Separation of magnetization precession in ^{3}He-B into two magnetic domains. Theory

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It is shown that even small deviations of the magnetic field from uniformity can substantially modify the magnetization precession in ^{3}He-B. Specifically, a two-domain structure forms if the magnetic-field non-uniformity is linear. The magnetization makes an angle $\sim 104°$ with the field in one of the domains and is parallel to it in the other. These domains can explain the anomalously long persistence of the induction signal in ^{3}He-B; moreover, the change in the induction-signal frequency with time discovered and investigated by Borovik-Romanov *et al.* [JETP Lett. **40**, 1033 (1984)] is a consequence of the relaxation of the domain structure.

1. INTRODUCTION

Recent experiments[1] have shown that our present understanding of the phenomena observed in pulsed NMR experiments in superfluid ^{3}He-B is incomplete.[1] An induction signal of anomalously long persistence (henceforth referred to as the slowly decaying signal, or SDS) was systematically studied in Refs. 1 and 2. The results indicate that for large initial tipping angles $\beta \sim 90°$ of the magnetization vector, the SDS in ^{3}He-B persists some 10–100 times longer than the characteristic dephasing time of the magnetization precession predicted from the magnitude of the spatial variations in the nearly constant magnetic field $\mathbf{H}_0$. Slowly decaying signals were first observed by Corruccini and Osheroff,[3] and somewhat later by Giannetta, Smith, and Lee.[4] Apart from this, the other experimental results were in good agreement with calculations carried out for a spatially uniform stationary precession of the nuclear spin.[5,6] This apparently explains why, in spite of their obvious interest, the unusual behavior of the slowly decaying signals did not seem to require any fundamental changes in the interpretation of pulsed NMR experiments; instead, it was attributed[3] to textural effects in the experimental cells.

In their experiments, Borovik-Romanov, Bun'kov, Dmitriev, and Mukharskiĭ[1] detected and analyzed some quantitative characteristics of the slowly decaying signal which suggest an explanation for why this signal is present.[7] In the present paper we will expand on the brief theoretical treatment of SDS's presented previously in Ref. 7. Assume that ^{3}He-B is present in a magnetic field with a small but nonzero gradient, and that the initial magnetization precession is spatially uniform; we will then show that the precession becomes modified shortly after the magnetization is deflected by an rf tipping pulse—the structure breaks up into two or more domains, depending on the specific configuration of the field. The subsequent behavior of the structure is consistent with the experimentally observed properties of the SDS. The two-domain structure is the easiest to study; in particular, it was investigated in Refs. 1 and 2 and will be analyzed in detail here. (This case already contains the essential physics, and the treatment generalizes straightforwardly to multidomain structures).

2. TWO-DOMAIN STRUCTURE OF THE MAGNETIZATION PRECESSION

We will not review all of the experimental results in Ref. 1, where the influence of spatial variations in the magnetic field $\mathbf{H}_0$ on spin precession was studied in detail. For our purposes, the principal findings were: 1) the induction signal persists even in gradients as large as ~ 10 Oe/cm; 2) in a nonzero-gradient field, the signal frequency changes with time at a rate that depends on $|\nabla H|$. The first observation shows that in order to interpret the SDS theoretically, one must seek solutions of the spin dynamic equations which describe a stationary precession of the nuclear spins in ^{3}He-B in nonuniform magnetic fields.

The order parameter in ^{3}He-B is a matrix $\hat{R}(\mathbf{n},\theta)$, which describes a rotation by an angle θ about the unit vector $\mathbf{n}$. It is helpful to parametrize $\hat{R}$ in terms of the Euler angles; by definition, we then have

$$\hat{R}(\mathbf{n},\ \theta)=\hat{R}_z(\alpha)\,\hat{R}_y(\beta)\,\hat{R}_z(\gamma), \tag{1}$$

where $\hat{R}_z(\alpha)$ represents a rotation by α about the z axis, etc. If we temporarily neglect the energy dissipation, we must then solve the Leggett equations.[8] The motion of the order parameter will be described in terms of the angels α, β, $\Phi=\alpha+\gamma$ and the canonically conjugate combinations $P=S_z-S_\zeta$, S_B, and S_ζ of the projections of the spin density vector S. Here S_z and S_ζ are the projections on the z and $\zeta=\hat{R}z$ axes, respectively, and S_β is the projection on the vector $z\times\zeta/|\mathbf{z}\times\zeta|$ (nodal line). In order to simplify the formulas we will choose the mass units so that the magnetic susceptibility χ of ^{3}He-B per unit volume is equal to g^2, where g is the gyromagnetic ratio for the ^{3}He nuclei. The energy per unit volume wil then have the dimensions of frequency squared, while the spin per unit volume will have the dimensions of frequency. in the spatially uniform case, the Leggett Hamiltonian takes the form[9]

$$\mathscr{H}^{(0)}=\frac{1}{1+\cos\beta}\left[S_\zeta{}^2+PS_\zeta+\frac{P^2}{2(1-\cos\beta)}\right] -\omega_L{}^{(0)}(P+S_\zeta)+\frac{1}{2}S_\beta{}^2+U_D(\alpha,\beta,\Phi). \tag{2}$$

