Dynamic bremsstrahlung of a relativistic charged particle scattered by an atom

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The cross section for the bremsstrahlung of a relativistic charged particle scattered by an atom is calculated. In contrast with the customary use of the screening approximation, the effect of an atomic electron on the bremsstrahlung is taken into account exactly. The atomic electron is treated as a moving particle interacting with the electromagnetic field, not simply as a source of a static external field. As a result, a new (dynamic) term appears in the transition amplitude in addition to the static term, which leads to the Bethe-Heitler expression. The corresponding cross section—the cross section for dynamic bremsstrahlung—is significantly larger than the cross section for static bremsstrahlung in certain intervals of the frequency and of the photon emission direction.

Bethe and Heitler[1,2] derived a theory for the bremsstrahlung of a relativistic charged particle in a given external field. Their formulation of the problem is exact for the bremsstrahlung of an electron in the field of a bare nucleus, since the large mass of the nucleus in comparison with that of the electron allows us to assume that the nucleus remains fixed before and after the collision. Bethe and Heitler also treated the effect of an atomic electron on the bremsstrahlung by a charged particle in a given field. They introduced the static field of an atomic electron which screens the nuclear charge $Z|e|$ in the relativistic Hamiltonian of the incident particle. This is the screening approximation. This formulation of the problem has the obvious shortcoming that it does not allow us to take the limit $Z = 0$ to deal with the bremsstrahlung of an incident particle in a collision with a free electron. (At $Z = 0$, when the field of the nucleus disappears, the scattering of the incident particle by an atom goes over to scattering by a free electron. This is of course also true of scattering accompanied by the emission of a photon, i.e., bremsstrahlung.) Just how important this shortcoming is can be seen from the following discussion. We assume that the incident particle (of mass m_1 and charge e_1) is heavy—a meson or a proton. In scattering in a given static field, classical electrodynamics and quantum electrodynamics then lead to identical bremsstrahlung cross sections, which are smaller by a factor of $(m_1/m)^2$ than the cross section for the bremsstrahlung of an electron (of mass m) in the same field if the electron velocity is the same as that of the incident heavy particle. We now transform to a frame of reference in which the heavy particle is at rest. In this frame the atom is incident on the heavy particle. Under certain conditions,[3] the atomic electron may be treated as free. The bremsstrahlung which arises when the atom collides with the heavy particle should then be roughly the same as the bremsstrahlung in the scattering of an electron in the prescribed Coulomb field of the heavy particle. The cross section which is calculated in the screening approximation for the bremsstrahlung of a heavy charged particle scattered by an atom thus turns out to be substantially too small by a factor of $(m_1/m)^2$.

The inadequacy of the screening approximation was recognized a long time ago, apparently first by Landau and Rumer.[4] In their 1938 paper,[4] the atomic electrons were treated as free, with the result the factor of Z^2 in the bremsstrahlung cross section is replaced by a factor of $Z(Z+1)$.

There is accordingly a need for a systematic formulation of the problem of the bremsstrahlung in scattering by an atom, with the atomic electron treated on an equal footing with the incident particle, i.e., as a moving particle which is interacting with a quantized electromagnetic field. The problem of the bremsstrahlung of a nonrelativistic charged particle scattered by an atom has been solved rigorously in several recent papers.[5–11] The familiar static term in the bremsstrahlung amplitude becomes supplemented with a new—dynamic—term. The bremsstrahlung amplitude now consists of two terms, which describe static bremsstrahlung and dynamic bremsstrahlung. In a certain frequency interval the dynamic term turns out to be larger than the static term, giving rise to new results in the theoretical description of bremsstrahlung. In particular, when the frequency of the bremsstrahlung photon is near an atomic absorption frequency resonant bremsstrahlung, a resonant structural feature in the bremsstrahlung cross section, occurs.[5] Particularly noteworthy in this regard are Refs. 12–14, where experimental data were interpreted as evidence of dynamic bremsstrahlung. Wendin and Nuroch[12] start from the equations of the nonrelativistic theory of resonant bremsstrahlung, while a more comprehensive theory is used in Refs. 13 and 14.

It should be noted that the dynamic theory is required not only for bremsstrahlung in scattering by an atom but also for bremsstrahlung in a plasma or a condensed medium. An effect analogous to this dynamic bremsstrahlung—transition bremsstrahlung in a plasma—was proposed and studied in Refs. 15 and 16. Ter-Mikaelyan[17] has pointed out a deficiency of the screening theory in condensed media.

With the tools which have been developed in quantum electrodynamics it is now possible to systematically formulate and solve the problem of the bremsstrahlung of a relativistic charged particle scattered by an atom.

We thus consider the relativistic problem of the bremsstrahlung in scattering by a hydrogen atom or a hydrogen-like ion. For definiteness we assume that the incident particle is described by the Dirac equation. We assume that the nucleus is immobile and is the source of a static Coulomb

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field, while the initial and final velocities of the incident particle satisfy the condition $v_{i,f} \gg Z\,|ee_i|$. (We are using a system of units with $\hbar = c = 1$.) The operators representing the electron-positron field and the field of the incident particle must then be expanded in the wave functions which are the exact solutions of the Dirac equation in the external field (the Furry representation). We will use this representation only for the atomic electron; we assume that the incident particle is free (i.e., we use the Born approximation).

