Tunnel-current oscillations in a transverse magnetic field

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(Submitted 25 December 1984)
Zh. Eksp. Teor. Fiz. **88**, 1898–1905 (May 1985)

A symmetric tunnel junction in a transverse magnetic field is analyzed for the case in which the characteristic electron energy at which the tunnel transmission of the barrier changes is low in comparison with the Fermi energy. The tunnel current oscillates as a function of the magnetic field and of the applied voltage. The period of the oscillations in the voltage is twice the ordinary period, while the period of the oscillations in the inverse magnetic field has the usual value. As the voltage is increased, these oscillations broaden and eventually disappear.

1. INTRODUCTION

The tunneling of electrons in metal-insulator-semiconductor structures in a quantizing magnetic field has been studied in several experiments (e.g., Refs. 1–3). These experiments have been carried out to determine how the magnetic field or the applied voltage influences the quantum oscillations of the tunnel current which stem from the particular features of the state density in the Landau levels of the semiconductor. Oscillations have been observed in both longitudinal (with respect to the current) and transverse magnetic fields. Under the conditions of these experiments the tunnel transmission of the barrier was apparently identical for all electrons whose tunneling was allowed by the conservation laws. In other words, the inequality $E_0 > E_F$ held, where E_F is the Fermi energy of the semiconductor, and E_0 is a characteristic value of the electron energy at which the tunnel transmission of the barrier changes. If this inequality does not hold, the picture of tunneling in a quantizing magnetic field changes substantially, as we will show below.

We will analyze tunneling in a quantizing magnetic field in the case of a symmetric junction (Fig. 1a). We assume $\hbar\Omega \ll E_0 \ll E_F$, where $\hbar\Omega$ is the distance between Landau levels. A decisive factor here is the strong dependence of the barrier transmission coefficient on the electron energy. Because of this strong dependence, the tunnel current is dominated by those electrons which are incident along the normal to the plane of the barrier at velocities near the Fermi velocity.

It is easy to see that under these conditions there should be no quantum oscillations in a longitudinal field. In fact, the oscillations are usually the result of a crossing of Landau levels by the Fermi level. The normal projections of the momenta of the electrons in these Landau levels are small, so that these electrons undergo essentially no tunneling. Most of the tunnel current is carried out by electrons from low-lying Landau levels ($N\hbar\Omega \sim E_0$), which lie well below the Fermi level.[1]

In contrast, in a transverse field (i.e., if the magnetic field vector lies in the plane of the barrier), most of the electrons which undergo tunneling are electrons which are moving in a "boundary layer" and which, as they move through the magnetic field, are reflected repeatedly from the plane of the barrier (Fig. 1b). Of all possible trajectories of these boundary-layer electrons, the optimum trajectories are evidently those on which an electron traces out a semicircle between two successive collisions with the barrier. Such electrons are incident normally on the barrier. It is also clear that the tunneling transmission is maximized when the energy of these electrons associated with the motion across the field is a maximum, i.e., is equal to the Fermi energy E_F.

We will show that in a transverse field and under the condition $\hbar\Omega \ll E_0 \ll E_F$ there are oscillations in the tunnel current. What makes these oscillations unusual is that their period in the voltage is twice the ordinary period, i.e., is given by $\Delta eV = 2\hbar\Omega$. The reason for this relation is that for the electrons which are effectively in a boundary layer the distance between the magnetic levels is twice the distance between Landau levels. The period of the oscillations in the inverse magnetic field, in contrast, has its usual value: $\Delta/\hbar\Omega = 1/E_F$.

2. DERIVATION OF AN EQUATION FOR THE TUNNEL CURRENT

The tunnel current which flows across the magnetic field, like the ordinary conduction current, differs from zero only because of collisions, since there exist no quantum-mechanical states with a nonzero expectation value of the current across the field. For the problem of the present paper,

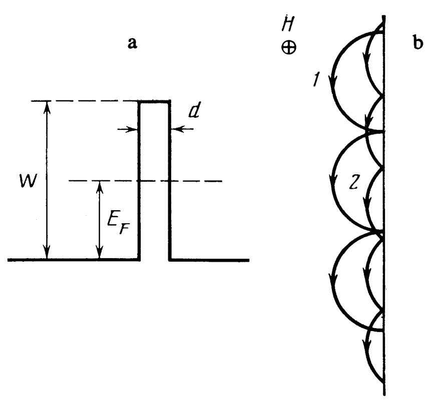

FIG. 1. a—Symmetric tunnel junction in the absence of an applied voltage; b—trajectories of boundary-layer electrons in a transverse magnetic field. The tunnel current is dominated by the trajectories of type 1.

