Dynamics of one-dimensional charge-density waves in disordered systems

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The influence of defects on the propagation of charge-density waves in quasi-one-dimensional systems is investigated. It is shown that, when disorder is present in the system, the current transported by the charge density waves may increase as a result of the increase in the rate of generation of solitons on fluctuations in the random field of defects.

It is well known that impurities present in quasi-one-dimensional compounds, whose electrical properties at low temperatures are due to the existence of charge-density waves, have an important influence on the behavior of the system. The static characteristics of charge-density waves (CDW) in systems with disorder (e.g. correlation length and static susceptibility) have been examined in papers by one of the present authors.[1] In the present work we analyze how disorder in quasi-one-dimensional systems affects the static CDW conductivity in systems with high viscosity.

Systems with high viscosity probably include most of the compounds with one-dimensional CDWs that are being investigated experimentally at present (for example, TaS_3, $NbSe_3$, and so on). The dynamic properties of such systems can be described by the equation

$$\frac{1}{\Gamma}\frac{\partial\varphi}{\partial t}=-\frac{\delta H}{\delta\varphi}+\xi(x,t). \qquad (1)$$

where $\varphi(x,t)$ is the CDW phase, x is the coordinate, H is the Hamiltonian of the system, Γ is the viscosity, $\xi(x,t)$ is the thermal noise,

$$\langle\xi(x,\ t)\xi(x',\ t')\rangle=2\Gamma T\delta(x-x')\delta(t-t'),$$

and T is the temperature. The CDW Hamiltonian is given by

$$H=\int dx\Big[{}^1/_2\bar v_F(\nabla\varphi-h)^2+V_0(1-\cos m\varphi)-F\varphi$$
$$+V_b\sum\delta(x-x_i)\cos(2k_Fx+\varphi)+V_f(x)\nabla\varphi\Big], \qquad (2)$$

where $\overline{v_F}=v_F/2\pi$, v_F is the Fermi velocity, $F=(e^*/\pi)E$, e^* is the soliton charge, E is the external electric field (henceforth, we shall refer to F as the external field), h is the incommensurability parameter, m is the commensurability order, and V_0 is the amplitude of the commensurability potential. The last two terms in the Hamiltonian describe random impurity fields that are responsible for forward and backward scattering, respectively.[2]

Let us begin by considering a commensurable CDW. In this case $h=0$ and, to simplify the formulas, we set m equal to unity (generalization to the case where $m\neq1$ is trivial). It is found that backward-scattering impurities have the greatest effect, so that the basic formulas reproduced below will refer to this particular case. We note that (1) with the Hamiltonian given by (2) (without the last two terms) will also describe the propagation of vortices in SNS Josephson junctions. In such systems, microbridges are typical disorder elements and are described by a small random additional term $f(x)$ in the amplitude of the commensurability potential V_0. It can be readily verified that they have the same qualitative effect on vortex dynamics as impurities in systems with charge-density waves. In this paper, we shall confine our attention to subthreshold fields $F<V_0$ and moderate temperatures $T\ll\Delta$, where Δ is the energy necessary to produce a soliton-antisoliton pair in the system.

The system characterized by Hamiltonian (2) admits of a simple and clear mechanical description if we use the model of an elastic string lying in a sinusoidal potential distribution. This representation is used, for example, to describe the motion of dislocations in crystals. A dislocation is then described by the elastic string model, and the potential distribution is called the Peierls distribution. The CDW phase can be interpreted as a transverse displacement, and the current

$$j=\langle\dot\varphi\rangle \qquad (3)$$

as the rate of transverse displacement of the string.

The motion of an elastic string in subthreshold fields for $T<\Delta$ occurs through the activated formation of a soliton-antisoliton pair (i.e., a fluctuational transition of a small portion of the string to a neighboring valley of the potential) and the subsequent propagation of this pair under the influence of external fields.[3,4] Random fields affect the motion of the string in two ways: by reducing, at some points, the energy necessary to produce nuclei, and by retarding the subsequent propagation of solitons. The resultant effect (increase or decrease in the velocity of the string) is determined by the competition between these two phenomena.

