Cooperative effects in radiative de-excitation of quantum systems by electrons

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A new mechanism of x-ray and γ-ray production by passage of relativistic electrons through an excited medium is suggested, whereby the radiation is due to de-excitation of the quantum system as it interacts with the electrons. It is shown that the radiation yield can be appreciably increased by the correlation between the excited quantum systems contained in the medium.

INTRODUCTION

Interest has greatly increased recently in generation of high-power electromagnetic radiation in the x-ray and γ-ray bands. This interest is mainly due to the ever expanding use of these rays in purely scientific and in applied research.

At present the electromagnetic-radiation source that is best from the viewpoint of intensity, directivity, and polarization is synchrotron radiation. The spectral density of synchrotron radiation in the x-ray band (2–100 keV) is higher by six orders than that of bremsstrahlung and by three orders than that of the characteristic radiation of the best x-ray tubes. Generation of synchrotron radiation in the x-ray band, however, calls for powerful beams of high-energy relativistic electrons. Thus, for example, a 2–5 GeV accelerator is needed to obtain 50–100 keV synchrotron radiation. Another possible mechanism of generating x and γ rays is spontaneous emission of electrons and positrons channeled in a single crystal. Generation by this method is hindered considerably by the very high requirements imposed on the monochromaticity and angular directivity of the beam, as well as on the crystal quality.[1,2]

We propose here a new mechanism of x-ray and γ-ray generation by passing relativistic electrons through an excited medium. The radiation yield in the medium is greatly increased by the onset of correlations between excited quantum systems (the Dicke effect[3]). An electron interacting with an excited quantum system de-excites the latter and emits a bremsstrahlung photon.[4]

We show in this paper that the cross section for this process has a kinematic singularity corresponding to emergence of a virtual photon to the mass shell. In the ultrarelativistic limit the cross section of this process coincides in the vicinity of the pole with the cross section for Compton backscattering of electrons from real photons.

In §§1 and 2 we obtain the cross section for bremsstrahlung from a phonon upon de-excitation of a single quantum system. In §3 we show that cooperative effects can result from interaction of a relativistic electron with a medium consisting of excited quantum systems, and demonstrate the possibility of obtaining hard electromagnetic radiation of high spectral density.

§1. MATRIX ELEMENTS OF THE PROCESS

Consider an ensemble of N quantum systems. Assume that each such system has two energy levels and consists of A elements (A nucleons in a nucleus, A electrons in an atom, etc.). To be definite, we take a quantum system hereafter to mean an atom. The results that follow can be used without restrictions for other quantum systems.

Assume that a relativistic electrons passes through a medium consisting of excited atoms. Consider the interactions of the electron with an individual atom. They can be described by the following diagrams: The diagram of Fig. 1 describes inelastic scattering of the electron by an atom. If the atom excitation energy E_N is low compared with the initial electron energy, the excitation and de-excitation probabilities are approximately equal and the number of excited atoms remains constant. Since the process has no directivity in this case, the interatomic correlation that leads to the onset of cooperative effects is then insignificant.[5]

The diagram of Fig. 2 describes bremsstrahlung via virtual excitations of the atom. This process modifies substantially the spectrum of the bremsstrahlung of nonrelativistic electrons.[6] We note that in this process the photon energy coincides with the energy of the transitions in the atom. Therefore the spectral distribution of the bremsstrahlung of relativistic electrons changes little.

The diagram of Fig. 3 describes the "usual" bremsstrahlung. The matrix element corresponding to this process can be written in the form

$$M=\int\sum_{j=1}^{N}\sum_{i=1}^{A}\Psi_i(\mathbf{r}_e)\hat{H}_\gamma(\mathbf{r}_e)G(\mathbf{r}_e,\mathbf{r}_e')\Psi_f(\mathbf{r}_e')\times\hat{H}_{int}(\mathbf{r}_e',\mathbf{r}_{ij})\chi_I(\mathbf{r}_{ij})\chi_F(\mathbf{r}_{ij})\,d\mathbf{r}_e\,d\mathbf{r}_e'\,d\mathbf{r}_{ij}. \tag{1}$$

The sumation here is over all N atoms and over the A electrons contained in the atom; $G(\mathbf{r}_e,\mathbf{r}'_e)$ is the electron Green's functions; $\hat{H}_\gamma$ is the 2-photon emission operator; $\hat{H}_{int}$ is the Hamiltonian of the electromagnetic interaction of the electron with the atom:

$$\hat{H}_{int}=-\frac{1}{c^2}\int J(\mathbf{r})\frac{e^{i\omega|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}j(\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}', \tag{2}$$

where $j(\mathbf{r}')$ is the incident-electron 4-current operator and

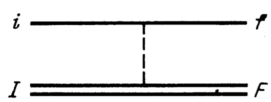

FIG. 1.

