Tunneling in a stochastic electric field: Effects of laser linewidth on nonresonant multiphoton ionization of atoms

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The transition probability amplitude is expressed as a Feynman path integral to derive concise asymptotic estimates for the ionization probability of a bound atom by a stochastic electromagnetic field of finite spectral width. The results are used to analyze: a) tunnel ionization of an atom by a stochastic field; b) multiphoton ionization by electromagnetic radiation of finite spectral width; c) ionization by a constant electric field superposed on a stochastic field. The expressions for ionization by a quasi-monochromatic (stochastic) field describe both tunnel and multiphoton ionization in a unified way. They imply in particular that when the spectral width is finite, the field amplitude must satisfy both upper and lower bounds in order for perturbation theory of arbitrary fixed order to be valid. As a result, the ionization probability does not obey a power-law dependence on the intensity for relatively weak fields. It is shown that weak superposed stochastic fields cause the ionization probability to increase rapidly (exponentially) in constant electric fields in a manner that depends significantly on the decay rate of the correlation function of the stochastic field.

The basic properties of the decay of bound charged-particle states (electrons in atoms, negative ions, etc.) in time-varying electric fields were determined first by Keldysh[1] and subsequently in Refs. 2–7. When a monochromatic field of frequency ω acts on a bound particle, the decay process is sensitive to the magnitude of the product $\gamma = \omega\tau_0$. Here $\tau_0 = (2mI_0)^{1/2}/e_0F_0$ has the dimensions of time (where m and e_0 are the atomic mass and charge, I_0 is the bound state energy, and F_0 is the amplitude of the electric field); it is equal to the absolute value of the "imaginary" tunneling time to which the equations of classical mechanics are continued analytically. This continuation arises naturally in semiclassical estimates of the transition probability, for which the semiclassical approximation has proven to be extremely effective.[4–7]

For $\gamma \ll 1$ the system responds quasistatically to the external field, i.e., the ionization probability can be found by averaging the tunneling decay probability of the bound state over the field period $2\pi/\omega$ for a specified ac field $F = F_0 \cos\omega t$. For $\gamma \gg 1$ the response is dynamic and many-photon transitions are involved. In this case $F(t)$ oscillates rapidly over the characteristic times for the quantum transition. The transition can be interpreted naturally as a photoelectric effect in which $n_0 = I_0/\hbar\omega$ photons are absorbed ($n_0 \gg 1$ in the semiclassical approximation), and the power-law field dependence $W \propto (F_0)^{2n_0}$ of the probability agrees with the result found from n_0th order perturbation theory.

Because the laser beams used in multiphoton ionization experiments are not monochromatic (the amplitude and phase vary randomly), the probability of multiphoton ionization was calculated in Refs. 8–16 with allowance for the random behavior of the ionizing field. However, the results in Refs. 8, 9, and 11–13 rest heavily on the assumption that the field fluctuations are quasistatic. The transition probability was calculated by averaging the result found in Refs. 1–7 for $F_0 = \text{const}$ over a Rayleigh distribution $P(F_0)$. Although the fluctuations in F were not assumed to be quasistatic in the formulation of the equations in Refs. 10 and 14, all the specific results there pertain to the quasistatic case because the decay of the field correlation function $\langle F(t)F(0)\rangle$ was neglected. The transition from tunneling to multiphoton ionization in a stochastic field was found to be describable in terms of a universal parameter $\bar{\gamma} = \overline{\omega}\bar{\tau}_0$, where $\overline{\omega}$ is the central frequency of the quasi-monochromatic radiation and $\bar{\tau}_0 = (\hbar m/e_0^2\overline{F}_2)^{1/3}$, where $\overline{F}^2$ is the mean-square field strength. It is remarkable that $\bar{\gamma}$ is independent of the atomic parameters. The transition probability $W \propto \exp\{-2n_0F(\bar{\gamma})\}$, where the function $F(\bar{\gamma})$ is also universal.[13]

However, in accordance with the general conclusions in Refs. 1–7 we expect that the response of a system to a stochastic field of central frequency $\overline{\omega}$ and spectral width σ should also depend on the additional dimensionless parameter $\xi = \sigma\bar{\tau}_0$ (here σ^{-1} is the characteristic decay time of the correlation function $\langle F(t)F(0)\rangle$). The results of the quasistatic theory[8–16] should be valid for $\xi \ll 1$. For $\xi \gtrsim 1$, the probability for direct (nonresonant) ionization should depend significantly on the spectral width of the ionizing radiation. In strong fields whose spectrum is so wide that the concept of central frequency $\overline{\omega}$ is meaningless, we expect that the behavior will depend only on the single universal parameter $\xi = \sigma(\hbar m/e_0{}^2\overline{F}{}_0^2)^{1/3}$.