To determine the mean velocity $\langle\dot\varphi\rangle$ (the angle brackets indicate an average over realizations of the random fields), we must find the mean time $\langle t\rangle$ for the transition of an arbitrary point on the string to a neighboring valley of the Peierls distribution. We then have

$$\langle\dot\varphi\rangle=2\pi/\langle t\rangle. \qquad (4)$$

The mean time $\langle t\rangle$ can be determined by the Kolmogorov method[5]:

$$\langle t\rangle=\left\langle\int_0^\infty dt\exp\Big[-\int_0^x \mathcal{J}(z)\,(t-\tau(z))\,dz\Big]\right\rangle, \qquad (5)$$

where $\tilde J(z)$ is the rate of generation of the nucleus whose center of gravity lies at the point z and $\tau(z)$ is the time taken by a soliton initially located at the point z to reach the point of observation (the origin), i.e., $t=\tau(x)$. The exponential in the

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integrand in (5) is then the probability that the point $x=0$ will be in the original valley at time t. The rate of generation of nuclei in a pure system was calculated by Büttiker and Landauer[4] and has the activated form

$$\mathcal{J}_0=\nu_0 \exp(-\Delta/T).$$

In a disordered system, the configuration and energy of a nucleus can be determined by minimizing the Hamiltonian (2) and solving the resulting equation of motion:

$$-\bar{v}_F\varphi''+V_0\sin\varphi-F+dV_i/d\varphi=0, \tag{6}$$

where V_i represents the random potential. Equation (6) cannot be solved in its general form but, if the impurities are weak, we can use a perturbation theory to treat the impurities. The increment U on the energy of a nucleus due to random fields is calculated in the Appendix for different types of disorder, and is given by

$$U(x)=\int dx' V_i(x')\Phi_n(x-x'), \tag{7}$$

where Φ_n depends on the shape of the nucleus and is different for different types of disorder. The condition for the validity of perturbation theory is that $\varphi_1(V_i)/\varphi_0 \ll 1$, where φ_0 is the configuration of the nucleus in a pure system, and φ_1 is the increment due to impurities. This condition is definitely satisfied if the characteristic fluctuation of the random field is $\delta V_i \ll \Delta_0 = 16(V_0\bar{v}_F)^{1/2}$. It can be shown, however, that, near the threshold ($F\simeq V_0$), we have $\varphi_1(\delta V_i)/\varphi_0 \sim [(V_0-F)/V_0]^{1/4} \ll 1$ even when $\delta V_i \sim \Delta$. This enables us to use (7) for the increment in the energy of the nucleus (with the corresponding function Φ_n) near the threshold even for large fluctuations in the random fields. We shall suppose that the impurities are weak: $\langle V_i^2\rangle^{1/2} \ll V_0$ and are frequently encountered ($cv_F \gg \langle V_i^2\rangle^{1/2}$), which enables us to describe the statistical properties of random fields by correlators of the form (the mean separation between the impurities is less than the size of the soliton)

$$\langle V_i(x)V_i(x')\rangle=\gamma\delta(x-x'). \tag{8}$$

Thus, the nucleation rate can be described

$$\mathcal{J}=\mathcal{J}_0\exp(-\beta U), \quad \beta=1/T.$$

We now proceed to specific calculations. Consider a commensurate wave near the threshold ($F\approx V_0$). Since the random fields are weak, $\gamma^{1/2} \ll V_0$, the impurities have little effect on the motion of the solitons along a line, and the soliton propagation velocity u may be considered to be the same as in a pure crystal. We then have

$$\langle t\rangle=\frac{\xi_0}{u}\int_0^\infty dx\left\langle \exp\left[-J_0\int_0^x dz z e^{-\beta U(z)}\right]\right\rangle, \qquad J_0=(\xi_0^2/u)\mathcal{J}_0, \tag{9}$$

where the coordinates are dimensionless and measured in units of ξ_0 (the size of a soliton in a pure system for $F=0$), and the integration variable is $t=\xi_0 x/u$. The properties of the fields $U(x)$ are determined by the correlator

$$\langle U(x)U(x')\rangle=2\gamma b^2\xi a^{-1}A(x-x'), \tag{10}$$

$$A(x-x')=\int_{-\infty}^{\infty}\frac{dy}{\cosh^2(y-x/2\xi)\cosh^2(y-x'/2\xi)} \tag{11}$$

$$b=3\sqrt{2}\left(\frac{V_0-F}{V_0}\right)^{1/2}, \quad \xi=\xi_0 3^{1/2}b^{-1/2}.$$