We use standard perturbation theory. Figure 1 shows the Feynman diagrams for the spontaneous emission of a single photon when a charged incident particle is scattered by an atom in first-order perturbation theory. Here a thin solid line corresponds to a free incident particle, and a thick solid line corresponds to an atomic electron in the nuclear field. The sloping lines represent the initial and final states of the incident particle ($|\,p_i\rangle$ and $\langle\, p_f\,|$) and of an atomic electron ($|n_i\rangle$ and $\langle n_f\,|$), and horizontal lines represent particle propagators. A thin line represents a free incident particle, and a thick line an electron in a nuclear field. The vertical dashed lines in the first two diagrams represent the interaction of the incident particle with the nuclear field, while those in the other diagrams represent the photon propagator. The vertical wavy line represents an emitted photon. The first four diagrams describe the "emission of a photon by the incident particle" in scattering by a nucleus (the first pair of diagrams) and in scattering by an atomic electron (the second pair). The last two diagrams describe the "emission of a photon by an atomic electron" in the scattering of the incident particle by this electron. When the incident particle is an electron, it is generally necessary to also consider the terms in the transition amplitude which are described by exchange diagrams ($\langle n_f\,| \leftrightarrow \langle\, p_f\,|$). In the Born approximation, however, with the velocity of the atomic electron significantly less than that of the incident electron, such diagrams need not be considered. If there is no atomic electron we are left with only the first two diagrams, which describe the bremsstrahlung in the field of the nucleus. If there is no nucleus ($Z = 0$), the last four diagrams describe the bremsstrahlung in the collision of an incident particle with an electron. For this case, the electron propagator in the nuclear field must be replaced by the propagator of a free electron. Consequently, the limiting case $Z = 0$ is included in the exact formulation of the problem.

Treating the nucleus as fixed before and after the collision (i.e., ignoring the recoil of the nucleus), and restricting the analysis to a nondegenerate initial state of the atomic electron, $|n_i\rangle$, we find the following expression for the differential bremsstrahlung cross section:

$$d\sigma_{fi}=|T_{fi}|^2\frac{\varepsilon_i}{(2\pi)^5|\mathbf{p}_i|}\delta(\varepsilon_i+E_{n_i}-\varepsilon_f-E_{n_f}-\omega)\,d\mathbf{p}_f\,d\mathbf{k}. \qquad (1)$$

Here ω and $\mathbf{k}$ are the energy and momentum of the photon, ε and $\mathbf{p}$ are those of the incident particle, k and p are the corresponding 4-momenta, and E_n is the energy of the atomic electron. The subscripts i and f specify the initial and final states. All quantities refer to the laboratory frame of reference, in which the nucleus is at rest. Here $d\sigma_{fi}$ is the differential cross section for the emission of a photon into the interval of states $d\,\mathbf{k}$ and for the simultaneous scattering of the incident particle into the interval of states $d\mathbf{p}_f$. The amplitude T_{fi} depends on the final state of the atom. Expression (1) refers to the case in which the atomic electron remains in its initial state or is excited into a state of the discrete spectrum in the course of the bremsstrahlung. If $|n_f\rangle$ is a state of the continuous spectrum of the atomic electron, then expression (1) must be changed in the usual way: The right side must be multiplied by the final-state density of the atomic electron.

The δ function in (1) expresses conservation of energy. If the atom is excited beforehand, e.g., by light, then the incident particle may, as it collides with the excited atom, emit a photon with an energy greater than its own energy ε_i by an amount equal to the atomic excitation energy $E_{ni} - E_{nf}$.

The amplitude T_{fi} corresponding to the diagrams in Fig. 1 is

$$T_{fi}=T_{fi}{}^{\rm st}+T_{fi}{}^{\rm dy},$$

$$T_{fi}{}^{\rm st}=4\pi e_1{}^2\frac{1}{q_1{}^2}\left[Zeg^{0\nu}\langle n_f|n_i\rangle-j^{\nu}_{n_f n_i}(\mathbf{q}_1)\right]\frac{\bar{u}_f}{(2\varepsilon_f)^{1/2}}$$

$$\times[\gamma_\nu G(p_2)\gamma_\mu+\gamma_\mu G(p_1)\gamma_\nu]\frac{u_i}{(2\varepsilon_i)^{1/2}}A_{\mathbf{k}}^{\alpha\mu*}, \qquad (2)$$