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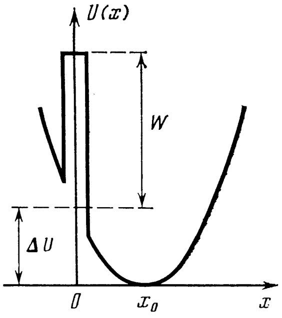

FIG. 2. Resultant potential energy for the transverse motion of an electron. Here x_0 is the center of the Landau oscillator.

this circumstance has the following implications. The transverse motion of an electron can be described as a 1D motion with a potential energy equal to the sum of the "magnetic potential"

$$U(x)=m\Omega^2(x-x_0)^2/2,$$

where x_0 is the center of the Landau oscillator, and the barrier potential (Fig. 2). Because of the finite transmission of the barrier, the electron wave function is a coherent superposition of states in the wells at the left and the right. An electron goes periodically from the well at the left to that at the right and back, so that no current flows in the absence of collisions. If, however, collisions occur at a rate far higher than the transition frequency (and we assume below that this condition holds, in accordance with the actual situation), a complete loss of coherence will result (the frequency of the transitions between the wells is determined by the transmission of the barrier). Under these conditions, collisions remove an electron from the vicinity of the barrier before it has time to tunnel back through the barrier. The tunnel current from one well to the other can then be expressed in the usual way in terms of the barrier transmission.

We assume that the barrier is in the yz plane (Fig. 2) and that the magnetic field is directed along the z axis. We characterize the states of an electron in the two half-spaces $x>0$ and $x<0$ by the quantum numbers $k_z, x_0=\lambda^2 k_y$, and N, where k_z and k_y are the projections of the wave vector onto the plane of the barrier, λ is the magnetic length, and N is the index of the magnetic energy level. The values of k_z and x_0 are evidently conserved during the tunneling, so we can write the following expression for the tunnel current:

$$J=C\int_{-\infty}^{\infty}dk_z\int_{-\infty}^{\infty}dx_0\sum_{N,N'}D_N(x_0)\,\delta(\varepsilon_N{}^+(x_0)-\varepsilon_{N'}{}^-(x_0)+eV)$$
$$\times\left[f\left(\varepsilon_N{}^+(x_0)+\frac{\hbar^2k_z{}^2}{2m}\right)-f\left(\varepsilon_{N'}{}^-(x_0)+\frac{\hbar^2k_z{}^2}{2m}\right)\right].\qquad(1)$$

The constant C will be defined below; $D_N(x_0)$ is the transmission coefficient of the barrier; $\varepsilon_N{}^+(x_0)$ and $\varepsilon_N{}'^-(x_0)$ are the magnetic energy levels of an electron in the right and left half-spaces, respectively; V is the applied voltage; and f is the Fermi function (we are assuming a zero temperature).

Let us examine the expression $D_N(x_0)$. This coefficient obviously depends on only the difference between the barrier height $W+\Delta U$ and the transverse electron energy $\varepsilon_N^+(x_0)$, where W is the barrier height in the absence of a field, and $\Delta U=m\Omega^2x_0^2/2$ is the extent to which the barrier rises in a magnetic field (Fig. 2). Within the coefficient of the exponent[21] function we can thus write

$$D_N(x_0)=\exp\{-F(W+\Delta U-\varepsilon_N{}^+(x_0))\},\qquad(2)$$

where the function F depends on the shape of the barrier. As we mentioned in the Introduction, we are considering the case in which the tunnel current is determined by electrons with transverse energies close to the Fermi energy E_F. For such energies we can write

$$D_N(x_0)=D_0\exp\left[-\frac{E_F-\varepsilon_N{}^+(x_0)+\Delta U}{E_0}\right],\qquad(3)$$

where

$$D_0=\exp(-F(W-E_F)),\quad E_0=[F'(W-E_F)]^{-1},$$

and the parameter E_0 determines the energy dependence of the transmission coefficient. In particular, for a square barrier of thickness d we would have

$$E_0=((W-E_F)\hbar^2/2md^2)^{1/2}.$$

We assume that the following conditions hold:

$$\hbar\Omega\ll E_0\ll E_F.\qquad(4)$$

In (2) and (3) we are ignoring the change caused in the transmission coefficient by the change in the shape of the barrier when the voltage V is applied, under the assumption that this change is quite small.

