# Electron and phonon kinetics in a nonequilibrium Josephson junction

A. M. Gulyan and G. F. Zharkov

*Institute of Physics Research, Academy of Sciences of the Armenian SSR: P. N. Lebedev Physics Institute, Academy of Sciences of the USSR*

(Submitted 1 November 1984)
Zh. Eksp. Teor. Fiz. **89**, 156–171 (July 1985)

The correct expression for a tunneling source is obtained taking account of the macroscopic phase coherence in a Josephson junction and the imbalance between the electron-hole excitation branches. The canonical forms of the electron-electron, electron-phonon, and phonon-electron inelastic collision operators are derived with allowance for the imbalance. The phenomenon of quantum oscillations of a tunneling source is discussed. This phenomenon gives rise to the oscillation of the electron distribution function in the nonequilibrium layer of the junction between two superconductors, as a result of which a chemical-potential-shift "jitter" arises and satellites are also formed in the radiation scattered by the junction. The spectral dependence of the phonon fluxes from a nonequilibrium junction is studied with the use of the results obtained in a numerical analysis of the kinetics of the electron-hole excitations. It is shown that the previously discussed phonon-deficit effect should occur even when the superconductors contain excess quasiparticles produced by the field from the pair condensate. The behavior of the current in a nonequilibrium junction is also analyzed. The current-voltage characteristics in the near-threshold voltage regime are calculated. Allowance for the nonequilibrium effects may affect the qualitative disagreement between the well-known experimental results obtained in the measurement of the "interference" (phase-difference dependent) conductivity of the Josephson junction and the predictions of the equilibrium theory of tunneling.

## §1. THE NONEQUILIBRIUM JOSEPHSON JUNCTION

The nonequilibrium phenomena that occur in Josephson junctions are of great interest from the standpoint of the microscopic theory. Not only has this subject not been sufficiently fully studied, it turns out on closer examination that many questions have not been touched upon at all. Among these questions is, for example, the question, considered in the present paper, of how the macroscopic phase coherence in the junction affects the kinetics of the single-particle excitations.

The existence of effects connected with the action of external fields on the Josephson junction is of great interest from the practical point of view as well. Let us note that, even in those cases when there are no external nonstationary fields, the voltage potential applied to the junction is an effective unbalancing agent. This is especially true of thin-film junctions with finite geometry. In this case the excitations produced during tunneling cannot be resorbed because of the fast diffusion process (which occurs in the presence of massive banks), and the relaxation into the equilibrium state occurs on account of the presence of a uniform kinetic mechanism. At the same time the phonon subsystem in a thin-film junction can be considered to be in a state of equilibrium, since the nonequilibrium phonons manage to effectively leave the film without exerting a reciprocal influence on the electron subsystem. Thus, the model with a phonon bath can be employed in the study of the kinetics of the electron subsystem of nonequilibrium junctions.

Furthermore, junctions obtained by the deposition technique usually contain a fairly large number of elastic-scattering centers, so that we can use the "dirty" limit approximation, and thus significantly simplify the analysis.

Under these conditions, which we shall assume below, it is possible to make considerable progress in the study of the nonequilibrium properties of Josephson junctions. In order to carry out a consistent investigation of the kinetics, we derive a kinetic equation (§§2, 3, and 4) for the single-particle excitations in the junction (the expressions given in the literature are not sufficiently complete and at times incorrect), after which we find it possible to study a number of effects that occur in the electron (§§5 and 6) and phonon (§7) subsystems.

## §2. BASIC RELATIONS

We shall use the approach proposed by Éliashberg[1] for describing the kinetics of nonequilibrium superconductors. The matrix Green's function $\hat{g}_{\varepsilon\varepsilon-\omega}(\mathbf{p},\mathbf{k})$, integrated over the energy variable $\xi$, is determined by the dynamical equations

$$\begin{pmatrix} (\omega-\mathbf{vk})g & (2\varepsilon-\omega-\mathbf{vk})f \\ (2\varepsilon-\omega+\mathbf{vk})f^{+} & -(\omega+\mathbf{vk})\bar{g} \end{pmatrix} = \hat{H}_1\hat{g}-\hat{g}\hat{H}_1+\hat{I}, \tag{1}$$

$$\hat{I}=\hat{g}\hat{\Sigma}^{A}-\hat{\Sigma}^{R}\hat{g}+\hat{g}^{R}\hat{\Sigma}-\hat{\Sigma}\hat{g}^{A},$$

where

$$\hat{g}^{(R,A)}=\begin{pmatrix} g & f \\ -f^{+} & \bar{g} \end{pmatrix}^{(R,A)}, \quad \hat{\Sigma}^{(R,A)}=\begin{pmatrix} \Sigma_1 & \Sigma_2 \\ -\Sigma_2^{+} & \bar{\Sigma}_1 \end{pmatrix}^{(R,A)},$$

$$\hat{H}_1=\begin{pmatrix} H_1 & 0 \\ 0 & \bar{H}_1 \end{pmatrix}; \quad H_1=-\frac{e}{c}\mathbf{vA}+e\varphi; \tag{2}$$

$$\bar{H}_1=\frac{e}{c}\mathbf{vA}+e\varphi; \quad v=v_F,$$

the causal functions being found from the diagrammatic expansions in which the propagators and the energy eigenval-

 0038-5646/85/070089-09$04.00  

ues are either all retarded, of all advanced; we do not give these expansions here so as not to encumber the discussion. The matrix product in (1) should be understood as a contraction over the internal variables. Thus, in the coordinate representation ($1\equiv \mathbf{r}, t$)

$$(\hat{\Sigma}\hat{g})(1,2)\equiv\int\hat{\Sigma}(1,3)\hat{g}(3,2)\,d3.$$

The self-energy parts $\hat{\Sigma}^{(R,A)}$ in (1) are additive quantities:

$$\hat{\Sigma}=\hat{\Sigma}^{(imp)}+\hat{\Sigma}^{(e-ph)}+\hat{\Sigma}^{(e-e)}+\hat{\Sigma}^{(T)} \tag{3}$$

and according to (3), describe the interaction of the electrons with the impurities and the phonons and with each other, as well as tunneling. They will be specified below. Separating out the virtual electron-phonon processes in (1), and introducing the order parameter $\Delta = (\Sigma_2^R+\Sigma_2^A)^{(e-ph)}/2$, we obtain up to unimportant renormalizations the following expression for the 11-component of $\hat{I}_{\varepsilon\varepsilon-\omega}$:

$$I_{\varepsilon\varepsilon-\omega}=\{-f\Delta^{*}+\Delta f^{+}\}_{\varepsilon\varepsilon-\omega}+\{-i(g\gamma+\gamma g)+i(-f\delta^{+}+\delta f^{+})$$
$$+g^R\Sigma_1^{(e-ph)}-\Sigma_1^{(e-ph)}g^A-f^R\Sigma_2^{+(e-ph)}+\Sigma_2^{(e-ph)}f^{+A}\}_{\varepsilon\varepsilon-\omega}+I'_{\varepsilon\varepsilon-\omega}. \tag{4}$$

Here

$$2i\gamma_{\varepsilon\varepsilon-\omega}=(\Sigma_1{}^R-\Sigma_1{}^A)^{(e-ph)},\quad 2i\delta_{\varepsilon\varepsilon-\omega}=(\Sigma_2{}^R-\Sigma_2{}^A)^{(e-ph)}, \tag{5}$$

where $I'_{\varepsilon\varepsilon-\omega}$ no longer contains $\Sigma^{(e-ph)}$ explicitly. The dissipation functions $\gamma$, $\delta$, and $\Sigma_{1,2}^{(e-ph)}$ in (4) have characteristic values of the order of the electron decrement in energy terms (e.g., $\gamma\sim T^3/\omega_D^2$), and smaller in order of magnitude than the modulus of the parameter $\Delta$ in almost the entire temperature region of existence of the latter. This requires that we make exact allowance for the contribution of the expression in the first curly brackets in (4) before going over to the kinetic approximation. We find on the basis of the equations for the off-diagonal elements in (1) that

$$2(\varepsilon-\omega)(f-f^{+})_{\varepsilon\varepsilon-\omega}=\mathbf{vk}(f+f^{+})_{\varepsilon\varepsilon-\omega}+\{i(f\bar{\gamma}+\bar{\gamma}f^{+})-i(\gamma f+f^{+}\gamma)$$
$$+i(\delta^{+}g-g\delta)+i(\bar{g}\delta^{+}-\delta\bar{g})+(g\Delta-\Delta^{*}g)+(\bar{g}\Delta^{*}-\Delta\bar{g})$$
$$+f^R\bar{\Sigma}_1+f^{+R}\Sigma_2{}^{+}-\Sigma_2{}^{+}g^A\}_{\varepsilon\varepsilon-\omega}+I''_{\varepsilon\varepsilon-\omega}. \tag{6}$$

[Here we have used the procedure employed in the derivation of (4), and I" also does not contain $\Sigma^{(e-ph)}$ explicitly.]