The dipole energy U_D in the B-phase depends only on β and

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Φ:

$$U_D(\beta,\ \Phi)={}^2/_{15}\Omega^2[\cos\beta-{}^1/_2+(1+\cos\beta)\cos\Phi]^2. \qquad (3)$$

If the system is not spatially uniform, we must replace the Larmor frequency $\omega_L^{(0)}$ by $\omega_L(\mathbf{r})$ in Eq. (2) and add a term F_∇ equal to the energy density of the spatial fluctuations in the condensate. We will assume that $\mathbf{H}_0$ is nearly constant and that ω_L depends only on z. This is true in typical experiments, because the small transverse fluctuations in $\mathbf{H}_0$ modify ω_L only to second order in $|\delta\mathbf{H}|/\mathbf{H}_0$ whereas the longitudinal fluctuations alter ω_L to first order. We will take ω_L to depend linearly on z in our calculations; special measures were taken to ensure this condition in the experiments in Refs. 1 and 2, where the measuring chamber was cylindrical with axis parallel to z. We will assume for definiteness that $z=0$ corresponds to the bottom of the chamber (which is located downfield), so that $\omega_L(z)=\omega_L(0)+z\nabla\omega_L$. We will use the expression derived in Refs. 9 and 10 for F_∇:

$$F_\nabla={}^1/_2c_\parallel^2[2(1-u)\alpha'(\alpha'-\Phi')+\Phi'^2+\beta'^2]$$
$$-(c_\parallel^2-c_\perp^2)[(1-u)\alpha'-\Phi']^2; \qquad (4)$$

where we assume at the outset that all variables depend only on z. Here the primes denote derivatives with respect to z; $u=\cos\beta$, and $c_\parallel^2$ and $c_\perp^2$ are phenomenological coefficients. For example, $c_\parallel^2$ and $c_\perp^2$ specify the dispersion law for the transverse spin waves inthe Leggett configuration (n parallel to $\mathbf{H}_0$) if the magnetic field is strong compared to the dipole energy U_D:

$$\omega=\omega_L+\frac{1}{\omega_L}(c_\parallel^2k_\parallel^2+c_\perp^2k_\perp^2).$$

Here $K_\parallel$ and $k_\perp$ are the components of the wave vector parallel and normal to $\mathbf{H}_0$.

We can then write the Hamiltonian as a functional

$$E=\int\mathscr{H}\,dV,$$

where the integration is over the entire volume of the helium in the chamber, and the Hamiltonian density is given by

$$\mathscr{H}=\mathscr{H}^{(0)}|_{\omega_L=\omega_L(z)}+F_\nabla. \qquad (5)$$

The equations of motion corresponding to (5) follow in the usual way:

$$\frac{\partial\alpha}{\partial t}=\frac{\partial\mathscr{H}}{\partial P},\quad \frac{\partial P}{\partial t}=-\frac{\delta\mathscr{H}}{\delta\alpha},$$
$$\frac{\partial\beta}{\partial t}=\frac{\partial\mathscr{H}}{\partial S_\beta},\quad \frac{\partial S_\beta}{\partial t}=-\frac{\delta\mathscr{H}}{\delta\beta}, \qquad (6)$$
$$\frac{\partial\Phi}{\partial t}=\frac{\partial\mathscr{H}}{\partial S_\zeta},\quad \frac{\partial S_\zeta}{\partial t}=-\frac{\delta\mathscr{H}}{\delta\Phi},$$

where, e.g.,

$$\frac{\delta\mathscr{H}}{\delta\alpha}=\frac{\partial\mathscr{H}}{\partial\alpha}-\frac{\partial}{\partial z}\left(\frac{\partial\mathscr{H}}{\partial\alpha'}\right), \qquad (7)$$

etc. We will require only the explicit form for the second equation, which reads

$$\frac{\partial P}{\partial t}+\frac{\partial}{\partial z}\{(u-1)[2c^2(u)\alpha'-c^2(-1)\Phi']\}=0. \qquad (8)$$

The abbreviation

$$c^2(u)=uc_\parallel^2+(1-u)c_\perp^2 \qquad (9)$$

will be used to denote various linear combinations of $c_\parallel^2$ and $c_\perp^2$ in what follows. Equation (8) expresses the fact that the quantity $\mathscr{P}=\int PdV$ is conserved, and the expression in the curly brackets is the flux density j_z^P of P in the z direction. ($\mathscr{P}$ is conserved because the coordinate α conjugate to P does not appear explicitly in the expression for the Hamiltonian density).

We now seek solutions of system (6) that describe a stationary spin precession with constant frequency ω_P, i.e., solutions such that

$$\frac{d\Phi}{dt}=\frac{d\beta}{dt}=\frac{dP}{dt}=\frac{dS_\zeta}{dt}=\frac{dS_\beta}{dt}=0, \qquad (10)$$
$$\frac{d\alpha}{dt}=-\omega_P. \qquad (11)$$

If we substitute (10) and (11) into the left-hand side of (6), the latter reduces to the stationarity condition

$$\mathscr{F}=\int(\mathscr{H}+\omega_PP)\,dV. \qquad (12)$$

The precession frequency ω_P is not known *a priori*; it can be regarded as a Lagrange multiplier corresponding to the constraint $\int PdV=$ const and will ultimately be determined by the value of $\int PdV$.