The energy levels $\varepsilon_N^\pm(x_0)$ in (1) should be calculated for an infinitely high barrier, i.e., under the assumption that the wave function vanish at the barrier (at $x=0$). It is clear that the relation $\varepsilon_N^+(x_0)=\varepsilon_N^-(-x_0)$ holds. As $x_0\to\infty$, the energy levels $\varepsilon_N^+(x_0)$ become the Landau volume levels $\varepsilon_N^+(\infty)=(N+1/2)\hbar\Omega$. At $x_0=0$, the values of $\varepsilon_N^+(0)$ and $\varepsilon_N^-(0)$ are the same, and the distances between adjacent levels are known to be equal to twice the distance between Landau levels. $\varepsilon_N^+(0)=\varepsilon_N{}^-(0)=(2N+3/2)\hbar\Omega$. Electron states with small values of x_0 [small in comparison with the Larmor radius $(2N)^{1/2}\lambda$] correspond to the classical trajectories of the boundary-layer electrons, consisting of arcs which are approximately semicircles. These are the states which dominate the tunnel current, by virtue of condition (4). We also see from this condition that the values of N which are importent are large values, $N\sim E_F/\hbar\Omega\gg1$, so that we can use the semiclassical approximation to calculate $\varepsilon_N^\pm(x_0)$.

To transform expression (1) for the tunnel current we substitute (3) into it and integrate over k_z and x_0, finding

$$J=2CD_0\left(\frac{2m}{\hbar^2}\right)^{1/2}\sum_{N,N'}\frac{(E_F-\varepsilon_N{}^+(x_0))^{1/2}-(E_F-\varepsilon_{N'}{}^-(x_0))^{1/2}}{|\partial\varepsilon_N{}^+/\partial x_0-\partial\varepsilon_{N'}{}^-/\partial x_0|}$$
$$\times\exp\left[-\frac{E_F-\varepsilon_N{}^+(x_0)+{}^1/{}_2\,m\Omega^2x_0{}^2}{F'}\right].\qquad(5)$$

For given values of N and N', the value of x_0 in this expression can be found from the condition

$$\varepsilon_N{}^+(x_0)-\varepsilon_{N'}{}^-(x_0)+eV=0,\qquad(6)$$

which corresponds to the vanishing of the argument of the δ-

function in expression (1). Since the value of x_0 which are important are small values, the derivatives $\partial\varepsilon/\partial x_0$ can be evaluated at $x_0=0$. This calculation, in the semiclassical approximation, yields

$$\left(\frac{\partial\varepsilon_N{}^+}{\partial x_0}\right)_{x_0=0}=-\left(\frac{\partial\varepsilon_N{}^-}{\partial x_0}\right)_{x_0=0}=-\frac{4\Omega}{\pi}(Nm\hbar\Omega)^{1/2}. \qquad (7)$$

Our next step is to calculate the energies $\varepsilon_N^+(x_0)$ and $\varepsilon_{N'}^-(x_0)$ at the value of x_0 which satisfies condition (6). We take the following approach. We write $N+N'=n$ and $N-N'=k$, where n and k are integers of identical parity. We also introduce

$$E_{n,k}=\varepsilon_N{}^+(x_0)+\frac{eV}{2}=\varepsilon_{N'}{}^-(x_0)-\frac{eV}{2}\,. \qquad (8)$$

In terms of this new notation, the semiclassical quantization conditions in the right and left wells take the following forms, respectively:

$$\int_0^a dx\,p\left(x,E_{n,k}-\frac{eV}{2}\right)=\left(\frac{n+k}{2}+\frac{3}{4}\right)\pi\hbar, \qquad (9)$$

$$\int_b^0 dx\,p\left(x,E_{n,k}+\frac{eV}{2}\right)=\left(\frac{n-k}{2}+\frac{3}{4}\right)\pi\hbar, \qquad (10)$$

where a and b are the turning points in the right and left well, respectively, and the semiclassical momentum is

$$p(x,\ \varepsilon)=[2m(\varepsilon-{}^1/{}_2m\Omega^2(x-x_0)^2)]^{1/2}. \qquad (11)$$