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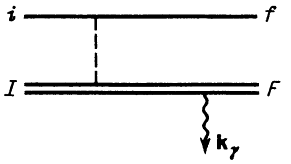

FIG. 2.

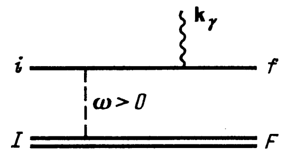

FIG. 3.

$J(\mathbf{r})$ is the atomic-electron 4-current operator.

We transform to the variables $\mathbf{r}'_{ij} = \mathbf{r}_{ij} - \mathbf{R}_j$, where $\mathbf{R}_j$, is the radius vector of the atom's centroid. The atomic wave functions can then be represented as products of the wave function $\Phi(\mathbf{R}_j)$ that describes the motion of the atom's centroid and $\varphi(\mathbf{r}'_{ij})$ that describes the motion of the ith electron contained in the j th atom:

$$\chi(\mathbf{r}_{ij}) = \Phi(\mathbf{R}_j)\varphi(\mathbf{r}_{ij}'). \tag{3}$$

Changing to the momentum representation, we obtain for the interaction between the electron and an ensemble of atoms (we use $\hbar = m = c = 1$)

$$M = \frac{4\pi i e^2}{(2\pi)^3}\left\{\int d\mathbf{q}\sum_{j=1}^{N}\langle\Phi_F(\mathbf{R}_j)|e^{i\mathbf{q}\mathbf{R}_j}|\Phi_I(\mathbf{R}_j)\rangle\right.$$
$$\times\frac{1}{q^2-\omega^2}\langle\varphi_f(\mathbf{r}_e)|\hat{H}_\gamma(\mathbf{r}_e)G(\mathbf{r}_e,\mathbf{r}_e')\alpha e^{i\mathbf{q}\mathbf{r}_e'}|\varphi_i(\mathbf{r}_e')\rangle$$
$$\times\sum_{i=1}^{A}\langle\varphi_F(\mathbf{r}_{ij}')|j^t e^{-i\mathbf{q}\mathbf{r}_{ij}'}|\varphi_I(\mathbf{r}_{ij}')\rangle$$
$$+\int d\mathbf{q}\sum_{j=1}^{N}\langle\Phi_F(\mathbf{R}_j)|e^{i\mathbf{q}\mathbf{R}_j}|\Phi_I(\mathbf{R}_j)\rangle\frac{1}{q^2}\cdot$$
$$\times\langle\varphi_f(\mathbf{r}_e)|\hat{H}_\gamma(\mathbf{r}_e)G(\mathbf{r}_e,\mathbf{r}_e')\rho(\mathbf{r}_e')e^{i\mathbf{q}\mathbf{r}_e'}|\varphi_i(\mathbf{r}_e')\rangle$$
$$\left.\times\sum_{i=1}^{A}\langle\varphi_F(r_{ij}')|\rho(r_{ij}')e^{-i\mathbf{q}r_{ij}'}|\varphi_I(r_{ij}')\rangle\right\}, \tag{4}$$

where j^t is the transverse part of the current of atomic electron, ρ is the charge density $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{k}$ is the vector momentum transferred to the atom, and α is a Dirac matrix. Equation (4) was derived under the condition that the velocity of the atomic electrons is much larger than the velocity of the atoms themselves.

The matrix element (4) describes the bremsstrahlung by an electron both with and without change of the state of the atom. Let us consider the kinematic relations for the case when the state of the atom is changed. The minimum momentum transfer is then $q = p_1 - p_2 - k$, where k is the energy of the bremsstrahlung photon; $E_2 = E_2 + \omega - k$, and ω is the transition energy in the atom (negative when the atom is excited and positive when it is de-excited). We use for simplicity the ultrarelativistic approximation (an exact analysis yields the same result):

$$p_1\approx E_1 - 1/2E_1,\ \ p_2\approx E_2 - 1/2E_2,\ \ q_{min}\approx(k-\omega)/2E_1 - \omega.$$

We see hence that the equality $q - \omega = 0$ can be satisfied only if $\omega > 0$, i.e., when the atom is de-excited. At $\omega < 0$ the matrix element in (4) has thus a pole singularity. The cause of the singularity is the possibility of emergence of a virtual photon to the mass shell, as a result of which the process in question can be represented as a set a two processes: radiative decay of the atom's excited state, followed by Compton scattering of the electron by the γ photon.

