Spin waves in narrow-gap ferromagnetic semiconductors

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The *s-d* exchange model and the narrow-gap Hubbard model are used to obtain the exact (in an approximation linear in the carrier distribution function) expressions for the spin Green functions at absolute zero. A study is made of the spectrum and damping of spin waves. It is shown that an optical magnon is not a well-defined mode, with the exception of cases of the extremely large or extremely small values of *s-d* exchange parameter. The existence of a nonpole part of the Green functions may give rise to nonresonant absorption of energy from an alternating magnetic field in a wide range of frequencies and to inelastic nonresonant neutron scattering.

1. INTRODUCTION

Narrow-gap ferromagnetic semiconductors, in which an indirect exchange interaction via conduction electrons ("double exchange"[1]) plays a significant role, are important representatives of non-Heisenberg magnetic materials.[2,3] Theoretical studies of their magnetic properties are therefore highly desirable. They are usually carried out within the framework of the *s-d* exchange model. The spectrum of spin excitations in wide-gap ferromagnetic semiconductors has been investigated repeatedly (see, for examples, Refs. 3–6) in the lowest orders of perturbation theory on the basis of the *s-d* exchange interaction. Moreover, the limit of a very narrow gap has been studied in the case of classical spins.[3] However, it is known that an analysis of the electron spectrum made under these approximations ignores a number of important (specifically quantum) effects such as the formation of a bound state between an electron and a spin wave or partial spin depolarization of electrons.[7–11] We can expect these quantum effects in the spectrum of spin excitation to result in major changes compared with the quasiclassical pattern.

Our aim will be to consider the spin wave spectrum of ferromagnetic semiconductors at absolute zero on the basis of a rigorous calculation of spin Green functions at a low carrier density.

2. ELECTRON AND MAGNON GREEN FUNCTIONS IN THE *s-d* EXCHANGE MODEL

We shall consider the Hamiltonian of the *s-d* exchange model

$$H=\sum_{ij\sigma} t_{ij} a_{i\sigma}^{+} a_{j\sigma} - I \sum_{i\sigma\sigma'} \mathbf{S}_i \sigma_{\sigma\sigma'} a_{i\sigma}^{+} a_{i\sigma'}, \tag{1}$$

where t_{ij} are the hopping integrals of the s electrons; I is the s-d exchange integral; $\mathbf{S}_i$ is the operator of a localized spin at a site i; $\boldsymbol{\sigma}$ are the Pauli matrices. One-electron Green functions can be calculated more exactly for the ferromagnetic ground state if the gas approximation is used.[8–10] At $T=0$, we have

$$\langle\langle a_{\mathbf{k}\uparrow} | a_{\mathbf{k}\uparrow}^{+} \rangle\rangle_E = (E+IS-t_{\mathbf{k}})^{-1}, \tag{2}$$

$$\langle\langle a_{\mathbf{k}\downarrow} | a_{\mathbf{k}\downarrow}^{+} \rangle\rangle_E = \left(E-t_{\mathbf{k}}+IS-\frac{2IS}{1-IR(E)} \right)^{-1}, \tag{3}$$

where $t_{\mathbf{k}}$ is the Fourier transform of the hopping integral ($\Sigma_{\mathbf{k}}\, t_{\mathbf{k}}=0$), and

$$R(E)=\sum_{\mathbf{p}} (E+IS-t_{\mathbf{p}})^{-1}. \tag{4}$$

We can see that, in contrast to the classical picture of spin-split subbands (which, however, is restored in the limit $S\to\infty$, $I\to 0$, $IS=\text{const}$), the density of electron states with the down spin ("incorrect" projection for $I>0$) differs from zero wherever the density of states with the up spin differs from zero. Therefore, mutual interpenetration of spin subbands takes place, i.e., states with different projections of the spin coexist at the same energy. Although the presence of a nonpole contribution to electron Green functions in the *s-d* exchange model has been known a long time, the problem of manifestation of such a nonquasiparticle behavior in the observed quantities has not been studied in detail. We shall show that the relevant effects are important in the magnon spectrum because in the case of the electron-magnon interaction we can expect elastic scattering between states with different spin projections.