Relations (9) and (10) constitute a system of equations determining $E_{n,k}$ and x_0. For our purpose below it is sufficient to find $E_{n,k}$ and x_0 to within terms of respectively first and zeroth order in V. Adding and subtracting Eqs. (9) and (10), and expanding their left sides in series in x_0 and V, we find

$$\frac{\pi E_{n,k}}{\Omega}-eVx_0\left(\frac{m}{2E_{n,k}}\right)^{1/2}=\left(n+\frac{3}{2}\right)\pi\hbar,$$

$$2x_0(2mE_{n,k})^{1/2}=k\pi\hbar. \qquad (12)$$

At the accuracy specified above we then find

$$E_{n,k}=\left(n+\frac{3}{2}\right)\pi\hbar+\frac{eVk}{4n},\quad x_0=\frac{k\pi\hbar}{(8mn\hbar\Omega)^{1/2}}. \qquad (13)$$

Expressions (13) hold under the conditions $k\ll n, n\gg 1$ and $eV\ll k\hbar\Omega$. As we will see below, the values of n and k which are important are those which satisfy

$$n\sim E_F/\hbar\Omega,\quad k\sim(E_0E_F)^{1/2}/\hbar\Omega.$$

Consequently, the applicability of expressions (13) is guaranteed by inequality (4) and the condition $eV\ll(E_0E_F)^{1/2}$.

We can now derive an explicit expression for the tunnel current from (5). Using (7), (8), and (13), and switching to a summation over n and k, we find

$$J=C_1\sum_{n,k}n^{-1/2}\left[\left(E_F-n\hbar\Omega+\frac{eV}{2}\left(1-\frac{k}{2n}\right)\right)^{1/2}\right.$$
$$\left.-\left(E_F-n\hbar\Omega-\frac{eV}{2}\left(1+\frac{k}{2n}\right)\right)^{1/2}\right]$$
$$\times\exp\left[-\left(E_F-n\hbar\Omega+\frac{\pi^2\hbar\Omega k^2}{16n}\right)E_0{}^{-1}\right], \qquad (14)$$

where C_1 is a new constant. In (14) we replaced $n+3/2$ by n by virtue of the condition $n\gg 1$, and we discarded a term of order eV/E_0 in the argument of the exponential function. The constant C_1 can be determined by examining the weak-field limit, $\Omega\to 0$. In this limit we have $J=V/R$, where R is the resistance of the tunnel junction. To relate the constant C_1 to this resistance, we switch from a summation over n and k to an integration (bearing in mind that n and k are of identical parity). We then find

$$C_1=(\hbar\Omega)^{3/2}/eE_0R. \qquad (15)$$

We can see from (14) that the derivative of the tunnel current with respect to the voltage in a transverse magnetic field has singularities corresponding to different values of n and k. At each value of n there are components in the tunnel current from a large number of terms corresponding to different values of k. The actual number is on the order of $(E_0E_F)^{1/2}/\hbar\Omega\gg 1$. The singularities associated with different values of k (at a fixed n) are packed very closely together. Along the voltage scale, for example, the distance between adjacent structural features, $e\Delta V$, is on the order of $(\hbar\Omega)eV/E_F\ll\hbar\Omega$, while along the scale of the inverse magnetic field this distance is $\Delta/\hbar\Omega\sim eV/E_F^2\ll 1/E_F$. Actually these singularities will be smeared out by the customary broadening factors which we are not discussing here. We can therefore switch from a summation over k in (14) to an integration. We then find the following expression for the differential conductance $G=\partial J/\partial V$:

$$G=\frac{\hbar\Omega}{2R(\pi E_0\Gamma)^{1/2}}\sum_n[\Phi(y_n{}^+)+\Phi(y_n{}^-)]\exp\left(\frac{n\hbar\Omega-E_F}{E_0}\right), \qquad (16)$$

$$y_n{}^\pm=\frac{E_F-n\hbar\Omega\pm eV/2}{\Gamma},\quad \Gamma=\frac{|eV|}{\pi}\left(\frac{E_0}{E_F}\right)^{1/2}. \qquad (17)$$

The function Φ is

$$\Phi(y)=\frac{1}{\sqrt{\pi}}\int_{-y}^{\infty}\frac{dz\,e^{-z^2}}{(y+z)^{1/2}}. \qquad (18)$$

