Resonant transformation of light by relativistic ion beams

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Resonant transformation of light by ions moving with velocities close to the velocity of light is investigated. Due to the Doppler effect the frequency of the light moving towards the ion beam is shifted into the far ultraviolet region and gets into resonance with an optically allowed ion transition. Most of the light scattered by the ions is concentrated into a narrow solid angle as a result of the aberration. This makes it possible for the transformation of laser radiation into the x-ray region of the spectrum to take place with high efficiency.

Existing accelerators allow ions to be accelerated up to energies of several hundred GeV,[1] i.e., several times the rest mass of light ions, $mc^2 \sim 10$ GeV (for Be, B, and C). The ions which are accelerated to such energies and which are not fully stripped, i.e., still have an electron, provide an interesting possibility of resonant transformation of laser radiation into the x-ray region of the spectrum. The present work is devoted to the analysis of such a possibility. Note that compared to the transformation of laser radiation by means of the inverse Compton scattering on relativistic electrons,[2–4] the resonant scattering via an optically allowed ion transition is significantly more efficient, since the cross section for the resonant scattering is many orders of magnitude larger than the Compton scattering cross section.

In order to keep an electron in an accelerated ion, it is necessary that the lifetime of the ion with respect to ionizations in collisions with the residual gas atoms be longer than the acceleration time, which, in modern accelerators, is equal to several seconds. The ionization cross section reaches its maximum when the relative velocity of colliding particles is of the order of the orbital velocity of an electron, i.e., $v \sim v_0 z_i$, where $v_0 = 2.2\times10^8$ cm/sec is the atomic velocity unit and z_i is the ion charge. In this case one can assume that the atom consists of a nucleus with charge z_a and the same number of independent free electrons. By using Rutherford's formula, one easily obtains the following expressions for the ionization cross section:

$$\sigma_i = 4\pi a_0^2 (v_0/v)^2 z_i^{-2}(z_a^2+z_a), \qquad (1)$$

where a_0 is the Bohr radius and v is the relative velocity of the particles. Equation (1) is applicable provided $v \gtrsim v_0 z_i$; for these velocities it yields slightly exaggerated estimates, as has been shown in Ref. 5. By substituting $z_a = 7$ (nitrogen), $z_i = 5$ and $v = v_0 z_i$ into Eq. (1), one can see that the majority of ions preserve an electron, provided the residual gas pressure in the accelerator chamber is of order 10^{-9}–10^{-10} Torr.

Suppose that a photon flux incounters the head of an ion beam moving along the x axis in the laboratory coordinate system with a velocity v close to the velocity of light. We will denote the coordinate system moving with the ion beam, by Greek letters ξ, η, ζ and will choose this system so that its axes are parallel to the axis x, y and z, respectively. Let θ_x be the angle between the direction of propagation of the light and the x axis. In transforming to the coordinate system moving with the ion beam, the light undergoes deflection, and the angle θ_ξ between the direction of propagation of the light and the ξ axis is related to the angle θ_x via[6]

$$\cos\theta_\xi = \left(\cos\theta_x - \frac{v}{c}\right)\left(1 - \frac{v}{c}\cos\theta_x\right)^{-1}. \qquad (2)$$

The frequency of the radiation also changes in the coordinate system associated with the ion beam (the Doppler effect):

$$\omega = \omega_l\left[\gamma\left(1+\frac{v}{c}\cos\theta_\xi\right)\right]^{-1} = \omega_l\gamma\left(1-\frac{v}{c}\cos\theta_x\right), \qquad (3)$$

where $\gamma = E/mc^2 = (1-v^2/c^2)^{-1/2}$ is the ratio between the ion energy and its rest energy, and ω_l, ω are the frequencies of the laser radiation in the laboratory frame and the rest frame of the ions, respectively.