Assuming that scattering by the impurities is the most rapid kinematic process, let us average over the angular variable in the expressions (1), (4), and (6); the self-energy parts corresponding to the interaction with the impurities then drop out from the diagonal (with respect to the energy variables) quantities $I'_\varepsilon$ and $I''_\varepsilon$, and we can go over directly to the derivation of the kinetic equations in the isotropic approximation. We first of all discuss the connection between the diagonal function $\hat{g}_\varepsilon$ and the nonequilibrium-electron distribution function.

Let us consider the normalization condition obtained in Ref. 2, which in our notation has the form

$$\check{g}_\varepsilon{}^2=-\pi^2\check{1},\quad \check{g}=\begin{pmatrix}\hat{g}^R & \hat{g}\\ \hat{0} & \hat{g}^A\end{pmatrix},\quad \check{1}=\begin{pmatrix}\hat{1} & \hat{0}\\ \hat{0} & \hat{1}\end{pmatrix}. \tag{7}$$

Using the fact that the condition (7) is identically fulfilled when we set

$$\hat{g}=\hat{g}^R\hat{a}-\hat{a}\hat{g}^A, \tag{8}$$

where $\hat{a}(\varepsilon)$ is an arbitrary (2×2) matrix function, which can be expanded in terms of the Pauli matrices, and the $\hat{g}^{R(A)}$ are defined by the relation (2), with

$$g_\varepsilon{}^{R(A)}=\frac{\varepsilon}{\Delta}f_\varepsilon{}^{R(A)}=i\pi\frac{\varepsilon}{\xi_\varepsilon^{R(A)}},$$
$$\xi_\varepsilon{}^R=-(\xi_\varepsilon{}^A)^{*}=\begin{cases}(\varepsilon^2-\Delta^2)^{1/2}\operatorname{sign}\varepsilon+i\delta, & \varepsilon^2>\Delta^2\\ i(\Delta^2-\varepsilon^2)^{1/2} & \varepsilon^2<\Delta^2\end{cases}, \tag{9}$$

we obtain

$$\begin{pmatrix} g & f\\ -f^{+} & \bar{g}\end{pmatrix}=f_1(\varepsilon)\begin{pmatrix} g^R-g^A & f^R-f^A\\ -(f^{+R}-f^{+A}) & \bar{g}^R-\bar{g}^A\end{pmatrix}+f_2(\varepsilon)$$
$$\times\begin{pmatrix} g^R-g^A & -(f^R+f^A)\\ -(f^{+R}+f^{+A}) & -(\bar{g}^R-\bar{g}^A)\end{pmatrix}, \tag{10}$$

where $f_1(\varepsilon)$ and $f_2(\varepsilon)$ are related to $a(\varepsilon)$. Using the properties

$$\bar{g}_\varepsilon=g_{-\varepsilon},\quad \bar{g}_\varepsilon^{R(A)}=-g_\varepsilon^{R(A)}, \tag{11}$$

which follow from (1), we find from (9) that

$$g_\varepsilon=f_1(\varepsilon)(g^R-g^A)_\varepsilon+f_2(\varepsilon)(g^R-g^A)_\varepsilon,$$
$$\bar{g}_\varepsilon=-f_1(\varepsilon)(g^R-g^A)_\varepsilon+f_2(\varepsilon)(g^R-g^A)_\varepsilon.$$

From this we can, bearing in mind (11), conclude that $f_1(\varepsilon)$ is an even function of $\varepsilon$, whereas $f_2(\varepsilon)$ should be an odd function. Thus, in the general case ($f_2\neq 0$) the $g_\varepsilon$ function has a part that is an even function of $\varepsilon$. This even—in energy terms—part is responsible for the electrochemical-potential shift $\delta\mu$ in the electron system under conditions of nonequilibrium, as can be seen from the expression

$$\delta\mu(\omega)=-\int_{-\infty}^{\infty}\frac{d\varepsilon}{4\pi i}\int\frac{dO_{\mathbf{p}}}{4\pi}g_{\varepsilon\varepsilon-\omega}(\mathbf{p},\mathbf{k}), \tag{12}$$

which is derived in Ref. 1 from the condition for electrical neutrality.

As follows from (10), the equality $f_\varepsilon=f_\varepsilon^{+}$ is satisfied when $f_2\propto\theta(\varepsilon^2-\Delta^2)$. Therefore, it is convenient to set

$$f_2(\varepsilon)=\varepsilon^{-1}(\varepsilon^2-\Delta^2)^{1/2}\theta(\varepsilon^2-\Delta^2)f_2'(\varepsilon)$$

(we shall hereinafter drop the prime). Introducing now the function $n_\varepsilon$ of arbitrary form, we can write

$$f_1(\varepsilon)=a_1(n_\varepsilon+n_{-\varepsilon}-1),\quad f_2(\varepsilon)=a_2(n_\varepsilon-n_{-\varepsilon}). \tag{13}$$

Since the function $n_\varepsilon$ is to be determined below, let us use the arbitrariness in the choice of the coefficients $a_{1,2}$ by defining them in such a way that the expressions for the $\hat{g}^{(R,A)}$ assume the form

$$\hat{g}_\varepsilon=(-2\pi i)\begin{pmatrix}u_\varepsilon\beta_\varepsilon+\alpha_\varepsilon & v_\varepsilon\beta_\varepsilon\\ -v_\varepsilon\beta_\varepsilon & -u_\varepsilon\beta_\varepsilon+\alpha_\varepsilon\end{pmatrix},$$
$$(\hat{g}^R-\hat{g}^A)_\varepsilon=2\pi i\begin{pmatrix}u_\varepsilon & v_\varepsilon\\ -v_\varepsilon & -u_\varepsilon\end{pmatrix},$$
$$u_\varepsilon=\frac{|\varepsilon|\theta(\varepsilon^2-\Delta^2)}{(\varepsilon^2-\Delta^2)^{1/2}},\quad v_\varepsilon=\frac{\Delta\operatorname{sign}\varepsilon\,\theta(\varepsilon^2-\Delta^2)}{(\varepsilon^2-\Delta^2)^{1/2}}. \tag{14}$$
$$\alpha_\varepsilon=(n_\varepsilon-n_{-\varepsilon})\theta(\varepsilon^2-\Delta^2)\operatorname{sign}\varepsilon,$$
$$\beta_\varepsilon=(n_\varepsilon+n_{-\varepsilon}-1)\theta(\varepsilon^2-\Delta^2)\operatorname{sign}\varepsilon.$$

With such a choice of the quantities, a choice which we can make without loss of generality, on the one hand we can obtain Éliashberg's expressions[1] by going over to the limit, and, on the other, the function $n_\varepsilon$ will have the same obvious meaning as the energy distribution function for the excitations in pure superconductors.

Proceeding from (14), we can arrive at the following formula:

$$u_\varepsilon \dot{n}_\varepsilon = -\frac{1}{8\pi i}\{(\dot{g}_\varepsilon - \dot{g}_{-\varepsilon}) + u_\varepsilon(\dot{g}_\varepsilon + \dot{g}_{-\varepsilon})\}\,\mathrm{sign}\,\varepsilon, \qquad (15)$$

where the dot denotes differentiation with respect to time. Thus, the right-hand side of (15) can be expressed in terms of the 11-element of (1), which, with allowance for the off-diagonal channel, contains the following effective collision integral:

$$I_{\mathrm{eff}}(\varepsilon) = I_{\mathrm{eff}}^{(e-ph)} + I_{\mathrm{eff}}^{(e-e)} + I_{\mathrm{eff}}^{(\mathrm{T})}, \qquad (16)$$

where the last two terms possess the structure

$$\begin{aligned} I_{\mathrm{eff}}(\varepsilon) &= I'(\varepsilon) - \frac{\Delta}{2\varepsilon} I''(\varepsilon) \\ &= g\Sigma_1{}^A - f\Sigma_2{}^{+A} - \Sigma_1{}^R g + \Sigma_2{}^R f^+ + g^R\Sigma_1 - f^R\Sigma_2{}^+ \\ &\quad - \Sigma_1 g^A + \Sigma_2 f^{+A} \\ &\quad - \frac{\Delta}{2\varepsilon}\{g\Sigma_2{}^A + f\bar{\Sigma}_1{}^A - \Sigma_1{}^R f - \Sigma_2{}^R \bar{g} + g^R\Sigma_2 + f^R\bar{\Sigma}_1 - \Sigma_1 f^A - \Sigma_2 \bar{g}^A \\ &\quad + f^+\Sigma_1{}^A + \bar{g}\Sigma_2{}^{+A} - \Sigma_2{}^{+R} g - \bar{\Sigma}_1{}^R f^+ + f^{+R}\Sigma_1 + \bar{g}^R\Sigma_2{}^+ - \Sigma_2{}^+ g^A - \bar{\Sigma}_1 f^{+A}\}. \end{aligned} \qquad (17)$$

A similar type of expression folows for the first term in (16); we do not give it here so as not to encumber the exposition.