We now remark that in all the experiments in which the SDS was observed, the spatial variations in the field were much smaller than the dipole energy U_D or the field $\mathbf{H}_0$ itself. Indeed, the total variation in the magnetic field over the length of the measuring chamber was usually less than a few Oersteds, while $\mathbf{H}_0$ was ~100 Oe and U_D was equivalent to a field 50–80 Oe. This suggests (and experimental results confirm) that the characteristic length of the spatial variations of the resulting stationary state is large compared with the dipole length $l_D\sim10^{-3}$ cm, so that also $F_\nabla\ll U_D$. We therefore split the Hamiltonian density (5) into two parts—a principal part which has no explicit dependence on z, and a small correction. It is helpful to express the frequency $\omega_L(z)$ in the form

$$\omega_L(z)=\omega_L(0)+z\nabla\omega_L$$
$$=\omega_L(z_0)+(z-z_0)\nabla\omega_L=\omega_P+(z-z_0)\nabla\omega_L,$$

i.e., so that the constant spatial component is given by $\omega_L=\omega_p$; the point z_0 is such that $\omega_L(z_0)=\omega_p$. In order to find solutions that describe stationary states, we must there minimize the functional

$$\int[(\mathscr{H}^{(0)}|_{\omega_L=\omega_p}+\omega_PP)-(z-z_0)(P+S_\zeta)\nabla\omega_L+F_\nabla]dV. \qquad (13)$$

This will be done in two steps by exploiting the smallness of the spatially nonuniform terms. We first find the extremals of the spatially uniform part to lowest order:

$$\delta\int(\mathscr{H}^{(0)}+\omega_PP)\,dV=0. \qquad (14)$$

This problem was already analyzed in Ref. 11, where the stationary solutions for the spatially uniform case were listed. The initial state is produced immediately after the rf tip-

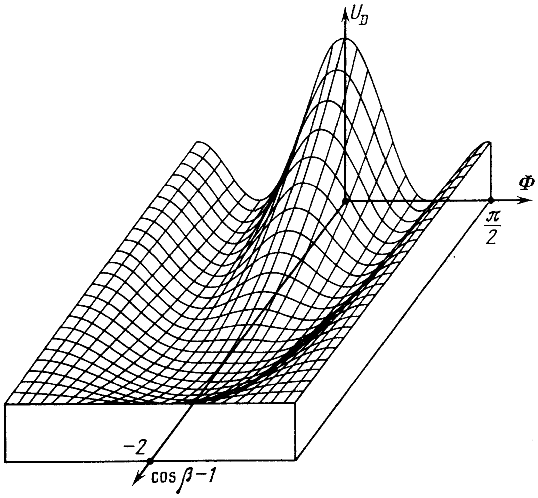

FIG. 1

ping pulse deflects the magnetization by an angle β; it corresponds to the solution

$$S_z=\omega_P\cos\beta,\ S_\zeta=\omega_P,\ S_\beta=0, \tag{15}$$

which belongs to the family denoted by 1b in Ref. 11. The angle Φ satisfies the equation

$$\cos\beta+\cos\Phi+\cos\beta\cos\Phi={}^1/_2, \tag{16}$$

which can be used to find Φ when β is specified. The angle α is unrestricted. For β and ϕ satisfying $-1/4\leqslant\cos\beta\leqslant1$ and $-1/4\leqslant\cos\Phi\leqslant1$, Eq. (16) describes a line in the β, Φ plane along which the potential U_D has a degenerate minimum as a function of β and Φ (Fig. 1). It is thus clear that the solutions (15), (16) form a degenerate family of extremals for the functional (14). To lowest order the angles α and β can vary arbitrarily in space (subject to $\cos\beta>-1/4$). This degeneracy is lifted only when the omitted terms are retained.

The corrections to the lowest-order Hamiltonian can be minimized to next higher order by substituting the constraints (15) and (16). This gives the extremum condition

$$\delta\int[F_\nabla-\omega_P\cos\beta\,(z-z_0)\nabla\omega_L]dV=0. \tag{17}$$

Taking α and $u=\cos\beta$ as the independent variables [with Φ expressed by (16) in terms of u], we readily find that

$$(d\Phi/du)^2=3/(1+4u)(1+u)^2.$$

The integrand in (17) can thus be recast in the form

$$T=(1-u)c^2(u)(\alpha')^2+\frac{1}{2}\left[\frac{3c^2(-1)}{(1+4u)(1+u)^2}+\frac{c^2(1)}{1-u^2}\right](u')^2 -(1-u)\frac{c^2(-1)}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}\alpha'u'-\omega_Pu(z-z_0)\nabla\omega_L. \tag{18}$$

