Bremsstrahlung of relativistic particles and dynamic polarizability of target atom

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The bremsstrahlung emitted during the collision of a relativistic charged particle with a complex atom is analyzed. The target atom is treated as a dynamic system having internal degrees of freedom. The basic characteristics of the emission are derived for photon frequencies on the order of the ionization potential of the atom. These characteristics are the spectrum of the emitted photons, the angular distributions of the electromagnetic radiation and of the scattered particles, and the polarization of the radiation. The bremsstrahlung cross section that is differential with respect to the photon energy increases logarithmically with the energy of the incident particle.

1. INTRODUCTION

Until recently, the electromagnetic radiation resulting from the collision of a particle with an atom has been described without consideration of the dynamic structure of the target atom. It has been assumed that the radiation occurs at a static atomic potential. The role played by the dynamics of the atom in bremsstrahlung was first studied in the case of the hydrogen atom.[1] It was later shown that the dynamic structure of the target atom is also important in more complex atoms.[2] A study[3] of the frequency region $\omega \gg I$ (I is the ionization potential of the atom) showed that in this case the bremsstrahlung occurs essentially at the bare nucleus. Extremely simple expressions have been derived[4–6] for the total cross section for the bremsstrahlung emitted in the collision of a particle with an atom, with the structure of the atom taken into account through the introduction of the dynamic polarizability of the atom.

These results, however, apply only at nonrelativistic energies of the incident particle. In the present paper we describe the bremsstrahlung emitted in collisions of a charged particle with an atom at arbitrary relativistic energies .

We find that in this case, too, it is possible to derive some rather simple expressions for the bremsstrahlung cross section at photon frequencies $\omega \gtrsim I$. The frequency region $\omega \gg I$ is studied separately for the ultrarelativistic case, where the situation differs substantially from the nonrelativistic case, because the composite system consisting of the incident electron and the atom cannot be treated in the dipole approximation. We find that the bremsstrahlung cross section increases logarithmically with the energy of the incident particle, which may be an electron, a positron, or a heavy charged particle. This effect is due entirely to allowance for the internal degrees of freedom of the target atom.

2. BREMSSTRAHLUNG AMPLITUDE FOR A RELATIVISTIC ELECTRON

We consider the collision of an electron with an energy $\varepsilon = (p^2 + m^2)^{1/2}$ (we set $\hbar = c = 1$) with an atom in its ground state. The interaction leaves the electron with an energy $\varepsilon_1 = (\mathbf{p}_1^2 + m^2)^{1/2}$ and results in the emission of a photon of frequency ω; the atom remains in its ground state. We assume that the energies of the electron and the photon satisfy the inequalities $\varepsilon, \varepsilon_1 \gg \omega$. We also assume that the Born condition holds for the incident and scattered electrons: $Ze^2/v \ll 1$, where Z is the atomic number of the atom. In this case we can restrict the analysis to first-order perturbation theory in the interaction of the incident electron with the atom and in the interaction of the atomic and incident electrons with the electromagnetic field. The bremsstrahlung amplitude contains two terms: the "electron" bremsstrahlung and the "atomic" bremsstrahlung.[2] The former describes the radiation emitted by the incident electron as it moves through the static field of the atom, while the latter results from the emission of a photon by the atom itself when excited by the incident electron.

We describe the electron before and after the collision by Dirac plane waves, while the motion of the atomic electrons is described by the nonrelativistic wave functions $\Psi_n(\mathbf{r}_1;\ldots;\mathbf{r}_z)$, which are the solutions of the Schrödinger equation for the atom.

The amplitude of the electron bremsstrahlung is well known (Ref. 7, for example) and is given by

$$S^{(\mathrm{el})} = -\frac{(2\pi)^{3/2} e^3}{(\varepsilon\varepsilon_1\omega)^{1/2}} \frac{Z-F(\mathbf{q}_1)}{\mathbf{q}_1^2} \bar{U}_{s_1}(p_1) \left[\hat{e} \frac{(\hat{p}_1+\hat{k})+m}{(p_1+k)^2-m^2} \gamma^0 + \gamma^0 \frac{(\hat{p}-\hat{k})+m}{(p-k)^2-m^2} \hat{e} \right] U_s(p). \quad (1)$$

Here $F(\mathbf{q}_1)$ is the form factor of the atom; $\hat{e} = \boldsymbol{\gamma}\cdot\mathbf{e}$, where $\mathbf{e}$ is the polarization vector of the photon; $U_s(p)$ is the bispinor amplitude; $\hat{a} \equiv a_\mu \gamma^\mu$ ($\mu = 0,1,2,3$); $\mathbf{q}_1 = \mathbf{p} - \mathbf{p}_i - \mathbf{k}$, $\mathbf{k}$ is the photon wave vector; and e is the electron charge.