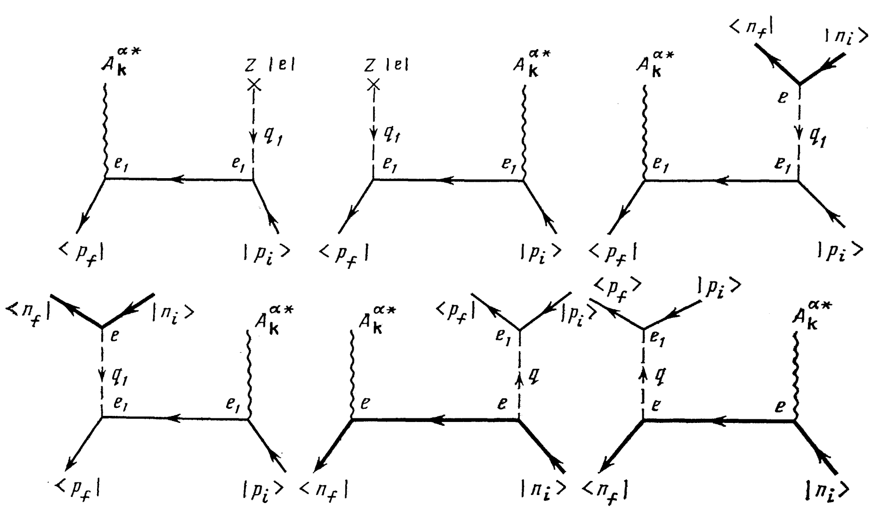

FIG. 1. Feynman diagrams describing the bremsstrahlung in scattering by an atom.

$$T^{\rm dy}_{fi}=\sum_n\left[\frac{j^{\bf k}_{n_f n}A^{\alpha *}_{\bf k}}{E_{n_f}+\omega-E_n\pm i\varepsilon}\,\frac{4\pi j^{\bf q}_{nn_i}J_{fi}}{q^2}+\frac{4\pi j^{\bf q}_{n_f n}J_{fi}}{q^2}\,\frac{j^{\bf k}_{nn_i}A^{\alpha *}_{\bf k}}{E_{n_i}-\omega-E_n\pm i\varepsilon}\right]_{\varepsilon\to+0},$$

$$q=p_f-p_i;\quad q_1=q+k;\quad p_2=p_f+k;\quad p_1=p_i-k.$$

Here

$$G(p_l)=\frac{\gamma p_l+m_1}{p_l^2-m_1^2},\qquad A_{{\bf k}\mu}{}^{\alpha}(x)=A_{{\bf k}\mu}{}^{\alpha}e^{-ikx},$$

$$A_{{\bf k}\mu}{}^{\alpha}=\left(\frac{2\pi}{\omega}\right)^{1/2}e_{{\bf k}\mu}{}^{\alpha},$$

$$|p\rangle\sim\psi_p(x)=\frac{u(p,s)}{(2\varepsilon)^{1/2}}e^{-ipx},\qquad J_{fi}{}^{\mu}=e_1\frac{\bar u_f\gamma^\mu u_i}{(2\varepsilon_f 2\varepsilon_i)^{1/2}},$$

$$j^{\varkappa\mu}_{n_2n_1}=e\langle n_2|\gamma^\mu e^{-i\varkappa r}|n_1\rangle=j^{\mu}_{n_2n_1}(\varkappa),\qquad(3)$$

$$ab=a^\mu b_\mu=a^0b_0-{\bf ab},\quad a=(a^\mu)=(a^0,{\bf a}),\quad x=(t,{\bf r}),$$

$$a\gamma=a^\mu\gamma_\mu,\quad a_\mu=g_{\mu\nu}a^\nu,\quad l=1,2;\quad \mu=0,1,2,3.$$

The metric, normalization, and notation in (3) are taken from Ref. 18: $g_{\mu\nu}$ is a metric tensor with zero off-diagonal terms and with the diagonal terms $g_{00}=1$ and $g_{11}{}^1=g_{22}=g_{33}=-1$; ab is the scalar product of the 4-vectors a and b; x is the 4-coordinate; γ^μ are the Dirac matrices; $u=u(p,s)$ is the bispinor of the relativistic incident particle (s is the spin variable); and $\bar u=u^+\gamma^0$. The normalization of the bispinors, $\bar u u=2m_1$, corresponds to the normalization of the wave function of the incident particle, $\psi_p(x)$, to a single particle in the main region with a unit volume. The photon wave function $A^{\alpha}_{\bf k}(x)$ is normalized to a single photon in the main region, and $e_k\,\alpha$ is the polarization 4-vector which corresponds to the polarization α, for which we will be using the $3D$-transverse gauge $e^{\alpha}_{\bf k}=(0,{\bf e}^{\alpha}_{\bf k})$, ${\bf e}^{\alpha}_{\bf k}{\bf k}=0$. To calculate the amplitude T_{fi} we express the propagator of the atomic electron as an expansion in the relativistic stationary states $|n\rangle$ of the electron in the nuclear Coulomb field (§27 in Ref. 19), where n is the set of quantum numbers determining the stationary state of the atom, and the sum is over all the states n and also over a repeated index μ or ν. For the photon propagator we use the Feynman gauge (§76 in Ref. 18). We first write the amplitude in the coordinate representation, after we have carried out all possible integrations over the time and the spatial coordinates we find that the amplitude has the form (2). The equations and the calculations can be simplified substantially, and the equations can be made considerably more graphic, by using the particle transition currents J_{fi} and $j_{n_1n_2}$: the transition current densities of the incident particle and of the atomic electron in the momentum representation without time factors [Eq. (43.11) in Ref. 18].