We get the equation

$$\frac{\partial}{\partial z}\left\{(1-u)\left[2c^2(u)\alpha'-\frac{c^2(-1)}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}u'\right]\right\}=0 \tag{19}$$

by varying α; this is just the condition $\partial j_z^P/\partial z=0$ [cf. Eq. (8)], with Φ' expressed in terms of u'. Equation (19) implies that $j_z^P=\text{const}$. Since the helium was contained in a closed vessel in the experiments in Refs. 1 and 2, j_z^P was equal to zero at the chamber walls; j_z^P const therefore implies that $j_z^P=0$ everywhere. Using (18), we thus find the relation

$$\alpha'=\frac{c^2(-1)}{2c^2(u)}\frac{1}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}u' \tag{20}$$

between α' and u'. An additional equation can be derived from (17) by considering variations with respect to u; it must be solved jointly with (20). It will be helpful to substitute (20) into (18) before calculating the variation with respect to u. We get

$$T=\frac{c_\parallel^2[8c^2(u)-3c_\parallel^2]}{4c^2(u)(1-u)(1+4u)}\left(\frac{du}{dz}\right)^2+\omega_P(1-u)(z-z_0)\nabla\omega_L, \tag{21}$$

where the term $\omega_p(z-z_0)\nabla\omega_L$ has been added in order to ensure that the energy is measured relative to the equilibrium value; this does not alter the equation for u. We define the dimensionless coordinate $\xi=(z-z_0)/\lambda$, where the characteristic length $\lambda=(c_\parallel^2/\omega_p\nabla\omega_L)^{1/3}$ is $\approx2\cdot10^{-2}$ cm if $\mathbf{H}_0\approx100$ Oe and $c_\parallel\approx5$ m/s. The functional then takes the form

$$\mathscr{F}=\lambda^2\omega_L^{(0)}\nabla\omega_L\int\left[\frac{8c^2(u)-3c_\parallel^2}{4c^2(u)(1-u)(1+4u)}\left(\frac{du}{d\xi}\right)^2+\xi(1-u)\right]d\xi. \tag{22}$$

The factor multiplying $(du/d\xi)^2$ has poles at the end points $u=-1/4$, 1 of the range of u. These singularities can be eliminated by defining the new variable v

$$\cos v={}^8/_5(u-{}^3/_8), \tag{23}$$

which ranges over the interval $[0,\pi]$. Expressed in terms of $v,\mathscr{F}$ becomes

$$\mathscr{F}=\frac{5}{8}\lambda^2\omega_L^{(0)}\nabla\omega_L\int\left[\frac{1}{2}m(\cos v)\left(\frac{dv}{d\xi}\right)^2+\xi(1-\cos v)\right]d\xi, \tag{24}$$

where

$$m(\cos v)=\frac{8c^2(\cos v)}{5c^2(\cos v)+3c^2(1)}.$$

The Euler-Lagrange equation

$$m(\cos v)\frac{d^2v}{d\xi^2}-\left[\frac{1}{2}\frac{dm}{d\cos v}\left(\frac{dv}{d\xi}\right)^2+\xi\right]\sin v=0 \tag{25}$$

follows from the requirement that (24) be an extremum. The boundary conditions must be specified before we can choose the required solution of (25). We note that since the length $L\sim1$ cm of the measuring chamber was large compared to the characteristic length λ, the boundary conditions may be imposed for $\xi\to\pm\infty$; we will require that the solution becomes spatially homogeneous in both limits (we will discuss the appropriateness of this condition below). It is clear from Eq. (25) that in order for the solution to be spatially homogeneous we must have $\sin v=0$, or $v=0$, π; we therefore consider neighborhoods of these points. For $v\to0$ we have the familiar Airy equation

$$\frac{d^2v}{d\xi^2} - \xi v = 0. \tag{26}$$

It has a solution that decays as

$$v(\xi) \sim \frac{1}{\xi^{1/4}} \exp\left(-\frac{2}{3}\xi^{3/2}\right) \tag{27}$$

for $\xi \to +\infty$ (we note that this solution oscillates for negative ξ).

Similarly, for $v \to \pi$ we set $v = \pi - \psi$, so that ψ satisfies the equation

$$\frac{d^2\psi}{d\zeta^2} + \zeta\psi = 0, \tag{28}$$

where $\zeta = \xi[m(-1)]^{-1/3}$. This equation is similar to (26) but has a solution that decays as

$$\psi \sim \frac{1}{|\zeta|^{1/4}} \exp\left(-\frac{2}{3}|\zeta|^{3/2}\right) \tag{29}$$

for $\zeta \to -\infty$. Relations (27) and (29) show that in order to satisfy our requirements, we must have $v(\xi \to +\infty) = 0$ and $v(\xi \to -\infty) = \pi$, which in terms of the original variables means that $\beta \to 0$ as $z \to +\infty$ and $\beta \to \theta_0$ as $z \to -\infty$.