We shall now calculate a magnon Green function for $I>0$. It is important to note that in spite of the presence of a tail of the density of states with the down spin, the ground state now contains only electrons with the up spin because in the case of degenerate statistics of conduction electrons the tail in question terminates on the Fermi surface. This is manifested formally by the fact that when we allow for the finite occupancy of the conduction band, the expression for the resolvent (4) becomes

$$R(E)=\sum_{\mathbf{p}} \frac{1-n_{\mathbf{p}\uparrow}}{E+IS-t_{\mathbf{p}}}, \tag{5}$$

where

$$n_{\mathbf{k}\uparrow} \equiv \langle a_{\mathbf{k}\uparrow}^{+} a_{\mathbf{k}\uparrow} \rangle = f(t_{\mathbf{k}}-IS)$$

is the Fermi distribution function, so that the imaginary part of Eq. (5) vanishes for $E<E_F$. (Conversely, if $I<0$ and also in the narrow-gap Hubbard model[12] the states with the "incorrect" spin projection appear below the Fermi surface.) Therefore, in calculating the magnon commutator retarded

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Green function we shall drop the contributions proportional to $\langle a_{k\downarrow}^{+} a_{k\downarrow}\rangle$. Going over from the spin operators to the Dyson-Maleev operators for ideal spin waves[13] b_q^{+} and b_q, deriving a chain of equations of motion, and ignoring the Green functions proportional to the magnon occupation numbers $\langle b_q^{+} b_q\rangle$ (and, consequently, the terms small on the temperature scale), we obtain the following closed system of equations which is exact in the approximation linear in the electron density:

$$\omega\langle\langle b_q | b_q^{+}\rangle\rangle_\omega = 1 - I(2S)^{1/2}\sum_k \langle\langle a_{k\uparrow}^{+} a_{k+q\downarrow} | b_q^{+}\rangle\rangle_\omega + I\sum_{kp}\langle\langle a_{k\uparrow}^{+} a_{k+q-p\uparrow} b_p | b_q^{+}\rangle\rangle_\omega, \quad (6)$$

$$(\omega - t_{k+q} + t_k - 2IS)\langle\langle a_{k\uparrow}^{+} a_{k+q\downarrow} | b_q^{+}\rangle\rangle_\omega = -I(2S)^{1/2}\sum_p \langle\langle a_{k\uparrow}^{+} a_{k+q-p\uparrow} b_p | b_q^{+}\rangle\rangle_\omega, \quad (7)$$

$$(\omega - t_{k+q-p} + t_p)\langle\langle a_{k\uparrow}^{+} a_{k+q-p\uparrow} b_p | b_q^{+}\rangle\rangle_\omega = \delta_{qp} n_{k\uparrow} - I(2S)^{1/2}\langle\langle a_{k\uparrow}^{+} a_{k+q\downarrow} | b_q^{+}\rangle\rangle_\omega + I\sum_r \langle\langle a_{k\uparrow}^{+} a_{k+q-r\uparrow} b_r | b_q^{+}\rangle\rangle_\omega. \quad (8)$$

Solving the systems (7) and (8) and substituting the results in Eq. (6), we obtain an expression of the type

$$\langle\langle b_q | b_q^{+}\rangle\rangle_\omega = \omega^{-1} + \omega^{-2}\Pi(q\omega), \quad (9)$$

$$\Pi(q\omega) = I\sum_k \frac{(t_{k+q} - t_k - \omega) n_{k\uparrow}}{2IS + (t_{k+q} - t_k - \omega)[1 - IR(k\omega)]}, \quad (10)$$

where

$$R(k\omega) = \sum_p (\omega + t_k - t_p)^{-1}. \quad (11)$$

Equation (9) represents the first two terms of the expansion of the magnon Green function in terms of the density c of the conduction electrons. A comparison of Eq. (9) with the Dyson equation

$$\langle\langle b_q | b_q^{+}\rangle\rangle_\omega = [\omega - \Pi(q\omega)]^{-1} \quad (12)$$

shows that $\Pi(q\omega)$ is the exact (in the approximation linear in c) self-energy part of the magnon. Our calculation method corresponds to perturbation theory applied to the mass operator in Ref. 13. It follows from Eq. (12) that the magnon frequency is

$$\omega_q = \mathrm{Re}\,\Pi(q0) = I\sum_k \frac{(t_{k+q} - t_k) n_{k\uparrow}}{2IS + (t_{k+q} - t_k)[1 - IR(k0)]}. \quad (13)$$