In this paper we develop a semiclassical theory for the decay of bound particle states in time-varying stochastic fields for a wide range of spectral widths, i.e., for arbitrary values of the parameter ξ, which depends on the decorrelation time of the field intensity. We express the transition probability amplitudes as Feynman path integrals to derive concise, exponentially accurate asymptotic expressions for the ionization probability of particles bound by short-range forces. In the limit $\xi \ll 1 (\sigma \to 0)$ these results reduce to the results of the semiclassical theory.[8–16] We consider the following three problems:

a) tunnel ionization of a particle by a stochastic electric

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field;

b) multiphoton ionization by electromagnetic radiation with a finite spectral width;

c) ionization by a constant electric field superposed on an additional stochastic field.

For ionization by a quasi-monochromatic (stochastic) field, our results yield a unified description of both tunnel and multiphoton ionization. In particular, they imply that if the finite spectral width is fixed, the results of fixed-order perturbation theory for the probability of multiphoton ionization are valid only if the field is bounded from below as well as from above. The leads to nonalgebraic dependence of the ionization probability on the intensity in relatively weak fields. We show that even a weak superposed stochastic field abruptly (exponentially) increases the ionization probability in constant electric fields. This effect depends significantly on the decay rate of the correlation function of the stochastic field.

The results obtained for ionization of particles bound by a short-range potential are known to accurately describe ionization processes in real atoms and ions (to within exponentially decaying terms).[6,15] Our results may therefore be useful in interpreting experiments on multiphoton ionization of atoms and ions by wide-band laser radiation. This area of research has received new impetus in recent years with the development of high-power multimode lasers which generate fields comparable to the intratomic fields.[11,15,17–19]

Our results may also be of interest in studies of ionization caused by widely fluctuating electric fields which have a broad spectral composition. Such fields are present in turbulent plasmas and are also of interest in astronomy.[20,21]

1. GENERAL EXPRESSIONS FOR THE IONIZATION PROBABILITY IN A STOCHASTIC FIELD

We consider a particle which is bounded by short-range forces with binding energy I_0 in a stochastic electric field $F(t)$, which we take to be a Gaussian stationary random process with the correlation function

$$\langle F(t_1)F(t_2)\rangle = B(t_1-t_2).$$

The probability amplitude for the particle to be in a free state of momentum p at time t is[6]

$$A(\mathbf{p},t)\infty\int d\mathbf{r}\int dt' G(\mathbf{r},t;0,t')\exp\left\{i\frac{I_0}{\hbar}t'-i\frac{\mathbf{p}}{\hbar}\mathbf{r}\right\}. \quad (1)$$

Here $G(r_1,t_1;r_2,t_2)$ is the Green's function for the particle in the field $F(t)$; it can be expressed as a Feynman path integral[22]

$$G(\mathbf{r},t;0,t')=\int\exp\left\{\frac{i}{\hbar}\int_{t'}^{t} dtL(\mathbf{r},\dot{\mathbf{r}},t)\right\}D(\mathbf{r}(t)), \quad (2)$$

$$L=\frac{m}{2}\dot{\mathbf{r}}^2+e_0xF(t),$$

where L is the Lagrangian for the particle for a field $F(t)$ directed along the x axis; $\mathbf{r}$ and $\dot{\mathbf{r}}$ are the spatial and velocity coordinates of the particle; the short-range binding potential is localized near $\mathbf{r}=0$. To get the ionization probability we must square (1), take the absolute value, and average over all possible instances of the random process $F(t)$. Since the integral in (2) is over all possible trajectories, which in general do not obey the classical equation of motion $m\ddot{x}=e_0F(t)$, we can perform the averaging by using the familiar expression for the generating functional for Gaussian random processes.[22] We find

$$\langle|A(\mathbf{p},t)|^2\rangle\infty\int d\mathbf{r}_1\int d\mathbf{r}_2\int dt'\int dt''\exp\left\{i\frac{I_0}{\hbar}(t'-t'')-i\frac{\mathbf{p}}{\hbar}(\mathbf{r}_1-\mathbf{r}_2)\right\}\iint D(\mathbf{r}_1(t))D(\mathbf{r}_2(t))\times\exp\left\{\frac{i}{\hbar}S([\mathbf{r}_1(t)],[\mathbf{r}_2(t)])\right\}; \quad (3)$$

$$S([\mathbf{r}_1(t)],[\mathbf{r}_2(t)])=\frac{m}{2}\int_{t'}^{t}dt\dot{\mathbf{r}}_1^2-\frac{m}{2}\int_{t''}^{t}dt\dot{\mathbf{r}}_2^2+\frac{ie_0^2}{2\hbar}\int_{t'}^{t}dt_1\int_{t'}^{t}dt_2B(t_1-t_2)x_1(t_1)x_1(t_2)+\frac{ie_0^2}{2\hbar}\int_{t''}^{t}dt_1\int_{t''}^{t}dt_2B(t_1-t_2)x_2(t_1)x_2(t_2)-\frac{ie_0^2}{\hbar}\int_{t'}^{t}dt_1\int_{t''}^{t}dt_2B(t_1-t_2)x_1(t_1)x_2(t_2). \quad (4)$$

