Interference of atoms in separated optical fields

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Interference of atoms due to the wavelike character of their motion is considered. Coherent interfering beams are produced when resonant atoms are diffracted by disjointed optical standing waves. In the general case, the wave acts on the atoms as an amplitude-phase grating. It is shown that the interference pattern can be observed with the aid of high-divergence atomic beams under echo conditions. The echo phenomenon in quantum-mechanical action of light on translational degrees of freedom is considered for the first time. In this case the phase memory can be transmitted over very large distances through the ground state of the atom. The interference manifests itself in formation of a periodic atom-density structure with a period smaller than the wavelength of the light. The modulation amplitude in various scattering regimes is found. It is shown that interference can be produced in a gas of resonant atoms by scattering that is selective in the particle velocities.

I. INTRODUCTION

One of the manifestations of the wave properties of particles is their interference. Great interest attaches to observation of the interference of heavy particles (atoms, molecules, ion). This may lead to development of interferometry in the sub-angstrom band. As in any interference experiments, this calls for obtaining coherent particle beams. In neutron interferometry[1] one uses for this purpose neutron diffraction by an array of silicon-crystal slabs. This mentod is not suitable for atoms, in view of their low penetrating power. The feasibility of an atomic interferometer, in which coherent beams are produced by scattering the atoms from a standing wave, was demonstrated in Ref. 2.

A resonant standing light wave is an effective diffraction grating for neutral atoms (the resonant Kapitza-Dirac effect). This was demonstrated in theoretical[3–10] and experimental[11–13] studies. In scattering by a standing wave, the recoil alters the atom momentum by an amount equal to the momentum $\pm \hbar k$ of the head-on photons (k is the standing-wave vector). The first standing wave (see Fig. 1) splits therefore the atom beam into two mutually coherent beams. Scattering by a second standing wave converges these beams and an interference pattern is produced at their intersection point. We emphasize that such an atomic interferometer is in principle feasible even now, in view of the recent progress in cooling of atomic beams (see Ref. 14) and obtaining coherent ultraviolet radiation sources.[15] Thus, if the object is a beam of light hydrogen atoms cooled to 2 K, and the standing waves are resonant with the $\lambda = 1215$ Å transition, an atom of momentum p is diffracted by an angle $\theta_d = 2\hbar k / p \gtrsim 1°$. At this diffraction angle, scattered coherent beams are spatially separated by as distance ~ 10 cm in a beam having a wide aperture (~ 0.1 cm).

As for the optical band, the radiation wavelength is here $\sim 1\,\mu$m, the objects are large-mass atoms (Na, Ca, Rb, etc.), and at a beam temperature 300 K the diffraction angle is $\theta_d \sim 10^{-4}$. Ordinary atomic beam with a thermal velocity spread and a large angular divergence $\theta \sim 10^{-2}$–10^{-4} cannot be spatially separated at so small a diffraction angle. The wave properties of the particles are revealed under these conditions by the interference of the periodic structure in the spatial distribution of the atom density, viz., density harmonics with period of the order of the wavelength. We have shown earlier[2] that despite the lack of coherence in the incident beam and the rapid loss of the coherence after diffraction by the standing wave, the periodic structure can be observed under echo conditions.

Echo in a gas of standing waves has a number of properties.[16] Processes similar to the echo lead to the appearance of a narrow resonance in separated fields.[17] This effect governs the present progress of ultrahigh-resolution spectroscopy in the optical band. It suffices for its purposes to quantize the internal degrees of freedom of the atom. If all degrees of freedom are quantized, however, the lines in the separated fields split into a number of components.[18] This splitting was theoretically investigated in Refs. 6 and 19 and was observed in Refs. 20 and 21. We investigate here the echo effect in a new dynamic situation, when only translational degrees of freedom are quantized.

A standing light wave produces spatially periodic perturbations of both the phase and the amplitude of the parti-

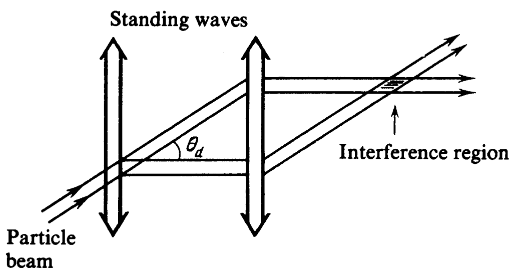

FIG. 1. Diagram of atomic interferometer.

cle wave function. The phase perturbation is easily realized even at short interaction times, owing to the high frequency of the induced transitions. The amplitude perturbation occurs for both elastic and inelastic scattering of the atoms by the standing waves. In the former case prolonged interaction between the atoms and the field is necessary. The latter case occurs when the upper working level has a large width due to processes such as ionization or decay to nonresonant states.

The density modulation under echo conditions manifests itself differently in phase and amplitude perturbations. In phase perturbation, the density modulation is connected with the quantum character of the atom motion and is zero if recoil is neglected. The production of a periodic atomic grating by amplitude perturbation is analogous to the classical shadow effect. It is known that superposition of shadows of two identical gratings, separated by distance L, on a screen located at a distance $2L$ (the echo effect) results in fringes of different brightness and having the period of the gratings. Even in this case, however, the features connected with the recoil effect turn out to be substantial. They manifest themselves most clearly in pulsed scattering of atoms and lead to quantum beats in the amplitudes of the density harmonics.

We report here detailed investigation of the interference pattern (IP) produced under echo conditions in standard and pulsed particle scattering regimes, for both joint and separate action of the phase and amplitude perturbations. In Secs. 3 and 4 we consider the case of short interaction times, when all the particles are scattered. If the interaction times are long (Secs. 6 and 7), the scattering becomes velocity-selective, so that IP can be observed in a resonant gas.

2. STATEMENT OF PROBLEM

We consider the conditions for observing interference of atoms scattered by a resonant field of a standing light wave $E(y)\cos kx \exp(-i\Delta t)$, which propagates along the x axis and whose frequency differs by a small amount Δ from that of the atomic transition. The smooth envelope $E(y)$ describes the field distribution in the light-beam cross section.