## §3. TUNNELING SOURCE OF THE DEVIATION FROM EQUILIBRIUM

The self-energy parts corresponding to tunneling in a SiS′ junction are obtained in Ref. 3, and have, in our notation, the form

$$\hat{\Sigma}^{(\mathrm{T})(R,A)}(t, t') = (\nu/\pi)\,\hat{g}'^{(R,A)}(t, t'), \qquad (18)$$

where $\nu$ is the "tunneling frequency" connected with the conductivity of the tunneling junction and the function $\hat{g}'$ pertains to the injector superconductor $S'$. We shall assume that the superconductor $S$ under investigation is maintained at potential $V = 0$, while the injector is maintained at a time-independent potential of $V$ $(e = \hbar = 1)$. Then on account of gauge invariance, the presence of $V$ gives rise to phase factors in the $g$ functions; as a result we find

$$\Sigma_1^{(R,A)}(t_1 t_2) = (\nu/\pi)\,g'^{(R,A)}(t_1 t_2)\exp[-iV(t_1 - t_2)],$$

$$\bar{\Sigma}_1^{(R,A)}(t_1 t_2) = (\nu/\pi)\,\bar{g}'^{(R,A)}(t_1 t_2)\exp[iV(t_1 - t_2)],$$

$$\Sigma_2^{(R,A)}(t_1 t_2) = (\nu/\pi)\,f'^{(R,A)}(t_1 t_2)\exp[-iV(t_1 + t_2)],$$

$$\Sigma_2^{+(R,A)}(t_1 t_2) = (\nu/\pi)\,f^{+\prime(R,A)}(t_1 t_2)\exp[iV(t_1 + t_2)].$$

We must substitute these expressions into (17) and carry out a Fourier transformation with respect to the time, assuming that the temporal dependence is quasiclassical. Using the following rules that then arise (here $a = g^{(R,A)}$, $f^{(R,A)}$, etc., $\varphi = 2Vt$):

$$\begin{aligned} &a\Sigma_2 \to \Sigma_{2\varepsilon-V} a_\varepsilon \exp(-i\varphi); \quad \Sigma_2 a \to \Sigma_{2\varepsilon+V} a_\varepsilon \exp(-i\varphi), \\ &\Sigma_2{}^+ a \to \overset{+}{\Sigma}_{2\varepsilon-V} a_\varepsilon \exp(i\varphi); \quad a\Sigma_2{}^+ \to \overset{+}{\Sigma}_{2\varepsilon+V} a_\varepsilon \exp(i\varphi), \\ &\Sigma_1 a = a\Sigma_1 \to a_\varepsilon \Sigma_{1\varepsilon-V}; \quad \bar{\Sigma}_1 a = a\bar{\Sigma}_1 \to a_\varepsilon \bar{\Sigma}_{1\varepsilon-V}, \end{aligned} \qquad (19)$$

we find, going over to the distribution function $n_\varepsilon$, the tunneling source in the form

$$u_\varepsilon \dot{n}_\varepsilon = \frac{\nu}{2}[Q_1(n_{\pm\varepsilon})\sin\varphi + Q_2(n_{\pm\varepsilon})\cos\varphi + Q_3(n_{\pm\varepsilon})], \quad \varepsilon \geqslant \Delta, \qquad (20)$$

where the factors $Q_i$ are equal to

$$\begin{aligned} Q_1(n_{\pm\varepsilon}) &= v_\varepsilon w_{\varepsilon-V}(2n_{\pm\varepsilon} - 1)\theta(\Delta' - \varepsilon + V)\theta(\Delta' + \varepsilon - V) \\ &\quad - v_\varepsilon w_{\varepsilon+V}(2n_{\pm\varepsilon} - 1)\theta(\Delta' + \varepsilon + V)\theta(\Delta' - \varepsilon - V), \\ Q_2(n_{\pm\varepsilon}) &= v_\varepsilon v_{\varepsilon-V}[(n_{\pm\varepsilon} - n_{\varepsilon-V}) + (n_{\pm\varepsilon} - n_{-\varepsilon+V})]\theta(\varepsilon - V - \Delta') \\ &\quad - v_\varepsilon v_{\varepsilon+V}[(n_{\varepsilon+V} - n_{\pm\varepsilon}) + (n_{-\varepsilon-V} - n_{\pm\varepsilon})]\theta(\varepsilon + V - \Delta') \\ &\quad + v_\varepsilon v_{V-\varepsilon}[(1 - n_{\pm\varepsilon} - n_{V-\varepsilon}) + (1 - n_{\pm\varepsilon} - n_{-V+\varepsilon})]\theta(V - \varepsilon - \Delta'), \\ Q_3(n_{\pm\varepsilon}) &= [(n_{\varepsilon-V} - n_{\pm\varepsilon})(u_\varepsilon u_{\varepsilon-V} \pm u_{\varepsilon-V} + u_\varepsilon \pm 1) \\ &\quad + (n_{-\varepsilon+V} - n_{\pm\varepsilon})(u_\varepsilon u_{\varepsilon-V} \pm u_{\varepsilon-V} - u_\varepsilon \mp 1)]\theta(\varepsilon - V - \Delta') \\ &\quad - [(n_{\pm\varepsilon} - n_{\varepsilon+V})(u_\varepsilon u_{\varepsilon+V} \mp u_{\varepsilon+V} - u_\varepsilon \pm 1) \\ &\quad + (n_{\pm\varepsilon} - n_{-\varepsilon-V})(u_\varepsilon u_{\varepsilon+V} \mp u_{\varepsilon+V} + u_\varepsilon \mp 1)]\theta(\varepsilon + V - \Delta') \\ &\quad + [(1 - n_{\pm\varepsilon} - n_{V-\varepsilon})(u_\varepsilon u_{V-\varepsilon} \pm u_{V-\varepsilon} - u_\varepsilon \mp 1) \\ &\quad + (1 - n_{\pm\varepsilon} - n_{-V+\varepsilon})(u_\varepsilon u_{V-\varepsilon} \pm u_{V-\varepsilon} + u_\varepsilon \pm 1)]\theta(V - \varepsilon - \Delta'), \end{aligned} \qquad (21)$$

with $w_\varepsilon = \Delta\theta(\Delta^2 - \varepsilon^2)/(\Delta^2 - \varepsilon^2)^{1/2}$. Let us emphasize that we have chosen the quantity $\varepsilon$ in the expressions (20) and (21) to be positive definite, and that, with this choice, $n_\varepsilon$ is the distribution function for the electronlike excitations, $n_{-\varepsilon}$ is the distribution function for the holelike excitations, and all the functions with shifted arguments pertain to the superconductor $S'$. Let us enumerate some properties of the source (20) and the consequences that follow from them.

1. In the $\Delta' = 0$ limiting case the source (20) goes over into the expressions obtained for the NiS junction in the equilibrium[5,3] and nonequilibrium[6] approximations. Typical of such a source is the property

$$Q(n_\varepsilon) \neq Q(n_{-\varepsilon}), \qquad (22)$$

as a result of which the nonequilibrium $n_\varepsilon$ function exhibits a branch imbalance, i.e.,$n_\varepsilon \neq n_{-\varepsilon}$, and there arises a chemical-potential shift, which, according to (12) and (14), is equal to

$$\delta\mu = \int_\Delta^\infty (n_\varepsilon - n_{-\varepsilon})\,d\varepsilon. \qquad (23)$$

2. The property (22) is maintained in the $\Delta' \neq 0$ case for both the equilibrium and nonequilibrium sources, this being the case even for the symmetric SiS junction. At the same time this property is not present in the source obtained by Kirichenko *et al.*,[7] which has figured in a number of papers (see Elesin and Kopaev's review paper[8]). As follows from (23), the property (22) can be directly established in experiment.

3. A nonequilibrium junction between superconductors is characterized by another interesting property, namely, the quantum oscillations of the excitation source in time—with the Josephson frequency in the case of a small deviation from

equilibrium and with frequencies that are multiples of the Josephson frequency in the case of a strong deviation from equilibrium. As a result, the chemical-potential shift (23) should also oscillate in time (for more details, see Ref. 9).

4. A large-scale effect can occur in the case of scattering by a junction of a high-frequency external field (electromagnetic or acoustic). Since the oscillating terms in (20) are nonzero even in the equilibrium approximation, the already linearized correction to the excitation distribution function undergoes oscillation with the Josephson frequency, as a result of which the single-particle excitation density oscillates, and satellites should appear in the radiation scattered by the junction (for greater details, see Ref. 10). Let us note that, if in the case of scattering of electromagnetic waves we can attempt to qualitatively explain the presence of the satellites as also being due to the presence of a variable Josephson current in the junction, in the case of scattering of acoustic waves the occurrence of satellites is unambiguously connected with the single-particle excitation density oscillations.