In the limit $y\to\infty$ we have $\Phi(y)\approx y^{-1/2}$, and as $y\to-\infty$ we have $\Phi(y)\sim e^{-y^2}/(2|y|)^{1/2}$. It can be seen from (16)–(18) that the broadening of a singularity is proportional to V. With $V=0$ we find from (16)

$$G=\frac{\hbar\Omega}{R(\pi E_0)^{1/2}}\sum_n(E_F-n\hbar\Omega)^{-1/2}\exp\left(\frac{n\hbar\Omega-E_F}{E_0}\right). \qquad (19)$$

DISCUSSION OF RESULTS

According to (16), the differential conductance of a tunnel junction in a transverse magnetic field undergoes quantum oscillations. There are two series of such oscillations, for which the positions of the maxima are determined by the conditions $E_F\pm eV/2=n\hbar\Omega$, where n is an integer. The period of oscillations along the voltage scale is thus $\Delta eV=2\hbar\Omega$, while that along the scale of inverse magnetic field has its usual value, $\Delta/\hbar\Omega=1/E_F$. With $V=0$, as we see from (19), G has square-root singularities at $E_F=n\hbar\Omega$ (we recall that we are ignoring the usual broadening factors: the thermal broadening and the collisional broadening). At nonzero values of V, the singularities are spread out, and the

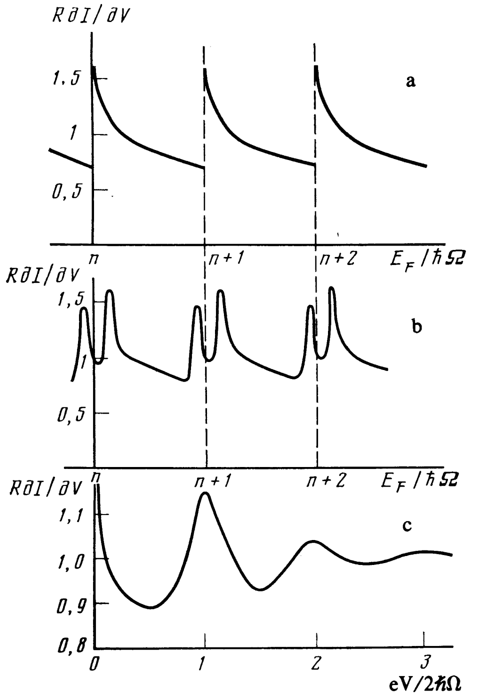

FIG. 3. Differential conductance as a function of (a,b) the inverse magnetic field and (c) the applied voltage. $E_F/E_0 = 10, E_0/\hbar\Omega = 10$. a—$V=0$; b—$eV/\hbar\Omega = 0.2$; c—$E_F$ is a multiple of $\hbar\Omega$.

widths of the peaks are determined by the quantity Γ, given by (17). At sufficiently high voltages $eV \gtrsim \hbar\Omega(E_F/E_0)^{1/2}$, with $\Gamma \gtrsim \hbar\Omega$, the widths of the peaks become comparable to the distance between peaks, so that the oscillations fade away.

Figure 3 shows the differential conductance G as a function of the inverse magnetic field and the voltage according to calculations from (16) and (19). Figure 3b demonstrates the splitting of the peaks at a nonzero value of V, while Fig. 3c demonstrates the broadening of these peaks and the disappearance of the oscillations with increasing voltage.

What is the physical meaning of these results? The singularities in the tunnel current stem from singularities in the state density in a magnetic field. These features arise when the Fermi level coincides with some magnetic level of an electron to the right or left of the barrier; the value of x_0 corresponding to this level is such that energy conservation law (6) is satisfied.

We first consider the simplest case, $V=0$. Figure 4a shows some schematic curves of $\varepsilon_N^+(x_0)$ and $\varepsilon_{N'}^-(x_0)$. According to the discussion above, the singularities in the tunnel current correspond to coincidences of the Fermi level with intersections of the $\varepsilon_N^+(x_0)$ and $\varepsilon_{N'}^-(x_0)$ curves. Remarkably, at a fixed energy $n = N + N'$ these intersections occur at the same value of the energy, $\hbar\Omega(n + 3/2)$, as can be seen from (13). This explains why the singularities in the differential conductance are not smeared at $V=0$.