Since for fields close to the threshold the size of the nucleus is the smallest length parameter, we may assume that

$$A\approx\delta(x-x')\cdot{}^8/_9\xi \tag{12}$$

[we use the dimensionless coordinate in (12)]. Using the local character of the nucleus $A(x-x')$, we readily find that, in (9),

$$\left\langle \exp\left[-J_0\int_0^x dz e^{-\beta U(z)}z\right]\right\rangle$$

$$=1+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}J_0^n\int_0^x dz_1 z_1\ldots\int_0^x dz_n z_n\left\langle\exp\left[-\beta\sum_{i=1}^n U(z_i)\right]\right\rangle$$

$$=\exp\left[-\frac{1}{2}J_0\langle\exp(-\beta U(z))\rangle x^2\right]. \tag{13}$$

When the mean of the exponential in (13) is evaluated, it must be remembered that negative fluctuations in the random field U, whose absolute magnitude exceed the energy Δ of a nucleus in a pure system, need not be taken into account because the activation formula for J is meaningful only when $\beta(U+\Delta) \gg 1$. We must therefore introduce a cutoff at $U=-\Delta$ and

$$\langle\exp(-\beta U)\rangle\sim\int_{-\Delta}^{\infty}dU\exp\left(-\frac{U^2}{2\tau}-\beta U\right), \quad \tau=48b\gamma\xi_0/a. \tag{14}$$

The nucleation rate will be written with exponential precision: $J\sim\exp(-\beta\Delta-\beta U)$ since, to calculate the factor multiplying the exponential, we must know the configuration of the nucleus in the presence of the random fields. The coefficient obtained in Ref. 4. cannot be used because translational invariance does not hold in a system with disorder and there are no Goldstone modes. The rigorous determination of the exponential factor is difficult and lies outside the scope of the present work. On evaluating the integrals, we readily find that

$$\langle\dot\varphi\rangle\sim j_0\exp({}^1/_4\beta^2\tau), \quad \beta\ll\Delta/\tau, \tag{15}$$

$$\langle\dot\varphi\rangle\sim\exp(-\Delta^2/4\tau), \quad \beta\gg\Delta/\tau, \tag{16}$$

where j_0 is the current in the pure system.

As can be seen, disorder leads to an effective increase in the current near the field threshold. There are then two distinct regimes: for high temperatures, $T\gg T^*=\tau/\Delta$, the main contribution is provided by the thermally activated solitons, and impurities provided only a correction ($\beta^2\tau \ll \beta\Delta$). However, since it is quite possible to have a situation where $\beta^2\tau \gg 1$ for sufficiently low temperatures (because of the original condition $\beta\Delta \gg 1$), the increase in the current may be very substantial. As the temperature is reduced, the principal contribution begins to be due to solitons induced by impurities at points where random field fluctuations become comparable (in absolute magnitudes) with the energy δ of a nucleus in the pure system. In that case ($T \ll T^*$), impurities will also produce a substantial rise in the current; in a pure system, $j_0 \sim \exp(-\frac{1}{2}\beta\Delta) \ll \exp(-\Delta^2/4\tau)$, since $\beta \gg \Delta/\tau$.

Substituting for τ in (15) and (16), we have

$$\langle\dot\varphi\rangle \sim j_0 \exp\left[36\sqrt{2}\beta^2\gamma\left(\frac{V_0-F}{V_0}\right)^{1/2}\right],$$

$$T \gg \frac{320\gamma}{2^{1/4}\Delta_0}\left(\frac{V_0-F}{V_0}\right)^{-3/4}, \qquad (15a)$$

$$\langle\dot\varphi\rangle \sim \exp\left[-C\frac{\Delta_0^2}{\gamma}\left(\frac{V_0-F}{V_0}\right)^2\right], \quad T \ll \frac{320\gamma}{2^{1/4}\Delta_0}\left(\frac{V_0-F}{V_0}\right)^{-3/4}, \qquad (15b)$$

$C \sim 10^{-2}$.