To derive the amplitude for the atomic bremsstrahlung we assume that the frequency of the photon emitted by the atom is such that the dipole approximation is valid; i.e., we assume $\mathbf{k}\cdot\mathbf{r}_i \sim \omega R_{\mathrm{at}} \ll 1$, where R_{at} is the size of the atom. Choosing the Green's function of the photon in the Coulomb gauge, and assuming that the atom is nonrelativistic, we write the amplitude of the atomic bremsstrahlung in second-order perturbation theory as the sum of three terms:

$$S^{(\mathrm{at})} = S^{(1)} + S^{(2)} + S^{(3)}. \quad (2)$$

Here

$$S^{(1)} = -\frac{(2\pi)^{3/2} e}{(\varepsilon\varepsilon_1\omega)^{1/2}} \sum_n \frac{1}{\omega-\omega_{n0}}$$

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$$\times\left\{\frac{b_0}{\mathbf{q}^2}\langle 0|-\frac{e^2}{m}\sum_{j=1}^{z}(\hat{\mathbf{p}}_{(j)}\mathbf{e})|n\rangle\langle n|\sum_{j=1}^{z}e^{i\mathbf{q}\mathbf{r}_j}|0\rangle\right.$$

$$+\frac{b^i(\delta_{il}-q_iq_l/\mathbf{q}^2)}{\omega^2-\mathbf{q}^2}\langle 0|-\sum_{j=1}^{z}\frac{e}{m}(\hat{\mathbf{p}}_{(j)}\mathbf{e})|n\rangle$$

$$\left.\times\langle n|-\frac{e}{m}\sum_{j=1}^{z}e^{i\mathbf{q}\mathbf{r}_j}\hat{p}_l^{(j)}|0\rangle\right\},$$

$$S^{(2)}=-\frac{(2\pi)^{3/2}e}{(\varepsilon\varepsilon_1\omega)^{1/2}}\sum_n\frac{1}{\omega+\omega_{n0}}\tag{2a}$$

$$\times\left\{\frac{b^0}{\mathbf{q}^2}\langle 0|\sum_{j=1}^{z}e^{i\mathbf{q}\mathbf{r}_j}|n\rangle\langle n|-\frac{e^2}{m}\sum_{j=1}^{z}(\hat{\mathbf{p}}_{(j)}\mathbf{e})|0\rangle\right.$$

$$+\frac{b^i(\delta_{il}-q_iq_l/\mathbf{q}^2)}{\omega^2-\mathbf{q}^2}\langle 0|-\frac{e}{m}\sum_{j=1}^{z}e^{i\mathbf{q}\mathbf{r}_j}\hat{p}_l^{(j)}|n\rangle$$

$$\left.\times\langle n|-\frac{e}{m}\sum_{j=1}^{z}(\hat{\mathbf{p}}_{(j)}\mathbf{e})|0\rangle\right\},\tag{2b}$$

$$S^{(3)}=\frac{(2\pi)^{3/2}e^3}{m(\varepsilon\varepsilon_1\omega)^{1/2}}\left(\mathbf{eb}-\frac{(\mathbf{eq})(\mathbf{qb})}{\mathbf{q}^2}\right)\langle 0|\sum_{j=1}^{z}e^{i\mathbf{q}\mathbf{r}_j}|0\rangle.\tag{2c}$$

In (2a)–(2c), $\mathbf{q}=\mathbf{p}-\mathbf{p}_1$ is the momentum transfer to the atom, the operator $\hat{\mathbf{p}}_{(j)}=-i\partial/\partial\mathbf{r}_{(j)}$ represents the momentum of the jth electron, b^μ is the 4-vector $b^\mu=\overline{U}_{s1}(p_1)\gamma^\mu U_s(p)$, a repeated index means summation, s and s_1 are the spins of the electron before and after the collision, and the summation over n also means integration over states of the continuum.