The first term in (2), $T^{\rm st}_{fi}$, corresponds to the sum of the first four diagrams in Fig. 1. It describes the "emission by the incident particle" in a collision with a nucleus and an atomic electron ($T^{\rm st}_{fi}\propto e_1^2e$). This is static bremsstrahlung. If the state of the atomic does not change as a result of the collision ("pure bremsstrahlung"), the cross section for the static bremsstrahlung is that given by the familiar Bethe-Heitler expression.[1,2] The second term in (2), $T^{\rm dy}_{fi}$ corresponds to the sum of the last two diagrams and describes the "emission by the atomic electron" in a collision with the incident particle ($T^{\rm dy}_{fi}\propto e_1e^2$). This term arises only in the systematic formulation of the problem of bremsstrahlung in scattering by an atom, in which the motion of the atomic electron is taken into account. This is dynamic bremsstrahlung. The bremsstrahlung cross section contains terms which corresponds to the square modulus of each of the components (the cross section for static bremsstrahlung and that for dynamic bremsstrahlung) and also an interference term. The structure of the amplitude for the dynamic bremsstrahlung is demonstrated clearly by the diagrams in Fig. 2, which correspond to the various components in the sum over n. The lower vertices in each diagram correspond to the atomic currents, and the upper vertices to the current of the incident particle. Relativistic expression (2) for the amplitude $T^{\rm dy}_{fi}$ can be found from the nonrelativistic expression (2) in Ref. 8 by replacing the charges and 3-currents in the latter expression by relativistic 4-currents, while replacing the 4-momentum transfer $\bf q$ in the denominator in expression (2) by the 4-vector q (in the normalization of the present paper, the charge density is numerically equal to the charge; in Ref. 8, it is necessary to transform to the system of units used here). When this replacement is made, the Fourier transform of the Coulomb potential $4\pi/{\bf q}^2$ becomes, to within a factor of $g_{\mu\nu}$,

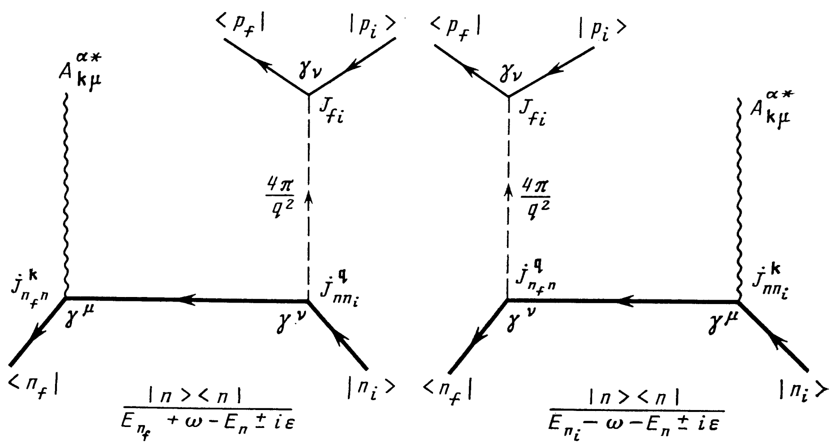

FIG. 2. Diagrams corresponding to the various terms in the amplitude for dyanmic bremsstrahlung.

the photon propagator in the momentum representation (the tensor $g_{\mu\nu}$ becomes the scalar product of 4-vectors). The product of currents and the product of the atomic electron and a $A_k^{\alpha *}$ (the "photon potential") must be understood as four-dimensional scalar products. The sum over the intermediate states n runs over both electron and positron states.

Formally, expression (2) is completely relativistic: Both the incident particle and the atomic electron are described by Dirac bispinors. We restrict the present paper to the approximation in which the atomic electron is nonrelativistic ($Z \ll 137$; $|E_{n_i} - m|$, $|E_{n_f} - m| \ll m$), while the incident particle is relativistic. If the condition $\omega \ll m$ also holds, the amplitude for the dynamic bremsstrahlung simplifes substantially:

$$T_{fi}^{dy} = \frac{4\pi e_1}{q^2}\left(\frac{2\pi}{\omega}\right)^{1/2}\left\{\sum_n \frac{\mathbf{e}_\mathbf{k}^{\alpha *}\mathbf{j}_{n_f n}(\mathbf{k})\,[j^0_{nn_i}(\mathbf{q}) - \mathbf{v}_i\mathbf{j}_{nn_i}(\mathbf{q})]}{E'_{n_f} + \omega - E'_n + i\varepsilon} + \sum_n \frac{[j^0_{n_f n}(\mathbf{q}) - \mathbf{v}_i\mathbf{j}_{n_f n}(\mathbf{q})]\,\mathbf{e}_\mathbf{k}^{\alpha *}\mathbf{j}_{nn_i}(\mathbf{k})}{E'_{n_i} - \omega - E'_n + i\varepsilon} - \frac{e}{m}\,e_\mathbf{k}^{\alpha *}\mathbf{v}_i j^0_{n_f n_i}(\mathbf{q}_1)\right\}_{\varepsilon\to+0}, \qquad (4)$$