Let us assume that the frequency ω is identical to the frequency ω_{12} of an optically allowed transition of the ion. In this case, the process of resonant light scattering becomes feasible. Let $\tilde\theta_\xi$ be the angle between the direction of the scattered light and the ξ axis. In the laboratory frame the scattered light will propagate at angle $\tilde\theta_x$, satisfying relation

$$\cos\tilde\theta_x = \left(\cos\tilde\theta_\xi + \frac{v}{c}\right)\left(1 + \frac{v}{c}\cos\tilde\theta_\xi\right)^{-1}. \qquad (4)$$

The frequency of the scattered light measured by an observer is given by

$$\tilde\omega = \omega\left[\gamma\left(1-\frac{v}{c}\cos\tilde\theta_x\right)\right]^{-1} = \omega\gamma\left(1+\frac{v}{c}\cos\tilde\theta_\xi\right). \qquad (5)$$

By substituting the value for ω from Eq. (3), we obtain

$$\tilde\omega = \omega_l\left(1-\frac{v}{c}\cos\theta_x\right)\left(1-\frac{v}{c}\cos\tilde\theta_x\right)^{-1}. \qquad (6)$$

The maximum possible frequency conversion is achieved for $\theta_x = \pi$ and $\tilde\theta_x = 0$. Then

$$\tilde\omega^{max} = 4\gamma^2\omega_l. \qquad (7)$$

Here and in the following we consider the case where $v \sim c$, which allows us to replace expressions of type $1 + nv/c$, where $n \neq -1$, by $1 + n$. For $\gamma = 10$, the frequency of the scattered light is increased by a factor of 400, as can be seen from Eq. (7).

Consider now the efficiency of such a conversion. Modern accelerators yield $\Delta p/p \sim 10^{-3}$ for the momentum spread of the accelerated particles.[7] Since $\Delta p = m\Delta v\gamma^3$, the corresponding uncertainty in the velocity is $\Delta v/v \sim 10^{-3}\gamma^{-2}$. For a fixed value of the frequency ω_l of the

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laser radiation and its angle of propagation θ_x ($\pi/2 < \theta_x < \pi$), the uncertainty in the velocity of an ion is the same as the uncertainty in the transition frequency $\Delta\omega_{12}/\omega_{12} \sim 10^{-13} v/c$ [see Eq. (3)]. Let us characterize the flux of the photons by the total number N_l of quanta in the light pulse, the length τ_l of the pulse and the average energy $\hbar\omega_l$ of a photon. If $\Delta\omega_l/\omega_l \sim \Delta\omega_{12}/\omega_{12}$, then practically all the photons and ions participate in the scattering process. We will assume that this is the case, that the light is propagating towards the ion beam ($\theta_x \sim \pi$), and that the cross section S of the light of the light beam is equal to the cross section of the ion beam.

The flux density q of the photons in the rest frame of the beam is related to the flux density $q_l = N_l/S\tau_l$ in the frame of the source as $q = q_l\omega/\omega_l$ (only time is transformed, since the quantities N_l, S and $\omega_l\tau_l$ are invariants of the transformation).

We assume that the frequency of the photons in the rest frame of the ions is in resonance with the frequency of an optically allowed ion transition $\omega_{12} = \omega$. The probability of absorption of a photon by an ion via the transition $1 \rightarrow 2$ is given by[8]

$$W_{12} = q\sigma_{12}, \qquad \sigma_{12} = \frac{g_2}{g_1}\frac{\pi^2 c^2}{\omega_{12}^2}\frac{A_{21}}{\Delta\omega_{12}}, \tag{8}$$

where σ_{12} is the absorption cross section, g_1 and g_2 are the statistical weights of the corresponding levels and A_{21} is the probability of the spontaneous emission. As long as the probability of induced emission is less than the probability of spontaneous emission, i.e.,

$$q \leqslant A_{21}/\sigma_{12}, \tag{9}$$

the scattering takes place due to the spontaneous reemission of photons. In this case Eq. (8) yields the probability of resonance fluorescence ($\Delta\omega_{12} > A_{21}$). Since we are considering backward scattering, condition (9) on the flux is, in fact, the condition for the optimum scattering (with respect to the ratio between the number of the backscattered photons and the number of incident photons).

