Resonant excitation and decay of autoionizing states in a strong electromagnetic field

A. I. Andryushin, A. E. Kazakov, and N. V. Fedorov

Institute of General Physics, Academy of Sciences of the USSR
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The photoionization of an atom accompanied by the resonant excitation of an autoionizing state is analyzed. The time evolution of the total ionization probability, the dependence of this probability on the frequency of the resonant radiation, and the energy spectrum of the photoelectrons are studied. It is found that the energy of the final state of the system may become localized either at $E\sim E_a$, where E_a is the energy of the autoionizing state, or at $E\sim E_a+\hbar\omega$, where $\hbar\omega$ is the photon energy of the resonant radiation. The energy spectrum of the photoelectrons corresponding to the region $E\sim E_a+\hbar\omega$ is basically similar to the spectrum of electrons during atom photoionization accompanied by the resonant excitation of a bound state. The spectrum of photoelectrons corresponding to the region $E\sim E_a$ is strongly affected by an interference between different pathways for the decay of the ground state in the resonant field, with the result that the spectrum acquires a characteristic Fano structure. The interference also influences the widths of both spectral curves, the ratio of the numbers of electrons in the two energy regions, and other characteristics of the ionization process. The additional presence of a noninterfering pathway for photoionization of the autoionizing state leads to finite widths and heights of the spectral curves and prevents a complete "confluence of coherences."

1. INTRODUCTION

The behavior of autoionizing states of atoms in a strong electromagnetic field has been the subject of a fair number of recent papers.[1–12] The quasienergy spectrum and the decay of a system containing an autoionizing state coupled by a resonant field to another state of this system were studied in Refs. 2 and 12. The spectra of the absorption of probing radiation by such a system (autoionization resonances) were studied in Refs. 1 and 4–6. The spectrum of photoelectrons formed during the photoionization of an atom accompanied by the resonant excitation of an autoionizing state was studied in Ref. 3. The effect of the spontaneous decay of an autoionizing state in a resonant electromagnetic field on the energy distribution of the photoelectrons and on the spontaneous-emission spectrum was studied in Refs. 7–11 and 13.

This research is of interest in its own right and also in connection with experiments on the two-electron ionization of alkaline earth atoms.[14–16] Two-electron excited states, including autoionizing states, may serve as intermediate states in two-electron ionization. In a typical situation, a strong external electromagnetic field (up to 10^{10} W/cm^2) is at resonance with transitions between certain autoionizing states or between an autoionizing state and discrete atomic levels.

In the present paper we analyze the photoionization of an atom accompanied by resonant excitation of an autoionizing state. We study the time evolution of the total ionization probability, its dispersive dependence on the frequency of the resonant radiation, and the energy spectrum of the photoelectrons. This process has much in common with ordinary ionization involving a resonance with a discrete intermediate level,[17] but there are also some qualitative differences. Specifically, the levels of autoionizing states differ from discrete levels in that they lie above the first ionization threshold of the atom and can decay spontaneously by virtue of an interelectron configuration interaction. For the same reason, photoelectrons arise not only at energies $E\sim E_a+\hbar\omega-E_f$, where E_a and E_f are the energies of the autoionizing state and of the final state of the ion, and $\hbar\omega$ is the energy of the photon of the electromagnetic field, but also at energies $E\sim E_a-E_f$. Furthermore, in this case there are several pathways for the decay of the system to given states of the continuum, $E\sim E_a$. An important role may be played here by interference between the corresponding transitions, while there would be no such interference in ordinary resonant ionization.

The photoelectron spectrum studied in Ref. 3 did not contain the energy region $E\sim E_a+\hbar\omega-E_f$, since the direct photoionization of the autoionizing state itself was not considered there. As we show below, under certain conditions this energy region may hold most of the photoelectrons. In addition, the existence of an ionization pathway for the decay of the autoionizing state reduces the role of interference phenomena and thus also influences the electron spectrum in the region $E\sim E_a-E_f$. For the same reasons, as we show below, the results derived in Ref. 3 are not correct.

2. STATEMENT OF THE PROBLEM; GENERAL EXPRESSIONS

We assume that an electromagnetic field of frequency $\omega\approx E_a-E_1$ (we are setting $\hbar=1$) acts on an atom that is initially in a state φ_1 with an energy E_1. This field causes Rabi oscillations in the E_1, E_a level system, and it also causes ionization of the atom from each of these states. The autoionizing state itself decays by virtue of an interelectron

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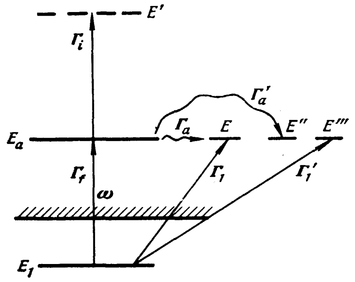

FIG. 1. Scheme of the pertinent transitions.

configuration interaction. Figure 1 shows the transitions considered.

States with a given energy E are always degenerate (for example, in the orbital quantum number L). We partition the set of degenerate continuum states into three subgroups as follows: Subgroup E consists of those states of the continuum to which the autoionizing state can decay and to which a dipole transition from the discrete level E_1 can also be induced by the external field. Subgroup E'' consists of those states of the continuum of energy E which are not dipole-coupled to the level E_1 but to which the autoionizing state E_a does decay. Subgroup E''' consists of those states of the continuum (with the same energy) which are coupled exclusively with the discrete E_1 and which are not populated during the autoionization decay of the autoionizing state. A partitioning of this sort is obviously possible in all cases. Let us assume, for example, that the initial state E_1 has an orbital quantum number $L = 1$ (a P state), while the autoionizing state E_a coupled with it by the resonant field is a D state ($L = 2$). If the orbital quantum number is conserved during the autoionization decay, autoionizing state E_a decays only to D states of the continuum; i.e., the state of continuum E have $L = 2$. During ionization of state E_1, on the other hand, both the D states of continuum E and the S states of continuum E''' are populated. In this case, there is no autoionization decay of the state E_a to the S states of continuum E''', so that the amplitudes for transitions to E and E''' do not interfere with each other. In precisely the same way, states of the continuum E''' may appear to which there is an autoionization decay of E_a but which are not populated from state E_1 by the electromagnetic field, e.g., in cases in which L is not a good quantum number. In certain special situations there may be no states in continua E'' and E''' in the partitioning described above. One such situation is that in which L is a good quantum number, E_1 is an S level, and E_a is a P level. The amplitudes for the transitions by the different pathways $E_1 \to E_a \to E$ and $E_1 \to E$ to given states of the continuum interfere with each other. Accordingly, in addition to the "interfering" transitions there are "noninterfering" transitions, including the transitions $E_1 \to E_a \to E''$, $E_1 \to E'''$, and the ionization transition upward from the autoionizing state involving the absorption of a photon, $E_a \to E' \approx E_a + \omega$.

To find the probability for ionizing the system we need to solve the Schrödinger equation for the atomic wave function $\Psi(t)$, which we take to be a superposition of the wave functions of the unperturbed atomic states $\varphi_1, \varphi_a, \varphi_E, \varphi_{E'}, \varphi_{E''}, \varphi_{E'''}$ with the respective energies $E_1, E_a, E, E', E'', E'''$:

$$\Psi(t) = a_1 \varphi_1 e^{-iE_1 t} + a_a \varphi_a e^{-iE_a t} + \int a_E \varphi_E e^{-iEt} dE + \int a_{E'} \varphi_{E'} e^{-iE't} dE' + \int a_{E''} \varphi_{E''} e^{-iE''t} dE'' + \int a_{E'''} \varphi_{E'''} e^{-iE'''t} dE'''. \qquad (1)$$

We assume that the field is turned on instantaneously at the time $t = 0$; i.e., we assume that the time required to turn the field on satisfies $\Delta t \ll t, 1/\Omega_R, 1/\gamma^{\pm}$ [see expression (7) below]. We adopt the initial conditions

$$a_1(0) = 1, \quad a_a(0) = 0, \quad a_E(0) = 0,$$
$$a_{E'}(0) = 0, \quad a_{E''}(0) = 0, \quad a_{E'''}(0) = 0. \qquad (2)$$