We note that the presence of a pole only at $\omega > 0$ makes the process directional: bremsstrahlung becomes preferred when the atom is de-excited. In a real situation the pole singularity is eliminated by introducing imaginary parts of the electron and atom energies. These widths result from the finite lifetimes τ_e and τ_A of the electron and atom, respectively, in a definite energy state. The value of τ_e is determined mainly by collisions of the electron with the atoms of the medium:

$$\tau_e = 1/\sigma_e v_e n,$$

where σ_e is the effective cross section for inelastic scattering of the electron, ve is its velocity, and n is the particle density in the medium. The atom lifetime depends on the probability of radiative spreading and the rate of energy transfer by collision. The total energy width contained in the pole denominator is thus given by

$$\Gamma = \Gamma_\gamma + \sigma_e v_e n + \sigma_A v_A n,$$

where σ_A is the cross section for inelastic collision of the atoms in the medium and v_A is their velocity.

Despite the presence of an initial width Γ, the process remains resonant ($\Gamma/\omega \ll 1$). We shall therefore consider hereafter only the bremsstrahlung accompanied by de-excitation of the atom.

Taking this into account, the matrix element (4) can be rewritten in the form

$$M = \frac{4\pi i e^2}{(2\pi)^3}\int d\mathbf{q}\sum_{j=1}^{N'}\frac{\langle\Phi_F(\mathbf{R}_j)|e^{i\mathbf{q}\mathbf{R}_j}|\Phi_I(\mathbf{R}_j)\rangle}{q^2-\omega^2-i\omega\Gamma}\mathfrak{M}_e$$
$$\times\sum_{i=1}^{A}\langle\varphi_F(\mathbf{r}_{ij}')|j^t e^{-i\mathbf{q}\mathbf{r}_{ij}'}|\varphi_I(\mathbf{r}_{ij}')\rangle. \tag{5}$$

The summation is now only over atoms in the excited state, and the electron matrix element is

$$\mathfrak{M}_e = \langle u_2|\Lambda_t|u_1\rangle,$$

where u_1 and u_2 are Dirac spinors and

$$\Lambda_t = \frac{\alpha e^*[E_2 + k + \alpha(\mathbf{p}_2 + \mathbf{k}) + \beta]\alpha}{2kd_2}$$
$$-\frac{\alpha[E_1 - k + \alpha(\mathbf{p}_1 - \mathbf{k}) + \beta]\alpha e^*}{2kd_1}. \tag{6}$$

Here β is a Dirac matrix, $\mathbf{e}$ the γ-photon polarization vector,

$$d_1=E_1-p_1\cos\theta_1,\quad d_2=E_2-p_2\cos\theta_2,$$

where θ_1 and θ_2 are the angles between the directions of the initial and final electron and γ photon.

It is convenient to write the matrix element in the form

$$M=\frac{4\pi ie^2}{(2\pi)^3}\int d\mathbf{q}\sum_{j=1}^{N'}\frac{\langle\Phi_F(\mathbf{R}_j)|e^{i\mathbf{q}\mathbf{R}_j}|\Phi_I(\mathbf{R}_j)\rangle}{q^2-\omega^2-i\omega\Gamma}M_j\mathfrak{M}_e,\tag{7}$$

where M_j is the atomic matrix element:

$$M_j=\sum_{i=1}^{A}\langle\varphi_F(\mathbf{r}_{ij}')|j^te^{-i\mathbf{q}\mathbf{r}_{ij}'}|\varphi_I(\mathbf{r}_{ij}')\rangle.$$

We represent it as a multipole expansion[7]

$$M_j=(2\pi)^{1/2}\sum_{i=1}^{A}\sum_{LM}\hat{L}\langle J_iM_iLM|J_fM_f\rangle\sum_{\mu=\pm1}\mu D_{M\mu}{}^L(\mathbf{q})$$
$$\times\,\xi_\mu[N_{if}(\mathbf{q};ML)+\mu N_{if}(\mathbf{q};EL)].\tag{8}$$