The remaining calculations are carried out in the following sequence. The functional integrals in (3) are evaluated by the method of stationary phase. The equations for the extremal trajectories can be found by equating the functional derivatives $\delta S/\delta\mathbf{r}_1(t)$ and $\delta S/\sigma\mathbf{r}_2(t)$ to zero, which yields

$$m\ddot{\mathbf{r}}_{1\perp}(t)=m\ddot{\mathbf{r}}_{2\perp}(t)=0,\quad \mathbf{r}_\perp=\{y,z\}, \quad (5)$$

$$m\ddot{x}_1=m\ddot{x}_2=\frac{ie_0^2}{\hbar}\int_{t'}^{t}dt_1B(t-t_1)x_1(t_1)-\frac{ie_0^2}{\hbar}\int_{t''}^{t}dt_1B(t-t_1)x_2(t_1). \quad (6)$$

Equations (5) imply that the components of $\dot{\mathbf{r}}_1(t)$ and $\dot{\mathbf{r}}_2(t)$ normal to the x axis are conserved: $\dot{\mathbf{r}}_{1\perp}(t)\equiv\dot{\mathbf{r}}_{1\perp}$, $\dot{\mathbf{r}}_{2\perp}(t)\equiv\dot{\mathbf{r}}_{2\perp}$.

The action functional S can be calculated from (5), (6), and the relation

$$\dot{\mathbf{r}}^2=\frac{d}{dt}(\mathbf{r}\dot{\mathbf{r}})-\mathbf{r}\ddot{\mathbf{r}}.$$

On the extremal trajectories we thus find that

$$S=\frac{m}{2}\dot{\mathbf{r}}_{1\perp}[\mathbf{r}_{1\perp}(t)-\mathbf{r}_{1\perp}(t')]-\frac{m}{2}\dot{\mathbf{r}}_{2\perp}[\mathbf{r}_{2\perp}(t)-\mathbf{r}_{2\perp}(t'')]+\frac{m}{2}[x_1(t)\dot{x}_1(t)-x_1(t')\dot{x}_1(t')]-\frac{m}{2}[x_2(t)x_2(t)-x_2(t'')\dot{x}_2(t'')]. \quad (7)$$

We can substitute (7) into (3) and use the saddle point method to evaluate the integrals containing rapidly varying exponentials. The estimate for the integrals over $\dot{\mathbf{r}}_1(t)$ and $\dot{\mathbf{r}}_2(t)$ gives the conditions

$$m\mathbf{r}_{1\perp}=m\mathbf{r}_{2\perp}=\mathbf{p}_\perp,\quad \mathbf{p}_\perp=\{p_y,p_z\},$$
$$m\dot{x}_1(t)=m\dot{x}_2(t)=p_x. \tag{8}$$

With (8) we can rewrite (7) as

$$S=\frac{1}{2m}\mathbf{p}_\perp^2(t''-t')+\frac{1}{2}p_x[x_1(t)-x_2(t)]$$
$$-\frac{m}{2}[x_1(t')\dot{x}_1(t')-x_2(t'')\dot{x}_2(t'')]. \tag{9}$$

The estimate for the integrals over t' and t'' gives the saddle-point condition

$$\dot{x}_1^2(t_0)=\dot{x}_2^2(t_0)=-\varkappa^2(\mathbf{p}_\perp)=-\frac{1}{m}\left(2I_0+\frac{1}{m}\mathbf{p}_\perp^2\right). \tag{10}$$

Moreover, if t_0 is a saddle point for the integration over t', then the complex conjugate t_0^* is a saddle point in the integral over t''. To within exponentially small terms, we obtain the final, concise expression

$$W=\langle|A(\mathbf{p},t)|^2\rangle\sim\exp\left\{-\frac{m}{\hbar}\varkappa^2(\mathbf{p}_\perp)\tau_0\right\}, \tag{11}$$
$$\tau_0=\operatorname{Im}t_0,$$

for the decay probability of a bound state in a stochastic electric field. It is easy to see that together with conditions (8) and (10), Eqs. (6) determine an essentially unique extremal trajectory $x_1(t)=x_2(t)=x(t)$.