In the resonant approximation we find for the amplitudes $a_1(b), \ldots, a_{E'''}(t)$ a system of linear first-order differential equations that can be solved by Laplace transforms, among other methods. The procedure for solving equations of this sort is described in detail in Refs. 4 for some other transition schemes, and we will not reproduce it here. As a result, we find the following expressions for the probability amplitudes for finding the system in the continuum states:

$$a_E(t) = \frac{1}{(2\pi)^{1/2}} (\Gamma_1^{1/2} b_1^{(+)} + \Gamma_a^{1/2} b_a^{(+)}) \times \left\{ \frac{\exp(-it[E_a - E + \varepsilon/2 - {}^1/_2 \operatorname{Re}\Omega - i\gamma^{(+)}/2]) - 1}{E_a - E + \varepsilon/2 - {}^1/_2 \operatorname{Re}\Omega - i\gamma^{(+)}/2} \right\} + \frac{1}{(2\pi)^{1/2}} (\Gamma_1^{1/2} b_1^{(-)} + \Gamma_a^{1/2} b_a^{(-)}) \times \left\{ \frac{\exp(-it[E_a - E + \varepsilon/2 + {}^1/_2 \operatorname{Re}\Omega - i\gamma^{(-)}/2]) - 1}{E_a - E + \varepsilon/2 + {}^1/_2 \operatorname{Re}\Omega - i\gamma^{(-)}/2} \right\}, \qquad (3)$$

$$a_{E'}(t) = \left(\frac{\Gamma_i}{2\pi}\right)^{1/2} \times \left\{ b_a^{(+)} \frac{\exp(-it[E_a + \omega - E' + \varepsilon/2 - {}^1/_2 \operatorname{Re}\Omega - i\gamma^{(+)}/2]) - 1}{E_a + \omega - E' + \varepsilon/2 - {}^1/_2 \operatorname{Re}\Omega - i\gamma^{(+)}/2} + b_a^{(-)} \frac{\exp(-it[E_a + \omega - E' + \varepsilon/2 + {}^1/_2 \operatorname{Re}\Omega - i\gamma^{(-)}/2]) - 1}{E_a + \omega - E' + \varepsilon/2 + {}^1/_2 \operatorname{Re}\Omega - i\gamma^{(-)}/2} \right\}.$$

The amplitudes $a_{E''}(t)$ and $a_{E'''}(t)$ are given by expressions similar to that for $a_{E'}(t)$, in which we must omit ω and use the substitutions $\Gamma_i \to \Gamma_a', E' \to E''$ for $a_{E''}(t)$ and $\Gamma_i \to \Gamma_1', E' \to E''', b_a^{(\pm)} \to b_1^{(\pm)}$ for $a_{E'''}(t)$.

In (3) we have used the notation

$$\Gamma_a = 2\pi |W_{aE}|^2|_{E=E_a},$$
$$\Gamma_a' = 2\pi |W_{aE''}|^2|_{E''=E_a} \qquad (4)$$

for the interfering and noninterfering, respectively, parts of the total autoionization width $\tilde{\Gamma}_a = \Gamma_a + \Gamma_a'$ of the level E_a caused by the decay of the autoionizing state to states of the continua E and E' under the influence of the interelectron interaction operator W. In addition,

$$\Gamma_1 = 2\pi |V_{1E}|^2|_{E=E_1+\omega}, \quad \Gamma_1' = 2\pi |V_{1E'''}|^2|_{E'''=E_1+\omega},$$
$$\Gamma_i = 2\pi |V_{aE'}|^2|_{E'=E_a+\omega} \qquad (5)$$

are the ionization widths of the levels due to the electromagnetic field; the operator $V = -\frac{1}{2}\vec{\mathbf{d}}\mathscr{E}$ represents the interaction of the atom with the field (in the dipole approximation); the operator **d** represents the dipole moment of the atom; and $\mathscr{E}$ is the electric field amplitude of the electromagnetic wave. The total ionization width $\widetilde{\Gamma}_1 = \Gamma_1 + \Gamma_1'$ of level E_1 also consists of an interfering part (Γ_1) and a noninterfering part (Γ_1'). The ionization width of the autoionization state, Γ_i, is due to the transitions $E_a \to E_a + \omega$. We see from (5) that, generally speaking, the ionization widths $\Gamma_1, \Gamma_1', \Gamma_i$ are of the same order of magnitude:

$$\Gamma_1 \sim \Gamma_1' \sim \Gamma_i \sim (\mathscr{E}/\mathscr{E}_{at})^2 E_{at}. \tag{6}$$

Here $\mathscr{E}_{at}$ and E_{at} are characteristic values of the intra-atomic electric field and the intra-atomic energy. As usual, we consider the case $\mathscr{E} \ll \mathscr{E}_{at}$, so that we may ignore transitions between states of the continuum.[17] It can be seen from (6) that the ionization widths are small in comparison with E_{at}. The quantity ε is the deviation from resonance, $\varepsilon = E_1 + \omega - E_a$, where the level energies E_1 and E_a have been corrected for the dynamic Stark effect in the fields $\mathscr{E}$ (Ref. 4). The quantities $\pm Re\Omega(\varepsilon)$ and $\gamma^{(\pm)}(\varepsilon)$ determine the positions and widths, respectively, of two quasienergy levels of the two-level system φ_1, φ_2 in the resonant field ω:

$$\Omega(\varepsilon) = \left\{ [\varepsilon + i(\Gamma^{(2)} - \Gamma^{(1)})/2]^2 + \Gamma_f^2 \left(1 - \frac{i}{2q}\right)^2 \Big/ 4 \right\}^{1/2}$$

$$= \{a(\varepsilon) + ib(\varepsilon)\}^{1/2},$$

$$\begin{gathered} a(\varepsilon) \equiv \varepsilon^2 + \Gamma_f^2/4 - (\Gamma^{(2)} - \Gamma^{(1)})^2/4 - \Gamma_{1a}^2, \\ b(\varepsilon) \equiv \varepsilon(\Gamma^{(2)} - \Gamma^{(1)}) - \Gamma_f\Gamma_{1a}, \\ \gamma^{(\pm)}(\varepsilon) = (\Gamma^{(1)} + \Gamma^{(2)})/2 \pm \mathrm{Im}\,\Omega(\varepsilon), \\ \mathrm{Re}\,\Omega(\varepsilon) = 2^{-1/2}(a(\varepsilon) + [a^2(\varepsilon) + b^2(\varepsilon)]^{1/2})^{1/2} \operatorname{sign} b(\varepsilon), \\ \mathrm{Im}\,\Omega(\varepsilon) = 2^{-1/2}(-a(\varepsilon) + [a^2(\varepsilon) + b^2(\varepsilon)]^{1/2})^{1/2}, \end{gathered} \tag{7}$$

where

$$\Gamma_f = 4|V_{1a}| \sim \frac{\mathscr{E}}{\mathscr{E}_{at}} E_{at} \gg \Gamma_1, \Gamma_i \tag{8}$$

is the field-induced width, which determines the frequency of the Rabi oscillations in the two-level system, $\Omega_R = (\varepsilon^2 + \Gamma_f^2/4)^{1/2}$. It is easy to show that the condition $\Omega_R \geqslant |Re\Omega(\varepsilon)|$ holds.

The quantity

$$\Gamma_{1a} = (\Gamma_1\Gamma_a)^{1/2} \tag{9}$$

is a cross width which describes the transition $E_1 \to E_a$ through states of the continuum, i.e., by the pathway $E_1 \to E \to E_a$, and which thus contains only the interfering parts of the width $\widetilde{\Gamma}_a$ and $\widetilde{\Gamma}_1$. The ratio

$$q = \Gamma_f/4\Gamma_{1a} \sim (E_{at}/\Gamma_a)^{1/2} \tag{10}$$

is the Fano parameter,[18] which is independent of the field $\mathscr{E}$ and which is large, $q \gg 1$, under typical conditions.

The widths $\Gamma^{(1)}$ and $\Gamma^{(2)}$ are the total widths of the isolated atomic levels E_1 and E_a in the field $\mathscr{E}$:

$$\Gamma^{(1)} = \Gamma_1 + \Gamma_1' + \Gamma_{r1}, \quad \Gamma^{(2)} = \Gamma_a + \Gamma_a' + \Gamma_i + \Gamma_{ra}. \tag{11}$$

These widths also include the natural widths of these levels, Γ_{41} and Γ_{4a}, which describe the spontaneous decay of these levels.

