Mutual excitation of acoustic and domain oscillations in a uniaxial ferromagnet, and domain-acoustic resonance

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Ferromagnet oscillations of a new type are investigated, a combination of quasioptic domain-structure oscillations and transverse sound waves of wavelength considerably exceeding the thickness of the domain wall. At a certain wavelength commensurate with the domain-structure period, the two oscillations are at resonance so that their amplitude can increase strongly. The quasiacoustic oscillations of the domain structure do not interact with the transverse sound in question.

In Ref. 1 we investigated the domain-structure natural-oscillation spectrum of a uniaxial ferromagnet, i.e., those deviations of the magnetization from its equilibrium distribution that correspond to wavelike displacements of a domain wall as a whole.

In this article we consider the interaction of the domain-structure oscillations with acoustic oscillations, which results in excitation of both the domain-structure oscillations and of the acoustic oscillations they induce via magnetostriction. No such effect was considered before; its existence, however is indirectly confirmed by theoretical and experimental results,[2–4] although for an entirely different situation. The studies cited were devoted to excitation of acoustic oscillations, with a wavelength that is a multiple of the domain-structure period, by an external uniform magnetic field that varied in time. The domain-structure oscillations had then an infinite wavelength. We shall show that wavelike domain-structure oscillations can excite sound waves, and conversely, sound waves can excite "domain" waves. This effect, which is resonant, was named by us domain-acoustic resonance.

Consider a uniaxial ferromagnet with large anisotropy constant $\beta \gtrsim 10$ in the form of a plate of thickness $D \gg \Delta = (\alpha/\beta)^{1/2}$ (where α is the exchange-interaction constant.[5] The plane xy of the plate is perpendicular to the anisotropy (z) axis. The dimensions L_x and L_y of the plate are much larger than its thickness, so that the influence of the plate edges can be neglected. A stripe domain structure is possible in such a sample. We shall assume that the domain walls are in planes parallel to the yz plane, so that the normal to them is parallel to the x axis. We confine ourselves hereafter to the interaction between the natural oscillations of the domain structure (quasi-optic and quasi-acoustic[1)] and the transverse sound oscillations polarized along the y axis. As will be shown below, it suffices here to consider sound oscillations with wavelength considerably exceeding the domain-wall thickness; this is equivalent to the inequality $k\Delta \ll 1$, where k is the wave vector of the oscillations.

The total energy, which is the sum of the energy of the intrinsic magnetic field (due to jumps of the magnetization on the crystal surface), the elastic energy, and the domain-wall energy, is equal to

$$W = W_H + W_y + W_i = -\frac{L_y}{2}\sum_n \int_{-D/2}^{D/2} dz \int_{na}^{(n+1)a} dx\, \mathbf{MH}_m + \frac{DL_y}{2}\int_{-L_x/2}^{L_x/2} dx\left[\rho \dot{u}_y^2 + c\left(\frac{\partial u_y}{\partial x}\right)^2\right]$$
$$+ DL_y \sum_n \int_{(n+1/2)a+X_n-\Delta}^{(n+1/2)a+X_n+\Delta} dx \left[\frac{\alpha}{2}(M_x'^2 + M_y'^2 + M_z'^2) + \frac{\beta}{2}(M_x^2 + M_y^2) + 2\pi M_x^2 + bM_xM_y\frac{\partial u_y}{\partial x}\right]. \quad (1)$$

Here M is the magnetization, M_x, $M_y = (M^2 - M_x^2)^{1/2} \sin\theta$, $M_z = (M^2 - M_x^2)^{1/2}$. its components, $\mathbf{H}_m$ the intrinsic magnetic field produced by the magnetization jumps on the crystal surface, n the number of the domain wall, x_n the location of its midplane, a the domain dimension along the x axis, u_{el} the elastic displacement, ρ the crystal density, c the elastic constant, and b the magnetostriction constant.

Assuming that each domain wall moves as a unit remaining plane and perpendicular to the x axis, we take M_x and θ to be dependent on the combination $x - \int v_n\, dt$, where v_n is the velocity of the nth wall. These relations satisfy for each domain wall the known equation of magnetization motion,[5] in which we have added the magnetostriction interaction and neglected the dissipation. $M_x = 0$ in a domain wall in the absence of elastic deformation.[5] In analogy with Ref. 6, it can be shown that M_x is proportional to the wall velocity v or to the elastic deformation $\partial u_{el}/\partial x$. The zeroth approximation in these small parameters leads to the known equation for the structure of an immobile domain wall[5]

$$\alpha\theta_0'' - \beta \sin\theta_0 \cos\theta_0 = 0 \quad (2)$$

(a prime denotes differentiation with respect to $x - X_n(t)$).