The dispersion law (13) is of the non-Heisenberg type. When the quantity $2IS$ is small, compared with the width W of the conduction band, Eq. (13) becomes identical with the results of a canonical transformation[3] and of the variational principle.[14] It should be pointed out that for arbitrary values of $2IS/W$ the exact formula (13) begins to deviate from the results of simple approximations, beginning from terms of the order of q^4 in the expansion of the magnon frequency. It should be pointed out that the coefficient of $n_{k\uparrow}$ in Eq. (13) is exactly equal to the coefficient of $\langle b_q^{+} b_q\rangle$ in the expression for the temperature correction to the energy of an electron with a quasimomentum $\mathbf{k}$ (Ref. 10) and also identical with the expression for the amplitude of the electron-magnon scattering found by the diagram method in Ref. 15.

We shall now consider the question of the existence of an optical magnon of frequency governed by the pole of the magnon Green function $\omega_0 \neq 0$ in the limit $q \to 0$. If $q = 0$, it follows from Eqs. (10) and (12) that

$$R(0\omega_0) = 1/I - 2S/\omega_0. \quad (14)$$

At high values of I this equation has the solution $\omega_0 = I(2S+1)$, which describes transitions between states with the total spin $S + 1/2$ and $S - 1/2$ at a site. In the opposite case of small values of I, we find that $\omega_0 \approx 2IS$, which corresponds to the frequency of transitions between spin-split subbands. For $0 < \omega < W$ the function $R(0\omega)$ has a nonzero imaginary part

$$\mathrm{Im}\,R(0\omega) = \pi g(t_{min} + \omega), \quad (15)$$

where $g(E)$ is the "bare" (unrenormalized) density of states of the s electrons. Therefore, if $I(2S+1) < W$, an optical phonon experiences a strong Landau-type damping (decay into single-particle excitations). This magnon cannot be regarded as a well-defined mode, since its damping is generally speaking of the order of its energy. If $IS \ll W$, the ratio of the damping γ_0 to the frequency deduced from Eqs. (14) and (15) is small:

$$\gamma_0/\omega_0 \approx \pi I g(t_{min} + 2IS) \sim (2S)^{-1}(2IS/W)^{3/2}. \quad (16)$$

Therefore, in the case of extremely large and extremely small values of the s-d exchange integral an "optical magnon" does indeed exist. However, in the case of real substances the situation is intermediate, so that at best an optical magnon may be manifested by a strongly broadened resonance in the scattering of neutrons or in the absorption of infrared radiation. This is probably true also of ferromagnetic metals. The above analysis may therefore account for the failure to observe optical magnons experimentally. The conclusion reached in Refs. 5 and 6 that there is no optical magnon damping outside the Stoner continuum is based on the use of the lowest approximations for the Green functions, which ignore the interpenetration of the subbands and the electron-magnon scattering that are responsible for the optical mode damping.

The imaginary part of the spin Green function which governs, for example, the absorption of the energy of a high-frequency magnetic field and the cross section for the inelastic scattering of neutrons, includes contributions not only of the poles corresponding to elementary excitations, but also of the cut corresponding to a transition between single-particle states with different spin projections. Compared with the usual pattern of transitions in the Stoner continuum, the nonpole contribution to the absorption considered here and

associated with the interpenetration of the spin subbands has an important singularity: spin-flip transitions are possible without loss of energy and with an arbitrary change in the electron quasimomentum, since states with the "incorrect" spin projection are not well-defined quasiparticles. If $\mathbf{q}=0$, the magnon Green function is

$$\langle\langle b_0 | b_0^+ \rangle\rangle_\omega = \omega^{-1}\left\{1+\frac{cI}{2IS-\omega[1-IR(0\omega)]}\right\}^{-1}. \quad (17)$$

The presence of the imaginary part of Eq. (17) in a wide band of frequencies $0<\omega<W$ means that energy from a homogeneous alternating magnetic field can be absorbed. Naturally, in the case of identical g factors of the s and d electrons, when the total magnetic moment is an integral of motion, absorption is impossible for $\mathbf{q}=0$ and the imaginary part found should cancel out if allowance is made for the contribution of conduction electrons. No such cancellation occurs when the g factors are different. The nonpole contribution to the magnon Green function can also give rise to inelastic nonresonant neutron scattering.