Interference effects can be observed in atomic beams and in a gas of resonant atoms. Let the atomic beam propagate along the axis and comprise an incoherent mixture of plane waves with momenta $p_x \ll p_y$. In scattering by a standing wave, the transverse momentum of the atom changes by an integer multiple of $\hbar k$. An atomic beam with a small transverse-momentum spread ($p_x \lesssim \hbar k$) can naturally be called coherent. Under ordinary conditions, however, the beams are weakly coherent and their angular divergence $\theta = p_x - p_y$ exceeds substantially the diffraction angle $\theta_d = 2\hbar k/p_y$.

Scattering of a resonant two-level atom by a standing-wave field is described by the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=\left\{\frac{\hat{\mathbf{p}}^2}{2M}-\begin{pmatrix}\hbar\left(\dfrac{\Delta}{2}+i\Gamma\right) & V(x,y)\\ V(x,y) & -\dfrac{\hbar\Delta}{2}\end{pmatrix}\right\}\psi \qquad (1)$$

for a column ψ of two components, ψ_b and ψ_a, which are the amplitudes of the upper (b) and lower ground (a) states of the atom, $V(x,y)=V(y)\cos kx$, $V=d(E)y$, and d is the dipole matrix element of the transition. The constant Γ in (1) takes into account the upper-working-level damping due to transitions to other nonresonant states. Obviously, these transitions produce near resonance amplitude modulation of the atom wave function on the working levels. It is assumed here, of course, that $\Gamma \gg \gamma$, where γ is the rate of the radiative decay to the ground state.

Spontaneous transitions between working levels upset the coherence of the interaction with the field and suppress the interference effects. We assume therefore that the following condition is met:

$$w\gamma\tau_0<1, \qquad (2)$$

when the spontaneous emission can be neglected. Here τ_0 is the time of interaction between the atoms and the field, and w is the population of the upper state.

For atoms with strong transitions (e.g., Na), the parameter $\gamma\tau_0$ varies in a wide range, from $\sim$1–10 for atoms that pass through a focused light beam, up to $\sim 10^3$ for passage through an unfocused beam. Clearly, to satisfy condition (2) in these cases the population of the upper level must be small. If the atoms are excited by a field pulse of duration $\tau_0 < 1/\tau \sim 10^{-8}$ s the population of the upper level can be arbitrary.

Atoms with strong transitions can transport spatial coherence over large distances only if they remain in the ground state after interacting with the field. This condition is realized for not too strong fields and when the interaction is adiabatic, with $\max(\Gamma,\Delta) \gg \tau_0^{-1}$, $V/\hbar$. In this case the real population of the upper level can be neglected and the system (1) reduces to a scalar equation for $\psi = \psi_a$

$$\begin{gathered} i\hbar\frac{\partial\psi}{\partial t}=\{\hat{p}^2/2M+U(x,y)\}\psi,\\ U(x,y)=U(y)(1+\cos qx),\qquad U(y)=\frac{V^2(y)}{2\hbar(\Delta+i\Gamma)},\\ q=2k \end{gathered} \qquad (3)$$

with a Hamiltonian that is non-Hermitian near resonance. A scalar Schrödinger equation with such an Hamiltonian was first considered in Ref. 22. It is shown in [23] that it is a particular consequence of the generalized two-level-system formalism developed in Refs. 24 and 23 for nonresonant ($\Delta \gg \Gamma$) and resonant ($\Delta \sim \Gamma$) excitation, respectively.

When diffracted by a standing wave located at $y=0$ and having a characteristic thickness a, a plane atomic wave $\exp(i\mathbf{p}\cdot\mathbf{r}/\hbar)$ is transformed at $y>a$ into a superposition of plane waves

$$\begin{gathered} \psi_{\mathbf{p}}(\mathbf{r})=\sum_n A_n(\mathbf{p})\exp(i\mathbf{p}_n\mathbf{r}/\hbar),\\ p_{nx}=p_x+n\hbar q,\qquad p_{ny}=p_y-\frac{n\hbar q}{p_y}\left(p_x+\frac{n\hbar}{2}q\right) \end{gathered} \qquad (4)$$

with momenta that satisfy the energy conservation law $\mathbf{p}_n^2/2M = \mathbf{p}^2/2M$. The amplitudes of the scattered waves depend on the momentum of the incident particle, on the field parameters, and on the excitation times. Under ordinary conditions the aperture of the atomic beam is several orders larger than the wavelength of the light. The scattered beams can therefore be regarded as overlapping even at large distances from the interaction region. As a result of the interference of the scattered waves, the atom density becomes spatially modulated

$$\langle|\psi_p(\mathbf{r})|^2\rangle=1+\sum_{n\neq0}\rho_n(y)\exp(inqx),$$

$$\rho_n(y)=\left\langle\exp\left(-\frac{iny}{l_c}\right)\sum_m A_{n+m}(\mathbf{p})A_m^*(\mathbf{p})\times\exp\left[-in(n+2m)\frac{y}{l_r}\right]\right\rangle, \quad (5)$$

$$l_c=p_y/qp_x,\quad l_r=p_y/M\varepsilon_r,\quad \varepsilon_r=\hbar q^2/2M.$$

Here $\hbar\varepsilon_r$ is the recoil energy, and the angle brackets denote averaging over the momenta in the incident beam.