The total number N_s of photons scattered during the time $\tau = \omega_l\tau_l/\omega$ is given by

$$N_s = N_i n_s, \qquad n_s = W_{12}\tau = (\sigma_{12}/S) N_l, \tag{10}$$

where n_s is the number of quanta scattered by a single ion and N_i is the total number of ions in the beam. Here one should note that since in every single photon scattering event, on the average, the change in the ion momentum is $\hbar\omega_{12}\gamma/c$ (the averaging is over the directions of the momentum of the scattered photon), the maximum number of quanta which can be scattered by a relativistic ion before it gets out of the resonance is equal to

$$n_s^{\max} = 10^{-3} mc^2/\hbar\omega_{12}. \tag{11}$$

Thus, one should satisfy the condition $n_s \leqslant n_s^{\max}$. This condition in principle, is not a limitation, because the decelerating force can be balanced by an appropriate acceleration. Moreover, this condition is always satisfied in practically interesting cases, as will be shown in the following.

Let us assume that the scattering is isotropic, i.e., $(N_s/2)d(\cos\tilde{\theta}_\xi)$ out of the total N_s photons, are scattered at an angle $\tilde{\theta}_\xi$ relative to the ξ axis. In principle, the angular distribution of the scattered radiation is defined by the scalar product of the polarizations of the incident and scattered light.[9] In practice, however, this fact should be taken into account only in considering the polarization characteristics of the scattered radiation. In the rest frame, the scattered quanta carry energy

$$dE = \frac{N_s}{2}\hbar\tilde{\omega}d(\cos\tilde{\theta}_\xi) = \frac{N_s}{2}\hbar\omega_{12}\gamma\left(1 + \frac{v}{c}\cos\tilde{\theta}_\xi\right)d(\cos\tilde{\theta}_\xi). \tag{12}$$

The total energy scattered into angle $\tilde{\theta}_\xi \leqslant \pi/2$ with corresponding $\tilde{\theta}_x \lesssim \gamma^{-1}$ is equal to

$$E = {}^3/_4 N_s\hbar\omega_{12}\gamma. \tag{13}$$

The frequency of the photons in this scattered light flux varies between $\gamma\omega_{12}$ and $2\gamma\omega_{12}$. It is interesting to find the fraction of the energy propagating in the interval $\Delta\omega_0/\omega_0 \sim 10^{-3}$, where $\omega_0 = 2\gamma\omega_{12}$. This frequency spread is characteristic of photons scattered at angle $\tilde{\theta}_\xi = (2\times10^{-3}/\gamma)$ with the corresponding $\tilde{\theta}_x = \tilde{\theta}_\xi/2\gamma$. These photons carry energy $E_0 = 10^{-3}N_s\hbar\omega_{12}$.

Now we can easily obtain an expression for the intensity of the scattered light. The duration of the pulse of the scattered radiation in the rest frame of the ion beam is either $\tilde{\tau} = \omega_{12}\tau/\tilde{\omega}$ or $\tilde{\tau} = \omega_{12}l/\tilde{\omega}c$, depending on which of these quantities is the large (here l is the length of the ion beam). The intensity of the light scattered into angle $\tilde{\theta}_\xi \leqslant \pi/2$ (assuming $\tilde{\tau} = \omega_{12}\tau/\tilde{\omega}$) is equal to

$$j = {}^7/_6 N_i q_l \sigma_{12}\hbar\omega_{12}(\omega_{12}/\omega_l)\gamma^2. \tag{14}$$

Let us write expressions for the efficiency of the resonant transformation with respect to various parameters of laser radiation.