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We shall need subsequently the solution of (2) in the form

$$\sin^2\theta_0 = \mathrm{ch}^{-2}\left[\frac{x-X_n(t)}{\Delta}\right].$$

The first approximation yields an equation for M_x:

$$M_x'' - \left(\frac{4\pi}{\beta} + \cos 2\theta_0\right) M_x = \frac{v}{\beta g\Delta}\theta_0' + \frac{bM}{\beta}\frac{\partial u_y}{\partial x}\sin\theta_0.$$

This equation has a solution that satisfies the boundary conditions at the points $\theta_0 = n\pi$ and $(n+1)\pi$, in the form

$$M_x = (-1)^{n+1}\frac{v_n}{4\pi g\Delta}\sin\theta_0 - \frac{bM}{4\pi}\frac{\partial u_y}{\partial x}\sin\theta_0. \quad (3)$$

Substituting (3) in the Lagrange equation corresponding to (1), we obtain, in an approximation quadratic in v and $\partial u_{el}/\partial x$, the Lagrangian terms connected with the inhomogeneity of the magnetization distribution within the domain walls:

$$\frac{m}{2}\sum_n v_n^2 + \frac{1}{4\pi} bM\frac{DL_y}{g\Delta}\sum_n (-1)^n v_n \times \sum_{(n+1/2)a-\Delta}^{(n+1/2)a+\Delta} dx\frac{\partial u_y}{\partial x}\sin^2\theta_0 + \frac{1}{8\pi} b^2M^2DL_y\sum_n \int_{(n+1/2)a-\Delta}^{(n+1/2)a+\Delta} dx\left(\frac{\partial u_y}{\partial x}\right)^2 \sin^2\theta_0,$$

where $m = DL_y/2\pi g^2\Delta$ is the effective mass of the domain wall; its value turned out to be the same as in Ref. 6, even though the anisotropy was not assumed to be small. The first term here is due to the motion of the domain wall and has the meaning of its kinetic energy, while the second and third are connected with the interaction between the moving domain wall and the elastic deformation of the crystal. The last term leads to a small renormalization of the sound velocity inside the domain wall, which will be neglected hereafter.

Expanding the first term of (1) in powers of the displacements X_n of the domain walls, up to and including quadratic terms, and using the results of Ref. 1, we obtain

$$W_H - W_{0H} = \frac{1}{2}\sum_{n,n'}\gamma_{n,n'}X_nX_{n'},$$

$$\gamma_{n,n'} = \gamma(|n-n'|) = 8M^2L_y(-1)^{n-n'}\ln\left[1+\frac{D^2}{(n-n')^2a^2}\right],$$

$$\gamma_{nn} = \gamma = 16M^2L_y\sum_{n=1}^{\infty}(-1)^{n+1}\ln\left[1+\frac{D^2}{n^2a^2}\right]$$

$$= 16M^2L_y\ln\left[\frac{\pi D}{2a}\mathrm{cth}\frac{\pi D}{2a}\right].$$

We can now write the Lagrangian for the elastic oscillations and for the domain-structure oscillations, in the form

$$\mathcal{L} = \frac{m}{2}\sum_n \dot{X}_n^2 - \frac{1}{2}\sum_{n,n'}\gamma_{nn'}X_nX_{n'} + \frac{DL_y}{2}\int dx\left[\rho\dot{u}_y^2 - c\left(\frac{\partial u_y}{\partial x}\right)^2\right] + \frac{bM}{4\pi}\frac{DL_y}{g\Delta}\sum_n(-1)^n\dot{X}_n\int_{(n+1/2)a-\Delta}^{(n+1/2)a+\Delta}\sin^2\theta\frac{\partial u_y}{\partial x}dx.$$