We will assume that the correlation function $\langle F(t)F(0)\rangle=B(t)$ remains a real-valued function of τ when t is replaced by $i\tau$. Since Eqs. (6) are linear and homogeneous, we can introduce a dimensionless "imaginary time" $s=-it/\tau_0$; this leads to the following integrodifferential equation for τ_0 and the extremal trajectory:

$$\frac{d^2q}{ds^2}=-\frac{1}{2}\left(\frac{\tau_0}{\bar{\tau}_0}\right)^3\int_{-1}^{1}ds_1f(s-s_1)q(s_1), \tag{12}$$

$$q(1)=q(-1)=0,\quad \left.\frac{dq}{ds}\right|_{s=-1}=-\left.\frac{dq}{ds}\right|_{s=1}=1. \tag{13}$$

Equations (12), (13) are equivalent to (6); $x=\varkappa\kappa\tau_0q$ and the correlation function $B(t)$ is of the form

$$B(t)=\frac{\bar{F}^2}{2}\mathrm{f}(t),$$

$f(0)=1$, and $\bar{F}^2$ is the mean square field amplitude;

$$f(s)=f(-it/\tau_0),\quad \bar{\tau}_0=(\hbar m/e_0{}^2\bar{F}^2)^{1/3}.$$

We note that $\varkappa$ does not appear in Eq. (12) or condition (13); τ_0 is therefore also independent of $\varkappa$ and is a universal function of the parameters of the ionizing field. This fact was noted in the quasistatic theory[10,13]; here we have extended it to the general case of a stochastic field of arbitrary spectral width.

The momentum distribution of the ionized particles is specified by the quantity $\varkappa(\mathbf{p}_\perp)$ appearing in (11), and the probability has an extremum $\varkappa=\varkappa_0=(2I_0/m)^{1/2}$ at $\mathbf{p}_\perp=0$ (cf. the similar situation in Refs. (1–7).

2. TUNNELING IN A STOCHASTIC FIELD

Assume that the particle is ionized by a stochastic electric field with the correlation function

$$B(t)=\frac{1}{2}\bar{F}^2\exp(-\sigma^2t^2). \tag{14}$$

We first examine the case $\sigma=0$, for which the right-hand side of (12) is independent of s. Writing

$$A_0=\frac{1}{2}\left(\frac{\tau_0}{\bar{\tau}_0}\right)^3\int_{-1}^{+1}ds_1q(s_1), \tag{15}$$

we then find that Eq. (12) describes a "uniformly retarded motion"

$$q(s)=-\frac{1}{2}A_0s^2+A_1, \tag{16}$$

where A_1 is a constant. The conditions (13) give

$$A_1=\frac{1}{2}A_0,\quad A_0=1. \tag{17}$$

Substituting (16) in (15), we get

$$\tau_0=(3)^{1/3}\bar{\tau}_0 \tag{18}$$

and hence

$$W\sim\exp\left\{-\frac{2I_0}{\hbar}\left(\frac{3\hbar m}{e_0{}^2\bar{F}^2}\right)^{1/3}\right\}. \tag{19}$$

Of course, the same result follows by integrating the expression

$$W\sim\exp\left\{-\frac{2}{3}\frac{(2I_0)^{3/2}m^{1/2}}{e_0\hbar F}\right\}$$

for the tunneling probability in a uniform constant electric field F over a Rayleigh distribution function

$$P(F)=\frac{2F}{\bar{F}^2}\exp\left\{-\frac{F^2}{\bar{F}^2}\right\}$$

(see Refs. 10, 13, 15).

We will now see how these results change when the correlation function (17) decays, i.e., $\sigma\neq0$. In this case the transition probability can be expressed in the form

$$W\sim\exp\left\{-\frac{2I_0}{\hbar}\left(\frac{3\hbar m}{e_0{}^2\bar{F}^2}\right)^{1/3}\Phi(\xi)\right\}, \tag{20}$$

where $\Phi(\xi)$ is a universal function of the dimensionless parameter $\xi=\sigma(\hbar m/e_0^2\bar{F}^2)^{1/3}$ and $\Phi(0)=1$. Asymptotic expressions for $\Phi(\xi)$ in the limits $\xi\ll1$ and $\xi\gg1$ follow immediately from Eq. (12) if we rewrite it as

$$\frac{d^2q}{ds^2}=-\frac{3}{2}\Phi^3(\xi)\int_{-1}^{1}ds_1q(s_1)\exp\{3^{2/3}\xi^2\Phi^2(\xi)(s-s_1)^2\}. \tag{21}$$

For $\xi\ll1$ we get

$$q(s)=q_0(s)+\xi^2q_1(s)+\ldots,\quad \Phi(\xi)=1+\xi^2\Phi_1+\ldots \tag{22}$$

Substituting (22) in (21), we find that $q_0(s)=1/2\cdot(1-s^2)$, while q_1 satisfies the equation.

$$\frac{d^2q_1}{ds^2}=-3\Phi_1-\frac{3}{2}\int_{-1}^{1}ds_1q_1(s_1)-\frac{1}{2}3^{5/3}\int_{-1}^{1}ds_1(s-s_1)^2q_0(s_1). \tag{23}$$