We shall now consider qualitatively the more complex case when $I<0$. There is no penetration of the upper spin single-particle subband into the lower one at $T=0$ (although it does occur at finite temperatures[11]). However, in addition to single-particle states with the down spin, there are also essentially nonband states such as a down-spin electron + spin wave combination, states described by the cut of the Green function

$$\langle\langle d_{\mathbf{k}} | d_{\mathbf{k}}^+ \rangle\rangle_E, \quad d_{\mathbf{k}}^+ = \sum_{\mathbf{q}} a_{\mathbf{k}-\mathbf{q}\downarrow}^+ b_{\mathbf{q}}^+$$

and playing a role similar to that of single-particle states with the "incorrect" spin projection when $I>0$. The same situation occurs in the narrow-gap Hubbard model which, as shown below, is equivalent to the s-d exchange model in the limit when $S=1/2$ and $I=-\infty$. These states have been considered in the Hubbard model for the one-dimensional case.[16] Therefore, all the main features of the spin Green functions mentioned above are retained also for $I<0$. A quantitative analysis of this case is difficult to carry out for arbitrary parameters of the model because of the appearance of a cumbersome system of integral equations. We shall consider only the narrow-gap limit when $I=+\infty$. It is then convenient to use the formalism of the Hubbard X operators.[17]

3. NARROW-GAP s-d EXCHANGE MODEL AND HUBBARD MODEL IN THE ATOMIC REPRESENTATION

The Hubbard operators are described by the expression

$$X_i^{\alpha\beta} = |i\alpha\rangle\langle i\beta|,$$

where $|i\alpha\rangle$ is the complete set of many-electron states at a site. Since they are generalized projection operators, they satisfy the multiplication rule at a site

$$X_i^{\alpha\beta} X_i^{\gamma\varepsilon} = \delta_{\beta\gamma} X_i^{\alpha\varepsilon} \quad (18)$$

and the completeness relationship

$$\sum_\alpha X_i^{\alpha\alpha} = 1. \quad (19)$$

We shall write down the Hamiltonian of the s-d exchange model (1) in the X operator (atomic) representation. Using the values of the coefficients in the addition of S and $1/2$, and also the relationship between the one-electron and X operators,[17] we find that

$$a_{i\uparrow}^+ = \sum_{M=-S}^{S}\left[\left(\frac{S+M+1}{2S+1}\right)^{1/2} X_i^{M+1/2,\alpha;M} - \left(\frac{S-M}{2S+1}\right)^{1/2} X_i^{M+1/2,\beta;M}\right],$$

$$a_{i\downarrow}^+ = \sum_{M=-S}^{S}\left[\left(\frac{S+M}{2S+1}\right)^{1/2} X_i^{M-1/2,\beta;M} + \left(\frac{S-M+1}{2S+1}\right)^{1/2} X_i^{M-1/2,\alpha;M}\right]. \quad (20)$$

Here, $|M\rangle$ is a state without conduction electrons but with the localized spin projection M, whereas $|M'\alpha\rangle$ and $|M'\beta\rangle$ are states with one conduction electron, total spin $S+1/2$ and $S-1/2$, and spin projection M'. The states with two conduction electrons per site can be dropped when the electron density is low. The s-d exchange model has been used earlier in the atomic representation[18] employing the diagram technique, but the results are not quite correct. In particular, complete agreement with the quasiclassical limit[13] has not been obtained and the problem of high-frequency behavior of the spin Green functions has been considered incorrectly.

The Hamiltonian of the s-d exchange interaction considered in the X-operator representation reduces to the diagonal form

$$H_{s-d} = I(S+1)\sum_{iM} X_i^{M,\beta;M,\beta} - IS\sum_{iM} X_i^{M,\alpha;M,\alpha}. \quad (21)$$

If $I=\pm\infty$, since the transitions $\alpha\leftrightarrow\beta$ are forbidden, the operator H_{s-d} commutes with all the operators that appear in the calculation of the spin Green functions and can be omitted; in the case of the electron Green functions it simply alters the point of reference from which energy is measured.