Finally, $b_1^{(\pm)}$ and $b_a^{(\pm)}$ are given by:

$$b_1^{(\pm)} = \frac{\Omega \pm [\varepsilon + \frac{1}{2}i(\Gamma^{(2)} - \Gamma^{(1)})]}{2\Omega}, \quad b_a^{(\pm)} = \mp \frac{\Gamma_f(1 - i/2q)}{2\Omega}. \tag{12}$$

This system has two characteristic decay constants, $\gamma^{(+)}$ and $\gamma^{(-)}$, which are generally not the same as the widths of the isolated levels, $\Gamma^{(1)}$ and $\Gamma^{(2)}$. As was shown in Refs. (4) and (12), only in the limit $|\varepsilon| \to \infty$, i.e., in the case $|\varepsilon| \gg \Gamma_f$, $q\Gamma_a$ [or for a field $\mathscr{E} \to 0$, i.e., in the case $\Gamma_1 \ll \Gamma_{r1}, \Gamma_f \ll (\Gamma_{r1}\Gamma_a)^{1/2}$], does the mixing of the states φ_1 and φ_a by the resonant field become inconsequential, so that we would have $\gamma^{(+)} \to \Gamma^{(2)}$, $\gamma^{(-)} \to \Gamma^{(1)}$ (with $\Gamma^{(2)} > \Gamma^{(1)}$). It was also shown in Refs. (4) and (12) that $\gamma^{(+)} + \gamma^{(-)} = \Gamma^{(1)} + \Gamma^{(2)}$ and that $\gamma^{(+)} > \gamma^{(-)}$ for arbitrary ε. The difference between the widths $\gamma^{(-)}$ and $\gamma^{(+)}$, on the one hand, and $\Gamma^{(1)}$ and $\Gamma^{(2)}$, on the other, results from an interference between the amplitude for the transitions $E_1 \to E_a \to E$ and $E_1 \to E$. This interference becomes most apparent at a certain deviation $\varepsilon = \varepsilon_{\min}$ at which the functions $\gamma^{(+)}$ and $\gamma^{(-)}$ have a maximum and a minimum, respectively:

$$\begin{gathered} \varepsilon_{min} = -q(\Gamma^{(2)} - \Gamma^{(1)}), \\ \gamma_{\substack{max \\ min}} \equiv \gamma^{(\pm)}(\varepsilon_{min}) = \tfrac{1}{2}[\Gamma^{(1)} + \Gamma^{(2)} \pm ((\Gamma^{(2)} - \Gamma^{(1)})^2 + 4\Gamma_1\Gamma_a)]^{1/2}. \end{gathered} \tag{13}$$

In the formation of the decay width of one of the quasienergy states, $\gamma^{(-)}$, in the region $|\varepsilon - \varepsilon_{\min}| \lesssim q\Gamma_a$, the interference of the amplitudes for the transitions $E_1 \to E_a \to E$ and $E_1 \to E$ is destructive and results in a decrease in the decay rate of this state and its width. In contrast, in the formation of the width of the other state, $\gamma^{(+)}$, in the same region of the deviation ε, the interference of the amplitudes increases the width and the decay rate of this state. In a field which is not too strong, such that

$$\Gamma_i, \ \Gamma_1 \ll \Gamma_a, \quad \text{or} \quad \Gamma_f \ll q\Gamma_a, \tag{14}$$

we have $|\varepsilon_{\min}| \approx q\Gamma_a \gg \Gamma_a$, and the probability per unit time for a decay of the state φ_1 by the pathway $E_1 \to E_a \to E$, estimated as the probability for resonant ionization,[17] is equal to the probability (Γ_1) for the direct decay $E_1 \to E$ of the state φ_1:

$$\Gamma_f^2\Gamma_a/16(\varepsilon^2 + \Gamma_a^2/4) = \Gamma_1.$$

It is easy to see from (13) that $\gamma^{(-)} \ll \gamma^{(+)}$. If the noninterfering parts of the widths are zero,

$$\Gamma_a' = \Gamma_1' = 0, \tag{15}$$

we find from (13)

$$\gamma_{max} \approx \Gamma_a, \ \gamma_{min} = \Gamma_i\Gamma_1/\Gamma_a + \Gamma_{r1} \ll \Gamma_1, \ \Gamma_i \ll \Gamma_a. \tag{16}$$

This case occurs when, for example, E_1 is an S state and E_a is a P state.

We assume below that condition (15) holds, and we also ignore the spontaneous widths. These assumptions do not qualitatively change the results; they are made exclusively to simplify the equations. We might note that if we adopted the typical value $10^{-8}E_{at}$ for the spontaneous widths, we would find ionization widths exceeding the spontaneous widths even at $\mathscr{E} \gtrsim 10^{-4}\mathscr{E}_{at} \sim 5 \times 10^5$ V/cm.

Under conditions (14) and with $|\varepsilon - \varepsilon_{\min}| \ll q\Gamma_a$, we can easily find the following expansions for $\mathrm{Re}\,\Omega(\varepsilon)$ and $\gamma^{(\pm)}(\varepsilon)$ from (7):

$$\mathrm{Re}\,\Omega(\varepsilon) = -q\Gamma_a(1+\Gamma_i\Gamma_1/\Gamma_a^2) + (1-2\Gamma_1/\Gamma_a)(\varepsilon-\varepsilon_{min}),$$

$$\gamma^{(-)}(\varepsilon) = \frac{\Gamma_1\Gamma_i}{\Gamma_a}\left[1+\frac{(\varepsilon-\varepsilon_{min})^2}{q^2\Gamma_a\Gamma_i}\right], \qquad \gamma^{(+)} = \Gamma_a+\Gamma_i+\Gamma_1-\gamma^{(-)},$$

$$\Omega_R \approx |\varepsilon_{min}| \approx q\Gamma_a. \qquad (17)$$

In stronger fields, with $\Gamma_f \gg q\Gamma_a$, the interference between pathways is inconsequential, and the widths of the two quasienergy states are identical in order of magnitude for arbitrary values of ε: $\gamma^{(-)} \sim \gamma^{(+)} \gtrsim \Gamma_a$ (Refs. 4 and 12).

The total probability for the ionization of an atom by the time t,

$$W(t) = \int |a_E(t)|^2 dE + \int |a_{E'}(t)|^2 dE' \equiv W_1(t) + W_2(t), \qquad (18)$$

can easily be found from expression (3):

$$W_1(t) = \int |a_E(t)|^2 dE$$

$$= \left\{ |C^{(+)}|^2 \frac{(-e^{-\gamma^{(+)}t}+1)}{\gamma^{(+)}} + |C^{(-)}|^2 \frac{(-e^{-\gamma^{(-)}t}+1)}{\gamma^{(-)}} \right.$$

$$\left. -2\,\mathrm{Im}\left[C^{(+)}C^{(-)} \frac{1-\exp\{i\,\mathrm{Re}\,\Omega t - t(\gamma^{(+)}+\gamma^{(-)})/2\}}{\mathrm{Re}\,\Omega + {}^1/_2 i(\gamma^{(+)}+\gamma^{(-)})} \right] \right\},$$

$$C^{(\pm)} = \Gamma_1^{1/2} b_1^{(\pm)} + \Gamma_a^{1/2} b_a^{(\pm)},$$

$$W_2(t) = \int |a_{E'}(t)|^2 dE'$$

$$= \frac{\Gamma_i\Gamma_f^2(1+1/4q^2)}{16|\Omega|^2} \left\{ \frac{-e^{-\gamma^{(+)}t}+1}{\gamma^{(+)}} + \frac{-e^{-\gamma^{(-)}t}+1}{\gamma^{(-)}} \right.$$

$$\left. +2\,\mathrm{Im}\left[\frac{-\exp\{i\,\mathrm{Re}\,\Omega - t(\gamma^{(+)}+\gamma^{(-)})/2\}+1}{\mathrm{Re}\,\Omega + {}^1/_2 i(\gamma^{(+)}+\gamma^{(-)})} \right] \right\}. \qquad (19)$$

The function $W_1(t)$ is the probability for ionization accompanied by a transition of the system to the energy region $E \approx E_a$, while the function $W_2(t)$ is the probability for ionization accompanied by a transition of the system to the energy $E \approx E_a + \omega$. The function $W_2(t)$ is completely analogous in form to function (23) of Ref. (17), which describes the probability for the ionization of an atom accompanied by a resonance in a discrete level. In the present case, however, because of the presence of several decay pathways, the positions and widths of the quasienergy levels, $\mathrm{Re}\,\Omega$ and $\gamma^{(+)}$, are more complicated functions of the field and the frequency.

In the following section of this paper we analyze the time evolution of the photoionization probabilities $W_{1,2}(t)$ and their dispersive dependence on the radiation frequency ω for various pulselengths of the electromagnetic field.