The solution thus describes a structure consisting of two domains (Fig. 2) separated by a narrow transitional region (wall) of width $\sim\lambda$ located at $z \approx z_0$. The magnetization is parallel to the field for $z > z_0$ but makes an angle $\theta_0 = \cos^{-1}(-1/4)$ with **H** for $z < z_0$. Here the position z_0 of the boundary is determined by the integral $\int P dV$ over the volume of the measuring chamber, i.e., by the initial tipping angle β_0. The magnetization precesses at the same frequency $\omega_P = \omega_L(z_0)$ throughout the ^{3}He [here $\omega_L(z_0)$ is the local Larmor frequency at the domain wall]. Substituting (29) for $z < z_0$ into the right-hand side of (20), we find that $\alpha' \to 0$ as we move away from the wall, i.e., the nuclear spins tipped at the angle θ_0 precess in phase even though the magnetic field is nonuniform. This phasing is responsible for the slow decay of the free-induction signal. In order to explicitly determine the form of the domain wall, we must solve (25) numerically and impose the required limiting behavior for $\xi \to \pm\infty$.

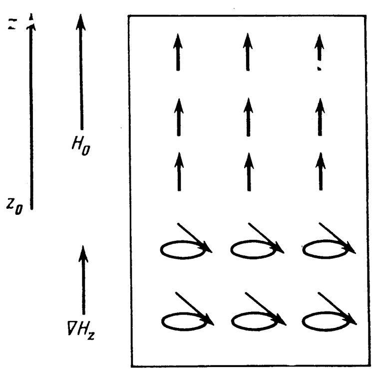

FIG. 2

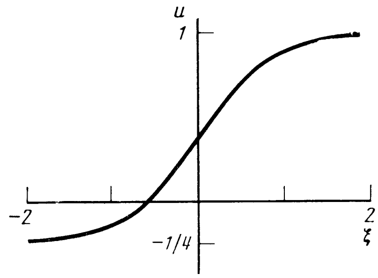

FIG. 3

Equation (25) contains the ration $c_\perp^2 c_\parallel^2$ as a parameter; Fig. 3 shows the solution obtained by setting $c_\perp^2/c_\parallel^2 = 3/4$, which corresponds to the limit $T \to T_c$. We can use Eqs. (16) and (20) to find the angles α and Φ as functions of z once $u(z)$ is known.

This solution is stable, because the two-domain structure minimizes the functional $\mathscr{F}$. This is clear from the fact that $\mathscr{F}$ decreases if the magnetization is redistributed so as to maximize the longitudinal component $M_\parallel$ where the field is strong and minimize it where the field is weaker. Since (16) implies that the tipping angle must satisfy $-1/4 \leqslant \cos\beta \leqslant 1$, the distribution of the magnetization is most favorable for the above two-domain solution. The energy of the domain wall is finite but its contribution to the total change in $\mathscr{F}$ is small of order $(\lambda/L)^2$ for the two-domain solution.

Further analysis is needed to determine the lower bound on $\cos\beta$. The bound $\cos\beta > -1/4$ was derived on the assumption that Eq. (16) is strictly satisfied, which is equivalent to assuming that U_D is infinitely large compared to the domain wall energy. The extent to which (16) holds can be assessed quantitatively by considering the frequency of the oscillations that results when Φ varies from the value $\Phi(\beta)$ given by (16). We showed previously[6] that when $\cos\beta \to -1/4$, the frequency of one of the oscillation modes tends to zero as $(\cos\beta + 1/4)^{1/2}$ and (16) cannot be used for $\beta \leqslant \theta_0$. We see from Eq. (3) that for $\beta \approx \theta_0$, violations of Eq. (16) will change U_D by $\sim\Omega^2(\cos\beta + 1/4)^2$, which must be large compared with the possible decrease in the gradient energy:

$$\Omega^2\left(\cos\beta + \frac{1}{4}\right)^2 \gg \frac{c^2}{\cos\beta + 1/4}\left[\frac{d(\cos\beta)}{dz}\right]^2. \tag{30}$$

If we now substitute the asymptotic expression (29) for the solution near $\beta = \theta_0$, we get the condition

$$\cos\beta + \frac{1}{4} \gg \frac{c^2}{\Omega^2\lambda^2}\frac{z}{\lambda} \tag{31}$$

for the solution derived using (16) to be valid. If we observe that $\cos\beta + 1/4 \approx 5\psi^2/16$ and use (29) once again, (31) takes the form

$$z/\lambda \leqslant 3 \quad \text{or} \quad \cos\beta + 1/4 \gg 10^{-3}$$

for typical parameter values. As $z < z_0$ decreases further, (16) starts to break down in a neighborhood $\beta \approx \theta_0$, and a transitional region forms in which β and Φ must be regarded as independent. Eventually (near the bottom of the chamber), β

becomes greater than θ_0. Although the behavior of β was not studied in detail for these z, it is clear that the departures from the solution considered above, in which $\beta < \theta_0$ everywhere, will be small. Indeed, the local shift in the precession frequency that occurs when $\beta > \theta_0$ can compensate the spatial variations of the Larmor frequency. The familiar expression[5]

$$\frac{16}{15}\frac{\Omega^2}{\omega_L}\left(\frac{1}{4}+\cos\beta\right)=(z-z_0)\nabla\omega_L$$

for the shift ($\beta > \theta_0$) may be used to determine when this can occur. As we move away from the wall ($z < z_0$), β should satisfy the above equation more and more clearly, i.e.,

$$\beta-\theta_0\approx\left(\frac{15}{16}\right)^{1/2}\frac{\omega_L\nabla\omega_L}{\Omega^2}(z_0-z)<\frac{\omega_L\nabla\omega_L}{\Omega^2}L.$$