The modulation amplitudes $\rho_n(y)$ contain two characteristic lengths: l_c and l_r. Over these distances, the atom's wave-function phase changes due respectively to the linear Doppler shift and to the recoil effect become of the order of unity. The length l_r changes in a rather wide range. For Na atoms with thermal velocity 10^5 cm/s it is small, $l_r\approx0.2$ cm. For iodine and methane molecules it reaches several centimeters. The coherence length l_c depends substantially on the transverse-momentum spread. It will be shown below (Sec. 3) that for short interaction times the amplitudes A_n depend little on p_x and the coherence length is determined by the initial angular beam divergence $l_c\bar{\lambda}/\theta$, where $\bar{\lambda}=q^{-1}$, so that for a weakly coherent atomic beam we have $I_c/I_r\sim\theta_d/\theta\ll1$. In this case the periodic structure in the atom distribution turns out to be localized at a distance of order I_c from the interaction region, which is large compared with a and small compared with the distance l_r typical of quantum effects. There exists an intermediate interaction-time range in which $\bar{\lambda}/\theta\lesssim a\ll l_r$. This case is typical of beams with large divergence, or of a gas of atoms. A major role is assumed here by the resonant structure of the scattering amplitudes as functions of p_x, and the standing wave separates a beam of resonant atoms with a divergence $\theta_0\sim\bar{\lambda}/a$, so that $l_c\sim a$. The periodic structure in the presence of amplitude modulation is also localized in the region of this size. Finally, at a long interaction time $a\gg l_r$ the length l_c coincides with l_r and the interference is confined to the interior of the light beam.

Thus, in a weakly coherent beam interference can occur in practice only in the region where the beam interacts with the field. With increasing distance from the field, the IP vanishes rapidly because of the large scatter of the free-motion phase shifts connected with the lengths l_c and l_r. These phase shifts can be cancelled by scattering the atoms from two standing waves, so that IP can be obtained at large distances from the exciting fields. At small l_r the phase shifts cancelled should be those connected both with the Doppler effect and with the recoil. In the case of weak recoil at distances shorter than l_r it suffices to cancel the Doppler phase shift.

3. CASE OF SHORT INTERACTION TIMES

Consider atoms scattering by two standing waves $V_{1,2}\cos kx$ separated by a distance $L\gg a$. The amplitudes of these waves can be different, and the detunings, the wave vectors, and the phase shifts can for simplicity be assumed equal. The interaction time is assumed to be short:

$$\tau_0^{-1}=v_y/a\gg kv_x, \quad (6)$$

so that when the particles pass through the light beam they are shifted by a distance shorter than the wavelength. In this case we can leave out in the field region the operator of the particle transverse (along the x axis) kinetic energy, and the stationary equation (3) takes the form

$$i\hbar v_y\frac{\partial\psi}{\partial y}=U(x,y)\psi. \quad (7)$$

Weak field

The main features of the IP under echo conditions can be explained using as an example weak fields, when few harmonics take part in the interference. In first order of perturbation theory in the field $U(1+\cos qk)$, the amplitudes of the transitions in which the momentum p_x changes by $\pm\hbar q$ is $A\pm1\approx-i\xi/2$, where

$$\xi=U\tau_0/\hbar. \quad (8)$$

In the next order, the momentum changes by $\pm2\hbar q$ and the amplitudes are $A\pm2\approx-\xi^2/8$. The phase shifts of the partial interfering waves are determined by the phase shifts of the free motion of the atom outside the light fields, in accordance with Eq. (4).

We consider the amplitude ρ_1 of the density modulation after scattering by two standing waves with field parameters ξ_1 and ξ_2 under echo conditions. Contributions to ρ_1 are made by the elementary scattering processes (see Fig. 2), for which the phase shifts of the free motion along the two interfering trajectories cancel out. For the processes shown in Fig. 2a, the "optical" path lengths $\int\mathbf{p}(\mathbf{r})\cdot d\mathbf{r}/\hbar$ along the trajectories 1 and 2 are equal and the phase shifts due to the Doppler effect and to the recoil are completely cancelled. For the processes shown in Fig. 2b, only the Doppler phase shift is cancelled. The summary contribution of all these processes to ρ_1 at $y=2L+l(l\ll L)$ is

$$\rho_1(l)=\frac{1}{2}\left\langle e^{-il/l_c}|\xi_1\xi_2^2|\sin\left(\varphi+\frac{l}{l_r}\right)\sin^2\left(\varphi+\frac{L}{l_r}\right)\right\rangle, \quad (9)$$

where $\tan\varphi=-\Gamma/\Delta$. At $L\gg l_r$, owing to averaging over the longitudinal velocities we have $(l_r\sim v_y)\sin^2(\varphi+L/l_r)\approx\frac{1}{2}$, which corresponds to the contribution from the processes of type 2a.

In the case of particles with low recoil energy, when l_r is large enough, interest attaches to distances $L\lesssim l_r$ and the modulation amplitude is determined by the general formula (9).

For weakly coherent atomic beams, the IP is localized near $y=2L$ with a width $l\sim l_c\ll l_r$. For amplitude perturbation ($\Gamma\neq0$), the modulation of ρ_1 becomes of the order of

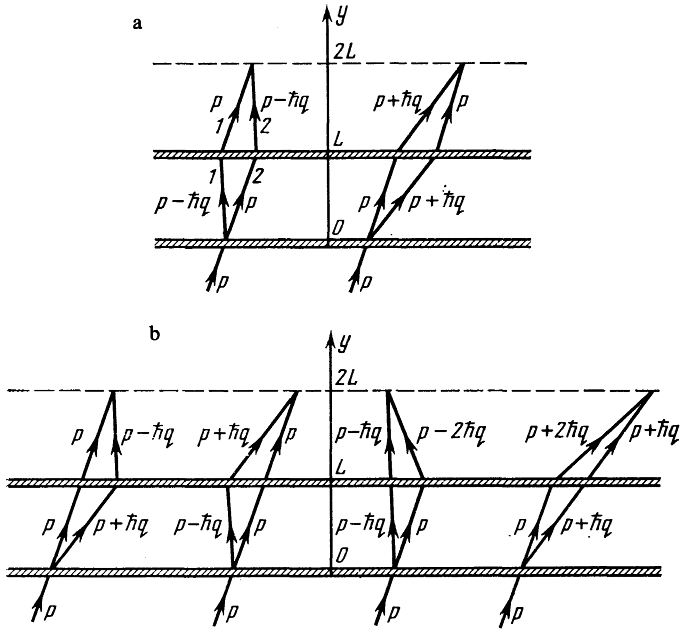

FIG. 2. Interfering trajectories for scattering by weak separated standing waves ($p\equiv p_x$). For processes of type a), under echo conditions, the free-motion phase shifts are completely cancelled. For processes of type b), the phase shift cancelled is the one connected with the Doppler effect.