5. The presence in (20) of oscillations connected with the macroscopic phase coherence in the superconductors deserves a special comment. It is usually assumed that, in the case of single-particle excitations, whose behavior is described by the kinetic equation, the role of the off-diagonal long-range order in the superconductors amounts to the appearance of coherence factors (in the transition matrix elements connecting one state with another) and the appearance of a singularity in the electron level density. As shown above, direct dependence of the excitation distribution function on the coherent phase difference can also occur in nonequilibrium superconductors. Of interest in this connection are experiments that would allow the detection of the effects enumerated in Subsections. 3 and 4 (a more detailed exposition can be found in Refs. 9–11).

## §4. CANONICAL COLLISION INTEGRALS

Using the expressions found in Ref. 1 for the self-energy parts, we can derive the inelastic collision integrals.

### 1. The inelastic electron-electron collisions

In this case the self-energy parts have the form

$$\begin{aligned}\Sigma_1^{(R,A)}&=L[A\{g_1g_2\bar{g}_3\}^{(R,A)}-B\{f_1f_2^+g_3\}^{(R,A)}],\\ \Sigma_2^{(R,A)}&=L[B\{g_1\bar{g}_2f_3\}^{(R,A)}-A\{f_1f_2f_3^+\}^{(R,A)}],\end{aligned}\tag{24}$$

where the operator $L$ is defined as

$$L=\left(\frac{mp_F}{2\pi^2}\right)^2\frac{1}{2\varepsilon_F}\iint_{-\infty}^{\infty}\frac{d\varepsilon_1\,d\varepsilon_2}{(4\pi i)^2}\iint\frac{dO_{\mathbf{p}_1}\,dO_{\mathbf{p}_2}}{(4\pi)^2}\delta\left(\frac{p_2}{p_F}-1\right),\tag{25}$$

the curly brackets have the structure

$$\begin{aligned}\{g_1g_2g_3\}^{R(A)}&\\ =g_1g_2g_3^{R(A)}&+g_1g_2^{R(A)}g_3+g_1^{R(A)}g_2g_3+g_1^{R(A)}g_2^{R(A)}g_3^{R(A)}\\ -g_1^{R(A)}g_2^{R(A)}g_3^{A(R)}&-g_1^{R(A)}g_2^{A(R)}g_3^{R(A)}-g_1^{A(R)}g_2^{R(A)}g_3^{R(A)},\\ \{g_1g_2g_3\}&=g_1g_2g_3+g_1(g_2^R-g_2^A)(g_3^R-g_3^A)\\ &+(g_1^R-g_1^A)g_2(g_3^R-g_3^A)+(g_1^R-g_1^A)(g_2^R-g_2^A)g_3,\end{aligned}$$

and the quantities $A$ and $B$ are connected with the scattering amplitudes for the normal excitations at the Fermi surface (their explicit form in the Born approximation is given in Ref. 1). Using (14), (15), (17), (24), and (25), we find after lengthy but straightforward transformations that

$$\begin{aligned}J^{(e-e)}(n_{\pm\varepsilon})=\frac{1}{16\varepsilon_F}\iiint_{\Delta}^{\infty}&\{E_1\delta(\varepsilon-\varepsilon_1-\varepsilon_2-\varepsilon_3)\\ &+3E_2\delta(\varepsilon+\varepsilon_1-\varepsilon_2-\varepsilon_3)+3E_3\delta(\varepsilon+\varepsilon_2+\varepsilon_3-\varepsilon_1)\}d\varepsilon_1\,d\varepsilon_2\,d\varepsilon_3,\end{aligned}$$

where the $E_i(\pm\varepsilon)$ have the form ($i=1, 2, 3$)

$$\begin{aligned}E_i=&M_i^1\alpha_i(\varepsilon_1,\varepsilon_2,\varepsilon_3)+M_i^2\alpha_i(-\varepsilon_1,\varepsilon_2,\varepsilon_3)\\ &+M_i^3\alpha_i(\varepsilon_1,-\varepsilon_2,\varepsilon_3)+M_i^4\alpha_i(\varepsilon_1,-\varepsilon_2,\varepsilon_3),\end{aligned}\tag{26}$$

with

$$\begin{aligned}\alpha_1\equiv&(1-n_{\pm\varepsilon})n_{\varepsilon_1}n_{\varepsilon_2}n_{\varepsilon_3}-n_{\pm\varepsilon}(1-n_{\varepsilon_1})(1-n_{\varepsilon_2})(1-n_{\varepsilon_3})\\ &+(1-n_{\pm\varepsilon})n_{\varepsilon_1}n_{-\varepsilon_2}n_{-\varepsilon_3}-n_{\pm\varepsilon}(1-n_{\varepsilon_1})(1-n_{-\varepsilon_2})(1-n_{-\varepsilon_3}),\\ \alpha_2=&\alpha_1(n_{\varepsilon_1}\to 1-n_{\varepsilon_1}),\quad \alpha_3=\alpha_1(n_{\varepsilon_2}\to 1-n_{\varepsilon_2},\ n_{\varepsilon_3}\to 1-n_{\varepsilon_3}).\end{aligned}$$

The coefficients $M_j^i(\pm\varepsilon)$ are given by the following expressions:

$$\begin{aligned}M_1^1=&a(uu_1u_2u_3-vv_1v_2v_3-uu_1\pm u_2u_3\mp 1)\\ &+b(uv_1v_2u_3-vu_1u_2v_3+vv_1\pm v_2v_3),\\ M_1^2=&a(uu_1u_2u_3+vv_1v_2v_3-uu_1\mp u_2u_3\pm 1)\\ &+b(uv_1v_2u_3-vu_1u_2v_3+vv_1\mp v_2v_3),\\ M_1^3=&a(uu_1u_2u_3-vv_1v_2v_3+uu_1\pm u_2u_3\pm 1)\\ &+b(uv_1v_2u_3-vu_1u_2v_3-vv_1\pm v_2v_3),\\ M_1^4=&a(uu_1u_2u_3-vv_1v_2v_3+uu_1\mp u_2u_3\mp 1)\\ &+b(uv_1v_2u_3-vu_1u_2v_3-vv_1\mp v_2v_3);\\ M_2^i=&-M_1^i(-\varepsilon_1);\quad M_3^i=M_1^i(-\varepsilon_2,-\varepsilon_3).\end{aligned}\tag{27}$$

The numbers $a$ and $b$ figuring in (27) are connected with $A$ and $B$ by the relation

$$a(b)=-2\pi\left(\frac{mp_F}{2\pi^2}\right)^2\iiint\frac{dO_{\mathbf{p}}\,dO_{\mathbf{p}_1}\,dO_{\mathbf{p}_2}}{(4\pi)^2}\delta\times\left(\frac{|\mathbf{p}-\mathbf{p}_1-\mathbf{p}_2|}{p_F}-1\right)A(B).$$

The elementary events described by (26) have quite an obvious meaning. Thus, in the addend proportional to $M_1^1$ in the expression for $E_1$ the first term describes, in the case when $\varepsilon$ has the positive sign, the process of fusion of three electronlike excitations into one excitation of the same type. In the case when $\varepsilon$ has the negative sign three electronlike excitations produce, on fusing, an excitation on the hole branch. As a result, the difference between the numbers of electrons and holes changes by two units in the first case and four units in the second. Similar processes involving a change in the difference in the excitation numbers are described by the other terms in the addend in question, which is why it vanishes in the normal metal ($M_1^1\equiv 0$ for $u_i=1$ and $v_i=0$). Such a clear representation of the collision integral is a consequence of our choice of the form of the function $n_\varepsilon$.