At $V \neq 0$ we have a different situation: Now the magnetic levels in the right and left half-spaces (Fig. 4b) are separated by a distance greater by an amount eV than in Fig. 4a. The curve intersection points $\varepsilon_N^+(x_0) + eV/2$ and $\varepsilon_{N'}^-(x_0) - eV/2$, where energy conservation law (6) holds at a fixed $n = N + N'$, now do not correspond to the same energy. At value of x small in comparison with the Larmor radius (these are the most important values for the tunneling) these points lie on a straight line whose slope is proportional to V [see (13)]. At $V \neq 0$ the Fermi level thus cannot pass through different intersection points simultaneously. Consequently, instead of the singularity which we formerly had with $V=0$ for some value of n, we now find a system of closely spaced singularities, whose envelope is a rounded maximum. The reason for the two series of oscillations is that the Fermi in the right and left half-spaces differ by an amount eV.

As we mentioned in the Introduction, quantum oscillations do not occur in a longitudinal magnetic field for the conditions assumed here. We are thus led to ask what deviation of the magnetic-field direction from the plane of the barrier discussed here disappear. Let us assume that the magnetic field makes a small angle φ with the plane of the barrier. If an electron is then initially incident on the barrier along the normal, then during thge next collision it is easy to

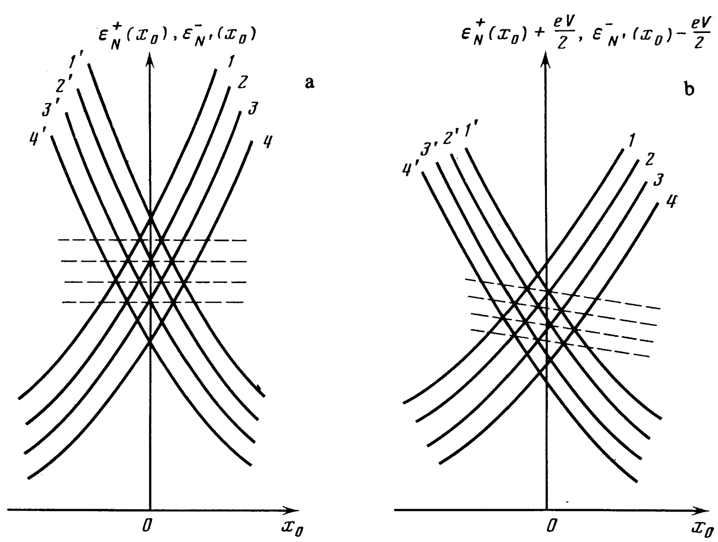

FIG. 4. The magnetic energies ε_N^+ (curves 1–4) and $\varepsilon_{N'}^-$ (curves 1'–4') as functions of the position of the center of the Landau oscillator, x_0. a—$V=0$; b—$V \neq 0$. At $x_0 = 0$, the values of ε_N^+ and ε_N^- are the same: $\varepsilon_N^+ = \varepsilon_N^- = (2N + 3/2)\hbar\Omega$.

see that the velocity vector of the electron will make an angle of 2φ with the normal. On the other hand, the only electrons which contribute to the tunnel current are those for which the angle of incidence does not exceed a value on the order of $(E_0/E_F)^{1/2} \ll 1$. We would thus expect that the quantum oscillations would disappear at $\varphi \gtrsim (E_0/E_F)^{1/2}$.

Finally, let us examine the conditions required for observing these oscillations of the tunnel in a transverse magnetic field. The conditions $E_F \gg E_0 \gg \hbar\Omega$ can be satisfied easily for a tunnel junction between two metals. However, the condition for specular reflection of electrons with energies on the order of the Fermi energy from the plane of the barrier—an important condition for the effect under consideration here—would apparently be impossible to satisfy with typical metals, since the de Broglie wavelength is comparable to the interatomic distance. More favorable in this regard would be a tunnel junction between two heavily doped semiconductors. As an example we consider heterostructure in which two n-type GaAs layers with electron densities of 10^{19} cm^3 are separated by a thin layer of a wide-gap $Al_x Ga_{1-x} As$ solid solution, which forms a barrier with a height $W = 300$ meV. We assume that the thickness of this barrier is $d = 100$ Å. We then have $E_F \sim 230$ meV and $E_0 \sim 20$ meV. In the magnetic field $H = 10$ kOe we would have $\hbar\Omega \sim 2$ meV. Consequently, at liquid-helium temperatures the conditions required for the onset of the quantum oscillations would be satisfied.

[1)]We will be ignoring the small effects which stem from oscillations of the Fermi level in a magnetic field and for which we would have $\Delta E_F \sim \hbar\Omega$.

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Translated by Dave Parsons