1) The ratio between the number of photons scattered into angle $\tilde{\theta}_x = \gamma^{-1}$ (the corresponding solid angle is $\Omega \sim \gamma^{-2}$) and the total number of photons is

$$\eta_N = (\sigma_{12}/2S)N_i. \tag{15}$$

Here, in contrast to the scattering on an ion at rest, we lose a factor $\Omega/4\pi$ due to the "projector" effect, i.e., because the light scattered at an angle $\tilde{\theta}_\xi = \pi/2$ is concentrated into an angle γ^{-1}.

2) The ratio between the energy of the scattered and incident laser radiation is

$$\eta_E = (3\sigma_{12}/2S)N_i\gamma^2. \tag{16}$$

The efficiency is increased by a factor γ^2 due to the Doppler effect.

3) The ratio between the corresponding intensities is

$$\eta_J = (14\sigma_{12}/3S)N_i\gamma^4. \tag{17}$$

An additional factor γ^2 in this formula due to the shortening of the duration of the pulse.

NUMERICAL ESTIMATES

Consider an ion beam, accelerated to an energy of 100 GeV (the rest mass of the ions is ~ 10 GeV and $\gamma = 10$) and having an optically allowed transition $1 \rightarrow 2$, with $\lambda_{12} = 100$ Å and the spontaneous transition probability $A_{21} = 10^{11}$ sec^{-1} (these parameters approximately correspond to Be

III, B IV ions and transitions $1s^2$–$1s2p$).

In order to have a resonance with the optically allowed transition, the wavelength of the laser radiation propagating towards the ion beam must be $\lambda_l = 20\lambda_{12} = 2000$ Å.

Condition (9) for the optimal scattering (with the photo-absorption cross section $\sigma_{12} = 4\times 10^{-16}$ cm^2) yields

$$q=(\omega/\omega_l)\,q_l\leqslant 2.5\cdot 10^{26}\ \text{quantum/cm}^2\text{sec} \quad . \tag{18}$$

Let $q = 10^{26}$, corresponding to $q_l = 0.5\times 10^{25}$ photons/cm^{2}sec and energy flux density $I_l = 0.5\times 10^7$ W/cm^2. The duration of the laser pulse will be assumed to be $\tau_l = 30$ nsec (the ion beam traverses 9 m, during this time interval). Then, in the beam frame, the duration of the pulse is $\tau = 1.5$ nsec, and the number of photons scattered by a single ion is $n_s = 60$. Let the total number of photons be $N_s = 60N_i$, where the number of the ions in the beam is assumed to be $N_i = n_i lS = 5\times 10^{10}S$, with $n_i = 10^9$ cm^{-3} and $l = 50$ cm. The efficiency with respect to the number of the scattered quanta [Eq. (15)] is $\eta_N = 10^{-5}$.

The total energy scattered into angle $\tilde{\theta}_\xi = \pi/2$ ($\tilde{\theta}_x = 0.1$) is equal $E = 4.2\times 10^{-4}S$ J, and the energy of the incident light in the pulse is $1.5\times 10^{-1}S$ J, i.e., $\eta_E = 3\times 10^{-3}$.

The intensity of the scattered light is $J = 0.5\times 10^7 S$ W and $\eta_J = 1$. The wavelength of the scattered radiation is $\tilde{\lambda} = 5$ Å and the duration of the pulse is $\tilde{\tau} = 0.075$ ns. The intensities of the laser and scattered radiation are equal. Note that the energy and intensity of the scattered light increase as the energy of the accelerated ions increases. The directionality of the radiation is also improved.

Thus, relativistic ion beams make the efficient resonant transformation of light into the x-ray region of the spectrum feasible. In the output of such a "transformer" one obtains a directed, polarized x-ray radiation in a relatively narrow frequency band. Also, there exists a possibility of tuning the frequency in the x-ray region, by varying the frequency of laser radiation and energy of the accelerated particles.

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