We find Φ_1 from the requirement that Eq. (23) be solvable with initial conditions $q_1(\pm 1) = 0$. To do this we multiply (23) by $q_0(s)$ and integrate over s from -1 to $+1$; q_1 then drops out of the equation and we find that $\Phi_1 = -2\cdot 3^{-1/3}\cdot 5^{-1}$. Consequently,

$$\Phi(\xi)=1-2\cdot 3^{-1/3}\cdot 5^{-1}\xi^2+\dots \quad . \tag{24}$$

The finite decay time of the correlation function $\langle F(t)\,F(0)\rangle$ thus causes the ionization probability to increase exponentially. This result is to be expected physically, because a faster decay enhances the contribution from the higher-frequency Fourier components of the random time-dependent field, and fewer high-frequency quanta are needed for effective multiphoton ionization.

We now discuss the opposite extreme case $\xi \gg 1$. Then since $\xi_2\Phi^2(\xi)\gg 1$ (see below), the main contribution to the integral in the right-hand side of (21) comes from the portions of the extremal trajectory near $s = \pm 1$. Since the function $q(s)$ is even, we therefore get the following expression for $q(s)$ near $s = -1$:

$$\frac{d^2q}{dz^2}=-B(\xi)\exp\{-\alpha(\xi)z\}, \quad z=s+1, \tag{25}$$

$$B(\xi)=\frac{3}{2}\Phi^3(\xi)\exp\{\alpha(\xi)\}\int_0^\infty dz_1 q(z_1)\exp\{-\alpha(\xi)z_1\}, \tag{26}$$

$$\alpha(\xi)=4\cdot 3^{2/3}\xi^2\Phi^2(\xi).$$

Hence

$$\frac{dq}{dz}=1-\frac{B(\xi)}{\alpha(\xi)}[1-\exp\{-\alpha(\xi)z\}]. \tag{27}$$

Imposing the requirement that $dq/dz\to 0$ and $\alpha(\xi)\to\infty$, we find that $B(\xi) = \alpha(\xi)$ so that

$$q(z)=\frac{1}{\alpha(\xi)}[1-\exp\{-\alpha(\xi)z\}]. \tag{28}$$

Substitution of (28) in (26) yields the equation

$$[\alpha(\xi)]^{-3/2}\exp\{\alpha(\xi)\}=2^5\xi^3 \tag{29}$$

for $\alpha(\xi)$, which for $\xi\gg 1$ implies that

$$\Phi(\xi)=\frac{1}{2\cdot 3^{1/3}\xi}[3\ln(2^{5/3}\xi)]^{1/2}\left\{1+\frac{\ln[3\ln(2^{5/3}\xi)]}{4\ln(2^{5/3}\xi)}+\dots\right\}. \tag{30}$$

The condition for (30) to hold is that $\xi^2\Phi^2(\xi)\sim \ln 2^{5/3}\xi\gg 1$. The above-noted tendency of the tunneling exponential to decrease as ξ increases thus naturally persists also for $\xi\gg 1$. Figure 1 plots the universal function $\Phi(\xi)$ for a wide range of ξ; the values were found by direct numerical solution of Eq. (12) $[\Phi(\xi) = (\tau_0/3^{1/3}\bar{\tau}_0)]$. For $\xi\ll 1$ and $\xi\gg 1, \Phi(\xi)$ reduces to (24) and (30), respectively. As $\xi\to\infty$, the tunneling exponential (20) tends to zero for fixed $\overline{F}^2$ due to the increase in σ. However, the assumptions used to derive the asymptotic estimate (11) break down under these conditions (the extremal trajectory is unbounded and the saddle-point method does not apply). Nevertheless, the exponential vanishing of the tunneling probability correctly reflects the circumstance that for $\sigma\to\infty$ the ionization probability can

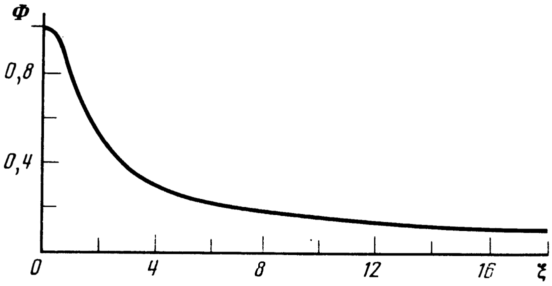

FIG. 1. The universal function $\Phi(\xi)$.

be calculated by perturbation theory to first order in the particle-field interaction for a stochastic field $F(t)$. This is because the high-frequency Fourier components of $F(t)$ give the dominant contribution to the ionization through the single-photon photoelectric effect. For $\xi\to\infty$ and fixed σ, Eqs. (20) and (30) are valid if $\overline{F}^2$ satisfies

$$\overline{F}^2\gg\frac{m\hbar\sigma^3}{e_0^2}\exp\left\{-\left(\frac{I_0}{\hbar\sigma}\right)^2\right\}, \tag{31}$$

which ensures that the stationary-phase method is valid. For fields violating condition (31) the ionization probability is described by first-order perturbation theory.