### 2. Collisions of the electrons with the phonons

The electron-phonon collision operator is also reduced to the canonical form when $n_\varepsilon$ is chosen by means of the relations (14). Let us immediately give here the result for this operator (it is obtained in the particle representation in

Ref. 6):

$$J^{(e-ph)}(n_{\pm\varepsilon})=\frac{\pi\lambda}{4(up_F)^2}\int_0^\infty \omega^2 d\omega \int_\Delta^\infty d\varepsilon'$$

$$\times[\Phi_1\delta(\varepsilon'-\varepsilon-\omega)+\Phi_2\delta(\varepsilon-\varepsilon'-\omega)+\Phi_3\delta(\varepsilon+\varepsilon'-\omega)],$$

$$\Phi_1=(u_\varepsilon u_{\varepsilon'}-v_\varepsilon v_{\varepsilon'}\pm1)[n_{\varepsilon'}(1-n_{\pm\varepsilon})(1+N_\omega)-n_{\pm\varepsilon}(1-n_{\varepsilon'})N_\omega]$$
$$+(u_\varepsilon u_{\varepsilon'}-v_\varepsilon v_{\varepsilon'}\mp1)[n_{-\varepsilon'}(1-n_{\pm\varepsilon})(1+N_\omega)-n_{\pm\varepsilon}(1-n_{-\varepsilon'})N_\omega], \quad (28)$$

$$\Phi_2=(u_\varepsilon u_{\varepsilon'}-v_\varepsilon v_{\varepsilon'}\pm1)[n_{\varepsilon'}(1-n_{\pm\varepsilon})N_\omega-n_{\pm\varepsilon}(1-n_{\varepsilon'})(1+N_\omega)]$$
$$+(u_\varepsilon u_{\varepsilon'}-v_\varepsilon v_{\varepsilon'}\mp1)[n_{-\varepsilon'}(1-n_{\pm\varepsilon})N_\omega-n_{\pm\varepsilon}(1-n_{-\varepsilon'})(1+N_\omega)],$$
$$\Phi_3=(u_\varepsilon u_{\varepsilon'}+v_\varepsilon v_{\varepsilon'}\mp1)[(1-n_{\pm\varepsilon})(1-n_{\varepsilon'})N_\omega-n_{\pm\varepsilon}n_{\varepsilon'}(1+N_\omega)]$$
$$+(u_\varepsilon u_{\varepsilon'}+v_\varepsilon v_{\varepsilon'}\pm1)[(1-n_{\pm\varepsilon})(1-n_{-\varepsilon'})N_\omega-n_{\pm\varepsilon}n_{-\varepsilon'}(1+N_\omega)],$$

where $N_\omega$ is the phonon distribution function and $\lambda$ is the dimensionless electron-phonon interaction constant.

### 3 Collisions of the phonons with the electrons

We shall also need the Phonon-electron collision operator. Using the general relations given in Ref. 12 and the expressions (14), we can obtain

$$J^{(ph-e)}(N_{\omega_q})=\frac{\pi\lambda}{8}\frac{\omega_D}{\varepsilon_F}\iint_\Delta^\infty d\varepsilon\, d\varepsilon'\{\delta(\varepsilon+\varepsilon'-\omega_q)T_1$$
$$+2\delta(\varepsilon-\varepsilon'-\omega_q)T_2\},$$

$$T_1=(u_\varepsilon u_{\varepsilon'}+v_\varepsilon v_{\varepsilon'}+1)\{[(N_{\omega_q}+1)n_\varepsilon n_{-\varepsilon'}-N_{\omega_q}(1-n_\varepsilon)(1-n_{-\varepsilon'})]$$
$$+[(N_{\omega_q}+1)n_{-\varepsilon}n_{\varepsilon'}+N_{\omega_q}(1-n_{-\varepsilon})(1-n_{\varepsilon'})]\}+(u_\varepsilon u_{\varepsilon'}+v_\varepsilon v_{\varepsilon'}-1)$$
$$\times\{[(N_{\omega_q}+1)n_\varepsilon n_{\varepsilon'}-N_{\omega_q}(1-n_\varepsilon)(1-n_{\varepsilon'})]+[(N_{\omega_q}+1)n_{-\varepsilon}n_{-\varepsilon'}$$
$$-N_{\omega_q}(1-n_{-\varepsilon})(1-n_{-\varepsilon'})]\}, \quad (29)$$
$$T_2=(u_\varepsilon u_{\varepsilon'}-v_\varepsilon v_{\varepsilon'}-1)\{[(N_{\omega_q}+1)n_\varepsilon(1-n_{-\varepsilon'})$$
$$-N_{\omega_q}(1-n_\varepsilon)n_{-\varepsilon'}]+[(N_{\omega_q}+1)n_{-\varepsilon}(1-n_{\varepsilon'})-N_{\omega_q}(1-n_{-\varepsilon})n_{\varepsilon'}]\}$$
$$+(u_\varepsilon u_{\varepsilon'}-v_\varepsilon v_{\varepsilon'}+1)\{[(N_{\omega_q}+1)n_\varepsilon(1-n_{\varepsilon'})-N_{\omega_q}(1-n_\varepsilon)n_{\varepsilon'}]$$
$$+[(N_{\omega_q}+1)n_{-\varepsilon}(1-n_{-\varepsilon'})-N_{\omega_q}(1-n_{-\varepsilon})n_{\varepsilon'}]\}.$$

The operators (26), (28), and (29) generalize the canonical form, obtained in Refs. 1 and 12, of the collision integrals to the case in which there is an imbalance in the populations of the electron-hole excitation branches in a nonequilibrium superconductor.

## §5. NONEQUILIBRIUM TUNNELING CURRENT

### 1. The nonequilibrium Josephson effect

The expression for the tunneling current in an equilibrium superconducting junction was derived by Josephson [13] (as well as by a number of authors in subsequent papers, in particular, Refs. 14–18), and has the form

$$j=j_0\sin\varphi+j_1\cos\varphi+j_{qp}. \quad (30)$$

Let us derive the expressions for the $j$ amplitudes of this current in the nonequilibrium case. Integrating the 11-component of Eq. (1) over $\varepsilon$, averaging over the angle variable, and integrating the resulting divergence of the total tunneling current over the volume $\mathscr{V}$ of the electrode, we find, using (14), that

$$j_0=-k\int_{-\infty}^{\infty}d\varepsilon[v_\varepsilon w_{\varepsilon+V}(1-n_\varepsilon-n_{-\varepsilon})\operatorname{sign}\varepsilon$$
$$+v_{\varepsilon+V}w_\varepsilon(1-n_{\varepsilon+V}-n_{-\varepsilon+V})\operatorname{sign}(\varepsilon+V)],$$
$$j_1=k\int_{-\infty}^{\infty}d\varepsilon v_\varepsilon v'_{\varepsilon+V}[(1-n_\varepsilon-n_{-\varepsilon})\operatorname{sign}\varepsilon$$
$$-(1-n_{\varepsilon+V}-n_{-\varepsilon-V})\operatorname{sign}(\varepsilon+V)], \quad (31)$$
$$j_{qp}=k\int_{-\infty}^{\infty}d\varepsilon\{u_\varepsilon u'_{\varepsilon+V}[(1-n_\varepsilon-n_{-\varepsilon})\operatorname{sign}\varepsilon$$
$$-(1-n_{\varepsilon+V}-n_{-\varepsilon-V})\operatorname{sign}(\varepsilon+V)]$$
$$+u_{\varepsilon+V}\theta(\varepsilon^2-\Delta^2)(n_\varepsilon-n_{-\varepsilon})\operatorname{sign}\varepsilon-u_\varepsilon\theta[(\varepsilon+V)^2-\Delta'^2]$$
$$\times(n_{\varepsilon-V}-n_{-\varepsilon-V})\operatorname{sign}(\varepsilon+V)\},$$

where $k=\nu\mathscr{V}mp_Fe/\pi^2$. In the equilibrium approximation these amplitudes go over into the well-known expressions obtained in Refs. 14–18; in the case $\Delta'=0$, into Bulyzhenkov and Ivlev's expression for the nonequilibrium tunneling current in a NiS junction.[6] Using the fact that (30), (31) should go over to Ohm's law at $T>T_c$, we find that $k=1/2eR$ and, consequently,

$$\nu=1/4e^2N(0)\mathscr{V}R,$$

where $R$ is the resistance of the junction in the normal state and $N(0)=mp_F/2\pi^2$.

The nonequilibrium amplitudes (31) allow us to describe a number of interesting effects. We shall show below that allowance for the deviation from equilibrium affects, in particular, the so-called sign paradox of the "interference" conductivity of the Josephson junction.

### 2. On the sign of the cos $\varphi$-term in the Josephson current

Let us recall that in (30) $j_0\sin\varphi$ is the nondissipative pair supercurrent, $j_{qp}$ is the dissipative normal-excitation current, and $j_1\cos\varphi$ is the dissipative "interference" term, which depends on the phase difference $\varphi$ across the junction. The presence of the $\varphi$-dependent dissipative part leads to a situation in which the $Q$-factor of the Josephson "plasma" oscillations[19] in the junction is phase dependent. This has allowed the presence in the Josephson current (30) of the second term to be experimentally established,[20] but, contrary to the prediction of the equilibrium microtheory,[13,18] $j_1$ turned out in the experiment to be opposite in sign to $j_{qp}$. On the one hand, the numerous experiments that have subsequently been performed have shown that this discrepancy is characteristic of almost all weakly coupled junctions (see, for example, Refs. 21–25), and, on the other, detailed investigations[26,27] have established a strong dependence of the interference conductivity on temperature: the conductivity changes sign in a very narrow temperature region, becoming positive in the vicinity of $T_c$

The attempts that have been made to date to explain this phenomenon (see, for example, Refs. 28 and 29) leave the question of the nature of the effect open. Hida and Ono's explanation[29] of the data of Pedersen *et al.*,[20] on the basis of the time-dependent Ginzburg-Landau equations is open to question mainly because of the subsequently established strong temperature dependence and reversal of the sign of the interference conductivity. The problem in question has therefore been considered "one of the most contradictory consequences of tunneling theory."[27]

