Magnetic field transport in an acoustic turbulence type flow

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(Submitted 23 August 1984)
Zh. Eksp. Teor. Fiz. **88**, 487–494 (February 1985)

We derive equations for the Green function and the correlation tensor of the magnetic field in the second approximation in the velocity amplitude of the conducting fluid, taking into account the temporal spectrum of the flow. For long times the average magnetic field is damped with time and its turbulent diffusion coefficient is determined by the magnitude of the spectrum at zero frequency. For sufficiently large Reynolds numbers the second moment of the field, particularly its energy, increases exponentially with time.

1. INTRODUCTION

The problem of the transport and amplification of a magnetic field by a given turbulent flow of a conducting fluid is of interest both from a general physics point of view and also for astrophysical, geophysical, and, possibly, laboratory-technical applications.

In a paper by one of the present authors[1] the behavior of a magnetic field was considered in short-range δ-correlated, mirror-symmetrical turbulent flow. In that approximation the damping of the average field is determined by the turbulent diffusion coefficient and one obtains for the correlation function a diffusion-type equation with a source. A detailed numerical study of this equation made it possible to establish criteria for the exponential growth of the field, to find the growth rate for a broad range of magnetic Reynolds numbers, and to find the structure of the correlation function which grows with time.[2]

The approximation of short-range correlated flow (white noise) is justified when the temporal spectrum of the velocity field is rather broad and large near the zero frequency (section 3) so that it can be approximated by a constant. When we take into account a small, but finite time-correlation we get for the correlation function an integral equation with similar spectral properties.[3,4]

For flows with more complicated spectra the short-range correlated approximation is no longer suitable. Acoustic turbulence can serve as an example[5,6] where the spectrum is concentrated in the shape of a sharp peak at $\omega = ck$ (c is the sound speed, k is the wavenumber) and has a very small value in the vicinity of the zero frequency.

In the present paper we derive equations for the Green function and the second moment of the magnetic field for flow with a temporal spectrum of general shape in the second approximation in the velocity field amplitude (more precisely, the Mach number). These equations [see (6), (11), and (12)] are valid for a velocity field which has both potential and rotational components. They can be simplified in divergenceless (incompressible turbulent flow) or potential (acoustic turbulence) velocity fields. For a flow with a power spectrum which is nearly a constant these equations generalize well-known results about the turbulent diffusion of an average magnetic field (or of a passive scalar) and on the generation of the second moment (section 3). In the case of a spectrum of acoustic-turbulence type we obtain for the second moment double-time equations which have solutions which grow with time (section 3).

We note that in the present paper we consider the transport of the magnetic field by a given flow, i.e., a problem which is linear in the field. For the non linear problem in weak acoustic turbulence, see, e.g., Ref. 7.

2. GREEN FUNCTION AND AVERAGE FIELD

The evolution of the magnetic field in a given flow of a conducting fluid is described by the induction equation

$$\partial \mathbf{H}/\partial t = \mathrm{rot}[\mathbf{v}\times\mathbf{H}] + \nu_m \Delta \mathbf{H}, \qquad \mathrm{div}\,\mathbf{H}=0,$$

where $\mathbf{v}(\mathbf{r},t)$ is the is the velocity of the flow, and ν_m the magnetic diffusion coefficient. In the Fourier representation the equation becomes an integral equation:

$$H_i(\mathbf{k},t) = g(\mathbf{k},t)H_i(\mathbf{k},0) + L_{ilm}(k)\int_0^t ds\, g(\mathbf{k},t-s) \times \int d^3p\, H_l(\mathbf{p},s)\, v_m(\mathbf{k}-\mathbf{p},s), \quad i,l,m=1,2,3. \tag{1}$$