We have $\beta_{\max} - \theta_0 < 1/10$ rad for typical parameter values. Moreover, the phase α of the precessing spins remains constant even though β depends on z when $\beta > \theta_0$. Indeed, Eq. (8) implies that

$$j_z^P=(u-1)\left[2c^2(u)\alpha'-c^2(-1)\Phi'\right], \tag{32}$$

and we have $\Phi \to 0$ in the limit, because any departure of Φ from 0 when $\beta > \theta_0$ will increase the energy by an amount comparable to the dipole energy U_D. Thus once again the condition $j_z^P = 0$ implies $\alpha' = 0$, or $\alpha =$ const.

The initially homogeneous precession is thus radically modified even by small field gradients; we can derive a corresponding lower bound on $\nabla\omega_L$ from the condition that $\lambda \lesssim L$, i.e.,

$$\nabla\omega_L \gtrsim c^2/\omega_L L^3.$$

Thus, gradients as small as $|\nabla H| \gtrsim 10^{-5}$–$10^{-6}$ Oe/cm can modify the precession in typical NMR experiments using ^{3}He. Since the residual gradients in these experiments are typically $\sim 10^{-2}$ Oe/cm, a precession domain structure should clearly be expected. Our initial assumption that $\nabla\omega_L =$ const is not important here; the above formulas will remain valid provided only that the characteristic length of the field variations is large compared to the thickness of the domain wall. However, we note that several domains may form and the domain walls may be curved if the spatial variations of the field are not controlled during the experiment.

3. DIPOLE ENERGY AND SPIN SUPERCURRENTS

The splitting of the magnetization precession into domains is a direct consequence of the spin supercurrents in ^{3}He-B. The possibility of spin transport unaccompanied by mass transport in superfluid ^{3}He-B was discussed in the literature shortly after it was established that triplet-state Cooper pairs are formed in ^{3}He (Ref. 12). Attempts were made to explain the unconventional features of the nuclear spin relaxation in the two ^{3}He superfluid phases by invoking simple spin-supercurrent models and drawing on the analogy with mass supercurrents (see Ref. 13 and the literature cited in Ref. 3). However, the analogy between spin and mass currents is quite limited. For one thing, spin the superfluid phases is generally violated because of the spin-orbit interaction. Furthermore, the equations of motion for the spin contain sources and sinks whose specific form depends on the dipole potential U_D. The B-phase is exceptional in this respect because, as we have already noted, U_D is independent of all the angles α, β, Φ; P is consequently a constant of the motion and obeys the usual conservation law (8). This fact permits us to consider the "flow" of P from one region of space to another.

We can use Eq. (8) to define the spin supercurrent velocity v_c^{sp} in a natural way. Recalling Eq. (32) for the z-component of the flux of P and defining $j_z^P = Pv_{sv}^{sp}$, we find that

$$v_{sz}{}^{sp}=\frac{1}{\omega_P}\left[2c^2(u)\alpha'-c^2(-1)\Phi'\right]. \tag{33}$$

The other components of v_s^{sp} vanish because the variables are independent of x and y. The velocity v_s^{sp} involves the difference of the gradients of the two angles α and Φ and can vanish even if α' and Φ' do not; this is another important difference between spin supercurrents and mass currents in HeII, for example. The spin-current states in ^{3}He-B are made up of various homogeneous states which are degenerate on a two-dimensional surface; the latter surface becomes a sphere if we make the change of variables (23). This constrasts with the case of mass currents in HeII, where the contributing states are initially degenerate on a circle.

It will also be helpful to discuss the formation of dynamic domains in ^{3}He-B from the viewpont of the stability of the spatially uniform spin precession. According to Ref. 10, the determinant

$$\Delta=\frac{\partial^2U_D}{\partial\Phi^2}\frac{\partial^2U_D}{\partial(\cos\beta)^2}-\left[\frac{\partial^2U_D}{\partial\Phi\,\partial\cos\beta}\right]^2$$

must be positive in order for the precession to be stable (here Δ is evaluated for β, Φ corresponding to a specified stationary solution). Since the intermediate case $\Delta = 0$ holds for the solutions given by (15) and (16), they describe a metastable precession which does not change spontaneously but is destablized even by very small nonuniform external perturbations.

We can picture the formation of a two-domain structure in a weakly nonuniform magnetic field as follows. The spins start to precess at the local Larmor frequency shortly after the application of the initial tipping rf pulse. Because ω_L depends on z, the magnetization spirals about the z axis and a gradient $\nabla\alpha$ is generated. The resulting current j_z^P [see Eq. (33)] carries p along the z direction. The chamber walls block the spin current, so that P increases at one wall and decreases at the opposite one. This continues until a state is reached in which j_z^P vanishes everywhere (cf. Sec. 2). The energy of the initial state is higher than for the final state; the latter is therefore not reached until the transient oscillations generated during the formation of the two-domain state have been damped.