$$J_{fi}{}^{\mu} = J_{ii}{}^{\mu} = e_1(1, \mathbf{v}_i);\quad E_n' = E_n - m;\quad \hat{\mathbf{v}}_a = -i\nabla/m;$$

$$j^0_{n_2n_1}(\boldsymbol{\varkappa}) = e\langle n_2|e^{-i\boldsymbol{\varkappa}\mathbf{r}}|n_1\rangle;$$

$$\mathbf{j}_{n_2n_1}(\boldsymbol{\varkappa}) = \frac{e}{2}\langle n_2|\hat{\mathbf{v}}_a e^{-i\boldsymbol{\varkappa}\mathbf{r}} + e^{-i\boldsymbol{\varkappa}\mathbf{r}}\hat{\mathbf{v}}_a|n_1\rangle.$$

In (4), $|n\rangle$ is the nonrealistic Schrödinger vector of the stationary state of the atom (without the time factor), the operator $\hat{\mathbf{v}}_a$ represents the velocity of the atomic electron, and $\mathbf{v}_i$ is the velocity of the incident particle. In the derivation of (4) from (2) it was assumed that J_{fi}, the transition 4-curent of the incident particle, can be replaced by the 4-current J_{ii} under the condition $|\mathbf{q}| \ll |\mathbf{p}_i|$, which we assume holds. The same approximation is used in the theory for the excitation of an atom by a relativistic particle (§82 in Ref. 18). In this approximation, the terms containing Pauli matrices drop out, so that the effects due to the spin of the incident particle also drop out. Expression (4) can be derived from (2) by breaking the sum over n in (2) into the two sums Σ^+ and Σ^-, over states with positive energy and over states with negative energy. The calculation of the sum Σ^- is exactly the same as that for the corresponding sum in relativistic dispersion theory (§27 in Ref. 19). Since $|E'_{n_i}|$, $|E'_{n_j}| \ll m$, we can assume that $|E'_n| \ll m$ also holds (other states do not make much of a contribution to Σ^- because of the small oscillator strengths of the corresponding transitions). The denominators in the various terms of the sum Σ^- are then equal to $2m$, and introducing a projection operator we find a convolution in n, which is equal to the last term in (4). To find an expression for the sum Σ^+ which is nonrelativistic in the atomic electron, we must replace the relativistic currents of the atomic transitions in it by non-relativistic currents [Eq. (82.6) in Ref. (18)]. The nonrelativistic current can be found from the relativistic current by replacing the bispinors by spinors and by ignoring the $1/c$ terms. In this manner we are ignoring effects associated with the spin of the atomic electron.

We wish to emphasize that in order to incorporate in the bremsstrahlung cross section the limiting case of a free electron ($Z = 0$) (mentioned at the beginning of this paper), we must carry out a summation over all the final states of the atomic electron, $|n_f\rangle$, including an integration over the continuum, in (1). Only this complete bremsstrahlung cross section allows the limit $Z = 0$. We now consider the case of dynamic "pure bremsstrahlung" ($|n_f\rangle = |n_i\rangle$). We first calculate the cross section for dynamic bremsstrahlung in the frequency range $\omega \ll p_a v_i$ ($p_a = Ze^2m$, where me^2 is the atomic unit of momentum; we have $p_a v_i \cong 3.7\cdot 10^3$ eV for $Z = 1$ and $v_i \cong 1$). Below we will prove rigorously that only under the condition $\omega < p_a v_i$ is the dynamic pure bremsstrahlung significant. In this case expression (4) can be simplified substantially by noting that the cross section for dynamic bremsstrahlung is dominated by small values of $|\mathbf{q}|$: $\omega/v_i \leqslant |\mathbf{q}| < p_a$. The lower boundary on the $|\mathbf{q}|$ range here is imposed by energy conservation. The matrix element $j^0_{n_in_i}(\mathbf{q}_1)$ from (4) falls off sharply at $|\mathbf{q}| > p_a$ because of the rapidly oscillating exponential function $e^{-i\mathbf{q}_1\mathbf{r}}$ in the integrand. As for the products $j^\nu_{n_in}(\mathbf{q})\cdot\mathbf{j}_{nn_i}(\mathbf{k})$ [like $\mathbf{j}_{n_in}(\mathbf{k})\cdot j^\nu_{nn_i}(\mathbf{q})$], which enter the sum over n in (4) in the case $\nu \neq 0$, we note that they also make a small contribution to the cross section at $|\mathbf{q}| > p_a$. In this case the transition current $j^\nu_{n_in}(\mathbf{q})$ is significant only for those intermediate states $|n\rangle = |p\rangle$ whose wave functions contain the "cancelling" factor $e^{i\mathbf{pr}}$, $\mathbf{p} \approx \mathbf{q}$ (§148 in Ref. [20]). For such states $|p\rangle$, however, the second factor, $\mathbf{j}_{\mathbf{nnj}}(\mathbf{k})$, is small ($|\mathbf{k}| \ll p_a$ for $\omega \ll p_a v_i$) and the intermediate state $|p\rangle$ contributes a rapidly oscillating exponential factor. We can therefore carry out a series expansion of all the exponential functions in the matrix elements $j^\mu_{n_1n_2}$ and retain only the first nonvanishing term. In calculating the integral cross section for dynamic bremsstrahlung over $\mathbf{q}$ we must then set $T^{dy}_{ii}(\mathbf{q}) = 0$ at $|\mathbf{q}| > p_a$. It then becomes possible to express the amplitude for the dynamic pure bremsstrahlung in terms of the polarizability $\beta_i(\omega)$ of the atom in state $|n_i\rangle$:

$$T_{ii}{}^{dy} = \frac{4\pi}{q^2}(2\pi\omega)^{1/2}e_1\beta_i(\omega)\,\mathbf{e}_\mathbf{k}{}^{\alpha *}(\mathbf{q} + \omega\mathbf{v}_i);\qquad |\mathbf{q}| < p_a; \qquad (5)$$

$$\beta_i(\omega) = \frac{e^2}{m}\sum_n \frac{f_{n_in}}{\omega^2_{nn_i} - \omega^2};\quad f_{n_in} = 2m\omega_{nn_i}|\langle n_i|\mathbf{x}|n\rangle|^2,$$

$$\omega_{nn_i} = E_n' - E_{n_i}';\qquad \Delta = |\omega - \omega_{nn_i}| \gg \Gamma_n. \qquad (6)$$

Here f_{n_in} is the oscillator strength of the transition $n_i \to n$, and Γ_n is the width of the discrete level n (here and below we are assuming that $|n_i\rangle$ is the ground state of the atom). Under the condition $\Delta \gg \Gamma_n$ we can ignore effects stemming from the level width Γ_n. In order to transform from expression (4), in which there is an expansion of exponential functions, to expression (5), we need to replace the matrix elements of the operator representing the velocity of the atomic electron by the matrix elements of the coordinate, making use of the completeness of the eigenvectors $|n\rangle$ and of the commutation rules for the momentum and coordinate operators. Making use of the spherical symmetry of the state

$|n_i\rangle$, we then find expression (5). Substituting (5) into (1), and summing over the polarizations of the photon [see expression (45.4b) in Ref. (18)], we find the frequency and angular distributions of the dynamic bremsstrahlung:

$$d\sigma_{ii}^{\rm dy}(\omega,\theta)=2\sigma_0\left(\frac{c}{v_i}\right)^2\left[\frac{m\omega^2}{e^2}\beta_i(\omega)\right]^2\frac{d\omega}{\omega}\ln\frac{p_a\gamma v_i}{\hbar\omega}(1+\cos^2\theta)\sin\theta\,d\theta, \tag{7}$$

$$\sigma_0=\left(\frac{e^2}{mc^2}\right)^2\frac{e_1^2}{\hbar c};\quad p_a=\frac{Ze^2m}{\hbar};\quad \gamma=\frac{\varepsilon_i}{mc^2};$$
$$v_i\gg Z|e_1e|/\hbar;\quad \hbar\omega\ll p_a v_i.$$

Here θ is the angle between $\mathbf{v}_i$ and $\mathbf{k}$; expression (7) and also expressions (8)–(10) below are written in the absolute system of units. It can be seen from (7) that the angular distribution of the dynamic bremsstrahlung does not depend on the velocity of the incident particle: For all $v_i \gg Z|e_1e|$, this distribution is determined by the slowly varying factor $1+\cos^2\theta$. As for the static bremsstrahlung, we note that the denominator of the corresponding cross section contains a factor $1-v_i\cos\theta$ (§28 in Ref. [19]), so that the static bremsstrahlung in the ultrarelativistic limit ($\gamma \gg 1$) is concentrated sharply in the forward direction, in a narrow cone around the direction of $\mathbf{v}_i$. The differential cross section for the dynamic bremsstrahlung at $\omega \ll p_a v_i$ and $\gamma \gg 1$ is significantly larger than the differential cross section for the static bremsstrahlung everywhere outside the cone in which the static bremsstrahlung is concentrated. The spectral cross section for dynamic bremsstrahlung corresponding to (7) takes the following form after integration over all θ:

$$d\sigma_{ii}^{\rm dy}(\omega)=\frac{16}{3}\sigma_0\left(\frac{c}{v_i}\right)^2\left[\frac{m\omega^2}{e^2}\beta_i(\omega)\right]^2\frac{d\omega}{\omega}\ln\frac{p_a\gamma v_i}{\hbar\omega}. \tag{8}$$