3. IONIZATION PROBABILITY AS A FUNCTION OF THE PULSE LENGTH AND FREQUENCY OF THE RADIATION

We first assume that the pulse length of the radiation is very short:

$$|\mathrm{Re}\,\Omega| t \ll 1, \quad \gamma^{(\pm)} t \ll 1. \qquad (20)$$

From these conditions we find $\Omega_R t \ll 1$ for both weak fields ($\Gamma_j \ll \Gamma_a$) and strong fields ($\Gamma_j \gg \Gamma_a$). From (19) we easily find

$$W_1(t) = |C^{(+)} + iC^{(-)}|^2 t, \qquad W_2(t) = \frac{\Gamma_i\Gamma_f^2}{48}\left(1+\frac{1}{4q^2}\right)t^3. \qquad (21)$$

As in the case of a resonance in a discrete level [expression (28) of Ref. 17], the function W_2 is proportional to t^3 at small values of t, since $E_1 \to E_a \to E' \approx E_a + \omega$ is a two-step transition. The probability W_1, on the other hand, is proportional to the pulse length t, because of the direct decay pathway $E_1 \to E \approx E_a$.

We now examine, for this range of pulse lenths, the dispersive dependence of the rate of absorption of radiation energy, dW_1/dt, on the frequency ω during transitions of the system of states $E \approx E_a$. From (19) we easily find that for weak fields, with $\Gamma_f \ll \Gamma_a$, we have

$$\frac{dW_1}{dt} = \Gamma_1 \frac{(q\Gamma_a)^2 + [\varepsilon + {}^1/_2(1+2q)\Gamma_a]^2}{\varepsilon^2 + \Gamma_a^2/4}. \qquad (22)$$

This is an expression of the Fano type[18] with a characteristic width $\Gamma = \Gamma_a$. The derivative dW_1/dt has a maximum at the point $\varepsilon = \varepsilon_1 = \Gamma_a/8q \ll \Gamma_a$, $(dW_1/dt)_{\max} = dW_1(\varepsilon_1)/dt = 8q^2\Gamma_1 \gg \Gamma_1$ and a minimum (due to the interference of different pathways for the decay of the state E_1) at the point $\varepsilon = \varepsilon_2 = -2q\Gamma_a$; here $|\varepsilon_2| \gg \Gamma_a$ and $(dW_1/dt)_{\min} = dW_1(\varepsilon_2)/dt \approx \Gamma_1/2$. Asymptotically, we have $dW_1/dt \to \Gamma_1$ as $\varepsilon \to \infty$ ($|\varepsilon| \gg q\Gamma_a$). In contrast with the standard Fano formula, however, we have $\varepsilon \to (dW_1/dt)_{\min} \neq 0$, and the interference minimum is not deep (Fig. 2).

Using (21) and (22) we can easily show that over the entire range of frequency deviations allowed [by (20)], $|\varepsilon| t \ll 1$, photoelectrons are formed primarily at energies $E \approx E_a - E_f$, $W_2 \ll W_1$.

In stronger fields satisfying the condition $\Gamma_f \gg \Gamma_a$ we have

$$\frac{dW_1}{dt} = \frac{\Gamma_1}{2}\left[1 + \frac{(\varepsilon+2q\Gamma_a)^2}{\varepsilon^2+\Gamma_f^2/4}\right]. \qquad (23)$$

The function (23) is of the form of the sum of Fano distributions with a width $\Gamma = \Gamma_f$ and a constant $\Gamma_1/2$; the maximum and the minimum are now reached at the respective points

$$\varepsilon_1 = \Gamma_f^2/8q\Gamma_a, \quad \varepsilon_2 = -2q\Gamma_a.$$

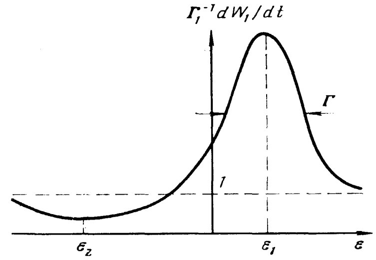

FIG. 2. Qualitative behavior of the energy absorption rate $\Gamma_1^{-1} dW_1/dt$ as a function of ε, the frequency deviation from resonance [expressions (22), (23), (27), (30), and (31)]. The characteristic width of the curve is $\Gamma = \Gamma_a$ in weak fields, $\Gamma_f \ll \Gamma_a$, and $\Gamma = \Gamma_f$ in stronger fields, $\Gamma_f \gg \Gamma_a$.

We again have $(dW_1/dt)_{\min} = dW_1(\varepsilon_2)/dt = \Gamma_1/2$, and asymptotically we have $dW_1/dt \to \Gamma_1$ as $\varepsilon \to \infty$ (i.e., $|\varepsilon| \gg \Gamma_f, q\Gamma_a$). The position of the maximum is shifted to the right by a factor $(\Gamma_f/\Gamma_a)^2 \gg 1$ in comparison with the case of a weak field, while the minimum remains at the same place. If the field satisfies the condition $q\Gamma_a \gg \Gamma_f$, we have $(dW_1/dt)_{\max} = dW_1(\varepsilon_1)dt \approx 8q^2(\Gamma_a/\Gamma_f)^2\Gamma_1 \gg \Gamma_1$; i.e., the maximum of the function dW_1/dt is still strongly pronounced, but it is lower than in the case of a weak field by a factor $(\Gamma_a/\Gamma_f)^2 \ll 1$. For very strong fields, with $q\Gamma_a/\Gamma_f$, the maximum of the function dW_1/dt is barely noticeable:

$$(dW_1/dt)_{\max} \approx (1+16(q\Gamma_a/\Gamma_f)^2)\Gamma_1 \gtrsim \Gamma_1.$$

Furthermore, $W_2(\varepsilon)/W_1(\varepsilon) \ll \Gamma_i/\Gamma_1 \lesssim 1$; i.e., $W_2 \ll W_1$ over the entire time interval in (20). At a longer pulse length t, at which several Rabi oscillations can occur in the E_1, E_a level system, we have

$$\Omega_R t \gtrsim |\mathrm{Re}\,\Omega| t \gg 1, \quad \gamma^{(\pm)} t \ll 1. \tag{24}$$

Both probabilities W_1 and W_2 are proportional to t:

$$W_1(t) = (|C^{(+)}|^2 + |C^{(-)}|^2)t,$$
$$W_2(t) = \frac{\Gamma_i \Gamma_f^2 (1+1/4q^2)t}{8(\mathrm{Re}\,\Omega)^2}. \tag{25}$$

For weak fields, when $\Gamma_f \ll \Gamma_a$, conditions (24) can hold simultaneously only for a large deviation from resonance: $|\varepsilon| t \gg 1$, $|\varepsilon| \gg \Gamma_a$. Here dW_1/dt is given by (22), in whch we must omit the width Γ_a in the denominator, and

$$dW_2/dt = \Gamma_i \Gamma_f^2 (1+1/4q^2)/8\varepsilon^2. \tag{26}$$

For stronger fields, with $\Gamma_f \gg \Gamma_a$, dW_1/dt is given by (23), wile dW_2/dt is a Lorentzian curve:

$$dW_2/dt = \Gamma_i \Gamma_f^2 (1+1/4q^2)/8(\varepsilon^2 + \Gamma_f^2/4). \tag{27}$$

It is easy to see that in moderately strong fields, $\Gamma_f \ll q\Gamma_a$, we have $dW_1/dt \gg dW_2/dt$ for arbitrary values of the deviation from resonance, ε. At $\Gamma_f \gg q\Gamma_a$, however, near the point $\varepsilon = -q\Gamma_a$, the ionization rate at energies $E \sim E_a + \omega$ may become comparable to the ionization rate at energies $E \sim E_a$ if Γ_i and Γ_i are approximately equal.

We now assume that the pulse length is such that one of the quasienergy states manages to decay:

$$\gamma^{(+)} t \gg 1, \quad \gamma^{(-)} t \ll 1. \tag{28}$$

The two inequalities can hold simultaneously only in fields that are not too strong, $\Gamma_f \ll q\Gamma_a$, for which we have $\gamma^{(-)} \ll \gamma^{(+)}$. From (19) we easily find

$$\frac{dW_1}{dt} = |C^{(-)}|^2 = \frac{\Gamma_1}{4|\Omega|^2}\left[(\mathrm{Re}\,\Omega(\varepsilon) + \varepsilon + 2q\Gamma_a)^2 + \left(\mathrm{Im}\,\Omega(\varepsilon) + \frac{\Gamma^{(2)} - \Gamma^{(1)}}{2} - \Gamma_a\right)^2\right],$$
$$\frac{dW_2}{dt} = \frac{\Gamma_i \Gamma_f^2 (1+1/4q^2)}{16|\Omega|^2}. \tag{29}$$

The case of weak fields, $\Gamma_f \ll \Gamma_a$, corresponds to the formulation of the problem in Fano's perturbation-theory calculation[18] of the absorption coefficient for weak radiation near an autoionization resonance. The expression for dW_1/dt becomes, in lowest order in the field, the ordinary Fano formula,[17]

$$\frac{dW_1}{dt} = \Gamma_1 \frac{(\varepsilon + q\Gamma_a)^2}{\varepsilon^2 + \Gamma_a^2/4}, \tag{30}$$

which describes an asymmetric absorption line under the condition $\Gamma_a t \gg 1$ [corresponding to the first of inequalities (28)]. The curve has a width $\Gamma = \Gamma_a$ and a zero interference minimum at $\varepsilon = \varepsilon_2 = -q\Gamma_a$. When we go to next order in the field, however, an additional positive term proportional to the width Γ_i appears in the numerator of the expression for dW_1/dt, so that the interference minimum is not zero. The maximum value of the function dW_1/dt as a function of ε is $4\Gamma_1 q^2 \gg \Gamma_1$ and is reached at $\varepsilon = \varepsilon_1 = \Gamma_a/4q$.