Here $H_i(\mathbf{k},0)$ is the Fourier transform of the initial field, $L_{ilm} = i(\delta_{im}k_l - \delta_{il}k_m)$, and $g(\mathbf{k},t) = \theta(t)\exp(-\nu_m k^2 t)$ is the Green function in the fluid at rest ($\theta(t)=0$, $t<0$, $\theta(t)=1$, $t>0$). We introduce the total Green function

$$H_i(\mathbf{k},t) = G_{ij}(\mathbf{k},t)H_j(\mathbf{k},0).$$

As we may assume that the velocity field and initial field distributions to be independent, the time-dependence of the average field is completely determined by the average value

 0038-5646/85/020285-05$04.00

of the Green function. From (1) we have

$$G_{ij}(\mathbf{k},t)=\delta_{ij}g(\mathbf{k},t)+L_{ilm}(\mathbf{k})\int_0^t dsg(\mathbf{k},t-s)\int d^3pv_m(\mathbf{k}-\mathbf{p},s)G_{lj}(\mathbf{p},s). \quad (2)$$

The velocity field is given by the correlator

$$\langle v_i(\mathbf{k},t)v_j(\mathbf{k}',t')\rangle=\delta(\mathbf{k}+\mathbf{k}')f_{ij}(\mathbf{k},t-t'), \quad (3)$$

and we assume that $\langle v_i(\mathbf{k},t)\rangle=0$. To obtain the average Green function an equation containing the velocity field correlator we put

$$G_{ij}=\langle G_{ij}\rangle+\delta G_{ij}, \qquad \langle\delta G_{ij}\rangle=0,$$

where the angle brackets indicate averaging over the velocity field. In first order in $\mathbf{v}(\mathbf{k},t)$ we have

$$\delta G_{ij}(\mathbf{k},t)=L_{ilm}(\mathbf{k})\int_0^\infty dsg(\mathbf{k},t-s)\int d^3pv_m(\mathbf{k}-\mathbf{p},s)\times\langle G_{mj}(\mathbf{p},s)\rangle. \quad (4)$$

Substituting this into (2) we find an equation for the average Green function in the second approximation:

$$\langle G_{ij}(\mathbf{k},t)\rangle=\delta_{ij}g(\mathbf{k},t)+L_{ilm}(\mathbf{k})\int_0^\infty dsg(\mathbf{k},t-s)\times\int_0^\infty d\sigma\int d^3pg(\mathbf{p},s-\sigma)L_{lrn}(\mathbf{p})f_{mn}(\mathbf{k}-\mathbf{p},s-\sigma)\langle G_{rj}(\mathbf{k},\sigma)\rangle,$$

or, in integro-differential form:

$$\left(\frac{\partial}{\partial t}+\nu_mk^2\right)\langle G_{ij}(\mathbf{k},t)\rangle=\delta_{ij}\delta(t)+L_{ilm}(\mathbf{k})\int_0^\infty d\sigma\int d^3pg(\mathbf{p},t-\sigma)L_{lrn}(\mathbf{p})f_{mn}(\mathbf{k}-\mathbf{p},t-\sigma)\times\langle G_{rj}(\mathbf{k},\sigma)\rangle. \quad (5)$$

It is natural to seek the average Green function in an isotropic velocity field in the form of a tensor that is invariant under rotations and reflections, i.e.,

$$\langle G_{ij}\rangle=G_\parallel k_ik_j/k^2+G(\delta_{ij}-k_ik_j/k^2).$$

Taking the components of Eq. (5) along the directions parallel to $\mathbf{k}$ and at right angles to it, we find that $G_\parallel$ is the same as $g(\mathbf{k},t)$ while the transverse part satisfies the equation

$$(\partial/\partial t+\nu_mk^2)G(\mathbf{k},t)=\delta(t)+\frac{1}{2}L_{ilm}(\mathbf{k})\int_0^t d\sigma\int d^3pg(\mathbf{p},t-\sigma)L_{lin}(\mathbf{p})f_{mn}\times(\mathbf{k}-\mathbf{p},t-\sigma)G(\mathbf{k},\sigma). \quad (6)$$

We consider a velocity field of potential form $v_i=\nabla_i\varphi$, for which

$$f_{mn}=\frac{k_mk_n}{k^2}f(k,t). \quad (7)$$

We can similarly consider the case of the flow of an incompressible fluid where $f_{mn}=(\delta_{mn}-k_mk_n/k^2)f(k,t)$.