We now turn to the case of weak fields ($F \ll V_0$). In the limit of fields that are weak in comparison with the threshold value, we have two possible situations, namely, $V_0 \gg F \gg \gamma^{1/2}$ and $F \ll \gamma^{1/2}$. In the former case, impurities again have no effect on the motion of solitons but, in the latter case, the mobility of solitons turns out to be $\sim\exp(-\gamma/2T^2)$ because they have to overcome the impurity potential in accordance with an activation-type formula. When $F \ll V_0$, there are again two temperature regimes: thermal solitons play the principal role for $T \gg T^* \sim \gamma/\Delta$, whereas solitons activated by impurities assume this function for $T \ll T^*$. Strictly speaking, our analysis is valid only for $T \gg T^*$ since the characteristic fluctuations in the random field that provide the principal contribution turn out to be of the order of $\gamma^{1/2} \ll \Delta$. At low temperatures, $T \ll T^*$, the main contribution is due to random field fluctuations $\sim\Delta$. Impurities can then produce an appreciable change in the equilibrium shape and size of the nucleus, and perturbation theory is no longer valid for the solution of (6). Hence, results referring to the case of low temperatures are, in fact, only qualitative.

Suppose, to begin with, that $V_0 \gg F \gg \gamma^{1/2}$. It is convenient to rewrite the expression for the mean migration time in the form

$$\langle t\rangle = \frac{\xi_0}{u}\int_0^\infty dx\left\langle \exp\left[-J_0\int_0^x dz(x-z)\right.\right.$$

$$\left.\left.\times\exp\left\{-\int_{-\infty}^{\infty}\Phi_n(z-y)\psi(y)\,dy\right\}\right]\right\rangle, \qquad (17)$$

where $\psi(z)$ is the dimensionless random-field potential ($\psi = V/T$):

$$\langle\psi(z)\psi(z')\rangle = \beta^2\gamma\delta(z-z').$$

Suppose that $\beta \ll \Delta/\gamma$. As before, we can then expand the exponetial into a series, and consider the mean

$$\left\langle \exp\left[-\sum_{\alpha=1}^{n}\int dy_\alpha\Phi_n(y_\alpha-z_\alpha)\psi(y_\alpha)\right]\right\rangle$$

$$= \frac{\int D\psi\exp(-S)}{\int D\psi\exp(-S_0)},$$

$$S = \frac{1}{2\beta^2\gamma}\int\psi^2(y)\,dy + \sum_{\alpha=1}^{n}\int dy_\alpha\Phi_n(y_\alpha-z_\alpha)\psi(y_\alpha),$$

$$S_0 = \frac{1}{2\beta^2\gamma}\int\psi^2(y)\,dy$$

(at high temperatures, the random field need not be cut off). Minimizing the action S, we obtain the extremal trajectory

$$\bar\psi(\xi) = -\beta^2\gamma\sum_{\alpha=1}^{n}\Phi_n(\xi-z_\alpha)$$

and the corresponding action

$$S[\bar\psi] = -\frac{1}{2}\beta^2\gamma\int dz\left[\sum_{\alpha=1}^{n}\Phi_n(z-z_\alpha)\right]^2.$$

Next, by evaluating the chain of integrals with respect to $z_1, \ldots, z_n$, we can readily verify that, when $\beta \ll \Delta/\gamma$, the principal contribution is due to points lying close to the surfaces $z_i = z_j$, i.e., it is due to regions for which $A(z_i - z_j) \simeq A(0) \equiv A_0$. Using the expressions for Φ_n and $A(z_i - z_j)$ in the limit of small fields, which are reproduced in the Appendix, we can readily show that

$$A_0 = {}^8/_3(1+4e^{-l}), \quad l \sim \ln(V_0/4\pi F),$$

where l is the equilibrium size of the nucleus in low fields in a pure system.[4] After this, we obtain

$$\langle t\rangle = \int_0^\infty dx\exp\left[-\frac{1}{2}J_0\exp\left\{\frac{4}{3}\beta^2\gamma(1+4\exp(-l))\right\}x^2\right]$$

and, consequently,

$$\langle\dot\varphi\rangle = j_0\exp\left[{}^2/_3\beta^2\gamma(1+4e^{-l})\right]. \qquad (18)$$