In calculating the spin Green functions at $T=0$ it is sufficient to retain in Eq. (20) only the terms with $M=S$ and $M=S-1$. We shall consider the limit $I=-\infty$. Then, the states α can be omitted from the hopping Hamiltonian. Introducing

$$|S\rangle = |+\rangle, \quad |S-1\rangle = |-\rangle, \quad |S-1/2,\ \beta\rangle = |2\rangle,$$

$$|S-3/2,\ \beta\rangle = |3\rangle,$$

we find that

$$H = (2S+1)^{-1}\sum_{\mathbf{k}} t_{\mathbf{k}}\{2SX_{-\mathbf{k}}^{2+}X_{\mathbf{k}}^{+2} + X_{-\mathbf{k}}^{2-}X_{\mathbf{k}}^{-2} + (2S-1)X_{-\mathbf{k}}^{3-}X_{\mathbf{k}}^{-3}$$

$$+[2S(2S-1)]^{1/2}(X_{-\mathbf{k}}^{2+}X_{\mathbf{k}}^{-3} + X_{-\mathbf{k}}^{3-}X_{\mathbf{k}}^{+2})\}. \quad (22)$$

We note that if $S = 1/2$ then the Hamiltonian (22) subject to the substitution $t_{\mathbf{k}} \to 2t_{\mathbf{k}}$ becomes identical with the Hamiltonian of the narrow-gap Hubbard model

$$H = \sum_{\mathbf{k}\sigma} t_{\mathbf{k}} X_{-\mathbf{k}}^{2\sigma} X_{\mathbf{k}}^{\sigma 2} \tag{23}$$

(compare with Ref. 12), where $|2\rangle$ refers to the states with two conduction electrons per site ("deuce"). It follows therefore that in this case the two models are exactly equivalent (this was already pointed out in Ref. 3).

In deriving a chain of equations of motion for the Green functions

$$G(\mathbf{q}\omega) = \langle\langle X_{\mathbf{q}}^{+-} | X_{-\mathbf{q}}^{-+} \rangle\rangle_{\omega}$$

we shall drop the terms proportional to the occupation numbers of magnons or to the electron density if present as a square or higher power. The first equation is of the form

$$\omega G(\mathbf{q}\omega) = 1 - c + (2S+1)^{-1} \sum_{\mathbf{k}} t_{\mathbf{k}} \langle\langle X_{-\mathbf{k}}^{2-} X_{\mathbf{k}+\mathbf{q}}^{+2} - 2S X_{\mathbf{q}-\mathbf{k}}^{2-} X_{\mathbf{k}}^{+2} + [2S(2S-1)]^{1/2} X_{-\mathbf{k}}^{2+} X_{\mathbf{k}+\mathbf{q}}^{+3} | X_{-\mathbf{q}}^{-+} \rangle\rangle_{\omega}. \tag{24}$$

Using the equation of motion for the function

$$F(\mathbf{q}\omega) = \left(\frac{2S-1}{2S}\right)^{1/2} \langle\langle X_{\mathbf{q}}^{23} | X_{-\mathbf{q}}^{-+} \rangle\rangle_{\omega}$$

and the identity which follows from Eq. (18),

$$X_{\mathbf{q}-\mathbf{k}}^{2-} = \sum_{\mathbf{r}} X_{-\mathbf{r}}^{2+} X_{\mathbf{q}-\mathbf{k}+\mathbf{r}}^{+-}, \tag{25}$$

we can transform Eq. (24) to

$$\omega G(\mathbf{q}\omega) = 1 - c + \sum_{\mathbf{kr}} \frac{t_{\mathbf{k}-\mathbf{q}} - t_{\mathbf{k}}}{2S+1} \Phi(\mathbf{k}|\mathbf{q}\omega\mathbf{r}) - \omega F(\mathbf{q}\omega), \tag{26}$$

where

$$\Phi(\mathbf{k}|\mathbf{q}\omega\mathbf{r}) = \langle\langle X_{-\mathbf{r}}^{2+} X_{\mathbf{q}-\mathbf{k}+\mathbf{r}}^{+-} X_{\mathbf{k}}^{+2} | X_{-\mathbf{q}}^{-+} \rangle\rangle_{\omega}.$$