Of most interest for applications is the case of small ν_m, so that it is natural when considering the behavior of the average magnetic field, to neglect the effect of the molecular magnetic diffusion, i.e., to put $\nu_m=0$. Introducing the variable $\mathbf{q}=\mathbf{k}-\mathbf{p}$ and integrating in (6) over the angles which determine the direction of $\mathbf{q}$

$$q^{-2}\int(\mathbf{pq})(\delta_{ij}\mathbf{kq}-q_ik_j)d\Omega=\frac{4\pi k^2}{3}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right),$$

we find

$$\frac{\partial G(\mathbf{k},t)}{\partial t}=\delta(t)-\frac{4\pi k^2}{3}\int_0^t d\sigma\int_0^\infty f(q,t-\sigma)q^2dqG(\mathbf{k},\sigma). \quad (8)$$

It is convenient to introduce instead of the correlation function of the velocity field the power spectrum $J(q,\omega)$:

$$f(q,t)=(2\pi)^{-1}\int_{-\infty}^\infty J(q,\omega)e^{-i\omega t}d\omega$$

and to take the Laplace transform of (8). We find

$$\tilde{G}(k,\xi)=\left(\xi+\frac{2k^2}{3}\int_0^\infty q^2dq\int_{-\infty}^\infty d\omega\frac{J(q,\omega)}{\xi+i\omega}\right)^{-1}. \quad (9)$$

It is well known that the asymptotic behavior of the Green function as $t\to\infty$ is determined by the asymptotic behavior of $\tilde{G}(\xi)$ as $\xi\to0$. As $f(t)$ does not increase at infinity the power spectrum is a decreasing function in the upper complex ω half-plane. The integral over $d\omega$ in (9) can thus be replaced as $\xi\to0$ by the residue at zero. As a result we have

$$G(k,t)\underset{t\to\infty}{\propto}\exp(-\nu_Tk^2t),\qquad \nu_T=\frac{4\pi}{3}\int_0^\infty J(q,0)q^2dq. \quad (10)$$

The average Green function, and hence the average magnetic field, is thus asymptotically damped at a rate determined by the value of the power spectrum at zero. Although this fact is known for a number of actual velocity fields (see, e.g., Refs. 8, 9) the derivation given here demonstrates its general character. The magnitude of the diffusion coefficient depends in an essential way on the form of the field. In the short-range correlated approximation ν_T is a maximum and is of the order of lv, where l is the correlation length and v the mean square velocity. In acoustic turbulence ν_T is very small, of order $M^2\varkappa$ where $M=v/c$ is the Mach number and $\varkappa$ the sound-wave damping coefficient defined by

$$f(k,t)=\frac{\pi c^2}{\varkappa}J(k,0)\exp(-\varkappa k^2t)\cos(ckt).$$

3. EQUATIONS FOR THE SECOND MOMENT

To obtain the equation for the correlator

$$\langle H_i(\mathbf{k},t)H_j(\mathbf{k}',t')\rangle=\delta(\mathbf{k}+\mathbf{k}')P_{ij}(\mathbf{k})\mathcal{H}(k,t,t'),$$
$$P_{ij}=\delta_{ij}-k_ik_j/k^2$$

it is sufficient to evaluate the expression

$$\langle G(\mathbf{k},t)H(\mathbf{k},0)G(\mathbf{k}',t')H(\mathbf{k}',0)\rangle$$
$$=\{\langle G(\mathbf{k},t)\rangle\langle G(\mathbf{k}',t')\rangle+\langle\delta G(\mathbf{k},t)\delta G(\mathbf{k}',t')\rangle\}\mathcal{H}(k,0,0),$$

using Eq. (4). The averaging is over the velocity field and the initial field distribution. We get