Let us find out what the occurrence of a state of nonequilibrium in the junction leads to. In the experiments reported in Refs. 26 and 27 the constant voltage potential across the junction was equal to zero ($j<j_0$), but the junction was located in a microwave electromagnetic field. Let us therefore go over in (31) to the limit $V\to 0$, and let us determine the equilibrium function $n_\varepsilon$ from the kinetic equation in the presence of an external high-frequency field. Under these conditions we can neglect the branch imbalance and set $n\varepsilon = n_{-\varepsilon}$ in (31). As a result, from (30) and (31), we obtain for the conductivities $[\sigma_i = \lim_{V\to 0}(j_i/V)]$ the expressions (in units of $2/R$)

$$\sigma_1=-\Delta^2\int_\Delta^\infty \frac{\partial n_\varepsilon}{\partial\varepsilon}\frac{d\varepsilon}{\varepsilon^2-\Delta^2},\quad \sigma_{qp}=-\int_\Delta^\infty \frac{dn_\varepsilon}{\partial\varepsilon}\frac{\varepsilon^2\,d\varepsilon}{\varepsilon^2-\Delta^2} \qquad (32)$$

(the junction is, for simplicity, assumed to be symmetric). The fact that the conductivities are functionals of the derivatives $\partial n_\varepsilon/\partial\varepsilon$ makes them very sensitive to the form of $n_\varepsilon$. This is enhanced by the presence of the resonance denominators in (32). From (32) we obtain, after simple transformations, the general relation

$$\sigma_{qp}=n_{\varepsilon=\Delta}+\sigma_1. \qquad (33)$$

Since near $T_c$ $n_{\varepsilon+\Delta}\approx 1/2$ and $\sigma_1$ is small $[\sigma_1(T_c)=0]$, it follows from (33) that the $\sigma_{qp}$ values are shifted as a whole into the region of positive values, and can become negative only under conditions of strong deviation from equilibrium.[1)] It remains to investigate the behavior of $\sigma_1(T)$. In the experiments reported in Refs. 20, 26, and 27 the microwave-field frequency $\omega_0$ was comparable to the Josephson plasma frequency and exceeded the characteristic damping rate (in energy terms) $\gamma$ of the quasiparticles (we assume $\gamma$ to be a constant), where the linear response of the junction was analyzed.

Therefore, it is sufficient to use here the linearized solution of the electron kinetic equation.[32] As a result the nonequilibrium correction to the distribution function can be written as

$$n_\varepsilon^{(1)}\approx\frac{\alpha}{\gamma}\frac{\omega_0}{T}\left(\frac{\Delta}{2}\right)^{1/2}\left[\frac{\theta(\varepsilon-\omega_0-\Delta)}{(\varepsilon-\omega_0-\Delta)^{1/2}}-\frac{1}{(\varepsilon+\omega_0-\Delta)^{1/2}}\right], \qquad (34)$$

where $\alpha=(e/c)^2D\,\mathbf{A}_{\omega0}\mathbf{A}_{\omega0}$ is the strength of the action of the field on the electrons, $D$ is the coefficient of diffusion of the electrons, and $\mathbf{A}_{\omega0}$ is the vector potential of the field, which is assumed to be monochromatic. Setting $n_\varepsilon = n_\varepsilon^{(0)} + n_\varepsilon^{(1)}$ ($n_\varepsilon^{(0)}$ is the Fermi distribution function for the electrons) in the expression (32) for $\sigma_1$ gives rise to divergent integrals for both the equilibrium and nonequilibrium parts of the conductivity, for the obvious reason that the attenuation is not taken into account. Smearing out the singularity in the density of single-particle states by virtue of

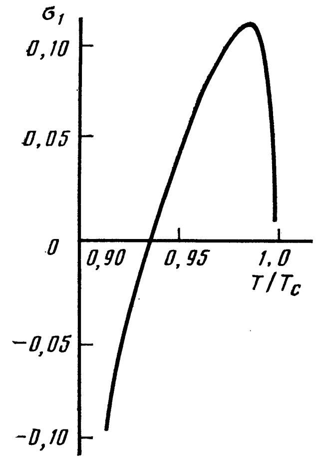


FIG. 1. Temperature dependence of the interference conductivity $\sigma_1$, (35) [in units of $\frac{1}{8}\ln(2\Delta/\gamma)$; we have set $\alpha^2/\gamma^2=2\omega_0/\Delta$].

the damping rate $\gamma$, we find

$$\sigma_1(T)\approx\frac{\Delta}{T}\left[1-\left(\frac{2\Delta}{\omega_0}\right)^{1/2}\frac{\alpha}{\gamma}\right]\frac{1}{8}\ln\frac{2\Delta}{\gamma}. \qquad (35)$$

The resulting dependence (35) is depicted in Fig. 1.

Thus, the Éliashberg mechanism,[32] by redistributing the equilibrium excitations in energy space, leads to a situation in which the derivative $\partial n_\varepsilon/\partial\varepsilon$ oscillates between positive and negative, and there occurs as a result a change in sign of the interference conductivity as the temperature is varied under conditions of fixed (and small) external-field intensity. (The quasiparticle conductivity behaves similarly in the process, but it does not change sign.) This conclusion is in accord with the experimentally observed picture.[26,27] From (35) we can find the temperature $T_0$ at which $\sigma_1(T)$ changes sign. Assuming that $\alpha^2/\gamma^2>\omega_0/2\Delta$ (otherwise the expression (35) does not change sign), we obtain

$$(T_c-T_0)/T_c\sim(\omega_0/2\tilde{\Delta})^2(\gamma/\alpha)^4 \qquad (36)$$

(here we have used the relation $\Delta=\tilde{\Delta}(1-T/T_c)^{1/2}$ for $T\sim T_c$, with $\tilde{\Delta}\approx 3.2T_c$). The strong dependence of $T_0$ on the external-field intensity $\alpha\omega_0^2$ is rather unexpected. References 26 and 27 contain no data for the direct determination of a dependence of the type (36), but the large spread in the experimental data does not exclude the existence of such a dependence. In this connection, the measurement of $T_0(\alpha)$ would be desirable, since it would help us to better understand the nature of the "interference" conductivity paradox.[2)]

## §6. KINETICS OF THE ELECTRONLIKE EXCITATIONS

Let us now consider the steady-state solutions to the kinetic equation

$$0=u_\varepsilon\dot{n}_\varepsilon=J^{(e-ph)}(n_\varepsilon)+Q^{\mathrm{T}}(n_\varepsilon) \qquad (37)$$

in the case when the voltage potential across the junction

$$V\sim 2\Delta, \qquad (38)$$

limiting ourselves to the case of the symmetric junction,

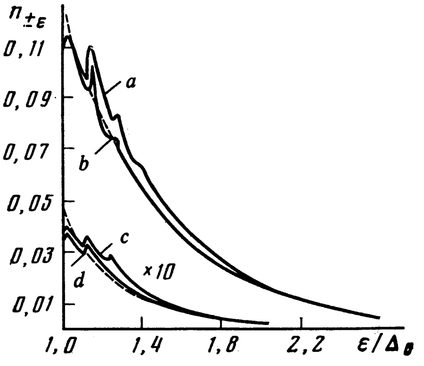


FIG. 2. Distribution functions for the electronlike $n_{+\varepsilon}$ (upper curves) and holelike $n_{-\varepsilon}$ (lower curves) excitations in the pre-threshold regime at different temperatures: $a$) and $b$) at $T = 0.4\,\Delta_0$; $c$) and $d$) at $T = 0.3\,\Delta_0$. In all cases $\nu/\gamma = 0.1$; $V/\Delta_0 = 0.1$. The dashed curve is a plot of the Fermi distribution; $\Delta_0 = \Delta(T=0,\ V=0)$.

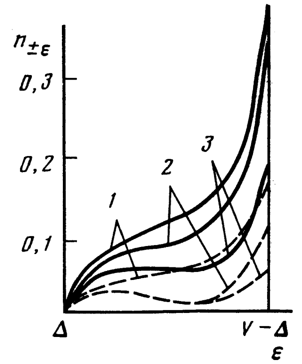


FIG. 4. Distribution functions for the electronlike (continuous curves) and holelike (dashed curves) excitations in the post-threshold regime at temperature $T = 0.1\,\Delta_0$; $\nu = 0.001\,\gamma$. The applied voltage potential is equal to: 1) $V = 1.9\,\Delta_0$; 2) $V = 2.01\,\Delta_0$; 3) $V = 2.5\,\Delta_0$. The degree of imbalance increases as the voltage potential is increased from the threshold value ($V = 2\,\Delta \approx 1.84\,\Delta_0$). The equilibrium-excitation distribution is not shown: under the present conditions the Fermi function is exponentially small. Also not shown is the small "tail" of the nonequilibrium electronlike-excitation distribution function in the region $\varepsilon > V - \Delta$.

when we can set in (20) $n_\varepsilon = n'_{-\varepsilon}$ and $n_{-\varepsilon} = n'_\varepsilon$, which simplifies the analysis.