In stronger fields, at $q\Gamma_a \gg \Gamma_f \gg \Gamma_a$, we have

$$\frac{dW_1}{dt} \approx \frac{\Gamma_1}{4} \frac{(\Omega_R\,\mathrm{sign}(b) + \varepsilon + 2q\Gamma_a)^2 + (|b|/2\Omega_R - \Gamma_a/2)^2}{\varepsilon^2 + \Gamma_f^2/4}, \tag{31}$$
$$b = \varepsilon\Gamma_a - \Gamma_f^2/4q, \qquad \frac{dW_2}{dt} = \frac{\Gamma_i \Gamma_f^2 (1+1/4q^2)}{16(\varepsilon^2 + \Gamma_f^2/4)}.$$

The function dW_1/dt has a maximum and a minimum as a function of ε at the points $\varepsilon = \varepsilon_1 = \Gamma_f^2/8q\Gamma_a$ and $\varepsilon = \varepsilon_2 = -q\Gamma_a \approx \varepsilon_{\min}$ respectively, given in (13); here

$$\frac{dW_1}{dt}(\varepsilon_1) \approx \frac{\Gamma_a}{4} \gg \Gamma_1, \qquad \frac{dW_1}{dt}(\varepsilon_2) = \frac{4\Gamma_1^2(\Gamma_1 + \Gamma_i)^2}{\Gamma_a \Gamma_f^2} \ll \Gamma_1,$$
$$\frac{dW_1}{dt} \to \Gamma_1$$

in the limit $\varepsilon \to \infty$ (i.e., $|\varepsilon| \gg q\Gamma_a$). The curve of dW_1/dt as a function of ε is again a Fano curve with a deep minimum. Its width is now determined by the field-induced width Γ_f, and the maximum is smaller than in the case of a weak field, (30), by a factor $(\Gamma_f/\Gamma_a)^2$. The curve of dW_1/dt as a function of the deviation from resonance, ε, is qualitatively a Fano curve (Fig. 2). Its characteristic width Γ is equal to the width of the autoionizing state, Γ_a, in the case of weak fields, $\Gamma_f \ll \Gamma_a$ [Eqs. (22) and (30)], and Γ is equal to the field-induced width Γ_f for stronger fields, with $\Gamma_f \gg \Gamma_a$ [Eqs. (23), (27), and (31)]. The curve of dW_2/dt as a function of ε is Lorentzian. It is easy to see that at the point of the maximum we have

$$\varepsilon = \varepsilon_1, \qquad \frac{dW_1}{dt} \Big/ \frac{dW_2}{dt} \approx \frac{\Gamma_a}{\Gamma_i} \gg 1.$$

The relation between dW_1/dt and dW_2/dt and that between the ionization probabilities W and W_2, however, at values of ε near the point $\varepsilon = \varepsilon_{\min}$ is more complicated. Using expansion (17), we find the following from (19) under the condition $|\varepsilon - \varepsilon_{\min}| \ll q\Gamma_a$:

$$W_1(t) = \frac{\Gamma_1}{\Gamma_a}\left[1 + \frac{(\varepsilon - \varepsilon_0)^2 + \Gamma_i^2/4}{(q^2 + 1/4)\Gamma_a} t\right], \qquad W_2(t) = \frac{\Gamma_i \Gamma_i}{\Gamma_a} t,$$
$$\varepsilon_0 = \varepsilon_{min} + q\Gamma_i. \tag{32}$$

The ratio of the decay rates to the states with the energies $E \approx E_a$ and $E \approx E_a + \omega$ is

$$\frac{dW_1/dt}{dW_2/dt} = f(\varepsilon) = \frac{(\varepsilon - \varepsilon_0)^2 + \Gamma_i^2/4}{(q^2 + 1/4)\Gamma_a \Gamma_i}. \tag{33}$$

At $|\varepsilon-\varepsilon_0| \gg q(\Gamma_a\Gamma_i)^{1/2}$ the function $f(\varepsilon)$ is much greater than unity. At $|\varepsilon-\varepsilon_0| \ll q(\Gamma_a\Gamma_i)^{1/2}$, the function $f(\varepsilon)$ is much smaller than unity, and the ionization rate is suppressed in the region $E\approx E_a$ by an interference of the decay channels of the state E_1. Conditions (28) can be rewritten in this case as $1/\Gamma_a \ll t \ll \Gamma_a/\Gamma_i\Gamma_1$.

From (32) we find

$$\begin{aligned} &W_1(t)\approx\Gamma_1/\Gamma_a \gg W_2(t) \quad \text{for} \quad \Gamma_i t\ll 1,\\ &W_1(t)\approx W_2(t)\approx\Gamma_1/\Gamma_a \quad \text{for} \quad \Gamma_i t\approx 1,\\ &W_2(t)\gg W_1(t) \quad \text{for} \quad 1\ll\Gamma_i t\ll\Gamma_a/\Gamma_1. \end{aligned} \tag{34}$$

At the boundary of the range of applicability of (34), with $t\approx\Gamma_a/\Gamma_i\Gamma_1$, we have $W_2\approx 1$ and $W_1\ll 1$. At $|\varepsilon-\varepsilon_0|\gg q(\Gamma_a\Gamma_i)^{1/2}$ the restrictions on the pulse length are more stringent:

$$1/\Gamma_a\ll t\ll (q\Gamma_a)^2/\Gamma_1(\varepsilon-\varepsilon_0)^2\ll\Gamma_a/\Gamma_i\Gamma_1.$$

The effect of the interference of pathways is now negligible, and we have $W_1(t)\gg W_2(t)$ over the entire allowed time interval. At the boundary of the range of applicability, at $t\approx(q\Gamma_a)^2/\Gamma_1(\varepsilon-\varepsilon_0)^2$, we now have, on the contrary, $W_1\approx 1$ and $W_2\ll 1$. For a very long radiation pulse length, at which the conditions for the decay of both of the quasienergy states are satisfied,

$$\gamma^{(+)}t\gg 1, \quad \gamma^{(-)}t\gg 1, \tag{35}$$

the atom decays completely during the pulse: $W_1+W_2=1$. The only question remaining to be studied is the relation between the probabilities W_1 and W_1. From (19) we easily find

$$W_2=W_2(t\to\infty)$$

$$=\frac{\Gamma_i\Gamma_f^2(1+1/4q^2)(\Gamma^{(1)}+\Gamma^{(2)})}{16\gamma^{(+)}(\varepsilon)\gamma^{(-)}(\varepsilon)[(\mathrm{Re}\,\Omega(\varepsilon))^2+{}^1/_4(\Gamma^{(1)}+\Gamma^{(2)})^2]}, \tag{36}$$

$$W_1=W_1(t\to\infty)=1-W_2.$$

Analysis of this expression shows that the function $W_2(\varepsilon)$ is a Lorentzian curve with a scale width Γ_f in strong fields, $\Gamma_f\gg q\Gamma_a$, for which the widths of the quasienergy states, $\gamma^{(+)}$ and $\gamma^{(-)}$, are approximately equal for any value of ε (the deviation from resonance) vary only slightly with ε (Ref. 12), and $(\mathrm{Re}\,\Omega)^2\approx\Omega_R^2=\varepsilon^2+\Gamma_f^2/4$. The probability ratio W_2/W_1 is equal in order of magnitude to the ratio of the ionization widths, Γ_i/Γ_1, in the case $|\varepsilon|\lesssim\Gamma_f$, while it is $W_2/W_1\ll 1$ at $|\varepsilon|\gg\Gamma_f$.