$$P_{ir}(\mathbf{k})\mathcal{H}(k,t,t')=\langle G_{rj}(\mathbf{k},t)\rangle\langle G_{il}(\mathbf{k},t')\rangle P_{jl}(\mathbf{k})\mathcal{H}(k,0,0)$$
$$-\int d^3q\int_0^t ds\int_0^{t'} d\sigma\langle G_{rj}(\mathbf{k},t-s)\rangle\langle G_{il}(\mathbf{k},t'-\sigma)\rangle L_{jmn}(\mathbf{k})L_{lpc}(\mathbf{k})$$
$$\times f_{nc}(\mathbf{k}-\mathbf{q},s-\sigma)P_{mp}(\mathbf{q})\mathcal{H}(q,s,\sigma).$$

To eliminate the initial field we act on this equation with the operator

$$\left(\frac{\partial}{\partial t}+\nu_m k^2\right)\delta_{ir}-L_{ilm}(\mathbf{k})\int_0^t d\sigma\int d^3p g(\mathbf{p},t-\sigma)$$
$$\times L_{lrn}(\mathbf{p})f_{mn}(\mathbf{k}-\mathbf{p},t-\sigma).$$

As a result we get an integro-differential equation for the second moment of the magnetic field

$$(\partial/\partial t+\nu_m k^2)\mathcal{H}(k,t,t')$$
$$=\frac{1}{2}L_{rlm}(\mathbf{k})\int_0^t d\sigma\int d^3pg(\mathbf{p},t-\sigma)L_{lrn}(\mathbf{p})$$
$$\times f_{mn}(\mathbf{k}-\mathbf{p},t-\sigma)\mathcal{H}(k,\sigma,t')-\frac{1}{2}\int_0^{t'} d\sigma\int d^3p$$
$$\times\langle G_{il}(\mathbf{k},t'-\sigma)\rangle L_{imq}(\mathbf{k})L_{lnp}(\mathbf{k})f_{qp}(\mathbf{k}-\mathbf{p},t-\sigma)$$
$$\times P_{mn}(\mathbf{p})\mathcal{H}(p,t,\sigma). \quad (11)$$

We get a similar equation for the derivative with respect to the second time:

$$(\partial/\partial t'+\nu_m k^2)\mathcal{H}(k,t,t')=\frac{1}{2}L_{rlm}(\mathbf{k})\int_0^{t'} d\sigma$$
$$\times\int d^3pg(\mathbf{p},t'-\sigma)L_{lrn}(\mathbf{p})f_{mn}(\mathbf{k}-\mathbf{p},t'-\sigma)\mathcal{H}(k,t,\sigma)$$
$$-\frac{1}{2}\int_0^t d\sigma\int d^3p\langle G_{il}(\mathbf{k},t-\sigma)\rangle L_{imq}(\mathbf{k})$$
$$\times L_{lnp}(\mathbf{k})f_{qp}(\mathbf{k}-\mathbf{p},\sigma-t')P_{mn}(p)\mathcal{H}(p,\sigma,t'). \quad (12)$$

In the case of short-range correlated incompressible flow with

$$f_{ij}(\mathbf{k},t-t')=P_{ij}f(k)\delta(t-t')$$

(11) changes into the equation for the equal time correlation function derived in Ref. 1:

$$\left\{\frac{\partial}{\partial t}+2\left(\nu_m+\frac{1}{3}\int f(p)d^3p\right)k^2\right\}\mathcal{H}(k,t)$$
$$=\int d^3q\mathcal{H}(p,t)f(q)\left[k^2-\frac{(\mathbf{kq})(\mathbf{kp})(\mathbf{qp})}{q^2p^2}\right],\quad \mathbf{p}=\mathbf{k}-\mathbf{q}. \quad (13)$$

Let the velocity field have a small but finite correlation time ε. For the sake of argument we put

$$f_{ij}=P_{ij}(\mathbf{k})f(k)\frac{1}{(4\pi\varepsilon^2)^{1/2}}\exp\left[-\frac{(t-t')^2}{4\varepsilon^2}\right].$$