unity $\xi_{1,2}\sim 1$. In the absence of amplitude perturbation ($\Gamma=0$) the situation changes in two respects. First, the amplitude ρ_1 of the density modulation is strictly zero at $y=2L$. This can be easily understood by recognizing that at this distance the echo effect reproduces the structure formed directly after the action of the first wave. And inasmuch as there is no density modulation for pure phase perturbation in the wave function, there is likewise none at $y=2L$. It is necessary to move away some distance from the point to observe the spatial bunching of the particles that have acquired a recoil momentum $\pm\hbar q$. This is the gist of the qualitative feature of the echo when translational degrees of freedom are quantized. Second, the depth of the IP modulation is lower for weakly coherent beams. Indeed, at $\varphi=0$ we have $\rho_1\sim l_c/l_r\ll 1$ even at the saturation $\xi_{1,2}\sim 1$. As shown above, an appreciable modulation depth due to quantum effects can be obtained only for large saturation, when $\xi_1\gg 1$.

Strong field

Integration of Eq. (7) is elementary for arbitrary fields. After scattering by a standing wave, the atom's wave function acquires a complex factor $\exp[-i\xi(1+\cos qx)]$. As a result, the amplitudes A_n and ρ_n in Eqs. (4) and (5) can be expressed in terms of Bessel functions

$$A_n=(-i)^n\exp(-i\xi)J_n(\xi), \tag{10}$$

$$\rho_n(y)=(-1)^n\left\langle\exp\left(-\alpha-\frac{iny}{l_c}\right)\times\left[\frac{\Delta\,\mathrm{tg}(ny/l_r)-\Gamma}{\Delta\,\mathrm{tg}(ny/l_r)+\Gamma}\right]^{n/2}J_n(z)\right\rangle,$$
$$\alpha=-2\,\mathrm{Im}\,\xi=\frac{V^2\tau_0\Gamma}{\hbar^2(\Gamma^2+\Delta^2)},$$
$$z=2|\xi|\left[\sin^2\left(\frac{ny}{l_r}\right)-\frac{\Gamma^2}{\Gamma^2+\Delta^2}\right]^{1/2}. \tag{11}$$

The scattered-wave amplitudes A_n are independent in this case of the particle transverse momentum. For a coherent atomic beam, the averaging in (11) can be omitted, and it can be seen from (5) and (11) that the IP becomes periodic in space, with a period πl_r. For a weakly coherent beam with angle divergence θ we have $y\sim l_c\sim\lambda\!\!\!^{-}/\theta\ll l_r$. At $\theta=10^{-3}$ we have $l_c\sim 10^{-2}$ cm.

Using (4) and (10) we easily obtain the modulation amplitude following scattering by two waves with parameters ξ_1 and ξ_2.

If only amplitude perturbation is present ($\Delta=0$, $\xi=-\xi^*$), ρ_1 is expressed at a distance $y=2L+l$ ($l\ll L$) in terms of modified Bessel functions and takes in the limiting cases $L\ll l_r$ and $L\gg l_c$ the respective forms

$$\rho_1(l)=-\langle\exp[-2(|\xi_1|+|\xi_2|)-il/l_c]\times I_1(2|\xi_1|)I_2(2|\xi_2|)\rangle, \tag{12}$$

$$\rho_1(l)=-\langle\exp[-2(|\xi_1|+|\xi_2|)-il/l_c]\times I_1(2|\xi_1|)I_1^2(|\xi_2|)\rangle. \tag{13}$$

Expression (12) determines the density modulation due to the shadow effect. We note that $\rho_1(l)$ is an even function and reaches a maximum at $l=0$. For the case of pure phase perturbation ($\Gamma=0$, $\xi=\xi^*$) we obtain at $L\gg l_r$

$$\rho_1(l)=\langle\exp(-il/l_c)J_1(2\xi_1 l/l_r)J_1^2(\xi_2)\rangle. \tag{14}$$

This shows that $\rho_1(l)$ is an odd function and $\rho_1\sim 1$ at $\xi_1\sim l_r/l_c\sim p_x/\hbar q\gg 1$ and $\xi_2\sim 1$. In other words, the first wave should scatter the atoms through angles of the order of the initial beam divergence.

In weak fields Eqs. (12)–(14) coincide with the perturbation-theory result (9).

4. PULSED SCATTERING REGIME

An IP can also be obtained using short pulses of standing light waves that are separated in space and in time. The first beam acts on the atom beam near $y=0$ at the instant $t=0$ and its duration is τ_0. The second pulse (of duration τ_0) at $y=L$ is applied at an instant $t=T\gg\tau_0$. Obviously, the space and time intervals between pulses must be so reconciled that $T=v_0T$, where v_0 is the characteristic thermal velocity of the atoms in the beam.

In the case of pulsed irradiation, the light beams need not be focused and the atoms can be regarded as immobile during the interaction time τ_0, if $kv_x\tau_0\ll 1$ and $v_y\tau_0\ll a$.

In contrast to the stationary regime, the particle longitudinal velocity v_y remains unchanged, and the kinetic energy changes by an amount $[(p_x+n\hbar q)^2-p_x^2]/2M$. The modulation amplitude ρ_1 at the instant $t=2T+\tau$ $(\tau\ll T)$ is obtained in the same manner as in Sec. 3. All that are needed are the following obvious substitutions:

$$l/l_c\to qv_x\tau,\quad l/l_r\to\varepsilon_r\tau,\quad L/l_r\to\varepsilon_rT,$$
$$\xi_1\to\eta_1(y)=\xi_1f(y-2v_yT),\quad \xi_2\to\eta_2(y)=\xi_2f(y-L-v_yT),$$

where the function $f(y)$ describes the spatial distribution of the intensity in the cross section of the light beam, has a natural width a, and is normalized to unity at the maximum.

