably lower than the electron Fermi energy, $\hbar\omega \ll \epsilon_F$. The phonon momentum $\hbar q = kT/s$ (s is the speed of sound) becomes comparable with the electron Fer-

mi momentum p at an electron density $N_s \sim 10^{11} \text{ cm}^{-2}$. In the temperature and density regions investigated by us it can be assumed that $\hbar q \ll 2p$. Possible violations of the last inequality will be discussed below.

When the foregoing inequalities hold, it is impossible to ascertain, by means of the temperature dependence of the energy relaxation time, which of the energy-transfer mechanisms predominates, since we have in both cases $\tau_e \propto T^{-3}$, and $W \propto T_e^5 - T_{ph}^5$. To verify this, we calculate the energy transferred to the electrons by emission from bulk acoustic phonons. The square of the matrix element that describes the interaction of an electron of a two-dimensional layer with the bulk phonons is given by²¹

$$|M_{pp'}|^2 = \frac{\hbar q B^2}{2\rho V} \left(N_q + \frac{1}{2} \pm \frac{1}{2} \right) \delta_{p,p' \pm \hbar q_{\parallel}} \delta_{e,e' \pm \hbar\omega} \quad (7)$$

$$\times \begin{cases} s_l^{-1} \left(D^2 + 2D \frac{q_z^2}{q^2} + \frac{q_z^4}{q^4} \right), & \mathbf{q} \parallel \mathbf{e}, \\ s_t^{-1} \left(\frac{q_z^2}{q^2} - \frac{q_z^4}{q^4} \right), & \mathbf{q} \perp \mathbf{e}, \end{cases}$$

where $B = \Xi_u$; $D = \Xi_d/\Xi_u$, $\Xi_{u,d}$ are the components of the strain-potential tensor, ρ is the density of silicon, V the crystal volume, N_q the phonon distribution function, and s_l and s_t the speeds of the longitudinal and transverse sound waves, respectively. It is assumed that the electron density in the z direction is given by

$$|\Psi_0(z)|^2 = \frac{1}{2} b^3 z^2 \exp(-bz),$$

with $q_z/b \ll 1$. (Typical values are $b \sim 10^7 \text{ cm}^{-1}$ and $q \sim 10^5 \text{ cm}^{-1}$ at $T \sim 1 \text{ K}$.) The electron and phonon systems have different temperatures T_e and T_{ph} and different distribution functions $f(\mathbf{p}, T_e)$ and $N_q(T_{ph})$.

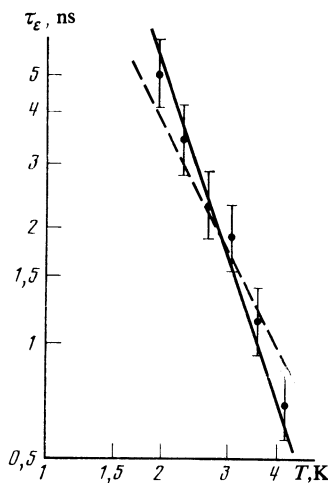


FIG. 5. Temperature dependence of energy-relaxation time. Sample No. 6, $N_s = 1.7 \cdot 10^{12} \text{ cm}^{-2}$. The solid and dashed curves correspond to $\tau_e \sim T^{-3}$ and $\tau_e \sim T^{-2}$, respectively. The results were obtained by the pulse technique.

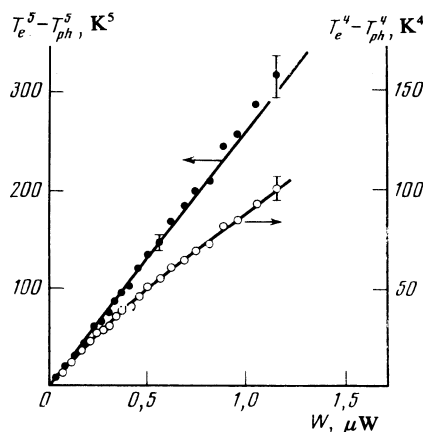


FIG. 6. Differences between the fifth and fourth powers of the electron and phonon temperatures versus the power input to the electron system. Sample No. 7, $N_s = 8.5 \cdot 10^{11} \text{ cm}^{-2}$.