In almost the entire temperature range from 0 to $T_c$, with the exception of a very narrow region around the transition point, the times corresponding to (38) are small compared to the times characterizing the single-particle excitation kinetics. Therefore, for the voltage potentials (38) the effects connected with the coherent phase difference in the excitation system are negligible (cf. §3). Nevertheless, the region of voltage potentials (38) is quite interesting, since the applied constant field is capable of breaking up the pairs in the course of the tunneling.

We numerically solved Eq. (37) with allowance for (21), (28), and the self-consistency equation for the order parameter Δ, which has the form

$$1=\lambda\int\limits_{\Delta}^{\omega_D}\frac{1-n_\varepsilon-n_{-\varepsilon}}{(\varepsilon^2-\Delta^2)^{1/2}}\,d\varepsilon, \tag{39}$$

(the computational procedure is described in Ref. 11). The nonintegrable singularities in $Q^T(\varepsilon)$ and $J(\varepsilon)$ were cut off by allowing for the damping $\gamma$ in the density of single-particle states.

The behavior of the excitation distribution function is illustrated in Figs. 2 and 3. Let us first of all point out the existence of an imbalance between the electronlike ($n_\varepsilon$) and holelike ($n_{-\varepsilon}$) excitations, which occurs in both regimes below ($V<2\Delta$) and above ($V>2\Delta$) the threshold. In the below-threshold case the excitation distribution function exhibits "spikes," and, furthermore, tends to undergo a global shift into the region of higher energies. This, in the final analysis, stimulates the superconductivity, and produces the phonon-deficit effect (an effect which is similar to what occurs under the action of a microwave field[32,34]). As the temperature is decreased, the relative degree of nonequilibrium increases, and an ever-increasing number of peaks separated by distances $V$ in $\varepsilon$ space appear on the distribution function (Fig. 3). In the region of very low $T$, where the thermal smearing is negligible, the distribution function acquires a "saw-toothed" character, and is different from zero in the region where the equilibrium Fermi function is negligible. Notice that the number of significant spikes increases as the voltage potential decreases (cf Figs. 2 and 3).

Beyond the threshold regime almost the entire bulk of the excess excitations is concentrated in the region above the gap (see Fig. 4), the "tail" adjoining this region being ex-

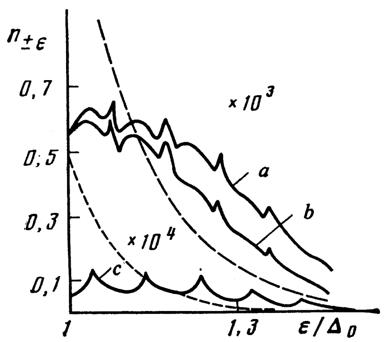


FIG. 3. Same as in Fig. 2, but for a different set of temperatures: $a$) and $b$) $T = 0.15\,\Delta_0$; $c$) $T = 0.1\,\Delta_0$ (in the last case the behavior of the electronlike excitation branch is practically the same as that of the holelike excitation branch).

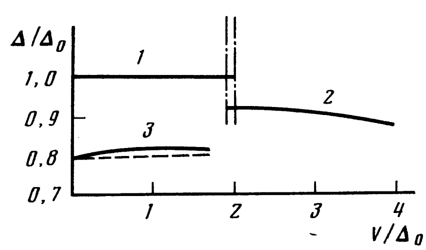


FIG. 5. Dependence of the nonequilibrium gap Δ on the voltage potential $V$. For the curves 1 and 2 $T/\Delta_0 = 0.1$ (the same curves are obtained at $T/\Delta_0 = 0.2$) and $\nu = 0.01\,\gamma$; for the curve 3 $T = 0.4\,\Delta_0$ and $\nu/\gamma = 0.1$; the dashed line indicates the equilibrium value.

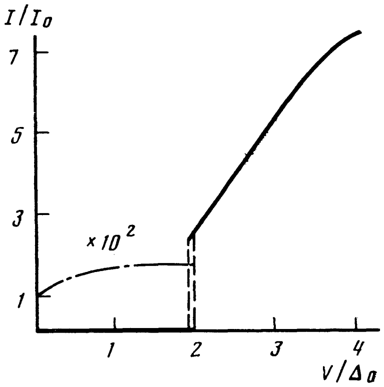


FIG. 6. The current-voltage characteristic with hysteresis ($T = 0.2\ \Delta_0$; $\nu = 0.01\ \gamma$).

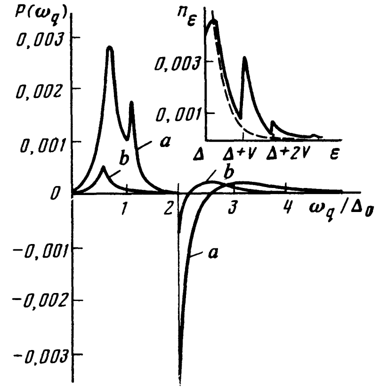


FIG. 8. Spectral dependence of the phonon emission in the pre-threshold regime: *a*) at $T = 0.2\ \Delta_0$, $V = 0.5\ \Delta_0$, and $\nu = 0.2\ \gamma$; *b*) same as in *a*), but with $V = 0.1\ \Delta_0$. The inset shows the electronlike-excitation distribution function corresponding to the case *a*): $P(\omega_q) = \omega_q^2\, J(N^0_{\omega q})(\pi\lambda\omega_D^2)/8\varepsilon_F\,\Delta_0^3)^{-1}$.

tremely small. Therefore, we can speak of a "quasilocalized" nature of the nonequilibrium quasiparticle distribution. This localization, which does not disappear when the thermal smearing is taken into account, leads to a remarkable manifestation of the phonon deficit effect (see §7) in superconductors with excess quasiparticles.

Let us note two characteristics of the behavior of the nonequilibrium gap $\Delta$. First, there exists a voltage-potential region where below-threshold and above-threshold values coexist (cf. curves 1 and 2 in Fig. 5). This leads to hysteresis in the current-voltage characteristics (Fig. 6), as well as in the dependence of the nonequilibrium chemical-potential shift on the applied voltage potential (Fig. 7). Secondly, in the region $T \sim T_c$ the curve $\Delta(V)$ rises slightly as the voltage V is increased, i.e., the superconductivity is stimulated. Compared to the case of the microwave field, when the stimulation was considerable,[35] and the "heating up" and other $\Delta$-suppressing factors of the electromagnetic influence had to be taken into account in order to achieve agreement with the experimental data,[36] the tunneling mechanism of extraction is quite weak (this is due to the smallness of the "tunneling frequency" $\nu$ in comparison with the decrement $\gamma$), and the stimulation does not exceed one percent of the values even when we allow for the "heating up" and other processes that lead to the suppression of the gap.

## §7. SPECTRUM OF THE PHONON RADIATION

Within the framework of the above-employed approach to the kinetics of the nonequilibrium junction we can use the scheme developed in Refs. 12 and 36 for the computation of the phonon radiation. The number of phonons emitted per unit time from a film of the junction in the spectral interval $d\omega_q$ is equal to

$$d\dot{N}_{\omega_q} = J(N_{\omega_q}{}^0)\,\rho(\omega_q)\,d\omega_q, \tag{40}$$

where $\rho(\omega_q) = \mathscr{V}\omega_q^2/2\pi^2u^3$ and $J(N^0_{\omega q})$ is the operator (29) in which $N^0_{\omega q}$ is the equilibrium Bose distribution function for the phonons at the thermostat temperature. The fact that the density of electronlike-excitation states enters into (29) twice makes the phonon radiation (40) very sensitive to the form of the electron distribution.

Figure 8 shows the phonon-radiation spectrum occurring below the threshold. The dip in the spectral dependence at $\omega_q \geqslant 2\Delta$ indicates the occurrence of a phonon-deficit effect in the system. It has the same nature as in the case of irradiation of a junction by a microwave field, and does not, therefore, need to be commented upon in detail (see Refs. 31, 12, and 37). Some difference is exhibited in the fact that there

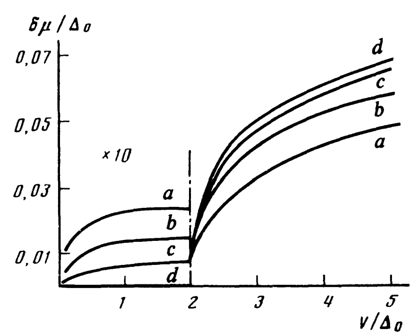


FIG. 7. Dependence of the nonequilibrium chemical potential shift $\delta\mu$ on the applied voltage potential $V$ at $\nu = 0.01\ \gamma$. The temperature $T$ is equal to: *a*) $0.4\ \Delta_0$; *b*) $0.3\ \Delta_0$; *c*) $0.2\ \Delta_0$; and *d*) $0.1\ \Delta_0$.