3. NONRESONANT MULTIPHOTON IONIZATION IN A STOCHASTIC FIELD

We now consider ionization of a bound particle by a stochastic field with the correlation function

$$B(t)={}^1/{}_2\overline{F}^2\cos(\varpi t)\exp(-\sigma^2t^2).$$

Here ϖ is the central frequency of the field intensity distribution, which is Gaussian with characteristic width $\sim\sigma$. We thus need to generalize the results in Refs. 1–7 to nonmonochromatic laser radiation of arbitrary spectral width.

Equation (12) in this case takes the form

$$\frac{d^2q}{ds^2}=-\frac{1}{2}\left(\frac{\tau_0}{\bar{\tau}_0}\right)^3\int_{-1}^{1}ds_1\,\mathrm{ch}[\varpi\tau_0(s-s_1)]$$

$$\times\exp[\sigma^2\tau_0^2(s-s_1)^2]q(s_1). \tag{32}$$

We first consider quasistatic field amplitude fluctuations, $\sigma\tau_0\to 0$. The kernel of Eq. (32) then factors and one readily finds the solution

$$q_k(s)=\frac{1}{\psi_k\,\mathrm{sh}\,\psi_k}(\mathrm{ch}\,\psi_k-\mathrm{ch}\,\psi_k s), \tag{33}$$

where ψ_k satisfies the transcendental equation

$$\frac{1}{2}\mathrm{sh}\,2\psi_k-\psi_k=2\bar{\gamma}^3, \quad \bar{\gamma}=\varpi(\hbar m/e_0^2\overline{F}^2)^{1/3}. \tag{34}$$

The ionization probability is

$$W\sim\exp\{-2n_0\psi_k(\bar{\gamma})\}, \quad n_0=I_0/\hbar\varpi. \tag{35}$$

Equation (34) and formula (35) were derived in Ref. 13 by directly averaging the Keldysh formula[1] for the ionization probability (derived for a field of fixed amplitude F) over a Rayleigh distribution $P(F)$.

We next examine how the formula for W changes when the spectral width is nonzero, $\xi = \sigma(\hbar m/e_0^2\,\overline{F}^2)^{1/3}\neq 0$. If ξ

is not too large, so that $(\xi/\bar{\gamma})\psi_k(\bar{\gamma})\ll 1$, Eq. (32) can be solved by perturbation theory as discussed in the previous section. We find that

$$W\sim\exp\{-2n_0\psi(\bar{\gamma},\xi)\}; \tag{36}$$

$$\psi(\bar{\gamma},\xi)=\psi_k(\bar{\gamma})-\frac{\xi^2}{\bar{\gamma}^2}\varphi(\bar{\gamma}), \tag{37}$$

$$\varphi(\bar{\gamma})=[\bar{\gamma}^3(\operatorname{ch}2\psi_k-1)]^{-1}J(\bar{\gamma}),$$

$$J(\bar{\gamma})={}^1/_4\psi_k^2\operatorname{sh}^2(2\psi_k)+{}^{23}/_{16}\operatorname{sh}^2(2\psi_k)-{}^3/_4\psi_k\operatorname{sh}(4\psi_k)$$
$$-{}^2/_3\psi_k^3\operatorname{sh}(2\psi_k)-{}^{17}/_4\psi_k\operatorname{sh}(2\psi_k)+{}^5/_2\psi_k^2\operatorname{ch}(2\psi_k)$$
$$+{}^1/_4\psi_k^2\operatorname{ch}^2(2\psi_k)+{}^1/_3\psi_k^4+3\psi_k^2.$$

Expression (36) is valid for arbitrary $\bar{\gamma}$. For $\bar{\gamma}\ll 1$ and $(\xi/\bar{\gamma})\psi_k(\bar{\gamma})\sim\xi\ll 1$, we obtain

$$\psi(\bar{\gamma},\xi)\approx 3^{1/3}\bar{\gamma}\left(1-\frac{3^{-1/3}}{5}\bar{\gamma}^2-\frac{2\cdot 3^{-1/3}}{5}\xi^2\right), \tag{38}$$

which of course agrees with (24), and

$$W\sim\exp\left\{-\frac{2\cdot 3^{1/3}I_0}{\hbar}\left(\frac{m\hbar}{e_0^2\bar{F}^2}\right)^{1/3}\right.$$
$$\left.\times\left[1-\frac{1}{3^{1/3}\cdot 5}\left(\frac{m\hbar}{e_0^2\bar{F}^2}\right)^{4/3}(\bar{\omega}^2+2\sigma^2)\right]\right\}. \tag{39}$$

The tunneling exponential in (39) contains corrections which depend both on the central frequency of the ionizing radiation and on the spectral width.