Here $\hat{L}=(2L+1)^{1/2}$; $D_{m\mu}{}^L$ is the Wigner D-function; N_{if} are the magnetic and electric interaction matrix elements defined in Ref. 7; ξ_μ are unit vectors of the spherical basis. We express now the probability of the investigated process in the form

$$dW=\frac{2\pi}{2J_i+1}\sum_{M_iM_f}|M|^2\frac{d\mathbf{p}_2\,dk}{(2\pi)^6},\tag{9}$$

where M_i and M_f are the magnetic quantum numbers of the initial and final states of the atom. The averaging and summation over the initial and final magnetic substates of the atom are easily performed and yield

$$dW=4\pi e^2\frac{2J_f+1}{2J_i+1}\frac{1}{(q^2-\omega^2-i\omega\Gamma)}$$
$$\times\sum_{LM}\left|\sum_{j=1}^{A}\langle\Phi_F(\mathbf{R}_j)|e^{i\mathbf{q}\mathbf{R}_j}|\Phi_I(\mathbf{R}_j)\rangle\right|^2\left[\mathfrak{M}_e{}^2-\frac{1}{q^2}(\mathfrak{M}_e\mathbf{q})^2\right]$$
$$\times[|N_{if}(\mathbf{q};ML)|^2+|N_{if}(\mathbf{q};EL)|^2]\frac{d\mathbf{p}_2\,d\mathbf{k}}{(2\pi)^6}.\tag{10}$$

Introducing the radiative width of an isolated atom

$$\Gamma_\gamma=\frac{2J_f+1}{2J_i+1}8\pi\omega\sum_L\{|N_{if}(\mathbf{q};ML)|^2+|N_{if}(\mathbf{q};EL)|^2\},\tag{11}$$

we obtain for the cross section of the process

$$dW=\frac{\Gamma_\gamma}{(q^2-\omega^2-i\omega\Gamma)^2}\frac{1}{\omega}\left|\sum_{j=1}^{N'}\langle\Phi_F(\mathbf{R}_j)|e^{i\mathbf{q}\mathbf{R}_j}|\Phi_I(\mathbf{R}_j)\rangle\right|^2$$
$$\times\left[\mathfrak{M}_e{}^2-\frac{1}{q^2}(\mathfrak{M}_e\mathbf{q})^2\right]\frac{d\mathbf{p}_2\,d\mathbf{k}}{(2\pi)^6}.\tag{12}$$

The cross section for scattering of an electron by an individual atom can be obtained from the probability in the usual manner:

$$d\sigma_\gamma=\frac{dW}{jN'}=\frac{dW_0}{jN'}P_k(\mathbf{q}),\tag{13}$$

where j is the incident-particle current density,

$$dW_0=\frac{1}{(2\pi)^2}\frac{\alpha^2}{k}\left[\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2-2k\left(\frac{1}{d_1}-\frac{1}{d_2}\right)\right.$$
$$\left.+k^2\left(\frac{d_2}{d_1}+\frac{d_1}{d_2}\right)\right]\frac{p_2}{p_1}\frac{1}{(q^2-\omega^2-i\omega\Gamma)^2}d\Omega_1\,d\Omega_2,\tag{14}$$
$$d\Omega_i=\sin\theta_i\,d\theta_i\,d\varphi_i\quad(i=1,2),\quad\alpha=1/137;$$

$$P_k(\mathbf{q})=\left|\sum_{j=1}^{N'}\langle\Phi_F(\mathbf{R}_j)|e^{i\mathbf{q}\mathbf{R}_j}|\Phi_I(\mathbf{R}_j)\rangle\right|^2.\tag{15}$$

The cross section of the process is thus represented by a single process multiplied by a correlation factor P_k, the latter determined by the cooperative properties of the medium and ranging from N to N^2, depending on the degree of correlation. We note that the presence of a sharp maximum of the cross section permits direct integration with respect to the angles φ, θ_1, and θ_2.