We then get instead of (13) the equation

$$\left\{\frac{\partial}{\partial t}+2\left(\nu_m+\frac{1}{3}\int f(q)\Theta(\nu_m\varepsilon p^2)d^3q\right)k^2\right\}\mathcal{H}(k,t)$$
$$=\int d^3q\mathcal{H}(p,t)f(q)\left[k^2-\frac{(\mathbf{kq})(\mathbf{kp})(\mathbf{qp})}{q^2p^2}\right]\Theta(\nu_T\varepsilon k^2), \quad (14)$$

where we have substituted

$$G_{ij}(\mathbf{k},t)=P_{ij}(\mathbf{k})\exp(-\nu_T k^2 t),\quad \nu_T=\frac{1}{3}\int f(q)d^3q,$$

ν_T is the turbulent-diffusion coefficient (10). In deriving (14) we considered the integrals in the right-hand side of (11) as integrals of the product of a rapidly changing exponent and a slowly changing function $\mathcal{H}(k,t,\sigma)$. In that approximation the kernel Θ can be estimated for small x as follows:

$$\Theta(x)\approx 1-2\pi^{-1/2}x+\dots.$$

Equations which have a very similar contents were obtained in Refs. 3, 4.

For small ν_m the kernel $\Theta(\nu_m\varepsilon p^2)$ may be set equal to unity. The integral on the left-hand side of (14) then simply gives the turbulent-diffusion coefficient. The behavior of the kernel $\Theta(\nu_T\varepsilon k^2)$ on the right-hand side is equivalent to an effective scale cutoff, i.e., it operates only for $k<k_\varepsilon\equiv(\nu_T\varepsilon)^{-1/2}$. One can understand this in the sense that what operates effectively is not the true correlation function $f(r)$ of the velocity field but a smoothed function that is close to 1-const $\cdot\, r^2$ up to scales of the order of k_ε^{-1}. We note that the hydrodynamic viscosity ν does not affect the form of this effective function when k_ε^{-1} is larger than the Kolmogorov scale $l\mathrm{Re}^{-3/4}$, i.e., when $\nu<\nu_T$, if $\varepsilon\sim l/v$. The condition for the excitation of the second moment of the magnetic field in a flow with a finite correlation time does in particular, in contrast to the δ-correlated case,[2] not depend on the ratio of the hydrodynamic and the magnetic viscosities, but only on the magnitude of ν_m, more precisely, on the dimensionless magnetic Reynolds number.

When the temporal spectrum of the flow differs considerably from white noise we no longer obtain from (11) a closed equation for the equal-time correlation function of the field (cf. the opposite conclusion reached in Ref. 10). Equations (11) and (12) turn out to be in principle integrals over time. This important point was not drawn attention to earlier. It is interesting to compare the situation with hydrodynamic turbulence. There one can obtain a closed equation for the equal-time correlation function of the velocity field (equation of the von Kármán-Howard kind) and equations for the double-time correlator (of the Chandrasekhar kind; see, e.g., the monograph by Monin and Yaglom[11]). In the problem considered for the induction equation even when an equal-time velocity-field correlator is specified the magnetic field correlator turns out, in general, to depend on two times.

Stationary solutions of Eqs. (11), (12) must depend only on the time difference $t-t'=\tau$. It is therefore convenient when studying stability to change from t and t' to τ and $T=(t+t')/2$. Bearing in mind that

$$t=T+\tau/2,\quad t'=T-\tau/2,\quad \partial/\partial t+\partial/\partial t'=\partial/\partial T,$$