The equations for the oscillations are

$$\ddot{X}_{2n+1} + \frac{1}{m}\left(\gamma X_{2n+1} + \sum_{n'\neq 2n+1}\gamma_{2n+1,n'}X_{n'}\right) = bgM\Delta\dot{u}_{yx}\left(\left(2n+\frac{3}{2}\right)a\right), \quad \ddot{X}_{2n} + \frac{1}{m}\left(\gamma X_{2n} + \sum_{n'\neq 2n}\gamma_{2n,n'}X_{n'}\right) = -bgM\Delta\dot{u}_{yx}\left(\left(2n+\frac{1}{2}\right)a\right), \quad \ddot{u}_y - S_t^2\frac{\partial^2 u_y}{\partial x^2} = -\frac{bM}{2\pi\rho}\frac{1}{g\Delta}\sum_n(-1)^n\dot{X}_n\frac{d}{dx}\sin^2\theta_0\left(x-\left(n+\frac{1}{2}\right)a\right).$$

It is recognized here (S_t is the speed of sound) that the quantity $\partial u_{el}/\partial x \equiv u_{yx}$ can be regarded as constant over the width of the domain wall ($k\Delta \ll 1$). Just as in Ref. 1, the equations for X_{2n} *and* X_{2n+1} are different, but are now connected with the equation for the elastic displacment u_{el}; the latter, in turn is connected with the equations for X_{2n} and X_{2n+1}. The expression in the right-hand side of the last equation is periodic in x, with a period that is a multiple of a. It can therefore be expanded in the Fourier series $\Sigma_k c_k e^{ikx}$ with $k = \pi n/a$. At the wave vector values $k\Delta \ll 1$ of interest to us, the Fourier series for it takes the form

$$-\frac{bM}{2\pi\rho}\frac{1}{g\Delta}\sum_k e^{ikx}\sum_n \dot{X}_n\frac{1}{2Na}\int_{-Na}^{Na}e^{-ikx}\frac{d}{dx}\sin^2\theta_0\,dx = -\frac{bM}{2\pi\rho}\frac{1}{ga}\sum_k(ik)e^{ikx}\frac{1}{N}\sum_n(-1)^n\dot{X}_ne^{-ik(n+1/2)a},$$

where $2N$ is the total number of the domains. Expanding all the quantities in the equations for the oscillations in terms of plane waves, we get

$$(-\omega^2 + k^2S_t^2)u_k = -\frac{bM}{2\pi\rho}\frac{k\omega}{ga}e^{-ika/2}(X_k^+ - X_k^-),$$

where $X_k^{\mp}$ are the Fourier components of the displacements of the even and odd domain walls, respectively. These equations lead to a dispersion relation for the coupled oscillation

$$(\omega_{ac}^2 - \omega^2)\left[(\omega_t^2 - \omega^2)(\omega_{opt}^2 - \omega^2) - \xi\omega^2\omega_t^2\right] = 0. \quad (4)$$

Here ω_t is the frequency of the transverse sound, ω_{ac} the frequency of the quasiacoustic oscillations, and $\omega_{\rm opt}$ the frequency of the quasioptic oscillations, for which we give asymptotic expressions: in the limit of low values of the wave vector, $k \to 0$, when

$$\omega_{opt}^2=\Omega^2 \frac{\Delta}{a}(2-|k|D), \quad \Omega=4\pi gM,$$

and in the limit of the largest values $k \to k_{\max} = \pi 2a$; in this case

$$\omega_{opt}^2=2\frac{\Omega^2}{\pi}\frac{\Delta}{D}\left[\ln 2+2\mathscr{G}\left(1-\frac{2ka}{\pi}\right)\right],$$

$$\mathscr{G}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}.$$

The coupling parameter of the acoustic and quasioptic oscillations is $\xi = (b^2M^2/\pi\rho S_t^2)(\Delta/a)$. At $M \approx 8.5 \cdot 10^2$ G, $\rho \approx 10$ g/cm^3, and $S \approx 3 \cdot 10^5$ cm/sec, $\Delta \approx 10^{-6}$ cm, $\beta \approx 15$, $b \approx 10$, and $D \approx 1$ cm we have $\xi \approx 10^{-8}$.