4. RELAXATION OF THE TWO-DOMAIN STRUCTURE

The states that correspond to a spatially uniform spin precession in the B-phase do not relax for $\beta < \theta_0$ (Refs. 14 and 11), and this fact is also of great significance for the formation and observation of the two-domain structure. Indeed, because such states make up the two-domain structure, the latter can relax only to the extent that the precession is not spatially uniform. The relaxation is therefore quite slow and involves an expansion of the region in which the magnetization is parallel to the field ($\beta = 0°$) at the expense of a corresponding shrinkage of the region where $\beta = \theta_0$. The consequent motion of the domain wall toward weaker fields causes the precession frequency in the entire structure to decrease. This mechanism explains the change in the SDS frequency with time noted experimentally by Borovik-Romanov *et al.*[1]

In order to compare our results quantitatively with experiment[1,2] we will calculate the time derivative of the precession frequency in the hydrodynamic approximation, i.e., we will assume that $\omega_P \tau \ll 1$ and $l \ll \lambda$, where τ is the reciprocal of the collision frequency for the quasiparticles and l is their mean free path. Recalling that the SDS frequency ω_P is equal to the Larmor frequency at the wall: $\omega_P = \omega_L(z_0)$, we find that

$$\frac{d\omega_P}{dt} = \frac{d\omega_L}{dz}\frac{dz_0}{dt}.$$

We will calculate dz_0/dt by using the expression

$$\frac{dE}{dt} = -\int \left\{ D_{ik33}\frac{\partial S_i}{\partial z}\frac{\partial S_k}{\partial z} + \tau_{eff}\left(\frac{d\mathbf{S}}{dt} + g[\mathbf{H}\times\mathbf{S}]\right)^2 \right\} dV \tag{34}$$

for the rate at which energy is dissipated by the entire structure. The dissipative function in the right-hand side is the leading term in the expansion of dE/dt with respect to the spatial and time derivatives of $\mathbf{S}$. The integration extends over the volume of the measuring chamber, and the first term in the integrand describes the energy dissipation due to spin diffusion ($D_{ik\xi\eta}$ is the spin diffusion tensor). By symmetry (cf. Ref. 10), we can write the diffusion tensor in the form[2)]

$$D_{ik\xi\eta} = D_1\delta_{ik}\delta_{\xi\eta} + D_2(A_{i\xi}A_{k\eta} + A_{i\eta}A_{k\eta}), \tag{35}$$

where $A_{i\xi}$ is the instantaneous value of the order-parameter matrix at z, t. If we substitute $A_{i\xi} = R_{i\xi}(\alpha,\beta,\Phi)$ for the B-phase into (35) then into (34), we see that if all variables depend only on the z coordinate, only the isotropic component D_1 of the diffusion tensor appears in the dissipative term. Indeed, we have $S_i = |\mathbf{S}|R_{iz}$ for the solution given by (15), so that

$$R_{iz}\frac{dS_i}{dz} = |\mathbf{S}|R_{iz}\frac{dR_{iz}}{dz} = \frac{1}{2}|\mathbf{S}|\frac{d(R_{iz}R_{iz})}{dz} = 0.$$

If l_s is the characteristic length over which S varies in some region, then the contribution of this region to the dissipative function will be $\sim 1/l_s$. The contribution from the neighborhood of the domain wall is therefore particularly important. If we substitute the expression for the S_i in terms of α, β into (34), we find that

$$\left(\frac{dE}{dt}\right)_{\text{walls}} = -D_1\int \frac{\partial S_i}{\partial z}\frac{\partial S_i}{\partial z}dz = -\frac{D_1\omega_P^2}{\lambda}\sigma \tag{36}$$

per unit cross sectional area, where

$$\sigma = \int_{-\infty}^{+\infty} \left[\sin^2\beta\left(\frac{\partial\alpha}{\partial\xi}\right)^2 + \left(\frac{\partial\beta}{\partial\xi}\right)^2\right] d\xi$$

is ~ 1 and depends on the shape of the wall. If $D\omega/c^2$ is $\ll 1$, then the dissipative terms in the equations of motion will be small compared to the gradient terms and we can take the position z_0 of the domain wall, found in Sec. 2 by neglecting dissipation, as a first approximation. Numerical integration then leads to the value $\sigma_0 \approx 1.1$. Unfortunately, this approximation is not very accurate. Because c^2 tends to zero as $T \to T_c$ and $\omega_P\tau \to \infty$ as $T \to 0$, the hydrodynamic approximation is inconsistent with a small diffusion tensor except in a very narrow temperature interval. The dissipative terms in the equations of motion must therefore be included in order to find the shape of the domain wall. The coefficient σ will then differ from σ_0 and in general will depend on $D\omega_p/c^2$. We also expect that S will vary over distances $\sim\lambda$ near the chamber wall which bounds the precessing domain on the opposite side. The contribution of the chamber wall to the dissipative function will then be comparable to the contribution from the domain wall itself, and this will also affect σ. Thus, in order to calculate σ accurately one must include the dissipative terms and also analyze the bundary conditions on the wall of the measuring chamber. However, this constitutes a separate problem which we will not pursue here.