In the nonrelativistic limit ($\gamma \approx 1$), expression (8) becomes the corresponding expression of the nonrelativistic theory, derived by Zon.[9] For comparison, here is the spectral cross section for the static bremsstrahlung of a particle of mass m_1 in scattering by a hydrogen-like ion for $v_i \approx 1$ and for the case of complete screening, i.e., under the condition $\omega < p_a\gamma^2$ (§93 in Ref. [18]):

$$d\sigma^{\rm st}(\omega)=\frac{16}{3}Z^2\sigma_0\left(\frac{m}{m_1}\right)^2\left(\frac{e_1}{e}\right)^2\frac{d\omega}{\omega}\ln\frac{m_1}{\alpha Zm},\quad \alpha=\frac{e^2}{\hbar c}. \tag{9}$$

From (8) and (9) we find some characteristic differences between the cross sections for dynamic and static bremsstrahlung:

1. There are differences in the dependence on the mass of the incident particle. The cross section for the dynamic bremsstrahlung does not depend on the mass m_1 at a given velocity v_i, while the cross section for the static bremsstrahlung is inversely proportional to the square of m_1.

2. The dynamic bremsstrahlung and the static bremsstrahlung have different spectra. The frequency dependence of the cross section for the dynamic bremsstrahlung is determined primarily by the factor $\beta_i^2(\omega)\omega^3$.

3. The cross section for the static bremsstrahlung, (9), has a strong dependence on the nuclear charge Z, in contrast with the cross section for the dynamic bremsstrahlung, (8).

4. In the ultrarelativistic limit ($\gamma \gg 1$) the cross section for the dynamic bremsstrahlung increases logarithmically with the energy of the incident particle, while the cross section for the static bremsstrahlung does not depend on the energy of the incident particle at $\omega < \gamma^2 p_a$.

It follows from (8) and (7) that the cross section for the dynamic bremsstrahlung increases significantly if there is a resonant intermediate level ($\Gamma_n \ll \Delta \ll \omega_{nn_i}$). A resonant bremsstrahlung arises. The situation here is analogous to that in the resonant two-photon absorption and the resonant scattering of light by an atom: The probabilities for these processes also increase significantly when there is a resonant intermediate level. This analogy can be pursued in the following way: Resonant bremsstrahlung, like resonant scattering and resonant two-photon absorption, is a "direct" process; i.e., it occurs without any real filling of an intermediate level. Level n begins to be filled under the condition $\Delta < \Gamma_n$: The atom is excited by electron impact, and then a photon is emitted. A "cascade process" occurs. This case requires a more complicated analysis, and we will not consider it further here. We do wish to emphasize that in the case of resonant bremsstrahlung ($\Delta \gg \Gamma_n$) energy is conserved for the direct process [the δ-function in (1)]. The relation between the direct and cascade processes in the resonant scattering of light and in resonant two-photon absorption was studied in Refs. (21–23).

Let us use (8) and (9) to estimate $d\sigma^{\rm RB}(\omega)/d\sigma^{\rm st}(\omega)$ for the case of the bremsstrahlung emitted by an electron as it is scattered by a hydrogen atom. From (8) and (6) we find an expression for resonant bremsstrahlung:

$$d\sigma^{\rm RB}(\omega)=\frac{4}{3}\sigma_0\left(\frac{c}{v_i}\right)^2 f_{nn_i}^2\left(\frac{\omega}{\Delta}\right)^2\frac{d\omega}{\omega}\ln\frac{p_a\gamma v_i}{\hbar\omega}. \tag{10}$$

In deriving (10) we used an expression for the resonant polarizability of the atom: $\beta_i(\omega)\approx e^2 f_{n_in}/2m\omega\Delta$. From (10) and (9), and for the parameter values corresponding to the $1s\to 2p$ transition in the hydrogen atom, with $\gamma=10$ and $\omega/\Delta=10^2$, we find the estimate $d\sigma^{\rm RB}/d\sigma^{\rm st}\approx 10^3$ for the ratio of the cross sections for resonant bremsstrahlung and static bremsstrahlung. For these particular parameter values, the condition $\Delta \gg \Gamma_n$ holds. The corresponding estimate for a nonrelativistic incident electron was made in Ref. (5).

We now consider the dynamic bremsstrahlung in the frequency interval $I < \omega \ll m$ where I is the ionization potential of the atom. In this case we can simplify (4), noting that the sum over n is dominated by transitions in the intermediate states nearest $|n_i\rangle$, for which the oscillator strength has a significant value. In the frequency interval under consideration here, we have $\omega_{nn_i} < \omega$ for these transitions, and we can expand (4) in powers of ω_{nn_i}/ω. In this case the transitions corresponding to the resonant terms are linked with states of the continuous spectrum of the atomic electron. The integration over intermediate states in (4) erases the resonance, and the corresponding terms make a negligible contribution to the large-ω limit because of the small numerator. An analogous situation arises in the theory of dielectric permeability, because the dielectric permeability of any medium is described in the high frequency limit by the expression for a free electron (§78 in Ref. [24]). Retaining only the zeroth term in the expansion in powers of ω_{nn_i}/ω, we find that the sums over n is eliminated by virtue of the completeness of the system of eigenvectors $|n\rangle$, and for the amplitude for the pure dynamic bremsstrahlung we find

$$T_{ii}{}^{\mathrm{dy}}=\frac{4\pi}{q^2}\left(\frac{2\pi}{\omega}\right)^{1/2}\frac{e_1 e}{m\omega}\mathbf{e}_{\mathbf{k}}{}^{\alpha*}(\mathbf{q}+\omega\mathbf{v}_i)\overset{0}{j}_{n_i n_i}(\mathbf{q}_1),$$
$$I=\frac{1}{2}Z^2\alpha^2 m;\qquad I<\omega\ll m. \tag{11}$$