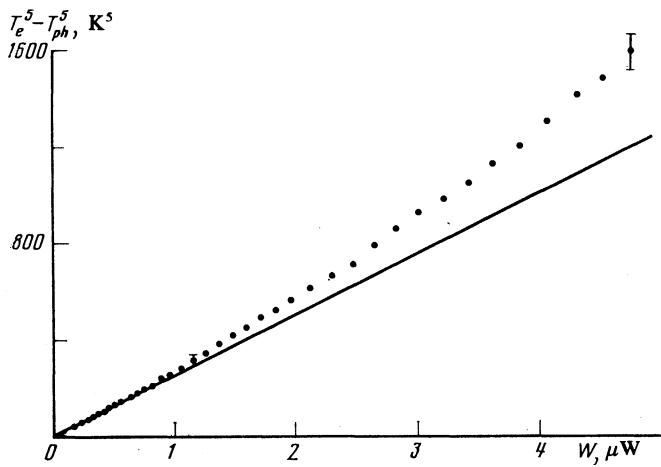


FIG. 7. Difference between the fifth powers of the electron and phonon temperatures versus power. Sample No. 7, $N_s = 8.5 \cdot 10^{11} \text{ cm}^{-2}$. The initial section is shown in Fig. 6

The power transferred by the electrons from a unit area to the lattice is equal to

$$W = \frac{B^2}{8\pi^4 \hbar^5 \rho} \int d\mathbf{p} d\mathbf{p}_1 d\mathbf{q} \left[\left(\frac{\hbar\omega_l}{s_l} \right)^2 \left(D^2 + 2D \frac{q_z^2}{q^2} + \frac{q_z^4}{q^4} \right) \times \delta(\varepsilon - \varepsilon_1 - \hbar\omega_l) \right. \\ \left. + \left(\frac{\hbar\omega_l}{s_l} \right)^2 \left(\frac{q_z^2}{q^2} - \frac{q_z^4}{q^4} \right) \delta(\varepsilon - \varepsilon_1 - \hbar\omega_l) \right] [(f(\mathbf{p}) - f(\mathbf{p}_1)) N_q + f(\mathbf{p}) (1 - f(\mathbf{p}_1))] \delta(\mathbf{p} - \mathbf{p}_1 - \hbar\mathbf{q}_{\parallel}). \quad (8)$$

Note that the momentum conservation law in (8) pertains only to the phonon momentum component parallel to the surface. Phonon emission at angles to the surface is limited exclusively by the energy conservation law. The latter means that an electron from the two-dimensional layer interacts with the three-dimensional phonon array.

It is convenient to eliminate the momentum δ function from (8) by integrating with respect to q_{\parallel} , introduce polar coordinates ε and φ for the electron position in p -space, and

then eliminate the energy δ function by integrating with respect to φ_1 . As a result, putting $x = \varepsilon - \varepsilon_1$ and $y = \varepsilon + \varepsilon_1$, we obtain

$$W = \frac{B^2 m}{\pi^3 \hbar^5 \rho} \int x^3 I(x, q_z) dq_z dx, \quad (9)$$

where

$$I(x, q_z) = \int \left[\varphi_l(x, q_z, y) s_l^{-4} \left(D^2 + 2 \frac{\hbar^2 s_l^2 q_z^2}{x^2} D + \frac{\hbar^4 s_l^4 q_z^4}{x^4} \right) + \varphi_l(x, q_z, y) s_l^{-4} \left(\frac{\hbar^2 s_l^2 q_z^2}{x^2} - \frac{\hbar^4 s_l^4 q_z^4}{x^4} \right) \right] \left[\frac{1}{2} \exp\left(-\frac{x}{2kT_e}\right) - N(x) \text{sh} \frac{x}{2kT_e} \right] \left[\text{ch} \frac{y - 2\varepsilon_F}{2kT_e} + \text{ch} \frac{x}{2kT_e} \right]^{-1} dy. \quad (10)$$

In expressions (9) and (10) m is the electron mass corresponding to electron motion in the two-dimensional layer plane, and $N(x)$ is the phonon distribution function corresponding to the temperature T_{ph} and to the energy $x = \varepsilon - \varepsilon_1$. The functions $\varphi_{l,i}$ are defined as

$$\varphi_i = \left[\frac{y}{ms_i^2} (x^2 - \hbar^2 s_i^2 q_z^2) - \frac{(x^2 - \hbar^2 s_i^2 q_z^2)^2}{4m^2 s_i^4} - x^2 \right]^{-1/2}. \quad (11)$$

The integration with respect to x is from zero to infinity. At fixed x , the integration is over the region

$$\hbar s_i |q_z| < x, \\ y > \frac{x^2 - \hbar^2 s_i^2 q_z^2}{4ms_i^2} + \frac{x^2 ms_i^2}{x^2 - \hbar^2 s_i^2 q_z^2}. \quad (12)$$

The main contribution to the integral (9) is made by the vicinity of the point $y = 2\varepsilon_F$, $x = 0$, $q_z = 0$. Since $ms_i^2 \sim 1 \text{ K}$, we can, in accordance with the second line of inequality (12), under the condition

$$2p_F \gg kT_e/s_i \quad (13)$$

integrate with respect to y in (10) from zero to infinity, and simplify the expression for φ_i as follows:

$$\varphi_i(x, q_z) = \left[\frac{2\varepsilon_F}{ms_i^2} (x^2 - \hbar^2 s_i^2 q_z^2) \right]^{-1/2}. \quad (14)$$