It is important to emphasize that the phase shift ε_rT due to the recoil is now independent of the particle velocity. Therefore ρ_1 receives contributions from all the trajectories that interfere under "classical" conditions, i.e., for which the Doppler effect is cancelled out.

Weak field

In a weak field ($\xi_{1,2}\ll 1$) it is necessary to take into account all the scattering processes shown in Figs. 2a and 2b. As a result we get

$$\rho_1(y,\tau)={}^1/_2|\xi_1\xi_2^2|\sin(\varphi+\varepsilon_r\tau)$$
$$\times\sin^2(\varphi+\varepsilon_rT)\langle\exp(-iqv_x\tau)f(y-2v_yT)f(y-L-v_yT)\rangle.\quad(15)$$

The function $\sin^2(\varphi+\varepsilon_rT)$ describes quantum oscillations caused by the recoil effect when T is changed. Obviously, these oscillations can be observed in broad light beams with $a\gg l_r$, when the T dependence contained in the functions f is a smooth one.

Strong field

For fields of arbitrary intensity, in the case of pure amplitude or phase perturbation, we have for ρ_1 the following respective expressions

$$\rho_1(y,\tau)=-\langle\exp[-2(|\eta_1|+|\eta_2|)-iqv_x\tau]$$
$$\times I_1(2|\eta_1|)I_2(2|\eta_2|\sin\varepsilon_rT)\rangle,\quad(16)$$
$$\rho_1(y,\tau)=\langle\exp(-iqv_x\tau)J_1(2\eta_1\varepsilon_r\tau)$$
$$\times J_2(2\eta_2\sin\varepsilon_rT)\rangle.\quad(17)$$

The amplitude ρ_1 as a function of y is localized near $y=2L$ in an interval of the order of a. To prevent attenuation of the signal by thermal spreading of the irradiated atom-beam region during the time T, it is advisable to use broad light beams with $a\lesssim L$. The IP exists as a function of time near $t=2T$ in a small interval of on the order of the coherence time $\tau_c=\lambda\!\!\!^{-}/v_0\theta$. At an angular divergence $\theta=10^{-3}$ and $v_0=5\cdot10^4$ cm/s we have $\tau_c=10^{-7}$ s.

Thus, in the pulsed regime (in contrast to the stationary one), the resultant atomic interference grating has a short lifetime τ_c, but occupies in space a larger region, of the order of the light-beam thickness. This offers definite advantages when such a grating is detected by backward reflection of a test signal.

5. NONLINEAR ATOMIC INTERFEROMETRY

Another possible way of studying interference makes use of nonlinear effects in the detection of an atomic beam. Whereas the modulation (11) of the average atom density exists only over distances on the order of l_c, the modulation in the density correlator is preserved over longer distances.

If the atoms are detected with the aid of two thin heated wires, the signal due to mixing of the currents of these detectors can be measured. It is proportional to the two-point correlator of the atom density

$$\varphi(\mathbf{r}_1,\mathbf{r}_2)=\langle|\psi_p(\mathbf{r}_1)|^2|\psi_p(\mathbf{r}_2)|^2\rangle$$

in the beam.

For stationary scattering of atoms by one standing wave we obtain with the aid of (4) and (10) at $y_1=y_2$ and $\Delta\gg\Gamma$

$$\varphi(x_1-x_2)=1-\left\langle\sum_n{}' J_n^4(\xi)\right\rangle+\langle J_0^2[2\xi\sin k(x_1-x_2)]\rangle.\quad(18)$$

This expression is independent of y, so that the phase memory is preserved in the correlation function φ over arbitrary large distances from the exciting field.

6. INFLUENCE OF SPONTANEOUS EMISSION ON THE IP

Near resonance and in the absence of inelastic processes ($\Gamma=0$), the IP can be substantially influenced by spontaneous transitions between the working levels.

In the presence of spontaneous relaxation the behavior of the atom is described by a density matrix

$$\hat\rho=\rho_{\alpha\beta}(x_1,x_2,y,p_y)\quad(\alpha,\beta=a,b),$$

in which we changed over to the Wigner representation for the motion along the y axis. Such a two-point (i.e., dependent on x_1 and x_2) density matrix was used in Ref. 10 to describe diffraction of atoms by a standing wave with allowance for spontaneous transitions.

Consider the case of exact resonance ($\Delta=0$) and short interaction time $\tau_0\ll 1/\gamma$, $1/kv_x$. The change of the diagonal elements of after scattering by a standing wave located near $y=0$ is described by the solutions of the two-component equation (1) at $\Delta=\Gamma=0$ and is of the form

$$\rho_{aa}(x_1,x_2,y=+0)=\cos(\xi\cos kx_1)$$
$$\times\cos(\xi\cos kx_2)\rho_{aa}(x_1,x_2,y=-0),\quad(19)$$
$$\rho_{bb}(x_1,x_2,y=+0)=\sin(\xi\cos kx_1)$$
$$\times\sin(\xi\cos kx_2)\rho_{aa}(x_1,x_2,y=-0),$$
$$\xi=V\tau_0/\hbar.$$