In the equation for the function Φ we have used Eq. (19) and also transformations similar to Eq. (25). When the resultant operator products are reduced to the "normal" form (in which the operators $X^{-\sigma}$ are on the left of the operators $X^{+\sigma}$) and dropping of the terms small on the temperature scale, we obtain

$$(\omega - t_{\mathbf{k}}^{\cdot} + t_{\mathbf{r}}^{\cdot})\Phi(\mathbf{k}|\mathbf{q}\omega\mathbf{r}) = (\delta_{\mathbf{kr}} - 1) n_{\mathbf{r}}^{\cdot} + (2S)^{-1} \sum_{\mathbf{f}} t_{\mathbf{f}}^{\cdot} [\Phi(\mathbf{q}-\mathbf{k}+\mathbf{r}+\mathbf{f}|\mathbf{q}\omega\mathbf{r}) - 2S\Phi(\mathbf{f}|\mathbf{q}\omega\mathbf{r})] + \left(\frac{2S-1}{2S}\right)^{1/2} t_{\mathbf{k}}^{\cdot} \langle\langle X_{-\mathbf{r}}^{2+} X_{\mathbf{q}+\mathbf{r}}^{+3} | X_{-\mathbf{q}}^{-+} \rangle\rangle_{\omega}. \tag{27}$$

Here,

$$n_{\mathbf{r}}^{\cdot} = \langle X_{-\mathbf{r}}^{2+} X_{\mathbf{r}}^{+2} \rangle = f(t_{\mathbf{r}}^{\cdot}), \quad t_{\mathbf{r}}^{\cdot} = \frac{2S}{2S+1} t_{\mathbf{r}}, \tag{28}$$

which can be demonstrated by calculating the relevant "electron" Green function. Writing down the chain equation

$$(\omega + t_{\mathbf{r}}^{\cdot}) \langle\langle X_{-\mathbf{r}}^{2+} X_{\mathbf{q}+\mathbf{r}}^{+3} | X_{-\mathbf{q}}^{-+} \rangle\rangle_{\omega} = \left(\frac{2S-1}{2S}\right)^{1/2} \sum_{\mathbf{f}} t_{\mathbf{f}}^{\cdot} \Phi(\mathbf{f}|\mathbf{q}\omega\mathbf{r}), \tag{29}$$

substituting it in Eq. (27), and introducing the function

$$\chi(\mathbf{k}|\mathbf{q}\omega\mathbf{r}) = \omega (n_{\mathbf{r}}^{\cdot})^{-1} \left[\Phi(\mathbf{k}|\mathbf{q}\omega\mathbf{r}) + \frac{2S-1}{\omega + t_{\mathbf{r}}^{\cdot}} \sum_{\mathbf{f}} t_{\mathbf{f}} \Phi(\mathbf{f}|\mathbf{q}\omega\mathbf{r}) \right],$$

we obtain the following closed equation for this function:

$$(\omega - t_{\mathbf{k}}^{\cdot} + t_{\mathbf{r}}^{\cdot}) \chi(\mathbf{k}|\mathbf{q}\omega\mathbf{r}) = (\delta_{\mathbf{kr}} - 1)\omega + (2S)^{-1} \sum_{\mathbf{f}} (t_{\mathbf{k}+\mathbf{f}-\mathbf{q}-\mathbf{r}}^{\cdot} - t_{\mathbf{f}}^{\cdot}) \chi(\mathbf{f}|\mathbf{q}\omega\mathbf{r}). \tag{30}$$

Calculating the function F with the aid of Eqs. (29) and (18), and applying the Dyson equation, we find that the expression for the self-energy part becomes

$$\Pi(\mathbf{q}\omega) = \sum_{\mathbf{kr}} n_{\mathbf{r}}^{\cdot} \chi(\mathbf{k}|\mathbf{q}\omega\mathbf{r}) \left[\frac{t_{\mathbf{k}-\mathbf{q}} - t_{\mathbf{k}}^{\cdot}}{2S} - \omega \frac{2S-1}{2S} \frac{t_{\mathbf{k}}^{\cdot}}{\omega + t_{\mathbf{r}}^{\cdot}} \right]. \tag{31}$$

It should be pointed out that an integral equation analogous to Eq. (30) appears in calculations of the temperature correction to the electron spectrum in the *s-d* exchange model considered in the limit $I = -\infty$ (Ref. 10) and also in the narrow-gap Hubbard model.[12] This equation was first solved by Nagaoka[19] who considered the problem of the scattering of a hole by a spin wave in an almost half-filled Hubbard model. Therefore, it should be pointed out that if we go to the limit of the narrow-gap Hubbard model as described above ($S = 1/2$, $t_{\mathbf{k}}^{*} \to t_{\mathbf{k}}$), then our results for the spin wave spectrum become identical with those of Nagaoka. In contrast to Nagaoka's method,[19] our approach allows us to find also the damping of spin waves (see below).