The result is

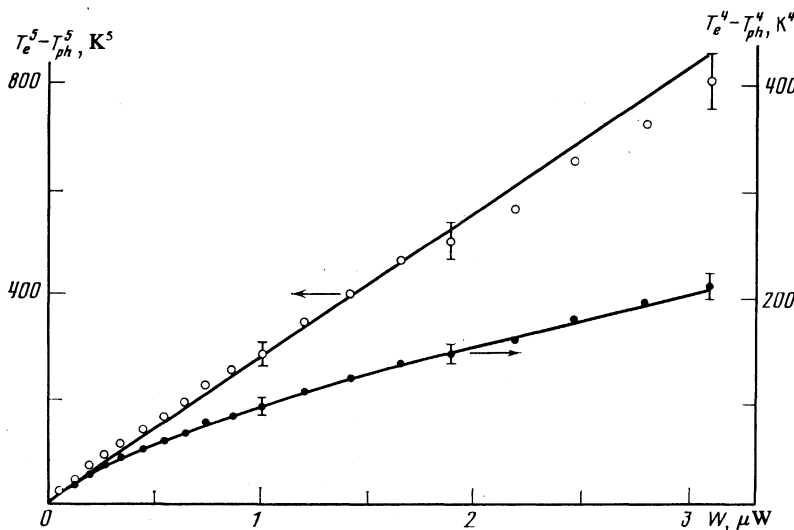


FIG. 8. Differences between the fifth and fourth powers of the electron and phonon temperatures versus power input to the electron system. Sample No. 7, $N_s = 1.34 \cdot 10^{12} \text{ cm}^{-2}$.

$$W = \frac{48\zeta(5)B^2m^2k^5}{\pi^2\hbar^6\rho s_i^4 p_F} \left[\left(D + \frac{1}{2} \right)^2 + \frac{1}{8} \left(1 + \frac{s_i^4}{s_i'^4} \right) \right] \times (T_e^5 - T_{ph}^5), \quad (15)$$

where

$$\zeta(5) = \sum_{n=1}^{\infty} n^{-5}.$$

For the energy-relaxation time we have

$$\tau_e = \frac{\pi^3 g_V \hbar^4 s_i^4 \rho p_F}{720 \zeta(5) B^2 m k^3 T^3 \left[(D + 1/2)^2 + \frac{1}{8} (1 + s_i^4/s_i'^4) \right]}. \quad (16)$$

If the electrons give up energy by emitting surface acoustic waves, similar calculations lead to the expression

$$W = \frac{96\zeta(5)Z_A^2 m^2 k^5}{\pi^2 K \hbar^6 \rho s_R^4 p_F} (T_e^5 - T_{ph}^5), \quad (17)$$

$$\tau_e = \frac{\pi^3 g_V \hbar^4 s_R^4 \rho p_F K}{1440 \zeta(5) Z_A^2 m k^3 T^3}, \quad (18)$$

where Z_A is expressed in terms of the strain potential and (just as K) the parameters that determine the damping of the longitudinal and transverse wave components in a direction perpendicular to the surface (see Ref. 7), and s_R is the wave velocity. It is assumed as before that $2p_F \gg kT_e s_R^{-1}$.

The experimentally observed law $\tau_e \propto T^{-3}$ can thus apply to emission of either surface or bulk phonons. The observed deviations from this law in the case of sample 7 are undoubtedly due to violation of inequality (13), inasmuch as the value of $T_e - T_{ph}$ starting with which the deviation becomes noticeable has increased with increasing N_s , and hence p_F . This probably explains also the lower exponent obtained in Ref. 8 in the temperature dependence of the energy relaxation time.

It can be seen from (16) and (18) that τ_e is expected to increase with increasing electron density, if the strain potential is independent of N_s . In our opinion the observed experimental decrease of τ_e with increasing N_s may be due to the fact that $B \sim Z_A \sim \epsilon_F$. In this case τ_e will be proportional to $N_s^{-3/2}$.

For interaction with bulk phonons, a deviation from the $\tau_e \sim T^{-3}$ law towards a larger exponent can occur in an MIS structure with a thin insulator of thickness $d < s\hbar/kT$. Under these conditions the phonon wavelength becomes comparable with or larger than the distance from the two-dimensional electron layer to the surface. The electron will interact not with a traveling but with a standing acoustic wave, and it is

this which increases the exponent. (The authors are indebted to A. V. Chaplik for the last remark.)

We note, finally, that the scatter of the absolute values of τ_e from sample to sample remains unexplained.

CONCLUSION

A procedure for measuring the conductivity of MIS structures with capacitive coupling permits direct measurement of the energy relaxation time. Under conditions when $kT/s > 2p_F$ the variation of τ_e with temperature corresponds to $\tau_e \propto T^{-3}$ and does not identify the phonons (bulk or surface) with which the electron interacts predominantly. In our opinion this problem can be resolved by measuring $\tau_e(T)$ in the opposite limiting case $kT/s \gg 2p_F$. What remains unclear is the decrease of τ_e with increasing density of the two-dimensional electrons, and the scatter of τ_e from sample to sample.

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