At low temperatures, the series representing the original exponential will diverge after integration with respect to z_1, which shows that there is a sharp change in the nature of the motion for $T \sim T^*$. In that case, we may proceed as follows. If the main contribution to the integral with respect to time is due to fairly large values of the time (equivalently, fairly large x), the integral in the exponential can be written in the form

$$\int_0^x dz(x-z)\exp\left[-\int_{-\infty}^{\infty}\Phi_n(z-y)\psi(y)\,dy\right]$$

$$\sim x\left\langle (x-z)\exp\left[-\int_{-\infty}^{\infty}dy\Phi_n(z-y)\psi(y)\right]\right\rangle$$

$$= \frac{1}{2}x^2\left\langle \exp\left[-\int_{-\infty}^{\infty}dy\Phi_n(z-y)\psi(y)\right]\right\rangle.$$

On evaluating the mean

$$\left\langle \exp\left[-\sum_{\alpha=1}^{n}\int dy_\alpha\Phi_n(y_\alpha-z_\alpha)\psi(y_\alpha)\right]\right\rangle$$

$$= \left\{\int D\psi e^{-S_0}\theta\left(\beta\Delta + \int\Phi_n(y-z)\psi(y)\,dy\right)\right\}^{-1}$$
$$\times\int D\psi e^{-S}\theta\left(\beta\Delta + \int\Phi_n(y-z)\psi(y)\,dy\right)$$

with the aid of the familiar representation of the θ function

$$\theta(x)=\frac{1}{2\pi i}\lim_{\varepsilon\to 0}\int dk\frac{e^{ikx}}{k-i\varepsilon},$$

we find that

$$\langle\dot{\varphi}\rangle\sim\exp\left(-\frac{\Delta^2}{2\gamma A_0}\right)=\exp\left(-\frac{3\Delta^2}{16\gamma(1+4e^{-l})}\right),$$

$$T\ll\gamma/\Delta,\quad \gamma^{1/2}\ll F\ll V_0. \tag{19}$$

We now consider the limiting case of weak fields for which $F\ll\gamma^{1/2}$. Taking into account the retardation of solitons by impurities, and transforming from integration with respect to t to integration with respect to x, we obtain

$$\langle t\rangle=\exp(-\beta F)\int_0^\infty dx\left\langle\exp\left[-J_0e^{-\beta F}\int_0^x dz\exp\left\{-\int\psi(y')\Phi_n\right.\right.\right.$$
$$\left.\times(y'-z)\,dy'\right\}$$
$$\left.\times\int_z^x dy\exp\left\{\int\Phi_s(y'-z)\psi(y')\,dy'\right\}\right]$$
$$\left.\times\exp\left\{\int\Phi_s(x-y')\psi(y')\,dy'\right\}\right\rangle. \tag{20}$$

Once again, we expand the exponential in a series, and perform the Gaussian integration over D (since we are concerned with high temperatures, we ignore the θ function):

$$\langle t\rangle=\int_0^\infty dx\sum_{n=0}^\infty\frac{(-1)^n}{n!}e^{\frac{1}{2}\beta^2\gamma B_0}\prod_{i=1}^n$$
$$\times\int_0^x dz_i\exp\left[\frac{1}{2}\beta^2\gamma\sum_{j,j'}(A(z_j-z_{j'})\right.$$
$$\left.+B(z_j-z_{j'})-2C(z_j-z_{j'})\right]\int_{z_j}^x dy_i$$
$$\times\exp\left[\beta^2\gamma\sum_j(B(y_j-x)-C(y_j-x))\right], \tag{21}$$

where

$$B(z_j-z_{j'})=\int dz\Phi_s(z_j-z)\,\Phi_s(z_{j'}-z),$$

$$C(z_j-z_{j'})=\int dz\Phi_s(z_j-z)\,\Phi_n(z-z_{j'}). \tag{22}$$

Next, we can again readily verify that, at high temperatures, for which $\beta\ll\Delta/\gamma$,

$$\langle t\rangle=e^{-\beta F}(\pi/2)^{1/2}\exp\left(\tfrac{1}{2}\beta^2\gamma B_0\right)\left[J_0\exp\left(\tfrac{1}{2}\beta^2\gamma(A_0+B_0)\right)\right]^{-1/2}$$

and

$$\langle\dot{\varphi}\rangle\sim j_0e^{\frac{1}{2}\beta F}\exp\left[\tfrac{1}{2}\beta^2\gamma(A_0-B_0)\right]. \tag{23}$$

Since $A_0>B_0$, impurities will also increase the current in low fields.