If $\bar{\gamma}\gg 1$ and $(\xi/\bar{\gamma})\psi_k(\bar{\gamma})\sim(\xi/\bar{\gamma})\ln(2\bar{\gamma})\ll 1$ we get

$$\psi(\bar{\gamma},\xi)\approx\frac{3}{2}\ln(2\bar{\gamma})-\frac{9}{2}\frac{\xi^2}{\bar{\gamma}^2}\ln^2(2\bar{\gamma}). \tag{40}$$

Consequently,

$$W\sim\left(\frac{1}{8\bar{\gamma}^3}\right)^{n_{\text{eff}}}=\left(\frac{e_0^2\bar{F}^2}{8m\hbar\bar{\omega}^3}\right)^{n_{\text{eff}}},$$

$$n_{\text{eff}}=n_0\left[1-\frac{3\xi^2}{\bar{\gamma}^2}\ln(2\bar{\gamma})\right]=n_0\left[1-\frac{\sigma^2}{\bar{\omega}^2}\ln\left(\frac{8m\hbar\bar{\omega}^3}{e_0^2\bar{F}^2}\right)\right]. \tag{41}$$

For $\bar{\gamma}\gg 1$ we can employ the method used to derive (30) to find an asymptotic expression for the universal function $\psi(\bar{\gamma},\xi)$ which is more general than (40) (i.e., which is also valid for $(\xi/\bar{\gamma})\psi_k(\bar{\gamma})\sim(\xi/\bar{\gamma})\ln(2\bar{\gamma})>1$, $\xi/\bar{\gamma}=\sigma/\bar{\omega}\ll 1$). In this case $\psi(\bar{\gamma},\xi)$ satisfies the equation

$$\psi^2(\bar{\gamma},\xi)+\frac{1}{2}\frac{\bar{\gamma}^2}{\xi}\psi(\bar{\gamma},\xi)$$
$$=\frac{3\bar{\gamma}^2}{4\xi^2}\ln\left\{(2\bar{\gamma})\left[1+\frac{4\xi^2}{\bar{\gamma}^2}\psi(\bar{\gamma},\xi)\right]\right\}, \tag{42}$$

which implies (40) for $(\xi/\bar{\gamma})^2\ln(2\bar{\gamma})^3\ll 1$. If $(\xi/\bar{\gamma})^2\ln(2\bar{\gamma})^3\gg 1$, we find from (42) that

$$\psi(\bar{\gamma},\xi)=\frac{1}{2}\frac{\bar{\gamma}}{\xi}[3\ln(4\xi)]^{1/2}\left\{1+\frac{\ln[3\ln(4\xi)]}{4\ln(4\xi)}+\ldots\right\}. \tag{43}$$

This result is not valid for $\bar{F}^2\to 0(\bar{\gamma}\to\infty)$ because $\bar{F}^2$ must satisfy a lower bound of the type (31) to ensure that the method of stationary phase is applicable.

According to (41), $W\sim(1/8\cdot\bar{\gamma}^3)^{n_0}$ for $\xi=0$, i.e., the transition probability is proportional to a power of the intensity and coincides in the limit $n_0\gg 1$ with the average transition probability calculated using n_0th order perturbation theory. However, if $\xi\neq 0$ and

$$\bar{F}^2<\bar{F}_{\text{cr}}^2=\frac{8m\hbar\bar{\omega}^3}{e_0^2}\exp\left\{-\frac{\bar{\omega}^2}{n_0\sigma^2}\right\}$$

the ionization probability no longer obeys a power-law dependence. The critical field $\bar{F}_{\text{cr}}$ depends nonanalytically on the spectral width σ.

It is remarkable that the result for the multiphoton transition probability derived by n_0th order perturbation theory is incorrect for weak fields, which are precisely the ones for which perturbation theory is generally regarded as unconditionally valid. The physical explanation is clear—in weak quasimonochromatic fields, the higher frequency components contribute significantly to the ionization in spite of their small "weight" in the spectral distribution of the intensity. The small statistical weight of these components is offset by the fact that fewer photons are needed for ionization. In the perturbation series for weak fields it is therefore incorrect to keep only the term of order $n_0=I_0/\hbar\bar{\omega}$ calculated at the central frequency. Indeed, other terms may be more important. The asymptotic estimates given in this paper provide an effective method for adding all the significant contributions in the perturbation series for the ionization probability.