§2. TOTAL CROSS SECTION PER ATOM

We integrate first with respect to the angle φ, and get

$$\frac{d\sigma}{dk}=\frac{\Gamma_\gamma\alpha^2}{k\omega2\pi}F(\theta_1,\theta_2)$$
$$\times\frac{p_2}{p_1}\int_0^{2\pi}\frac{d\varphi}{(q^2-\omega^2)^2+\Gamma^2\omega^2}\sin\theta_1\sin\theta_2\,d\theta_1\,d\theta_2,\tag{16}$$

where

$$F(\theta_1,\theta_2)=\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2-2k\left(\frac{1}{d_1}-\frac{1}{d_2}\right)+k^2\left(\frac{d_1}{d_2}+\frac{d_2}{d_1}\right).$$

Transforming to the variable q^2 and integrating with respect to it we obtain

$$\frac{d\sigma}{d\Omega_1\,d\Omega_2}=\frac{\Gamma_\gamma}{\Gamma}\frac{\alpha^2}{k\omega^2}\frac{F(\theta_1,\theta_2)}{(A+Bd_2+Cd_2{}^2)^{1/2}}dk,\tag{17}$$

where the coefficients A, B, and C are given by

$$A=-d_1{}^2[(E_2+k)-1]-2\omega d_1,$$
$$B=2[E_1E_2d_1+(kd_1-1)(\omega+d_1)],$$
$$C=1-(E_1-k)^2-2kd_1.$$

The region of integration over θ_2 is governed by the positiveness of the radicand in (17), and from the condition that the integral exist we obtain a restriction on the ranges of the variable d_1:

$$E_1/2\leqslant d_1\leqslant2\omega E_1/k\tag{18}$$

and of the angle θ_1:

$$\frac{p_1{}^2+k^2-(p_2+\omega)^2}{2p_1k}\leqslant\cos\theta_1\leqslant\frac{p_1{}^2+k^2(p_2-\omega)^2}{2p_1k}.\tag{19}$$

The energy of the bremsstrahlung γ photon should satisfy in this case the condition

$$\omega\leqslant k\leqslant4E_1{}^2\omega/(1+4E_1\omega).\tag{20}$$

Conditions (18)–(20) coincide with the requirements imposed on θ_1 and k in the inverse Compton effect. This in not accidental. The point is that we have used the pole approximation, in which a virtual photon emerges to the mass shell and becomes real. Kinematic relations that are valid for the real process of γ-quantum scattering by a moving electron are therefore satisfied in our process. Integration of the cross section (17) with respect to θ_2 is elementary and yields

$$\frac{d\sigma}{dk}=\frac{\Gamma_\gamma\alpha^2\pi}{\omega^2\Gamma k p_1^2}\left\{\frac{1}{(-C)^{1/2}}\left(\frac{1}{d_1^2}-\frac{2k}{d_1}\right)\right.$$

$$+\frac{1}{(-A)^{1/2}}\left(2k-\frac{2}{d_1}+k^2d_1\right)$$

$$\left.+\frac{B}{2}\left[\frac{1}{(-A^3)^{1/2}}+\frac{k^2}{d_1}\frac{1}{(-C^3)^{1/2}}\right]\right\}. \tag{21}$$

In the ultrarelativistic approximation, assuming E_1, $E_2 \gg 1, \omega E_1 \ll 1, k \ll E_1E_2$, we easily obtain from (21)

$$\frac{d\sigma}{dk}=\frac{\Gamma_\gamma\pi\alpha^2}{\Gamma\omega^2kp_1^2}\left\{\left(\frac{1}{d_1^2}-\frac{2k}{d_1}\right)\right.$$

$$+\left(2k-\frac{2}{d_1}+k^2d_1\right)\left(1+\frac{\Delta}{2B^2}\right)\frac{1}{x_0}$$

$$\left.+\frac{k^2}{d_1}x_0+\frac{1}{x_0^2}\left(1+\frac{\Delta}{4AC}\right)\left(1+\frac{\Delta}{2B^2}\right)\right\}I_0\,d\Omega_1, \tag{22}$$

where

$$I_0=\int\frac{d(d_2)}{(A+Bd_2+Cd_2^2)^{1/2}}=\frac{\pi}{(-C)^{1/2}},\qquad \Delta=B^2-4AC,$$

$$\Delta=4p_1^2\sin^2\theta_1[p_2^2\omega^2-(E_2\omega-kd_1)^2]\approx 2d_1(k\omega E_1E_2)^{1/2},$$

$$x_0=-B/2C$$

and the conditions $\Delta/B^2 \ll 1, \Delta/AC \ll 1$ are satisfied. Integration of (22) with respect to the angle θ_1, between the limits determined by (19) is elementary and yields

$$\frac{d\sigma}{dk}=\frac{\Gamma_\gamma\alpha^2\pi}{\Gamma\omega^2\cdot 2p_1^2}\left\{\frac{2k}{p_1}-\frac{k}{\omega p_1^3}+\frac{4k}{p_1}\ln\frac{k}{4\omega p_1^2}+8\omega p_1\right\}. \tag{23}$$