$$\partial/\partial t-\partial/\partial t'=2\partial/\partial\tau,$$

by adding and subtracting (11) and (12) we get in the new variables

$$\left(\frac{\partial}{\partial T}+2\nu_m k^2\right)\mathscr{H}(k,T,\tau)=\frac{1}{2}L_{rlm}(k)\int_0^{T+\tau/2}ds\int d^3q$$
$$\times g(\mathbf{k}-\mathbf{q},s)L_{lrn}(\mathbf{k}-\mathbf{q})f_{mn}(\mathbf{q},s)\mathscr{H}(k,T-s/2,\tau-s)$$
$$+\frac{1}{2}\int_0^{T-\tau/2}ds\int d^3q g(\mathbf{k}-\mathbf{q},s)L_{lrn}(\mathbf{k}-\mathbf{q})f_{mn}(\mathbf{q},s)$$
$$\times\mathscr{H}(k,T-s/2,\tau+s)-\frac{1}{2}\int_0^{T-\tau/2}ds\int d^3q\langle G_{il}(\mathbf{k},s)\rangle$$
$$\times L_{imq}(\mathbf{k})L_{lnp}(\mathbf{k})f_{qp}(\mathbf{q},\tau+s)P_{mn}(\mathbf{p})\mathscr{H}(k-q,T-s/2,\tau+s)$$
$$-\frac{1}{2}\int_0^{T+\tau/2}ds\int d^3q\langle G_{il}(\mathbf{k},s)\rangle L_{imq}(\mathbf{k})L_{lnp}(\mathbf{k})f_{qp}(\mathbf{q},\tau-s)$$
$$\times P_{mn}(\mathbf{p})\mathscr{H}(k-q,T-s/2,\tau-s)\tag{15}$$

and a similar equation for $\partial\mathscr{H}(k,t,\tau)/\partial\tau$.

In the limit of small ν_m the center of gravity of the required solution $\mathscr{H}(k,T,\tau)$ is positioned at large k. It is therefore natural to study Eq. (15) as $T\to\infty$ using in the integrands the fact that $k\gg q$. We get

$$(\partial/\partial T+2\nu_m k^2)\mathscr{H}(k,T,\tau)=-k_ik_j\int_0^\infty ds g(\mathbf{k},s)$$
$$\times\left(\int f_{ij}(\mathbf{q},s)d^3q\right)[\mathscr{H}(k,T-s/2,\tau-s)+\mathscr{H}(k,T-s/2,\tau+s)]$$
$$+{}^1/_2P_{il}(\mathbf{k})k_mk_n\int_0^\infty ds\langle G_{il}(\mathbf{k},s)\rangle$$
$$\times\left[\int f_{mn}(\mathbf{q},\tau+s)d^3q\,\mathscr{H}(k,T-s/2,\tau+s)\right.$$
$$\left.+\int f_{mn}(\mathbf{q},\tau-s)d^3q\mathscr{H}(k,T-s/2,\tau-s)\right].$$

For an incompressible (rotational) flow one can easily check that the right-hand side vanishes. Hence, in that case the large-k approximation does not apply and the contribution from large scales is always important. The right-hand side of this equation vanishes at $\tau=0$ also in the case of an arbitrary δ-correlated type of compressible flow.

We consider the acoustic turbulence kind of flow (7) and introduce the notation: $F(s)=\int f(q,s)d^3q$. The equation then becomes

$$(\partial/\partial T+2\nu_m k^2)\mathscr{H}(k,T,\tau)$$
$$=-\frac{k^2}{3}\int_0^\infty ds\{g(\mathbf{k},s)F(s)$$
$$\times[\mathscr{H}(k,T-s/2,\tau-s)+\mathscr{H}(k,T-s/2,\tau+s)]$$
$$-G(k,s)[F(\tau-s)\mathscr{H}(k,T-s/2,\tau-s)$$
$$+F(\tau+s)\mathscr{H}(k,T-s/2,\tau+s)]\}.\tag{16}$$