The spontaneous emission causes the element ρ_{bb} as well as the off-diagonal elements of the density matrix to attenuate over a distance of the order of $l_0 = v_y/\gamma$. For atoms with strong transitions this distance is much shorter than l_r relative to the parameter $l_0/l_r \sim \varepsilon_r/\gamma \ll 1$. At $v_y = 5\cdot 10^4$ cm/s and $\gamma = 10^8\ \text{s}^{-1}$ we have $l_0 \approx 5\cdot 10^{-4}$ cm. As a result, at $y \gg l_0$ all the atoms turn out to be in the ground state, with a density matrix

$$\rho_{aa}(x_1, x_2, y) = \exp\left[-i\frac{y}{v_y}(\hat{T}_1 - \hat{T}_2)\right]\{\rho_{aa}(x_1, x_2, +0) + f(x_1 - x_2)\rho_{bb}(x_1, x_2, +0)\}, \qquad (20)$$

where $\hat{T}$ is the transverse kinetic energy operator. The first term in the curly brackets describes the atoms that remain in the ground state after being scattered by the standing waves, and constitutes a mixture, incoherent in p_x, of atomic beams each of which is a coherent superposition of partial waves with momenta $p_x + 2n\hbar k$. The second term corresponds to arrival from the excited state as a result of spontaneous relaxation. The function $f(x_1 - x_2)$ describes the atom's transverse recoil due to spontaneous emission, and its Fourier components $f(q)$ differ from zero only if $|q| \leqslant k$. The reason for factoring this term $f\rho_{bb}$ is due to the fact that its relaxation rate (over the length l_0) is much faster than the rate at which the recoil disturbs the spatial coherence produced in the excited state after the scattering (this takes place over the length l_r). The second term is therefore also an incoherent mixture, with respect to $p_x + \hbar q$, of beams of which each is a coherent superposition of waves with momenta $p_x + \hbar q + (2n+1)\hbar k$. Owing to the scatter over the recoil momentum $\hbar q$, there is no coherence between the first and second terms.

After scattering by two waves spaced $L \gg l_r$ apart, we obtain under the echo conditions the following density-modulation amplitude:

$$\rho_1(l) = \langle \exp(-il/l_c) J_2(\xi_1 l/l_r) J_2^2(\xi_2)\rangle. \qquad (21)$$

In the short-interaction-time approximation the standing wave, as can be seen from (19) does not lead to density modulation, i.e.,

$$\text{Sp}\,\hat{\rho}(x, x, y=+0) = \text{Sp}\,\hat{\rho}(x, x, y=-0).$$

We therefore have under echo conditions $\rho_1(l=0) = 0$. In contrast to (14), however now ρ_1 is an even function of l.

7. CASE OF LONG INTERACTION TIMES

The preceding results were based on the assumption that the interaction time (6) is short for all atoms. This is equivalent to the condition $a \ll \lambda/\theta$. In this case the standing wave "has no time" to sort out the particles by transverse velocity and all of them take part in the interference.

For atomic beams with large divergence or in a gas of resonant atoms, where $\theta \sim 1$, the condition $a < \lambda/\theta$ does not hold.

We consider long enough interaction times, such that

$$\lambda/\theta \leqslant a \ll l_r. \qquad (22)$$

In this case the amplitudes A_n in (4) depend substantially on p_x and the scattering becomes selective in the transverse velocities of the particles. If the velocity change δv_x during the time τ_0 is small, so that $k\tau_0\delta v_x \ll 1$, the scattering amplitudes A_n can be obtained as before by using the given-motion approximation and let $v_y\partial/\partial y \to v$ in Eq. (7).[6,25] The A_n differ then from (1) by an additional phase factor $\exp[in\alpha(v_x/v_y)]$ and in that the Bessel functions have different arguments:

$$\xi \to \xi f\left(\frac{v_x}{v_y}\right), \quad f\left(\frac{v_x}{v_y}\right) e^{i\alpha(v_x/v_y)} = \int \frac{dy}{a}\exp\left(iqy\frac{v_x}{v_y}\right) f(y), \qquad (23)$$

where the function $f(y)$ describes the intensity profile in the light beam.

The modulation amplitude ρ_1 after scattering of the atoms by two standing waves of like profile is determined by Eqs. (12)–(14), in which an additional phase factor $\exp[i\alpha(v_x/v_y)]$ appears in the average over the velocities.

It can be seen from (23) that under the conditions of the inequality (22) only atoms with sufficiently low transverse velocities, $v_x/v_y \lesssim \theta = \lambda/a$, are effectively scattered. The standing wave separates thus from the initial beam with divergence θ a narrower beam with divergence $\theta_0 \lesssim \theta$, and it is this which determines the IP.

The size of the IP localization region is $l_c \sim a$, and its amplitude decreases and becomes of the order of $\rho_1 \sim \lambda/a\theta = \theta_0/\theta$. In a resonant gas ($\theta \sim 1$) at $a = 0.1$ cm this amounts to $\rho_1 \sim 10^{-4}$, while for atomic beams it is several orders larger. If the resultant atomic grating with period $\pi\lambda$ is used to observe the reflection of a test resonant field, the smallness of ρ_1 compared with the average atom density is of no importance, since only the alternating part of the density contributes to the backscattering.

8. BRAGG SCATTERING

A different situation obtains in weak fields at $a \gg l_r$.[25] If

$$V \ll \hbar\varepsilon_r, \qquad (24)$$

the only effectively scattered atoms are those whose momentum satisfies the Bragg condition $|p_x| \approx p_x \pm 2\hbar k|$.

Let $p_x = -\hbar k + \delta p$ ($\delta p \ll \hbar k$). The only two harmonics that matter in the expansion (4) are then

$$\psi_p(\mathbf{r}) = A_0(y)\exp(i\mathbf{pr}/\hbar) + A_2(y)\exp(i\mathbf{p}'\mathbf{r}/\hbar), \qquad p'_x = \hbar k + \delta p, \quad p_y' = p_y - 2\hbar k\delta p/p_y, \qquad (25)$$

and their amplitudes satisfy at $\Gamma = 0$ the equations

$$i\frac{dA_0}{dt} = \frac{V^2(t)}{4\hbar^2\Delta}\exp\left(-i\frac{2k\delta p}{M}t\right)A_2,$$
$$i\frac{dA_2}{dt} = \frac{V^2(t)}{4\hbar^2\Delta}\exp\left(+i\frac{2k\delta p}{M}t\right)A_0, \quad t = y/v_y \qquad (26)$$

with the initial condition $A_0(-\infty) = 1, A_2(-\infty) = 0$.