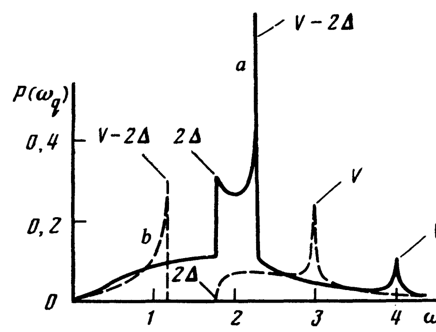


FIG. 9. Phonon-radiation spectrum in the above-threshold regime: $T = 0.1\ \Delta_0$, $\nu = 0.01\ \gamma$. and *a*) $V = 3\ \Delta_0$; *b*) $V = 4\ \Delta_0$. The scale along the abscissa axis is in units of $\Delta_0$.

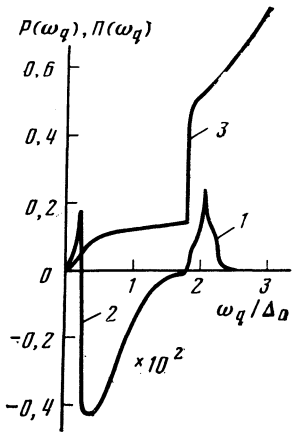


FIG. 10. The phonon deficit effect in the post-threshold regime: 1) recombination peak; 2) relaxation-related dip (magnified a hundred times); 3) spectral dependence of the absorption coefficient $\Pi(\omega_q)$ (in arbitrary units). The parameters $V = 2.05\ \Delta_0$, $T = 0.2\ \Delta_0$, and $\nu = 0.01\ \gamma$.

are in the present case two (and not one) relaxation-related peaks (in the case $a$; in the case $b$ the peaks are indistinguishable on the scale used) whose origin is connected with the "serrated nature" of the nonequilibrium excitation distribution function (see the inset in Fig. 8).

The behavior of the phonon-radiation spectrum in the above-threshold regime (Fig. 9) is interesting. Let us note that similar curves are obtained in Ref. 38 (in a simplified model, in which the imbalance is neglected). But there is no indication in Ref. 38 that the phonon fluxes are negative in the region of small $\omega_q$ (see Fig. 10).

The phonon-deficit effect, when it occurs under conditions when the system contains excess quasiparticles produced by the field from the condensate, is not trivial. It is apparently a consequence of the "quasilocalization" of the excess-excitation distribution in energy space near the gap edge, as a result of which, in the process of phonon scattering by the excess electrons, scattering events accompanied by the emission of phonons with frequency higher than the limiting frequency turn out to be impossible. At the same time events accompanied by the absorption of a phonon are possible, and therefore the scattering mechanism brings about a phonon deficit in a definite spectral interval. There arises here the pertinent question: Is the deficit not connected with the phonon instability (i.e., with the reversal of the sign of the phonon-absorption coefficient) at the relaxation frequencies? As the computations showed (the curve 3 in Fig. 10), the absorption coefficient does not change sign. Apparently, the reason for this is that even though its contribution to the phonon radiation is equal to zero, the small (equilibrium) distribution-function "tail" makes a nonzero contribution to the absorption coefficient, thereby compensating for the small negative dip that arises as a result of the deviation from equilibrium.

Let us note that effects similar to those shown in Figs. 9 and 10 occur in a broad range of junction-parameter values. We shall not, for lack of space, carry out further quantitative analysis of these effects here.

[1)]This circumstance, like the vanishing of $\sigma_1(T)$, can lead to the onset of current instability in the nonequilibrium junction.[30,31]

[2)]Let us note that, in Ref. 33, the sign variability of $\sigma_1(T)$ is explained on the basis of an allowance for the interaction of the tunneling electrons with the boson modes of the barrier. It remains, however, to explain the sign of $\sigma_1$ in other weakly connected structures,[21–25] the existence of boson modes in which is highly hypothetical.


[1]G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)].
[2]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP **46**, 155 (1977)].
[3]A. F. Volkov, Zh. Eksp. Teor. Fiz. **68**, 756 (1975) [Sov. Phys. JETP **41**, 376 (1975)].
[4]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, Moscow, 1962 (Eng. Transl., Prentice-Hall, Englewood Cliffs, N.J., 1963).
[5]M. Tinkham, Phys. Rev. B **6**, 1747 (1972).
[6]I. É. Bulyzhenkov and B. I. Ivlev, Zh. Eksp. Teor. Fiz. **74**, 224 (1978) [Sov. Phys. JETP **47**, 115 (1978)].
[7]I. K. Kirichenko, S. A. Peskovatskiĭ, and V. P. Seminozhenko, Preprint No. 64, Institute of Radiophysics and Electronics, Academy of Sciences of the Ukrainian SSR, Khar'kov, 1976.
[8]V. F. Elesin and Yu. V. Kopaev, Usp. Fiz. Nauk, **133**, 259 (1981) [Sov. Phys. Usp. **24**, 116 (1981)].
[9]A. M. Gulyan and G. F. Zharkov, Phys. Lett. **103A**, 283 (1984).
[10]A. M. Gulyan and G. F. Zharkov, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 134 (1984) [JETP Lett. **40**, 885 (1984)].
[11]A. M. Gulyan and G. F. Zharkov, Tr. Fiz. Inst. Akad. Nauk SSSR Vol. 174, Nauka, Moscow, 1986.
[12]A. M. Gulyan and G. F. Zharkov, Zh. Eksp. Teor. Fiz. **80**, 303 (1981) [Sov. Phys. JETP **53**, 154 (1981)].
[13]B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[14]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **51**, 1535 (1966) [Sov. Phys. JETP **24**, 1035 (1967)].
[15]N. R. Werthamer, Phys. Rev. **147**, 255 (1966).
[16]A. V. Svidzinskiĭ and V. A. Slyusarev, Zh. Eksp. Teor. Fiz. **51**, 177 (1966) [Sov. Phys. JETP **24**, 120 (1967)].
[17]S. B. Nam, Phys. Rev. **156**, 470 (1967).
[18]R. E. Harris, Phys. Rev. B **10**, 84 (1974).
[19]B. D. Josephson, in: Quantum Fluids (Ed. by D. F. Brever), Amsterdam, 1966.
[20]N. F. Pedersen, T. F. Finnegan, and D. N. Langenberg, Phys. Rev. B **6**, 4151 (1972).
[21]D. A. Vincent and B. S. Deaver, Jr., Phys. Rev. Lett. **32**, 212 (1974).
[22]A. Callegari, H. A. Atwater, B. S. Deaver, Jr., Phys. Lett. **59A**, 55 (1976).
[23]C. M. Falco, W. H. Parker, and S. E. Trullinger, Phys. Rev. Lett. **31**, 933 (1973).
[24]M. Nisenoff and S. Wolf, Phys. Rev. B **12**, 1712 (1975).
[25]O. P. Balkashin and I. K. Yanson, Fiz. Nizk. Temp. **2**, 289 (1976) [Sov. J. Low Temp. Phys. **2**, 143 (1976)].
[26]N. F. Pedersen, O. H. Soerensen, and J. Myggind, Phys. Rev. B **18**, 3220 (1978).
[27]S. Rudner and T. Claeson, J. Appl. Phys. **50**, 7070 (1979).
[28]A. B. Zorin, I. O. Kulik, K. K. Likharev, and J. R. Shrieffer, Fiz. Nizk-.Temp. **5**, 1138 (1979) [Sov. J. Low Temp. Phys. **5**, 537 (1979)].
[29]K. Hida and Y. Ono, J. Low Temp. Phys. **29**, 355 (1977).
[30]A. G. Aronov and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 218 (1975) [JETP Lett. **22**, 101 (1975)].
[31]A. I. Larkin and Yu. N. Ovchinnikov, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 556 (1984) [JETP Lett. **39**, 681 (1984)].
[32]G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 186 (1970) [JETP Lett. **11**, 114 (1970)].
[33]P. F. Michelson, Phys. Lett. **77A**, 362 (1980).
[34]A. M. Gulyan and G. F. Zharkov, Phys. Lett. **80A**, 79 (1980).
[35]B. I. Ivlev and G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 464 (1971) [JETP Lett. **13**, 333 (1971)].
[36]A. M. Gulyan and G. F. Zharkov, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 524 (1981) [JETP Lett. **34**, 498 (1981)].
[37]A. M. Gulyan and G. F. Zharkov, Tr. Fiz. Inst. Akad. Nauk SSSR, Vol. 148, Nauka, Moscow, 1983.
[38]J.-J. Chang and D. J. Scalapino, J. Low Temp. Phys. **31**, 1 (1978).


Translated by A. K. Agyei