We look for a solution of the form

$$\mathscr{H}(k,T,\tau)=h(\tau)\exp(2\gamma T),$$

where γ and h depend on k. Substituting this into (16) and using (10) we get

$$\Gamma_m h(\tau)=-\frac{k^2}{6}\int_{-\infty}^{\infty}ds[\exp(-\Gamma_m|s|)F(|s|)h(\tau-s)$$
$$-\exp(-\Gamma_\tau|s-\tau|)F(s)h(\tau)],\tag{17}$$

where

$$\Gamma_m=\gamma+\nu_m k^2,\quad \Gamma_\tau=\gamma+\nu_\tau k^2.$$

For a further study of Eq. (17) we must choose the actual velocity field spectrum. We consider the simple acoustic turbulence model, in which case

$$f(k,t)=\frac{2(\omega_k^2+\lambda_k^2)}{\lambda_k}J(k,0)\exp(-\lambda_k t)\cos\omega_k t.$$

It is especially easy to study the case when

$$J(k,0)=\mathrm{M}^2\varkappa k_0^{-2}\delta(k-k_0),$$

i.e., isotropically distributed interacting waves which have wavelengths. We assume that $\lambda_k=\varkappa k^2$ and $\omega_k=ck$, where $\varkappa$ is the damping coefficient and c the sound speed, and we introduce the Mach number $\mathrm{M}=v_0/c$, where v_0 is a characteristic amplitude of the acoustic turbulence velocity which is determined by the complete spectrum. According to (10) we have then $\nu_T=4\pi\mathrm{M}^2\varkappa/3$. Changing to the Fourier transform $h(\omega)$ we get

$$\left\{\Gamma_m+\frac{k^2}{6}\Gamma_m\left[\frac{1}{(\omega-\omega_0)^2+(\Gamma_m+\lambda_0)^2}\right.\right.$$
$$\left.\left.+\frac{1}{(\omega+\omega_0)^2+(\Gamma_m+\lambda_0)^2}\right]\right\}h(\omega)$$
$$=\frac{k^2}{3}\frac{\Gamma_\tau}{\omega^2+\Gamma_\tau^2}\int_{-\infty}^{\infty}F(\omega-\Omega)h(\Omega)d\Omega,\tag{18}$$

where λ_0 and ω_0 are values at $k=k_0$. In this integral equation $\omega_0\gg\lambda_0$, Γ_m, Γ_T.

We can show that the spectrum of the solution which increases in time is concentrated in the frequency region $\omega<\omega_0$. In that approximation (18) reduces to the simpler equation

$$\frac{3}{k^2}\frac{\Gamma_m}{\Gamma_\tau}(\omega^2+\Gamma_\tau^2)h(\omega)=\int_{-\infty}^{\infty}F(\omega-\Omega)h(\Omega)d\Omega,$$

which is equivalent to a Schroedinger equation type equation

$$\partial^2h(\tau)/\partial\tau^2+2(E-U)h(\tau)=0$$

with $E=-\Gamma_T^2/2$ and a potential

$$U=-\frac{k^2}{3}\frac{\Gamma_\tau}{\Gamma_m}F(\tau)=-U_0e^{-\lambda_0\tau}\cos\omega_0\tau,$$
$$U_0\equiv\frac{8\pi}{3}\frac{\Gamma_\tau}{\Gamma_m}k^2v_0{}^2\sim M^2\omega_0{}^2. \qquad (19)$$

It is well known that the spectrum of the problem consists of bands formed by close lying levels. Bound states appear when $U_0>\omega_0^2$, i.e., $\Gamma_T\Gamma_m^{-1}k^2v_0^2>\omega_0^2$ or close to the generation threshold ($\gamma=0$) when

$$\nu_T/\nu_m>(\omega_0/kv_0)^2\sim M^{-2}$$

($\varkappa/\nu_m>M^{-4}$ in the simple model considered here). The growth rate γ of the magnetic field can be estimated from the equation $U_0\approx\omega_0^2$ which gives ($\nu_m\to 0$)

$$\gamma\approx(M^2\nu_T-\nu_m)k^2\to M^2\nu_Tk^2\sim M^4\varkappa k^2.$$

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Translated by D. ter Haar