It is also easy to show that in weaker fields, $\Gamma_f\ll q\Gamma_a$, the probability (W_2) for ionization of the system in the energy region $E\sim E_a+\omega$ is small for any ε except in a small neighborhood of $\varepsilon_{\min}$. In this region of deviations we can write (36) in the following form, using expansion (17):

$$\begin{aligned} W_2={}&q^2\Gamma_a^2\Gamma_i(1+1/4q^2)[1+(\Gamma_i+\Gamma_1)/\Gamma_a]\\ &\times[\Gamma_i+(\varepsilon-\varepsilon_{min})^2(1+(\Gamma_i+\Gamma_1)/\Gamma_a)/\Gamma_a q^2]^{-1}\{[q\Gamma_a(1\\ &+(\Gamma_i+\Gamma_1)/\Gamma_a)-(\varepsilon-\varepsilon_{min})]^2+{}^1/_4\Gamma_a^2(1+(\Gamma_i+\Gamma_1)/\Gamma_a)^2\}^{-1}. \end{aligned} \tag{37}$$

From this expression we find

$$\begin{aligned} &W_1=(\Gamma_i+\Gamma_1)/\Gamma_a-2(\varepsilon-\varepsilon_{min})/q\Gamma_a\ll 1,\\ &W_2=1-(\Gamma_i+\Gamma_1)/\Gamma_a+2(\varepsilon-\varepsilon_{min})/q\Gamma_a\approx 1 \quad \text{for} \quad |\varepsilon-\varepsilon_{min}|\ll q\Gamma_i, \end{aligned} \tag{37a}$$

$$\begin{aligned} &W_1=(\varepsilon-\varepsilon_{min})^2/q^2\Gamma_i\Gamma_a\ll 1, \quad W_2=1-(\varepsilon-\varepsilon_{min})^2/q^2\Gamma_i\Gamma_a\approx 1\\ &\quad \text{for} \quad q\Gamma_i\ll|\varepsilon-\varepsilon_{min}|\approx|\varepsilon-\varepsilon_0|\ll q(\Gamma_a\Gamma_i)^{1/2}, \end{aligned} \tag{37b}$$

$$\begin{aligned} &W_1=1-\Gamma_i\Gamma_a q^2/(\varepsilon-\varepsilon_{min})^2\approx 1, \quad W_2=\Gamma_i\Gamma_a q^2/(\varepsilon-\varepsilon_{min})^2\ll 1\\ &\quad \text{for} \quad q(\Gamma_a\Gamma_i)^{1/2}\ll|\varepsilon-\varepsilon_{min}|\approx|\varepsilon-\varepsilon_0|. \end{aligned} \tag{37c}$$

At $|\varepsilon-\varepsilon_0|\ll q(\Gamma_a\Gamma_i)^{1/2}$ the ionization of the atom occurs for the most part at energies $E\approx E_a+\omega$, while the ionization in the region $E\approx E_a$ is suppressed by the destructive interference of the different pathways for the decay of the state E_1. At $|\varepsilon-\varepsilon_0|\gg q(\Gamma_a\Gamma_i)^{1/2}$, on the other hand, the interference is insignificant, and we have $W_1\gg W_2$. These conclusions were drawn qualitatively above from (32) for the upper boundary of the interval of times determined by (28).

4. PHOTOELECTRON SPECTRUM

Let us examine the energy spectrum of the photoelectrons resulting from the ionization of the atom by radiation pulses of substantial duration satisfying condition (35). If, during the decay of an autoionizing state, an ion is in a strictly definite state (e.g., the ground state), with energy E_f, as we will assume below, then the probability $W(E)$ for finding the system in a state of the continuum with the energy E is at the same time the probability for finding a photoelectron with an energy $E-E_f$. For brevity, we will refer to the function $W(E)$ as the "photoelectron distribution function." This function can be written as the sum

$$W(E)=W_1(E)+W_2(E), \tag{38}$$

where $W_1(E)$ and $W_2(E)$ are the photoelectron distribution functions localized in the energy regions $\sim(E_a-E_f)$ and $\sim(E_a+\omega-E_f)$. From (3) we easily find

$$W_1(E)\equiv|a_E(t\to\infty)|^2|_{E\sim E_a}=\frac{\Gamma_1}{2\pi}\frac{(E-E_a+q\Gamma_a)^2+{}^1/_4\Gamma_i^2}{[(E-E_a-\varepsilon/2+{}^1/_2\,\mathrm{Re}\,\Omega)^2+\gamma^{(+)2}/4][(E-E_a-\varepsilon/2-\mathrm{Re}\,\Omega/2)^2+\gamma^{(-)2}/4]},$$

$$\begin{aligned} W_2(E)\equiv{}&|a_E(t\to\infty)|^2|_{E\approx E'\sim E_a+\omega}\\ ={}&\Gamma_i\Gamma_f^2(1+1/4q^2)/32\pi[(E-E_a-\omega-\varepsilon/2+\mathrm{Re}\,\Omega/2)^2+\gamma^{(+)2}/4]\ [(E-E_a-\omega-\varepsilon/2-\mathrm{Re}\,\Omega/2)^2+\gamma^{(-)2}/4]. \end{aligned} \tag{39}$$

In terms of its behavior as a function of the energy (the product of two Lorentzians), the function $W_2(E)$ is similar to the electron distribution function found in Ref. 17 for the ionization of an atom accompanied by the resonant excitation of a bound state. This function has two peaks, with widths $\gamma^{(+)}$ and $\gamma^{(-)}$, which agree with the widths of quasienergy states of the system.

As for the function $W_1(E)$, we note that it is the product of a Lorentzian distribution of width $\gamma^{(+)}$ and a function $\Phi(E)$ of the Fano type, (17), with a width $\gamma^{(-)}$:

$$\Phi(E)=\frac{(E-E_a+q\Gamma_a)^2+{}^1/_4\Gamma_i^2}{\left(E-E_a-\frac{\varepsilon}{2}-{}^1/_2\operatorname{Re}\Omega\right)^2+{}^1/_4\gamma^{(-)2}}. \quad (40)$$

A minimum is caused in the function $\Phi(E)$ by the destructive interference of the amplitudes for the transitions $E_1\to E$ and $E_1\to E_a\to E$. However, in contrast with the ordinary Fano function, the minimum is not zero in this case, because of the existence of an ionizational pathway for the decay of the autoionization state.

We restrict the analysis below to a study of the functions $W_{1,2}(E)$ in moderately strong fields, satisfying condition (14), $\Gamma_f\ll q\Gamma_a$, and we restrict the analysis to the region of deviations of the electromagnetic field from resonance satisfying $|\varepsilon-\varepsilon_{\min}|\ll q\Gamma_a$, in which the interference phenomena are most obvious. Using expansions (17), we find the following expression for the denominator of the function $W_1(E)$:

$$[(E-E_a-q\Gamma_1)^2+{}^1/_4\gamma^{(+)2}(\varepsilon)][(E-E_a+q\Gamma_a-(\varepsilon-\varepsilon_0))^2+{}^1/_4\gamma^{(-)2}(\varepsilon)]. \quad (41)$$

It is clear that the position of the maximum $E^{(+)}_{\max}$ of the broad peak of the function $W_1(E)$, which has a width $\gamma^{(+)}\sim\Gamma_a$, does not depend on ε: $E^{(+)}_{\max}=E_a+q\Gamma_1$. Before we study the behavior of the function $W_1(E)$ near the narrow peak of width $\gamma^{(-)}$, we find the extrema of the Fano function $\Phi(E)$. Their positions are given by

$$(E-E_a+q\Gamma_a)^2(\varepsilon-\varepsilon_0)-(E-E_a+q\Gamma_a)[(\varepsilon-\varepsilon_0)^2-\Gamma_i^2/4+\gamma^{(-)2}(\varepsilon)/4]-{}^1/_4\Gamma_i^2(\varepsilon-\varepsilon_0)=0. \quad (42)$$

We thus see that in the region

$$|\varepsilon-\varepsilon_{\min}|\ll\min(q\Gamma_a,\ q\Gamma_a(\Gamma_i/\Gamma_1)^{1/2}), \quad (43)$$

for which $\gamma^{(-)}(\varepsilon)\ll\Gamma_i$ [see (17)], the function $\Phi(E)$ has a maximum $\Phi_{\max}$ and an interference minimum $\Phi_{\min}$ at the points $E^{(-)}_{\max}$ and $E^{(0)}_{\min}$:

$$E^{(-)}_{\max}=E_a-q\Gamma_a+(\varepsilon-\varepsilon_0),\qquad \Phi_{\max}=\frac{(\varepsilon-\varepsilon_0)^2+\Gamma_i^2/4}{\gamma^{(-)2}(\varepsilon)/4}\gg 1,$$
$$E^{(0)}_{\min}=E_a-q\Gamma_a-\frac{\Gamma_i^2}{4(\varepsilon-\varepsilon_0)}, \quad (44)$$
$$\Phi_{\min}\simeq\frac{\Gamma_i^2}{4[(\varepsilon-\varepsilon_0)^2+\Gamma_i^2/4]}<1.$$

The minimum distance between the points $E^{(0)}_{\min}$ and $E^{(-)}_{\max}$ is reached at $\varepsilon-\varepsilon_0=\pm\Gamma_i/2$; this minimum is $\Gamma_i\gg\gamma^{(-)}(\varepsilon)\neq 0$. The point $E^{(-)}_{\max}$ is on the remote wing of the Lorentz distribution with width $\gamma^{(+)}$: $|E^{(-)}_{\max}-E^{(+)}_{\max}|\approx q\Gamma_a\gg\gamma^{(+)}$. Near the point $E^{(-)}_{\max}$, the Lorentz function varies slowly, so that the point $E^{(-)}_{\max}$ in (44) is also a maximum of the function $W_1(E)$. The value of this maximum is $W_1(E^{(-)}_{\max})=\Gamma/2\pi q^2\Gamma_a^2\Phi_{\max}$.