Calculations show that the correction to (11) is of order $(I/\omega)^2$. Substituting (11) into (1), and summing over the polarizations of the photon, we find the following expression for the spectral cross section of the dynamic bremsstrahlung:

$$d\sigma_{ii}{}^{\mathrm{dy}}=\sigma_0\frac{d\omega}{\pi^2 v_i\omega^5}\int\int\frac{|[\mathbf{k},\mathbf{q}+\omega\mathbf{v}_i]|^2\delta\ (\omega+\mathbf{q}\mathbf{v}_i)}{(\mathbf{q}^2-\omega^2)^2(1+\mathbf{q}_1{}^2/4p_a{}^2)^4}d\mathbf{q}\,d\mathbf{k},$$
$$I<\omega\ll m. \tag{12}$$

Expanding the integral in (12) in a series in the small parameter $\omega(1+v_i)/2p_a v_i$, and retaining the leading term of the expansion, we find

$$d\sigma_{ii}{}^{\mathrm{dy}}(\omega)=\frac{16}{3v_i^2}\sigma_0\frac{d\omega}{\omega}\ln\frac{2\gamma p_a v_i}{\omega(1+v_i)},$$
$$I<\omega<\frac{2p_a v_i}{1+v_i}. \tag{13}$$

Let us compare (13) and (8). If we replace $\beta_i(\omega)$ in expression (8) at $\omega>I$ by the polarizability of a free electron, $\beta_i\approx -e^2/m\omega^2$, we find (13) from (8). The condition $\omega\ll p_a v_i$ allows us to ignore the factor of $2/(1+v_i)$ in the logarithm in (13). For $I<\omega\ll p_a v_i$, expression (13) is thus the same as (8). It follows from (13) that the cross section for the pure dynamic bremsstrahlung of an electron has a significant value, comparable to that of static bremsstrahlung, in the frequency interval $I<\omega<2p_a v_i/(1+v_i)$. Analysis of (12) shows that at $\omega>2p_a v_i/(1+v_i)$ the cross section for the dynamic bremsstrahlung is small, since it is proportional to small parameters: In the interval $2p_a v_i/(1+v_i)<\omega<\min\{m,2p_a v_i/(1-v_i)\}$ the cross section is

$$d\sigma_{ii}{}^{\mathrm{dy}}/d\omega\sim(2p_a v_i/\omega(1+v_i))^2;$$

while in the interval $2p_a v_i/(1-v_i)<\omega\ll m$ it is

$$d\sigma_{ii}{}^{\mathrm{dy}}/d\omega\sim(4p_a{}^2v_i^2\gamma/\omega^2(1-v_i))^4.$$

It can be shown that at $\omega>2p_a v_i/(1+v_i)$ the cross section for the dynamic bremsstrahlung is dominated by another process: bremsstrahlung accompanied by simultaneous excitation of the atom.

In summary, a systematic dynamic theory of bremsstrahlung in scattering by an atom reveals some important new results. This derivation can be generalized to the case of large Z, where relativistic effects are important for the atomic electron. At large Z we cannot ignore the effect of the nuclear field on the incident particle. The amplitude for the dynamic bremsstrahlung is described again in this case by the diagrams in Fig. 1, but the Furry representation now applies not only to the atomic electron but also to the incident particle.

The dynamic bremsstrahlung has characteristics which distinguish it from other types of radiation. There is a clear need for experiments designed especially to observe dynamic bremsstrahlung in atomic and molecular gases and also in condensed media.

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Note added in proof (31 March 1985). In a paper just published {Zh. Eksp. Teor. Fiz. **88**, No. 2, 383 (1985) [Sov. Phys. JETP **61**, 224 (1985)]}, M. Ya. Amus'ya, M. N. Kuchiev, A. V. Korol', and A. V. Solov'ev give an expression [their Eq. (6)] which is the same as the corresponding expression in the present paper [Eq. (8)]. These expressions describe dynamic bremsstrahlung not involving excitation of the atom. At frequencies $\omega>p_a v_i$, the dynamic bremsstrahlung is dominated by processes involving the emission of a photon and the simultaneous excitation and ionization of the atom [Yu. A. Krotov, Tormoznoe izluchenie i rasseyanie fotonov pri mnogochastichnykh stolknoveniyakh (Bremsstrahlung and Scattering of Photons in Many-Body Collisions), Author's Abstract, Candidate's Dissertation, Moscow Physicotechnical Institute, Moscow, 1981).

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