The second (volume) term in the dissipative function (34) describes the bulk relaxation of the structure by the Leggett-Takagi mechanism[14]; τ_{eff} is a phenomenological constant. Writing $\omega_L(z) = g\mathbf{H}_0(z)$, we get the expression

$$\left(\frac{dE}{dt}\right)_{LT} = -\tau_{eff}\int_0^L [\omega_P - \omega_L(z), \mathbf{S}]^2 dz = -\tau_{eff}S^2\int_0^L \sin^2\beta[\omega_P - \omega_L(z)]^2 dz \tag{37}$$

for the bulk dissipation. For a field with $|\nabla H| =$ const we have

$$\left(\frac{dE}{dt}\right)_{LT} = -\frac{5}{16}\tau_{eff}S^2z_0^3(\nabla\omega_L)^2 \tag{38}$$

if the width of the domain wall is neglected. To within the same approximation, the energy of the structure (relative to the equilibrium value) is given by

$$E = \int_0^L S^2(1-\cos\beta)dz = \frac{5}{4}S^2z_0. \tag{39}$$

Combining (36), (38), and (39), we get the equation

$$\frac{dz_0}{dt} = -\frac{4}{5}\frac{D_1}{\lambda}\sigma - \frac{1}{4}\tau_{eff}z_0^3(\nabla\omega_L)^2$$

for the velocity of the domain wall; multiplying by $\nabla\omega_L$, we obtain

$$\frac{d\omega_P}{dt} = -\frac{4}{5}\frac{D_1}{\lambda}\sigma(\nabla\omega_L) - \frac{1}{4}\tau_{eff}[\omega_P - \omega_L(0)]^3, \tag{40}$$

where small terms $\sim L(\nabla\omega_L)/\omega_p$ have been neglected.

An estimate reveals that the second term in Eq. (40) is approximately equal to $(z_0/\lambda)^2(z_0\nabla\omega_L/\omega_p)$ times the first; their ratio may thus be small or of order unity, depending on $\nabla\omega_L$ and the position z_0 of the domain wall. The first (diffusion) term becomes dominant as the structure relaxes and z_0 decreases, and the wall velocity approaches the constant value

$$\frac{4}{5}\sigma\frac{D_1}{c^{2/3}}(\nabla\omega_L)^{4/3}\omega_L^{1/3}.$$

The maximum wall velocity thus increases with $\nabla\omega_L$ as $(\nabla\omega_L)^{4/3}$.

We have $D\omega_p/c^2 \gg 1$ for T very close to T_c. No slowly decaying signals or two-domain structures were observed for $T \approx T_c$ in Refs. 1–4, and further study is needed in this case to determine how the magnetization moves in a weakly nonuniform magnetic field.

5. DISCUSSION

The formation of the dynamic domain structure can explain the basic features of the slowly decaying signal, as may be seen from the fact that Eq. (40) gives a satisfactory description of the time change of the SDS frequency found experimentally by Borovikov-Romanov *et al.*,[1] whose direct observations also demonstrated conclusively that two-domain structures do indeed exist.

In addition to generating the SDS, the relaxing dynamic domain structure provides a mechanism for magnetization relaxation in ^{3}He-*B* for tipping angles $\beta < \theta_0$. No other mechanisms leading to relaxation of a spatially uniform precession are currently known. Dynamic domains can also form in continuous-wave NMR experiments if the pump power is high enough to replenish the energy dissipated at the domain wall. This suggests that the formation of a domain structure may have been responsible for Webb's[16] and Osheroff's[17] experimental results, in which behavior similar to the nonlinear ferromagnetic resonance was observed in ^{3}He-*B*. These experiments detected a shift in the precession frequency which depended on the magnitude of the longitudinal component of the magnetization. Such shifts are not found in ^{3}He-*B* when the precession is spatially uniform but do occur during domain formation. The required shift agrees in order of magnitude with the estimated field gradients in the Webb-Osheroff experiments.[16,17] The results found there also revealed a lack of symmetry in the direction of the frequency shift, which can readily be understood by postulating a domain structure. Unfortunately, it is difficult to interpret these experiments unambiguously because no special measures were taken to regulate the field gradient.

In this paper we have analyzed in detail only the stationary two-domain structure in ^{3}He-*B*; neither the formation process nor the possible motions of the domain wall have been considered. Dynamic studies are of particular interest because of the important role played by the spin supercurrents. Further theoretical and experimental work is thus needed to analyze the oscillations of the domain structures and the motion of the domain walls.

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[1)]The reader may consult the paper by Borovik-Romanov, Bun'kov, Dmitriev, Mukharskiĭ, and Flachbart in the current issue of JETP for a more detailed discussion of these and other experimental results.
[2)]In the terminology of Ref. 15, $D_1 = D_\perp$ and $D_2 = D_\parallel - D_\perp$.

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