The positions of all of the extrema of the function $W_1(E)$ are determined by the real roots of the fifth-degree equation $dW_1/dE=0$. To find the roots of this equation which do not coincide with $E^{(+)}_{\max}$, we assume that for any value of ε satisfying condition (43) the distance from these roots to $E^{(+)}_{\max}$ is much greater than the corresponding widths $\gamma^{(+)}$. This assumption is justified by the result. We can then ignore the widths $\gamma^{(+)}$ and $\gamma^{(-)}$ in the denominator of the function $W_1(E)$ in (41), and we find the following third-degree equation:

$$(E-E_a+q\Gamma_a)^3+[\Gamma_i^2/2-(\varepsilon-\varepsilon_0)q\Gamma_a](E-E_a+q\Gamma_a)-{}^1/_4\Gamma_i^2q\Gamma_a=0. \quad (45)$$

Analysis of the Cardano formula, which determines the solutions of this equation, shows that under the condition $\varepsilon-\varepsilon_0<\frac{3}{4}(\Gamma_i/q\Gamma_a)^{1/3}\Gamma_i$ it has a single real root, while at $\varepsilon-\varepsilon_0>\frac{3}{4}(\Gamma_i/q\Gamma_a)^{1/3}\Gamma_i>0$ it has three different real roots. Under the strong inequality

$$|\varepsilon-\varepsilon_0|\gg(\Gamma_i/q\Gamma_a)^{1/3}\Gamma_i \quad (46)$$

the function $W_1(E)$ has a minimum at the point $E_{\min}=E^{(0)}_{\min}$ which is determined by (44). If $\varepsilon-\varepsilon_0>0$, then $W_1(E)$ has two other extrema, $E_{(\pm)}$, at which a maximum and a minimum, respectively, are reached:

$$E_{(\pm)}\simeq E_a-q\Gamma_a\mp((\varepsilon-\varepsilon_0)q\Gamma_a)^{1/2}. \quad (47)$$

Under the strong inequality inverse to (46), the function $W_1(E)$ has a single minimum $E_{\min}$, given by

$$E_{\min}\approx E_a-q\Gamma_a+\left(\frac{\Gamma_i}{2}\right)^{2/3}(q\Gamma_a)^{1/3},$$
$$\Phi(E_{\min})\approx 1. \quad (48)$$

It is easy to show that, in accordance with the assumption made in the derivation of Eq. (45), the following conditions hold for the roots in (44), (47), and (48):

$$|E^{(0)}_{\min}-E^{(\pm)}_{\max}|\gg\gamma^{(\pm)},$$
$$|E_{(\pm)}-E^{(\pm)}_{\max}|\gg\gamma^{(\pm)},\quad |E_{\min}-E^{(\pm)}_{\max}|\gg\gamma^{(\pm)}.$$

Figure 3 shows the Fano function $\Phi(E)$ and the photoelectron distribution function $W_1(E)$ near the narrow maximum $E^{(-)}_{\max}$. At a deviation from resonance $|\varepsilon-\varepsilon_0|\gg\Gamma_i$, we have $\Phi_{\min}\ll 1$, as can be seen from (44). The minimum of the function $W_1(E)$ at the point $E=E_{\min}$ is seen as a deep dip in a region in which there is a smooth variation of this function, determined by a Lorentzian function with a width $\gamma^{(+)}$ (the dashed line in Fig. 3b). At $|\varepsilon-\varepsilon_0|\lesssim\Gamma_i$, on the other hand, we have $\Phi(E_{\min})\approx 1$ [see (44) and (48)], and there is almost no dip in the function $W_1(E)$ at the point $E=E_{\min}$. As can be seen from Fig. 3b, as $(\varepsilon-\varepsilon_0)>3/4(\Gamma_i/q\Gamma_a)^{1/3}\Gamma_i$ decreases, the points $E_{(+)}$ and $E_{\min}$ move counter to each other, merging at $\varepsilon-\varepsilon_0=3/4(\Gamma_i\ q\Gamma_a)^{1/3}\Gamma_i$, and the interference dip at the point $E_{\min}$ disappears. At this value of $(\varepsilon-\varepsilon_0)$, the $W_1(E)$ curve has only a single minimum, $E_{(-)}$, which moves to the point $E_{\min}$ in (48) as $(\varepsilon-\varepsilon_0)$ decreases further. At this point, a new interference minimum (44) forms with increasing

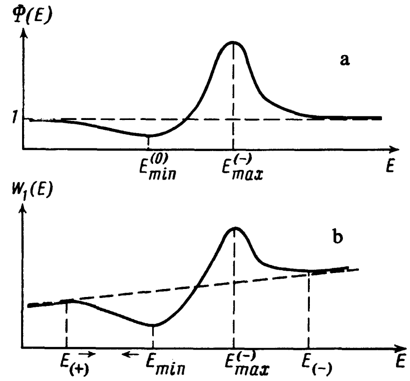

FIG. 3. The Fano function $\Gamma(\varepsilon)$ and the photoelectron distribution function $W_1(E)$ near the narrow peak, $E=E^{(-)}_{\max}=E_a-q\Gamma_a+(\varepsilon-\varepsilon_0)$, $\varepsilon-\varepsilon_0>3/4(\Gamma_i/q\Gamma_a)^{1/3}\Gamma_i$. The arrows show the directions in which the points $E_{(+)}$ and $E_{\min}$ move with decreasing $\varepsilon-\varepsilon_0$, until they merge at $\varepsilon-\varepsilon_0=3/4(\Gamma_i/q\Gamma_a)^{1/3}\Gamma_i$.

$|\varepsilon-\varepsilon_0|$ under the condition $\varepsilon-\varepsilon_0<0$; this minimum becomes deep when condition (46) becomes satisfied.

The maximum value of the function $\Phi(E)$, which determines the height of the narrow peak in the function $W_1(E)$, depends on the deviation from resonance, ε: $\Phi_{\max}=\Phi_{\max}(\varepsilon)$. The function $\Phi_{\max}(\varepsilon)$ has a minimum at $\varepsilon=\varepsilon_0$ and maxima at $\varepsilon=\varepsilon_{\min}\pm q(\Gamma_a\Gamma_i)^{1/2}\approx\varepsilon_0\pm q(\Gamma_a\Gamma_i)^{1/2}$, given by, respectively,

$$(\Phi_{max})_{min\ \varepsilon}=\Phi_{max}(\varepsilon_0)\simeq(\Gamma_a/\Gamma_1)^2.$$
$$(\Phi_{max})_{max\ \varepsilon}=q^2\frac{\Gamma_a^3}{\Gamma_i\Gamma_1^2}\gg(\Phi_{max})_{min\ \varepsilon}. \quad (49)$$

At $|\varepsilon-\varepsilon_0|\gg q(\Gamma_a\Gamma_i)^{1/2}$, the function $\Phi_{\max}(\varepsilon)$ falls off as $1/(\varepsilon-\varepsilon_0)^2$ (Fig. 4). The amplitude of the narrow peak, $W_1(E^{(-)}_{\max})$, also depends on ε. This behavior of $\Phi_{\max}(\varepsilon)$ is due to the joint effects of two factors: the interference between the different pathways for the decay of the states E_1 and the opening up of an ionization pathway for the decay of the autoionizing state. In the region $|\varepsilon-\varepsilon_0|\ll q(\Gamma_a\Gamma_i)^{1/2}$, where the function $\Phi_{\max}(\varepsilon)$ has a dip, most of the photoelectrons lie at energies $E\approx E_a+\omega-E_f$, $W_2\gg W_1$ [expression (37a)], as was shown above. The same factors influence the ratio of the heights of the peaks in the functions $W_1(E)$ and $W_2(E)$.

