Heating of free electrons by photorecombination in a plasma

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We consider the heating that accompanies photorecombination directly and is due to depletion of the cold part of the free-electron velocity distribution function. We show that this heating exceeds electron cooling by bremsstrahlung in a wide energy interval.

One of the most important characteristics of a plasma is the electron temperature T_e. It governs the ionization and recombination rates, the thermal conductivity, and others. Great interest attaches therefore to the mechanisms that influence T_e. No investigations have been made so far of the effect exerted on the electron temperature by photorecombination, i.e., by the process whereby the electrons enter into bound states after emitting energy. Both the internal energy and the number of free electrons are decreased thereby. It is shown in the present article that photorecombination is accompanied by direct heating of the electrons. This heating exceeds in a large temperature interval the cooling due to bremsstrahlung.

Let one electron with kinetic energy ε recombine per unit volume. This changes the electron temperature by $\Delta T_e = 2(\bar{\varepsilon} - \varepsilon)/3N_e$, where N_e and $\bar{\varepsilon} = (3/2)T_e$ are the density and average kinetic energy of the free electrons. The cross section $\sigma_n^r(\varepsilon)$ for recombination on the nth level of the ion usually decreases with increasing ε (Ref. 1). The energy-distribution region in which the electrons have small ε and for which $\Delta T_e > 0$ is then depleted, and photorecombination leads to a rise of T_e. Summing over the levels, we obtain for the rate of change of temperature

$$\dot{T}_e^{\,r} = \frac{2}{3} N \sum_n \int_0^\infty v(\varepsilon)\sigma_n^{\,r}(\varepsilon) F(\varepsilon)\left(\frac{3}{2}T_e - \varepsilon\right) d\varepsilon. \qquad (1)$$

Here $v(\varepsilon) = (2\varepsilon/m)^{1/2}$ is the free-electron velocity, $F(\varepsilon)$ a Maxwellian distribution function with temperature T_e, and N the density of the recombination centers.

We consider quantitatively photorecombination in a plasma consisting of free electrons and of nuclei of charge z. The cross section for photorecombination on a hydrogen-like-ion level with principal quantum number n is given in the quasiclassical approximation by the Kramers formula[1]

$$\sigma_n^{\,r}(\varepsilon) = \frac{32\pi}{3\sqrt{3}} \alpha^3 a_0^{\,2} \frac{1}{n^3} \frac{J^2}{\varepsilon(\varepsilon + J/n^2)}, \qquad (2)$$

where $J = \mathrm{Ry}z^2$ is the ion-ionization potential, α the fine-structure constant, and a_0 the Bohr radius. From (1) and (2) we obtain for the heating rate

$$\dot{T}_e^{\,r} = \frac{64}{9}\left(\frac{2\pi}{3}\right)^{1/2} \frac{\alpha^3 a_0^{\,2}}{m^{1/2}} JNT_e^{1/2} S\left(\frac{J}{T_e}\right),$$

$$S(x) = 2x \sum_{n=1}^{\infty} \frac{1}{n^3}\left[\left(\frac{3}{2} + \frac{x}{n^2}\right) e^{x/n^2} E_1\left(\frac{x}{n^2}\right) - 1\right], \qquad (3)$$

$$E_1(y) = \int_y^\infty \frac{e^{-t}}{t} dt.$$

At $T_e < 7.4J$, i.e., in all cases of practical interest, a good approximation (with accuracy 2%) for $S(J/T_e)$ is

$$S(J/T_e) = 3^{1/2} + 0.54 \ln (J/T_e). \qquad (4)$$

It was assumed above that in photorecombination the energy distribution of the free electrons corresponds to equilibrium with temperature T_e. This requires that the time τ_r between recombination events exceed substantially the electron mean free path time τ_e. At $T_e < 7.4$J we have $\tau_r/\tau_e \gg 1$ and the distribution is close to Maxwellian.

Equation (3) yields the photorecombination-heating rate. Is it large or small? This question can be answered by comparing $\dot{T}_e^{\,r}$ with the rates of temperature change in other processes. The closest "relative" of photorecombination heating is cooling by bremsstrahlung. These two processes, differing only by the final state of the electron (bound or free), lead to temperature effects of opposite sign. Let us compare them. According to Ref. 1 the bremsstrahlung-cooling rate is

$$\dot{T}_e^{\,\tau} = -\frac{64}{9}\left(\frac{2\pi}{3}\right)^{1/2} \frac{\alpha^3 a_0^{\,2}}{m^{1/2}} JNT_e^{1/2} \qquad (5)$$

and is the same, apart from the sign and the factor S, as the rate of recombination heating. $S > 1$ at $T_e < 4.0J = 54.4z^2$ eV, i.e., the heating exceeds the bremsstrahlung cooling in a wide temperature interval. Therefore, if account must be taken of the bremsstrahlung when the plasma behavior is considered, neglect of photorecombination heating can lead to results "accurate to reversal." Photorecombination heating is most substantial in a supercooled plasma, when no effect is produced by the familiar electron-cooling mechanism due to collisional excitation and ion ionization in conjunction with line or photorecombination radiation (Refs. 2 and 3). The conclusion that photorecombination heating

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must be taken into account along with bremsstrahlung cooling is the main result of the present article.

The estimates presented were based in the Kramers formula. More accurate calculations call for the introduction of the Gaunt factor[1] by which the cross section (2) must be multiplied. The results obtained for photorecombination on completely stripped nuclei change quantitatively when photorecombination is considered on ions with finite numbers of electrons. It can be assumed, however, that the relation $\dot{T}_e^r > |\dot{T}_e^t|$ will also hold in in this case.

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