At low temperatures $T\ll\gamma/\Delta$, we should have $\langle t\rangle\sim\exp(\Delta^2/2\gamma)$, but the mean distance traversed by a soliton during this time is found to be $\sim\exp(-\beta^2\gamma/4)\exp(\Delta^2/4\gamma)\ll 1$. This means that, at low temperatures and in weak fields, the solitons accumulate near impurities, and the above method is no longer valid.

We note that, as shown in Ref. 4, this analysis is valid provided the force is not too small, i.e.,

$$F\gg n_sT, \tag{24}$$

where n_s is the density of solitons in the system. When the force is smaller, the motion of the solitons after the formation of the nuclei becomes diffusive, and the velocity $\langle\dot{\varphi}\rangle$ is given by

$$\langle\dot{\varphi}\rangle=2\pi\langle n_s u\rangle, \tag{25}$$

where n_s is the equilibrium density of solitons in the system for $F=0$. Before we proceed to the analysis of the case of weak forces, which do not satisfy condition (24), we must establish the consequences of commensurability in the system.

If the system is not subject to cyclic boundary conditions (for example, if the ends of the string are free, which corresponds to a free CDW phase at the ends of the specimen), the so-called "geometric" solitons of the same sign appear in the system, and their density $n_s(h)$ depends on the proximity to the commensurability threshold. When the external fields are not too small, i.e., $F\gg\gamma^{1/2}$, the contribution of such solitons to the CDW current is

$$j_n\approx 2\pi u n_s(h); \tag{26}$$

The density of geometric solitons in one-dimensional disordered systems was calculated in a previous paper by one of the present authors[6]:

$$n_s(h)\sim\exp\left[-\beta E_s(1-h/h_c)+\tfrac{1}{4}\beta^2\gamma\right],\quad T\gg T_h,$$
$$n_s(h)\sim\exp\left[-E_s^2(1-h/h_c)^2/8\gamma\right],\quad T\ll T_h,$$
$$T_h=\gamma/E_s(1-h/h_c),\quad h_c=\frac{4}{\pi}(V_0/\bar{v}_F)^{1/2}; \tag{27}$$

where h_c is the critical (threshold) value of the incommensurability parameter. Geometric solitons provide the main contribution to the current if their density is high in comparison with some effective density of "dynamic" solitons, which can be defined as having $n_s(F)\sim(J/u)^{1/2}$. If, on the other hand, $n_s(F)\gg n_s(h)$, the current in the system is given by (15)–(19) and (23). We can now construct a diagram in the F, h plane, which shows the regions of predominant contribution of dynamic and "geometric" solitons, respectively. Figure 1 shows such diagrams for three temperature intervals, namely,

a) $T\gg\gamma^{1/2}$; b) $\gamma^{1/2}\gg T\gg\gamma^{1/2}(\gamma/\Delta_0^2)^{1/8}$; c) $\gamma^{1/2}(\gamma/\Delta_0^2)^{1/8}\gg T$.

To the left, above the curve, $n_h(h)\gg n_s(F)$, whereas, to the right, below the curve, $n_s(h)\ll n_s(F)$. The dot-dash lines define the region in which the soliton densities are low: $n_s(F)$, $n_s(h)\ll 1$. These results are not valid in the immediate neighborhood of the thresholds $F=V_0$ and $h=h_c$. The line of "equal density" near the threshold in Fig. 1a is described by the equation

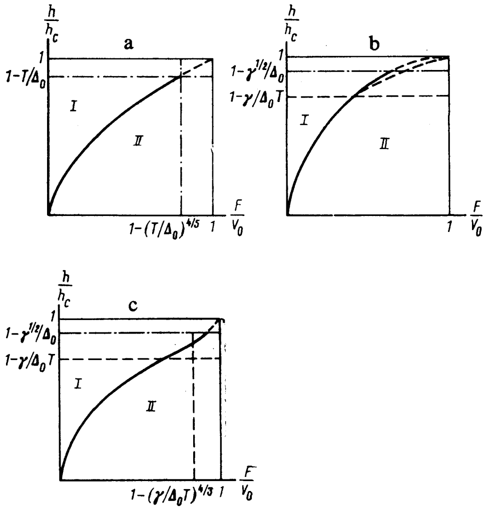

FIG. 1.