Under real conditions the field has a smooth envelope. For a potential that varies like

$$V^2(t) = V^2/\text{ch}(t/\tau),$$

Eqs. (26) can be integrated exactly, and at $t > \tau$ we obtain

$$A_0 = \cos(\pi\xi)\,[1+(\mathrm{th}(\pi\mu)\,\mathrm{tg}(\pi\xi))^2]^{1/2}\exp(i\alpha),$$
$$A_2 = i\sin(\pi\xi)/\mathrm{ch}(\pi\mu), \quad (27)$$
$$\xi = V^2\tau/4\hbar^2\Delta, \quad \mu = k\delta p\tau/M,$$
$$\alpha = \arg\frac{\Gamma^2(1/2+i\mu)}{\Gamma(1/2+\xi+i\mu)\,\Gamma(1/2-\xi+i\mu)},$$

where Γ is the gamma function. The intensities of the diffraction maxima satisfy the normalization condition $|A_0|^2 + |A_2|^2 = 1$.

The width δp of the Bragg resonance is determined by the time of flight

$$\delta p/\hbar k \sim 1/\varepsilon_r\tau \sim l_r/a \ll 1, \quad (28)$$

and its amplitude depends on the field and reaches a maximum at $\xi \sim 1$.

A symmetric diffraction pattern is produced obviously also at $p_x = \hbar k + \delta p$. The scattering amplitudes A_0 and A_{-2} are determined by Eqs. (25) and (27) with the substitution $k \to -k$.

A weak standing wave of thickness $a \gg l_r$ can thus select with high resolution the atoms in accordance with their transverse velocities, and "cuts out" from their broad initial distribution two narrow beams with momenta $p_x = \pm\hbar k$ and width $\delta p \ll \hbar k$.

The contributions of these particles to the IP are determined by the amplitude products $A_0 A_2^*$ for $p_x = -\hbar k$ and $A_{-2}A_0^* = -A_0 A_2^*$ for $p_x = \hbar k$. The density modulation is therefore proportional to the asymmetry of the atoms' distribution function $F_0(p_x, p_y)$ in the region of small transverse momenta:

$$F(p_y) = F_0(\hbar k, p_y) - F_0(-\hbar k, p_y) \approx 2\hbar k \left.\frac{\partial F_0}{\partial p_x}\right|_{p_x=0}.$$

We have then at $y > a$

$$\rho_1(y) = \int d(\delta p)\,dp_y F(p_y) A_0 A_2^* \exp(-2ik\delta p y/p_y). \quad (29)$$

Since the effective transverse momenta are $|p_x| \approx \hbar k$, the coherence length is $l_c \approx l_r$. The large phase shifts connected with the lengths l_c and l_r cancel out accurate to the width of the Bragg resonance. The phase shift connected with this width is of order unity only within the limits of the light beam $y \lesssim a$, and becomes large outside the beam, so that the IP is located practically in the interaction region.

We consider now Bragg scattering by two standing waves. The amplitudes for scattering by the first and second waves, A_0, $A_{\pm 2}$ and B_0, $B_{\pm 2}$ are determined by Eq. (27). The density modulation at $y = 2L + l$ ($l \ll L$) takes then the form

$$\rho_1(l) = \int d(\delta p)\,dp_y F(p_y)\,|B_2|^2 A_0 A_2^* \exp(-2ik\delta p l/p_y). \quad (30)$$

Interference, as already noted, exists if the atoms have an asymmetric distribution in the transverse velocities. This asymmetry can be obtained, for example, by oblique incidence of the atomic beam on the standing wave as a gas of resonant atoms flows through the interaction region. It is possible also to introduce a small difference between the frequencies of the opposing waves that form the standing waves; this moves the light-field profile along the x axis at a certain velocity.

The coefficients A and B can be of the order of unity even in weak fields. The modulation depth ρ_1 is therefore limited by two factors: by a distribution-function asymmetry of the order of $\hbar k/p_x \sim \hbar k/p_0\theta$ and by the scattered-particle fraction, which is of the order of $\delta p/p_x \sim \lambda/a\theta$, so that $\rho_1 \sim \hbar/p_0 a\theta^2$. For an atom beam with divergence $\theta \sim 10^{-2}$–10^{-3} and an incidence angle (with the normal) of the same order, at a light-beam thickness $a \sim 0.1$ cm, we have $\rho_1 \sim 10^{-4}$–10^{-2}. For a gas of resonant atoms this quantity is very small ($\sim 10^{-8}$), unless special measures are taken to produce a local nonequilibrium structure in the velocity distribution. The size of the coherence region in which an IP exists is determined by the light-beam thickness $l \sim p_y/k\sigma p \sim a$.

In a weak field ($\xi \ll 1$ at $p_y \sim p_0$) Eq. (30) takes the form

$$\rho_1 = \left(\frac{V^2 M a}{4\hbar^2\Delta}\right)^2 \frac{F(0)}{ka}\,\Phi\left(\frac{l}{a}\right),$$

$$\Phi\left(\frac{l}{a}\right) = -i\int_{-\infty}^{+\infty} d\mu \int_0^{\infty} \frac{d\xi}{\xi^3}\sin^3(\pi\xi)\cos(\pi\xi)$$

$$\times\,\mathrm{ch}^{-3}(\pi\mu)\,[1+\mathrm{th}^2(\pi\mu)\,\mathrm{tg}^2(\pi\xi)]^{1/2}\exp[-2il\mu/a + i\alpha(\xi,\mu)], \quad (31)$$

where the function Φ, which describes the spatial localization of the IP on the y axis, is of the order of unity at $l \lesssim a$. The coefficient of Φ determines the scale of the modulation amplitude. The integrand in (31) describes the distribution with respect to the longitudinal ($\sim 1/\xi$) and transverse ($\sim\mu$) momenta of the interfering particles. The distribution in the transverse momenta comprises two narrow peaks of width δp near $\pm\hbar k$. At $a \gg l_r$ the effective transverse temperature

$$T = (\delta p)^2/2M \sim \hbar\varepsilon_r (l_r/a)^2 \quad (32)$$

can be considerably lower than the recoil energy.