The expression obtained for the cross section can be represented as a product of the factor Γ_γ/Γ by the cross section for the inverse Compton effect in the case of scattering of ultrarelativistic electrons by low-energy γ rays.[8]

Integration of (23) with respect to the γ-ray energy yields

$$\sigma=\sigma_T\Gamma_\gamma/\Gamma, \tag{24}$$

where σ_T is the Thompson-scattering cross section. We must note here once more that this behavior of the cross section is the result of the resonant character of the process. Comparing expression (23) for the cross section with the cross section for ordinary bremsstrahlung, it should be noted that the latter is proportional to $r_0^2 \propto Z^2$ whereas the cross section of the investigated process is proportional to r_0^2 and therefore exceeds the cross section for ordinary bremsstrahlung in the case of small $Z \lesssim 10$.

§3. INFLUENCE OF MEDIUM ON RADIATIVE DE-EXCITATION

The influence of the medium on the investigated process is determined by the presence of the correlation factor P_k in the cross section (13). If the distance between the atoms $a_0 > \lambda$ (λ is the atom's spontaneous-emission wavelength), there is no correlation between the atoms and $P_k = N$. We shall therefore assume that $a_0 < \lambda$ and that spontaneous superradiance can occur in the system.

Let the ensemble of excited atoms have a volume V in the form of a right parllelepiped with sides $L_x, L_y, L_z (L_y = L_z = D$. It is known[3] that no cooperative properties are manifested in a large-size system, so that it is of interest to consider two cases.

1. $L_x, D \ll \lambda, e^{i\mathbf{qR}_j} \approx 1$. In this case the electromagnetic field of the electron exerts the same action on all the atoms of the sample.

The wave function of a system of N particles is characterized by quantum numbers r and m, where r is the "cooperative" quantum number, and $m = (n_+ - n_-)/2$ (n_+ and n_- are the numbers of the atoms in the excited and ground states). In this case we can write for the radiation cross section

$$\sigma=\sigma_0(\Gamma)(r+m)(r-m+1), \tag{25}$$

where $\sigma_0(\Gamma)$ is the integral bremsstrahlung cross section in the case of de-excitation of an isolated atom, and is given by Eq. (24). This cross section is a function of the total width Γ of the radiation energy. As already noted, Γ is determined by the sum of the radiative and collisional widths. The radiative width of the system considered is in turn also dependent on the quantum numbers r and m:

$$\Gamma_\gamma=\Gamma_{\gamma_0}\frac{1}{N}(r+m)(r-m+1), \tag{26}$$

where Γ_{γ_0} is the radiative width of the isolated atom. Substituting (24) in (25) and taking (26) into account, we obtain

$$\sigma=\sigma_0=\frac{1}{N}\frac{\Gamma_{\gamma_0}(r+m)(r-m+1)}{\Gamma_e+\Gamma_{\gamma_0}(r+m)(r-m+1)/N}. \tag{27}$$

It can be seen that the maximum of the cross section is reached at $r = N/2$ and $m = 0$:

$$\sigma_{max}=\sigma_k\frac{\Gamma_{\gamma_0}N^2/4}{\Gamma_e+\Gamma_{\gamma_0}N/4}\frac{1}{N}. \tag{28}$$

If N is large enough, we can neglect Γ_e in the denominator and the cross section of our process coincides with the cross section of the inverse Compton effect.

The dependence on n drops out. Thus, allowance for the cooperative effects makes the cross section in this case no longer dependent on the width Γ_e and Γ_γ or on the density of the medium. The reason is that for a system of small size the spontaneous-emission probability and the probability of our process have the same cooperative amplification coefficients. They are therefore cancelled out in the numerator and denominator of (28).