The oscillation equations have according to (4) two types of solution, viz., quasiacoustic oscillations of the domain structure and coupled elastodomain oscillations. For the latter, the amplitude of the sound generated by the domain-structure oscillations is

$$u_k=\pm\frac{bM}{\pi\rho}\frac{k\omega}{ga}\frac{e^{-ika/2}X_k^{\mp}}{\omega_t^2-\omega^2}, \tag{5a}$$

and conversely

$$X_k^{+}=-X_k^{-}=-\frac{bMg\Delta k\omega e^{ika/2}u_k}{\omega_{opt}^2-\omega^2}. \tag{5b}$$

For the analysis of the coupled elastodomain oscillations it is convenient to introduce a characteristic wave vector k_0 for which the quasiacoustic-oscillation frequency, which decreases with increasing k, crosses the sound frequency. For the case considered here k_0 is close to $k_{\max}$ and it can be easily shown that

$$k_0=\frac{\pi}{2a}-\frac{\pi}{4a}\frac{\omega_t^2(k_{max})-\omega_{opt}^2(k_{max})}{\omega_t^2(k_{max})+\mathscr{G}\omega_{opt}^2(k_{max})}\approx 3.7\cdot 10^2\,\mathrm{cm}^{-1}.$$

If

$$\xi\omega_t^2(\omega_t^2+\omega_{opt}^2)\ll(\omega_t^2-\omega_{opt}^2)^2,$$

then

$$\omega_a^2=\omega_{opt}^2\left(1+\xi\frac{\omega_t^2}{\omega_{opt}^2-\omega_t^2}\right),$$

$$\omega_b^2=\omega_t^2\left(1-\xi\frac{\omega_t^2}{\omega_{opt}^2-\omega_t^2}\right).$$

At k close to k_0 so that $\xi\omega_t^2(\omega_t^2+\omega_{opt}^2) \gg (\omega_t^2-\omega_{opt}^2)^2$, we have

$$\omega_a^2=\omega_{opt}^2(k)(1+\xi^{1/2}), \quad \omega_b^2=\omega_t^2(k)(1-\xi^{1/2}).$$

Thus, the two branches of the coupled elastodomain oscillations behave in the following manner. At small k these are almost exactly the earlier[1] quasioptic oscillations of the domain structure and the transverse acoustic oscillations. When k approaches k_0 (for the numerical example considered, this is almost tantamount to $k_{\max} = \pi/2a$) a qualitative change takes place in the spectra of both branches, viz., they become separated by a gap, i.e., by a region of forbidden frequencies. In this case the branch (a) which coincided at small k with the quasioptic branch has at the upper limit a higher frequency than branch (b), which has at small k the frequency of transverse sound.

Solutions (5) with account taken of the last expressions show that generation of sound by domain oscillations and the converse phenomenon take place in a "quasiresonant" manner. At $k = k_0$, when the amplitude of the excited oscillations becomes particularly large, a phenomenon occurs, which can be called arbitrarily domain-acoustic resonance. At this quasiresonance the amplitude of the excited oscillations, e. g., the sound amplitude, is larger than the amplitude of the exciting domain oscillation in a ratio $1/\xi^{1/2} \approx 10^4$; a similar situation obtained when domain walls are excited by sound.

We note finally an intersting circumstance, namely, that only the quasioptical oscillations interact with the sound, but not the quasiacoustic ones. First, the phase velocity of the quasiacoustic oscillations increases with increasing wave vector k to a value

$$2\Omega a\left(\frac{2\Delta}{\pi^3 D}\ln 2\right)^{1/2}\approx 1.5\cdot 10^5\ \mathrm{cm/s}$$

and therefore remains smaller than the speed of sound at all values of k. At the same time, the velocity of the quasioptic oscillations decreases with increasing k, and crosses the ω/k curve for the sound. This explains why the quasiacoustic oscillations do not interact with the sound. Second, this peculiarity can be explained also as follows: the elastic-stress forces due to the action of the sound wave on neighboring domain walls turn out, at small values of the wave vector ($k \to 0$), to have the same direction as the displacements of the domain walls in quasioptic oscillations but the opposite phase. On the contrary, at k close to $k_{\max} = \pi/2a$ these forces have the same direction, which coincides with the displacements of the domain walls in quasioptic oscillations. It is this equality which makes possible the interaction of the domain-structure quasioptic oscillations with the transverse sound, whereas the quasiacoustic oscillations behave differently and the quasielastic forces they exert on the domain walls do not coincide with the forces produced by the sound. As for the neglect of the quantity $k\Delta \ll 1$, it can be seen from the equations of motion that it does not influence the indicated result.

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