9. COMBINED SCATTERING REGIME

We see that under Bragg scattering conditions the independent contributions to the IP from two groups of atoms with transverse momenta $\pm\hbar k$ cancel each other. This can be avoided by changing the condition for the scattering by the first field.

Let the first field, which acts near $y = 0$, be a superposition of two traveling waves:

$$V_1(\mathbf{r}) = u_1 e^{i\mathbf{k}_1\mathbf{r}} + u_2 e^{i\mathbf{k}_2\mathbf{r}}, \quad k_1 = k_2, \quad \mathbf{k}_1 - \mathbf{k}_2 = k\mathbf{e}_x,$$

where $\mathbf{e}_x$ is a unit vector along the x axis. At large detunings from resonance the potential of the atom in the scalar equation (3)

$$U(r) = (u_1^2 + u_2^2 + 2u_1u_2\cos kx)/\hbar\Delta$$

has then a period double that of the case of a standing wave.

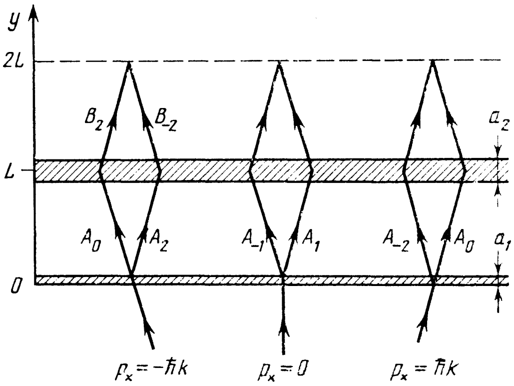

FIG. 3. Scheme of interference in Bragg scattering. In the case when the scattering by the first standing wave (at $y=0$) is selective in the transverse velocities, contributions (of opposite sign) are made to the IP by particles with momenta $p_x = \pm\hbar k$. In the combined scattering regime, contributions come from three groups of particles with momenta $p_k = 0, \pm\hbar k$.

We assume that the time $\tau_1 = a_1/v_y$ of interaction with this field is short ($kv_x\tau_1 \ll 1$), so that the wave function of the atom acquires after the scattering a phase factor

$$\exp(i\xi_1\cos kx) = \sum_n A_n \exp(inkx),$$
$$A_n = (-i)^n J_n(\xi_1), \quad \xi_1 = 2u_1u_2\tau_1/\hbar^2\Delta. \tag{33}$$

Scattering by the second field (near $y=L$ with thickness $a_2 \gg a_1$) takes place, as before, under Bragg conditions, i.e., only particles with transverse momenta $\pm\hbar k$ are scattered. The IP at the distance $y = 2L + l$ is then determined by three groups of particles with initial momenta $p_x = 0$, $\pm\hbar k$ and with a spread $\delta p \ll \hbar k$. The trajectories of these particles and the corresponding scattering amplitudes are shown in Fig. 3.

The contributions made to ρ_1 by these groups of particles are

$$|B_2(\xi_2)|^2 \begin{cases} A_{-1}A_1^* = J_1^2(\xi_1), & p_x = \delta p \\ A_0A_2^* = A_{-2}A_0^* = -J_0(\xi_1)J_2(\xi_1), & p_x = \pm\hbar k + \delta p, \end{cases}$$

where the amplitude of scattering by the second wave is determined at $a_2 \gg l_r$ by the expression (27), the distributions in the transverse momenta no longer cancel each other even if the initial function is symmetric.

For an atomic beam with large angular divergence or in a gas of resonant atoms, we then obtain for ρ_1

$$\rho_1(l) = \frac{2l}{\pi k a_2^2 \,\mathrm{sh}(l/a_2)} \langle p_y \sin^2(\pi\xi_2)[J_1^2(\xi_1) - 2J_0(\xi_1)J_2(\xi_1)]\rangle. \tag{34}$$

The angle brackets denote here averaging over the longitudinal velocities of the atoms, with a distribution function $F_0(p_y) = F_0(p_x = 0, p_y)$. The grating amplitude is a maximum at $l = 0$ and exists in a region of l on the order of the thickness a_2 of the second exciting field. If the field parameters ξ_1 and ξ_2 for the thermal velocities are of the order of unity, the modulation depth decreased, as in the case of intermediate interaction times, by a factor $\lambda\!\!\!^{-}/a_2\theta$.

10. CONCLUSION

Thus, using the high atom-scattering efficiency in a resonant light field, it is possible to obtain interference of heavy neutral particles. The IP can be observed in atomic beams having a sufficiently large angular divergence: $\theta \sim 10^{-2}$–10^{-3}, and also in a gas of resonant atoms. Under the echo condition, the IP is localized at large distances from the interaction region.

The IP parameters can vary in a wide range, depending on the characteristics of the light field and of the atoms.

At short interaction times, all the particles participate in the interference, so that the density modulation amplitude can be of the order of unity. Under stationary conditions, the half-width of the resultant atomic grating is determined by the angular divergence of the particle beam $l_c \sim \lambda\!\!\!^{-}/\theta$. Inasmuch as under adiabatic scattering conditions the atoms are in the ground state, the spatial coherence can be transported in an atom beam over very large distances, $\sim 10^2$–10^3 cm. For pulsed irradiation one can use unfocused light beams. In this case the size of the IP becomes large, of the order of the beam thickness, and its lifetime is of the order of l_c/v_y.

At long interaction times, scattering in a weak field becomes selective in the particle velocities. The atomic grating is made up of atoms having a very small transverse-momentum scatter. The corresponding effective temperature can become lower than the recoil energy. The interference can be realized in a large region with linear dimension of the order of the thickness of the light beam. It is important that in this case one can use beams that have a large angular divergence and even an ordinary resonant gas of atoms.

Atom diffraction in light waves thus promises feasibility of atomic interferometry. This is of interest for high-precision experiments in laser spectroscopy, as well as for applications, such as obtaining submicron structures in sputtering atoms on a plate, recording and transmitting information with the aid of long atomic beams, and others.

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