$$1-h/h_c\sim(1-F/V_0)^{5/4}. \tag{28}$$

The curve near the threshold in Fig. 1b is given by the equations

$$1-h/h_c\sim(1-F/V_0)^{5/4},\quad 1-h/h_c\gg\gamma/\Delta_0 T,$$

$$1-\frac{h}{h_c}\sim\frac{\gamma}{\Delta_0 T}\left(1-\frac{F}{V_0}\right)^{5/4},\quad \frac{\gamma^{1/2}}{\Delta_0}\ll 1-\frac{h}{h_c}\ll\frac{\gamma}{\Delta_0 T}. \tag{29}$$

Finally, the curve in Fig. 1c is described by

$$1-\frac{h}{h_c}\sim\left(1-\frac{F}{V_0}\right)^{5/4},\quad 1-\frac{h}{h_c}\gg\frac{\gamma}{\Delta_0 T},$$

$$1-\frac{h}{h_c}\sim\frac{\gamma}{\Delta_0 T}\left(1-\frac{F}{V_0}\right)^{5/8},\quad 1-\frac{h}{h_c}<\frac{\gamma}{\Delta_0 T},$$

$$1-\frac{F}{V_0}\gg\left(\frac{\gamma}{\Delta_0 T}\right)^{4/3},$$

$$\frac{h}{h_c}\sim\frac{F}{V_0},\quad 1-\frac{F}{V_0}\ll\left(\frac{\gamma}{\Delta_0 T}\right)^{4/3}. \tag{30}$$

For low values of the field and the incommensurability parameter ($F\ll V_0$, $h\ll h_c$), and $n_s(F)\sim n_s(h)$ lines are described by the following equation in all three diagrams:

$$h\sim h_c\frac{F}{\Delta_0}\ln\frac{V_0}{4\pi F}. \tag{31}$$

Thus, in a system with incommensurability, we have two possible situations: if the system is in the region of the phase diagram, the current in the system is described by (15), (16), (18), (19), and (23), and is characterized by a complex nonlinear dependence on the applied field F. As the field is reduced, the system enters region I and, when $h_c - h\ll h_c$, the current becomes proportional to the field and is given by (32). We note that, as the field is reduced, the system passes from region II to region I before (24) is violated, so that the current is determined by "geometric" solitons in the region in which the current is a linear function of the force.

Possible production of solitons on the boundary of the specimen was not taken into account in our calculation of the current in systems with CDWs. Burkov and Pokovskiĭ[7,8] have calculated the current in an impurity-free CDW, produced as a result of the formation of solitons on the specimen boundary. The conditions for the existence of this current are the boundary conditions corresponding to a string with free ends. If the boundary conditions in a practical system correspond to a free phase on the boundaries, then, in systems that are close to the incommensurability threshold ($h_c - h\ll h_c$), the current in fields approaching the threshold value for temperatures $T<\Delta$ will be largely determined by the nucleation of solitons near the boundary.[7,8] When the field F is small, the influence of boundary effects will be slight, as before, since, according to Ref. 8, solitons will not then be created on the surface.

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APPENDIX

Interaction of solitons with impurities

Let us suppose that the impurity density is high, so that

$$c^{-1}\ll\xi_0, \tag{A1}$$

where ξ_0 is the size of a soliton. In a pure system, the soliton configuration is

$$\varphi_s=4\,\mathrm{arctg}\,\exp\left(\frac{x-x_0}{\xi_0}\right), \tag{A2}$$

$$\xi_0=(\bar v_F/V_0)^{1/2}. \tag{A3}$$

The impurity increments on the energy density are

$$V_f(x)\,\nabla\varphi, \tag{A4a}$$

$$V_b\sum_i\delta(x-x_i)\cos(2k_F x+\varphi)$$

$$\equiv V_{b1}(x)\,(1-\cos\varphi)-V_{b2}(x)\sin\varphi, \tag{A4b}$$

$$f(x)\,(1-\cos\varphi). \tag{A4c}$$

The increment (a) describes forward scattering, (b) backward scattering, and (c) random variations in the commensurability potential. The statistical properties of random potentials are determined by correlators of the form

$$\langle V(x)\,V(x')\rangle=\gamma\delta(x-x'). \tag{A5}$$

We are interested in weak impurities, so that, in any case,

$$\gamma^{1/2}\ll V_0. \tag{A6}$$

We can then use the soliton configuration (A2) for a pure system, and consider the soliton to be a particle moving in a random potential:

$$U_f(x)=\frac{2}{\xi_0}\int_{-\infty}^{\infty}\frac{dz\,V_f(z)}{\mathrm{ch}[(z-x)/\xi_0]}, \tag{A7a}$$

$$U_b(x)=2\int_{-\infty}^{\infty}dz\left[V_{b1}(z)\,\mathrm{ch}^{-2}\left(\frac{z-x}{\xi_0}\right)+V_{b2}(z)\,\mathrm{ch}^{-2}\left(\frac{z-x}{\xi_0}\right)\mathrm{sh}\left(\frac{z-x}{\xi_0}\right)\right], \tag{A7b}$$

$$U_c(x)=\int_{-\infty}^{\infty} dz f(z)\,\mathrm{ch}^{-2}\left(\frac{z-x}{\xi_0}\right). \qquad \text{(A7c)}$$

The corresponding correlators now assume the form

$$\langle U_f(x)U_f(x')\rangle=4\gamma_f A_f(x-x'), \qquad \text{(A8a)}$$

$$\langle U_b(x)U_b(x')\rangle=4\gamma_b A_b(x-x'), \qquad \text{(A8b)}$$

$$\langle U_c(x)U_c(x')\rangle=4\gamma_c A_c(x-x'), \qquad \text{(A8c)}$$

where

$$A_f=\int_{-\infty}^{\infty}\frac{dz}{\cosh(x-z)\cosh(x'-z)},$$

$$A_b=\int_{-\infty}^{\infty}dz\left[\frac{1}{\cosh^2(x-z)\cosh^2(x'-z)}+\frac{\sinh(x-z)\sinh(x'-z)}{\cosh^2(x-z)\cosh^2(x'-z)}\right], \qquad \text{(A9)}$$

$$A_c=\int_{-\infty}^{\infty}\frac{dz}{\mathrm{ch}^2(x-z)\,\mathrm{ch}^2(x'-z)}.$$

The condition for constant soliton shape in the presence of impurities is

$$\gamma\ll v_F^{3/2}V_0^{1/2},\ \text{(a)}$$
$$\gamma\ll v_F^{1/2}V_0^{3/2},\ \text{(b), (c)} \qquad \text{(A10)}$$

Interaction of nuclei with impurities.

In low fields $F\ll V_0$, a nucleation is a soliton-antisoliton pair, the separation b in which is much greater than the size ξ_0 of a single soliton. The configuration of this king of nucleation can readily be described by

$$\varphi_n(x)=\varphi_s(x-l/2)+\varphi_s(x+l/2), \qquad \text{(A11)}$$

and the shape functions Φ_n, Φ_s and the correlators of the random fields $U(x)$ are easily calculated from this.

Near the threshold $F=V_0$, the shape of the nucleus is described by[4]

$$\varphi_n=b\cosh^{-2}(x/2\xi), \qquad \text{(A12)}$$

where

$$b=3\sqrt{2}\left(\frac{V_0-F}{V_0}\right)^{1/2},\quad \xi=\xi_0 3^{1/2}b^{-1/2}. \qquad \text{(A13)}$$

The random increment on the energy of a nucleus in the impurity field is therefore of the form

$$U_f^{(n)}(x)=\frac{2b}{\xi}\int_{-\infty}^{\infty}dy\,V_f(y)\frac{\sinh[(x-y)/2\xi]}{\cosh^3[(x-y)/2\xi]},$$

$$U_b^{(n)}(x)=\int_{-\infty}^{\infty}dy\left[V_{b1}(y)\frac{1}{2}b^2\cosh^{-4}\left(\frac{x-y}{2\xi}\right)+V_{b2}(y)\,b\cosh^{-2}\left(\frac{x-y}{2\xi}\right)\right], \qquad \text{(A14)}$$

$$U_c^{(n)}(x)=\frac{b^2}{2}\int_{-\infty}^{\infty}dy f(y)\cosh^{-4}\left(\frac{x-y}{2\xi}\right).$$

Using the second formula in (A14), we readily obtain [in the lowest order in the small quantity (b)] the equation given by (10) in the main text for $\langle U_b^{(n)}(x)U_b^{(n)}(x')\rangle$ [the indices b and (n) are omitted from the text].

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