Nonresonant multiphoton ionization is usually described by the statistical factor η, which is defined as the transition probability in a nonmonochromatic field divided by the corresponding probability for a monochromatic field of equal intensity.[12] The above discussion shows that η depends not only on the intensity distribution, as is generally assumed in the quasistatic theory,[12,15,16] but also on the form and decay rate of the field correlation functions, i.e.,

$$\eta(\bar{\gamma},\xi)=\eta_{qs}(\bar{\gamma})\eta_{corr}(\bar{\gamma},\xi). \tag{44}$$

Here $\eta_{qs}(\bar{\gamma})$ is the ordinary quasistatic statistical factor, and $\eta_{\text{corr}}(\bar{\gamma},\xi)$ is the additional "correlation statistical factor." When Eq. (37) is valid we have

$$\eta_{corr}(\bar{\gamma},\xi)=\exp\left\{\frac{2n_0\sigma^2}{\bar{\omega}^2}\varphi(\bar{\gamma})\right\}. \tag{45}$$

Figure 2 plots the universal function $\varphi(\bar{\gamma})$.

4. TUNNELING IN A WEAK STOCHASTIC FIELD SUPERPOSED ON A CONSTANT FIELD

We now examine ionization of a particle by a constant electric field E on which a stochastic field $F(t)$ with correla-

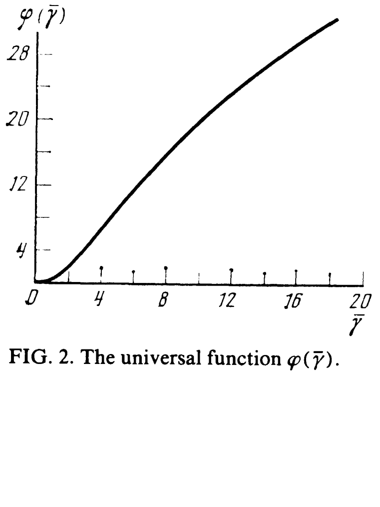

FIG. 2. The universal function $\varphi(\bar{\gamma})$.

tion function (14) is superposed. The procedure discussed in Sec. 1 can be used to derive the asymptotic formula

$$W\sim\exp\left\{-\frac{m\varkappa^2}{\hbar}\tau_0+\frac{e_0E}{2\hbar}\int_{-\tau_0}^{+\tau_0}d\tau x(\tau)\right\} \tag{46}$$

for the ionization probability. The extremal trajectory $x(\tau)$ and the quantity τ_0 are given by the equations

$$\frac{d^2q}{ds^2}=-\Phi-\alpha\Phi^3\int_{-1}^{+1}ds_1q(s_1)\exp[\nu^2\Phi^2(s-s_1)^2],$$

$$q(-1)=q(1)=0,\quad \left.\frac{dq}{ds}\right|_{s=-1}=-\left.\frac{dq}{ds}\right|_{s=1}=1, \tag{47}$$

$$x=\varkappa\tau_0q,\quad \tau_0=\frac{m\varkappa}{e_0E}\Phi,\quad \alpha=\frac{1}{2}\frac{\varkappa^3m^2\bar{F}^2}{e_0\hbar E^3},\quad \nu=\frac{\sigma m\varkappa}{e_0E}.$$

We assume that the stochastic field is weak ($\bar{F}^2/E^2\ll 1$) and retain only the first nonvanishing corrections in Eq. (47) involving the small parameter α. We find that

$$W\sim\exp\left\{-\frac{2}{3}\frac{m^{1/2}(2I_0)^{3/2}}{e_0\hbar E}\left[1-\frac{3}{16}\frac{\varkappa^3m^2\bar{F}^2}{e_0\hbar E^3}Q(\nu)\right]\right\}, \tag{48}$$

$$Q(\nu)=\int_{-1}^{1}dx\int_{-1}^{1}dy\int_0^y dz(1-x^2)y\exp[\nu^2(x-z)^2]$$

$$=\frac{\pi^{1/2}}{3\nu}\operatorname{erfi}(2\nu)\left(\frac{4}{\nu^2}+\frac{8}{5}\right)$$

$$-\frac{1}{5\nu^2}\exp(4\nu^2)\left(\frac{4}{3}+\frac{1}{\nu^2}+\frac{1}{10\nu^3}\right)+\frac{1}{3\nu^4}\left(1+\frac{1}{6\nu^2}\right). \tag{49}$$

Here

$$\operatorname{erfi}(x)=\frac{2}{\pi^{1/2}}\int_0^x dte^{t^2}.$$

For $\nu\ll 1$

$$Q(\nu)\approx\frac{8}{9}+\frac{16}{45}\nu^2, \tag{50}$$

while for $\nu\gg 1$

$$Q(\nu)\approx\frac{1}{2^6\nu^8}\exp(4\nu^2). \tag{51}$$

According to (50) and (51), the addition of the stochastic field abruptly increases the probability for ionization by the constant electric field. As we have already observed several times, the high-frequency Fourier components of the random time-dependent force are responsible for this. The corrections to the argument of the tunneling exponential [see (51)] become larger when the correlation function decays more quickly (for $\nu=\sigma m\varkappa/e_0E\gg 1$). A similar effect was noted recently in Ref. 23, where tunneling in a weak harmonic field was studied.

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