For example, the ratio of the heights of the peaks in the function $W_1(E)$ depends on ε in the same way as the function $\Phi_{\max}(\varepsilon)$:

$$\frac{W_1(E^{(-)}_{max})}{W_1(E^{(+)}_{max})}=\frac{1}{4q^2}\Phi_{max}(\varepsilon)=\begin{cases}4\Gamma_a^3/\Gamma_1\Gamma_f^2, & \varepsilon=\varepsilon_0\\ \Gamma_a^3/4\Gamma_1^2\Gamma_i\gg1, & \varepsilon=\varepsilon_0\pm q(\Gamma_i\Gamma_a)^{1/2}.\end{cases} \quad (50)$$

At the same time, for the function $W_2(E)$ the narrow peak is always higher than the wide peak by a factor $(\gamma^{(+)}/\gamma^{(-)})^2\gg1$. The ratio of the heights of the narrow peaks in the functions $W_1(E)$ and $W_2(E)$ also depends strongly on ε:

$$W_1(E^{(-)}_{max})/W_2(E^{(-)}_{max}+\omega)=[(\varepsilon-\varepsilon_0)^2+\Gamma_i^2/4]/\Gamma_i\Gamma_a q^2. \quad (51)$$

This ratio is small at $|\varepsilon-\varepsilon_0|\ll q(\Gamma_a\Gamma_i)^{1/2}$ and large in the opposite case. The interference between pathways does not, on the other hand, affect the heights of the wide peaks. For this reason we always have

$$W_1(E^{(+)}_{max})/W_2(E^{(+)}_{max}+\omega)=(\Gamma_a/\Gamma_i)^2\gg1, \quad (52)$$

which corresponds to the ratio of the probabilities for the decay of the autoionizing state by the autoionization and ionization pathways.

Let us briefly examine the relationship between these results and those of Ref. 3. Rzążewski and Eberly[3] ignored the ionization of the autoionization state, i.e., the width Γ_i, and there were no photoelectrons with energies $E\sim E_a+\omega-E_f$ in the spectrum. Actually, as was shown above, it is this energy region which holds most of the photoelectrons in certain cases [expression (37)]. At $\Gamma_i=0$, the width of the narrow maximum on the $W_1(E)$ curve approaches zero as $(\varepsilon-\varepsilon_{\min})^2$ in the limit $\varepsilon\to\varepsilon_{\min}$, and the limiting positions of the points $E^{(-)}_{\max}$ and $E^{(0)}_{\min}$ are the same [see (17) and (44)]. Rzążewski and Eberly[3] called these circumstances a "confluence of coherences" of the configuration interaction and the interaction with the external field. It should be noted, however, that at $\Gamma_i=0$ the value of the function $W_1(E^{(-)}_{\max})$ is not defined at $\varepsilon=\varepsilon_{\min}$; it depends on the order in which the two limits $E\to E^{(-)}_{\max}$ and $\varepsilon\to\varepsilon_{\min}$ are taken. It is easy to see from (39)–(41) that at $\Gamma_i=0$ we have $W_1(E^{(-)}_{\max})\sim1/(\varepsilon-\varepsilon_{\min})^2\to\infty$ as $\varepsilon\to\varepsilon_{\min}$ (see the dashed lines in Fig. 4), while we find $W_1(E)|_{\varepsilon-\varepsilon_{\min}}\to$const as $E\to E^{(-)}_{\max}$. Actually, this purely mathematical uncertainty has no physical meaning, and it disappears when the ionization with Γ_i is taken into account. It should also be noted that the conditions in Ref. 3 for a confluence of coherences, in the sense that the narrow peak on the $W_1(E)$ curve can contract, can be realized either as the field frequency ω is varied or as the field strenth $\mathscr{E}$ is varied. These results show that when ionization of the autoionizing state is taken into account there is still the possibility of a contraction (to certain limits) of the width $\gamma^{(-)}$ of the narrow maximum of the $W_1(E)$ curve as the frequency ω is varied at $\mathscr{E}=$ const [see (17)]. This is all that remains of the confluence of coherences. On the other hand, as the field strength $\mathscr{E}$ is varied at $\omega=$ const, the width $\gamma^{(-)}(\mathscr{E}^2)$ increases monotonically with increasing field, as is easily shown through an analysis of (13) and (17). The only exceptional case is that in which numerical factors make the ionization width Γ_i of the autoionizing

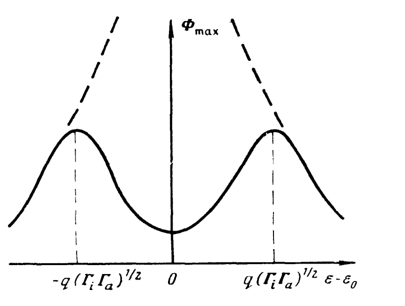

FIG. 4. The function $\Phi_{\max}(\varepsilon)$. The dashed line shows the function $\Phi_{\max}(\varepsilon)|\Gamma_i=0\sim(\varepsilon-\varepsilon_0)^{-2}$.

state anomalously small in comparison with the ionization width Γ_1 of the level E_1. In this case, there is a region on the $\gamma^{(-)}(\mathscr{E}^2)$ curve where the condition $d\gamma^{(-)}/d\mathscr{E}^2 < 0$ holds, and there is a minimum, $d\gamma^{(-)}/d\mathscr{E}^2 = 0$, at $\omega = \text{const}$. Typically, however, the two ionization widths Γ_i and Γ_1 are comparable in magnitude, and we have $d\gamma^{(-)}/d\mathscr{E}^2 > 0$.

For these reasons, the results of Ref. 3 are not correct.

5. CONCLUSION

Let us summarize the results of this study.

1) A resonant electromagnetic field gives rise to photoelectrons which are localized both at energies $\sim E_a - E_f$, because of a decay of the autoionizing state through a configuration interaction, and at energies $E_a + \omega - E_f$, because of the ionization of the autoionizing state by the radiation field.

2) The general expressions for $W_2(t)$ and $W_2(E)$, the probabilities for the ionization of an atom in the energy region $E \sim E_a + \omega$, and for the photoelectron distribution function are similar to the corresponding expressions for the ionization of an atom accompanied by the resonant excitation of a discrete level [expressions (19) and (39)]. On the other hand, the positions and widths of the quasienergy states in these expressions depend in a more complicated way on the strength and frequency of the electromagnetic field. At the same time, the interference between different pathways for the decay of the ground state (on the one hand) and the photoionization of the autoionizing state (on the other) strongly affect the ionization probability $W_1(t)$ in the region $E \sim E_a$. For this reason, the photoelectron spectrum $W_1(E)$ has a characteristic Fano structure [expression (40)] in this region.

3) An interference structure of the Fano type is also seen in the dispersive dependence on the radiation frequency ω (or on the deviation from resonance, ε) of the ionization rate dW_1/dt 1 in the energy region $E \sim E_a$ [expressions (22), (23), (30), and (31)], while the dispersive dependence of dW_2/dt is Lorentzian. For weak fields, $\Gamma_f \ll \Gamma_a$, the characteristic width of these curves is equal to the autoionization width Γ_a, while for stronger fields, $\Gamma_f \gg \Gamma_a$, the characteristic width is equal to the field-induced width Γ_f, which determines the Rabi frequency.

4) The interference between different pathways determines the ε-dependent relations among the ionization rates, the numbers of photoelectrons in the regions $E \sim E_a + \omega - E_f$ and $E_a - E_f$, and the height of the narrow peak in the function $W_1(E)$. In particular, for long radiation pulses and moderately strong fields, with $\Gamma_f \ll q\Gamma_a$, the probability is $W_2(t) \gg W_1(t)$ in the region of the most effective interference, $|\varepsilon - \varepsilon_{\min}| \ll q(\Gamma_a \Gamma_i)^{1/2}$ [see (37)]. In the same region of ε, the height of the narrow peak in the function $W_2(E)$ is much smaller than the height of the narrow peak in the function $W_1(E)$ [see (51)].

5)The existence of a noninterfering pathway for the ionization decay of the autoionizing state is the main reason for the finite heights and widths of the narrow peaks in the photoelectron distribution functions, i.e., for the absence of a complete confluence of coherences.

It would be worthwhile to carry out an experimental study of the spectrum of photoelectrons in a resonant field. Such experiments might yield information on the positions and widths of the autoionizing states, the transition matrix elements, the Fano parameter, etc., which are extremely difficult to calculate for complex atoms.

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