The presence of the second term in the brackets of Eq. (31) does not affect the spectrum of spin waves since the frequency of these waves is proportional to c. Equation (30) can be solved exactly for the simplest model dispersion laws. In the case of a simple cubic lattice, using the approximation of the nearest neighbors, we find that the expression for the spin rigidity constant $D = \lim_{\mathbf{q}\to 0}(\omega_{\mathbf{q}}/q^2)$ is (compare with Refs. 19 and 12):

$$D=\varkappa_S\frac{c|t|}{2S+1}\quad \varkappa_S=\frac{1-A/2S}{1+A/2S},$$

$$A=\int\limits_{-\pi}^{\pi}\int\int\frac{dk_1dk_2dk_3}{(2\pi)^3}\frac{\sin^2 k_i}{\gamma_{\mathbf{k}}}\approx 0{,}208,\quad \gamma_{\mathbf{k}}=3-\sum_{i=1}^{3}\cos k_i. \tag{32}$$

Equation (30) can also be solved by an iteration method which corresponds to expansion in terms of the parameter $1/zS$ (where z is the coordination number). If we ignore the integral term, we obtain the expression

$$\omega_{\mathbf{q}}=(2S)^{-1}\sum_{\mathbf{k}}(t_{\mathbf{k}-\mathbf{q}}-t_{\mathbf{k}}^{*})n_{\mathbf{k}}^{*}, \tag{33}$$

obtained by us earlier from the variational principle.[14] The corresponding Nagaev result[3] differs from Eq. (33) by the substitution $t_{\mathbf{k}}^{*}\to t_{\mathbf{k}}$, which is unimportant only in the quasiclassical limit. The presence of the integral term in Eq. (30) gives rise to strong non-Heisenberg corrections to Eq. (33).

A similar calculation in the $I=+\infty$ case gives the following expression for the spin Green function:

$$G(\mathbf{q}\omega)=(1-c)[\omega-\Pi(\mathbf{q}\omega)]^{-1},$$
$$\Pi(\mathbf{q}\omega)=\sum_{\mathbf{k}}\frac{[t_{\mathbf{k}+\mathbf{q}}-t_{\mathbf{k}}-(2S+1)\omega][1-\omega R(\mathbf{k}\omega)]}{2S+R(\mathbf{k}\omega)(\omega-t_{\mathbf{k}+\mathbf{q}}+t_{\mathbf{k}})}n_{\mathbf{k}}, \tag{34}$$

where $n_{\mathbf{k}}$ is the Fermi distribution function and the function R is defined by Eq. (11). As expected, the magnon spectrum governed by the poles of the Green function (34) is identical with the results of §2 in the limit $I\to+\infty$ (although the function G differs from the magnon Green function, since the operators $X_{\mathbf{q}}^{+-}$ and $b_{\mathbf{q}}$ are not proportional to one another). We can demonstrate the non-Heisenberg nature of the dispersion law by quoting the expression for the magnon frequency in the case of a simple cubic lattice when $I=+\infty$. It follows then from Eq. (34) that

$$\omega_{\mathbf{q}}=2c|t|\gamma_{\mathbf{q}}(2S+0{,}505\gamma_{\mathbf{q}})^{-1}, \tag{35}$$

where the numerical value of the Watson integral[19] is used.

An analysis of Eqs. (34) and (31) shows that in the limit $I\to\pm\infty$ the Green functions $G(0\omega)$ have no poles for $\omega\neq 0$. However, these functions have an imaginary part in a wide range of frequencies due to the cuts that originate from the resolvent R at $I=+\infty$ and from the integral term in Eq. (30) at $I=-\infty$. The physical meaning of this behavior is discussed at the end of §2. It is worth noting that the nontrivial contribution to Im $G(0\omega)$ disappears at $I=-\infty$ and $S=1/2$ (and also in the narrow-gap Hubbard model). In such cases a carrier (which is a single "deuce") and a spin wave cannot be located at the same site and this is the special property of these cases.