2. $L_x \ll \lambda, D \gg \lambda$. In this case the spontaneous-emission line width is given by[9]

$$\Gamma_\gamma=\frac{3\Gamma_{\gamma_0}}{8\pi}\frac{N\lambda^2}{D^2}.$$

Equation (13) can in the general case not be integrated for a system of arbitrary size. We consider therefore a definite range of the angles θ_1 and θ_2.

The point is that for an extended system coherence sets in only when the condition $\mathbf{q}\cdot\mathbf{R} \ll \pi$ is satisfied (for all R). We resolve $\mathbf{q}\cdot\mathbf{R}$ into two components, parallel and perpendicular to the sample length L_x: $\mathbf{q}\cdot\mathbf{R} = q_\perp R_\perp + q_\parallel R_\parallel$. The presence of a pole leads to $|q| = \omega$ and for the superradiant state to set in the condition $q_\parallel \lambda \lesssim \pi$ or $\omega \sin\delta D \lesssim \pi$ must be satisfied (δ is the angle between the direction of the vector **q** and the Z axis). Thus, the existence of superradiance is determined in this case by the direction of the momentum-transfer vector **q** ($\sin\delta \leqslant \pi/\omega D$). The direction of the vector **q** depends in turn on the energy of the emitted γ photon.

From elementary kinematic considerations we can write

$$E_\gamma = \frac{\omega[1-(v/c)\cos\theta_1]}{1-(v/c)\cos\theta_2 + \omega(1-\cos\theta)/E_1}, \quad (29)$$

where θ_1, θ_2 and θ are respectively the angles between the vectors $\mathbf{p}_1$ and **q**, $\mathbf{p}_1$ and **k**, and **q** and **k**. In our case, when a relativistic electron is incident on a system consisting of two-level atoms, expression (29) can be simplified ($\omega/E_1 \ll 1$):

$$E_\gamma = \frac{\omega[1-(v/c)\cos\theta_1]}{1-(v/c)\cos\theta_2}. \quad (30)$$

Assume that the initial momentum p_1 of the electron is directed along the Z axis. The energy band ΔE_γ in which the coherent properties will manifest themselves will then depend on the angle in the following manner:

$$\frac{\omega[1-(v/c)\cos(\arcsin(\pi/\omega D))]}{1-(v/c)\cos\theta_2} \leqslant E_\gamma \leqslant \frac{\omega(1-v/c)}{1-(v/c)\cos\theta_2}. \quad (31)$$

Provided that $D\omega \gg \pi$, the expression for the energy range ΔE_γ can be written in the form

$$\Delta E_\gamma = \pi^2 [D^2 \omega [1-(v/c)\cos\theta_2]]^{-1}. \quad (32)$$

In the energy interval ΔE_γ the cross section for our process increases thus by $N\Gamma_{\gamma_0}/\Gamma_\gamma$ times. In turn, we have the ratio $\Gamma_{\gamma_0}/\Gamma = (8\pi/3)(D^2/\lambda^2)$ (we assume here as before that the collision width Γ_e can be neglected). Thus, the cross section for radiative de-excitation in an extended object is decreased by a factor D^2/λ^2 compared with the case when there is no correlation between the atoms.

Knowing the cross section, it is easy to determine the total number of photons emitted by one electron

$$N_\gamma = \sigma(\omega)\lambda n \frac{8\pi}{3}\frac{D^2}{\lambda^2}\Delta E_\gamma. \quad (33)$$

The spectral density is

$$S_\gamma = \frac{8\pi}{3}\sigma(\omega) n\lambda \frac{D^2}{\lambda^2}. \quad (34)$$

The number of photons emitted at a wavelength $\lambda = 1$ Å on account of radiative de-excitation is higher by several orders than the number of synchrotron-radiation photons of the same frequency, viz., 10^{19} photons/mA·rad·Å·s. It was assumed in the calculations that the energy of the incident electrons is $E_1 = 150$ MeV and the atom excitation energy is $\omega = 0.1$ eV; the sampledimensions are $D = 10$ cm and $L_x = 10^{-3}$ cm, and the particle density in the medium is $n = 10^{19}$ cm^{-3}.

The radiation yield was calculated for one target. It should be noted that the number of photons can be increased by using a packet of targets with spacing greatly exceeding the radiation wavelength. In this case the condition for the onset of cooperative effects in each layer is not violated, and the total photon yield increases appreciably.[10]

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