We shall now consider the problem of magnon damping. It is related to the presence of a resolvent in Eqs. (10) and (34), and of an integral term in Eq. (30). However, the damping cannot be studied by a linear approximation with respect to the electron distribution function, because calculations of the imaginary parts give rise to factors of the $n_{\mathbf{k}}(1-n_{\mathbf{r}})$ type with $t_{\mathbf{k}}\approx t_{\mathbf{r}}$, which differ greatly from $n_{\mathbf{k}}$ in the degenerate statistics case. We shall calculate the damping including an expansion in terms of $1/z$, which can be carried out without recourse to the linear approximation and without assuming that $T=0$. Each additional order with respect to $1/z$ corresponds to additional summation with respect to the wave vector in the argument of the Fourier transform of the hopping integral. We shall consider the specific simpler case of the Hubbard model of Eq. (23). Applying successively the equations of motion, we obtain

$$\omega G(\mathbf{q}\omega)=\langle X^{++}-X^{--}\rangle$$
$$+\omega^{-1}\Big[\sum_{\mathbf{k}}(t_{\mathbf{k}-\mathbf{q}}-t_{\mathbf{k}})\langle X_{-\mathbf{k}}^{2-}X_{\mathbf{k}}^{+2}-X_{\mathbf{q}-\mathbf{k}}^{2-}X_{\mathbf{k}-\mathbf{q}}^{-2}\rangle$$
$$+\sum_{\substack{\mathbf{kr}\\ \mathbf{k}'\mathbf{r}'}}(t_{\mathbf{k}-\mathbf{q}}-t_{\mathbf{k}})(t_{\mathbf{k}'-\mathbf{q}}-t_{\mathbf{k}'})$$
$$\ll X_{-\mathbf{r}}^{2+}X_{\mathbf{q}-\mathbf{k}+\mathbf{r}}^{+-}X_{\mathbf{k}}^{+2}|X_{-\mathbf{k}'}^{2-}X_{\mathbf{k}'-\mathbf{q}-\mathbf{r}'}^{-+}X_{\mathbf{r}'}^{+2}\gg_{\omega}\Big]. \tag{36}$$

Calculation of the Green function in Eq. (36) gives the self-energy part accurate to within the second order in $1/z$. Its imaginary part describes the damping

$$\gamma_{\mathbf{q}}=\mathrm{Im}\,\Pi(\mathbf{q}\omega_{\mathbf{q}})$$
$$=\pi\sum_{\mathbf{kp}}(t_{\mathbf{k}+\mathbf{q}}-t_{\mathbf{k}})^2[n_{\mathbf{k}+\mathbf{p}}(1-n_{\mathbf{k}+\mathbf{q}})+N_{\mathbf{p}}(n_{\mathbf{k}+\mathbf{p}}-n_{\mathbf{k}+\mathbf{q}})]$$
$$\times\delta(\omega_{\mathbf{q}}-\omega_{\mathbf{p}}+t_{\mathbf{k}+\mathbf{p}}-t_{\mathbf{k}+\mathbf{q}}). \tag{37}$$

Here $\omega_{\mathbf{p}}$ is the magnon frequency in the lowest approximation in $1/z$, whereas $N_{\mathbf{p}}=N(\omega_{\mathbf{p}})$ is the Bose distribution function. Allowance for $\omega_{\mathbf{p}}$ (which is formally a third-order correction) in (37) is necessary for the correct calculation of the damping. It should be pointed out that the dependences of the damping defined by Eq. (37) are identical with those derived using a phenomenological theory of a ferromagnetic Fermi liquid.[20] At $T=0$ for $\mathbf{q}$ much smaller than the Fermi momentum, we find that

$$\gamma_{\mathbf{q}}=\pi^{-3}(qv_0^{1/3})^4\omega_{\mathbf{q}}(6\pi^2c)^{2/3}\sim c^{5/3}q^6, \tag{38}$$

where v_0 is the unit cell volume and c is the density of "deuces" (or holes).

4. CONCLUSIONS

The spectrum of spin waves of ferromagnetic semiconductors was considered at absolute zero. It was found that the magnons obey essentially a non-Heisenberg dispersion law. Moreover, magnons exhibit finite damping at $T=0$. In the case of the optical branch this damping is very strong, with the exception of the cases of very large or very small values of the *s-d* exchange parameter.

The expressions obtained for the spin Green functions are exact in the approximation linear in the carrier density c. The more general case can be studied using an expansion in $1/z$. It can be shown that the description of the magnon spectrum provided above remains qualitatively valid at all electron densities and model parameters that satisfy the condition of stability of a saturated ferromagnetic state.

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