Calculations of the bremsstrahlung cross section with the help of amplitude (2) lead to exact but complicated results. We can simplify $S^{(at)}$ by expressing it in terms of the dynamic polarizability of the atom. For this purpose we need to determine the range of the momentum transfer $\mathbf{q}$. In the amplitude for the atomic bremsstrahlung $\mathbf{q}$ appears in the matrix elements between the wave functions of the atoms, $\langle 0|e^{i\mathbf{q}\cdot\mathbf{r}_j}|n\rangle$. For values of $\mathbf{q}$ such that $\mathbf{q}\cdot\mathbf{r}_j\sim qR_{at}\gg 1$, these matrix elements contain a rapidly oscillating function and are therefore very small. The minimum value of q is related to the change in the energy of the incident electron, i.e., to the photon frequency ω, by

$$q_{min}=(\varepsilon^2-m^2)^{1/2}-(\varepsilon_1^2-m^2)^{1/2}\approx\omega/v.\tag{3}$$

For values of ω satisfying the condition $\omega\sim\omega_{at}$ (where ω_{at} is a typical atomic frequency), the inequality $q_{min}R_{at}\ll 1$ holds, since $\omega_{at}\sim v_{at}/R_{at}$ and $q_{min}R_{at}\sim v_{at}/v\lesssim v_{at}/c\ll 1$.

There thus exists a region of $q\gtrsim q_{min}$ in which we can legitimately use the expansion

$$e^{i\mathbf{q}\mathbf{r}_j}=1+i\mathbf{q}\mathbf{r}_j.$$

We will show below that, with logarithmic accuracy, this region dominates the cross section for the atomic bremsstrahlung so that large distances between the incident electron and the atom turn out to be important. Expanding the exponential function, we can put amplitude (2) in the form

$$S^{(at)}=\frac{(2\pi)^{3/2}e}{(\omega\varepsilon\varepsilon_1)^{1/2}}\left[b^0\frac{(\mathbf{eq})}{\mathbf{q}^2}+\left(\mathbf{eb}-\frac{(\mathbf{bq})(\mathbf{eq})}{\mathbf{q}^2}\right)\frac{\omega}{\omega^2-\mathbf{q}^2}\right]\omega\alpha_d(\omega),\tag{4}$$

where $\alpha_d(\omega)$ is the dynamic polarizability of the atom. The total bremsstrahlung amplitude is the sum of expressions (1) and (4).

3. CHARACTERISTICS OF THE ATOMIC BREMSSTRAHLUNG

Taking into account the atomic term in the amplitude, we can write the bremsstrahlung cross section as the sum of three terms:

$$d\sigma^{(b)}=d\sigma^{(el)}+d\sigma^{(at)}+d\sigma^{(int)};$$

the electron part of the cross section, $d\sigma^{(el)}$, has been studied thoroughly, so we will focus on the atomic and interference parts of the cross section.

a) Bremsstrahlung spectrum

To derive an expression for the distribution of photons in frequency ω, we first note that the interference term, $d\sigma^{(int)}$, is negligibly small in comparison with the two other terms. The physical reason for this small value is that the atomic bremsstrahlung is dominated by the region of large distances between the incident electron and the atom ($R\gg R_{at}$), while the electron bremsstrahlung is instead dominated by the short distances. The interference term can therefore be ignored in the total cross section.

According to Ref. 7, the part of the cross section due to the atomic bremsstrahlung is

$$\frac{d\sigma^{(at)}}{d\omega}=\frac{q\omega^2}{2v^2}\sum_{s;s_1;\sigma}|S^{(at)}|^2\frac{d\varphi_{\mathbf{p}_1}d\Omega_{\mathbf{k}}}{(2\pi)^5}d\mathbf{q},\tag{5}$$

where $d\Omega_{\mathbf{k}}$ is the solid angle into which the photon is emitted, and $\varphi_{\mathbf{p}1}$ is the azimuthal angle of $\mathbf{p}_1$.

We substitute expression (4) into (5), sum over the polarizations of the photon and over the spins of the electron in the initial and final states, integrate over angles and over the momentum transfer between $q=q_{min}$ and some $q_{max}\sim 1/R_{at}$. Because of the uncertainty in the upper limit on q, we write the result with logarithmic accuracy:

$$\frac{d\sigma^{at}}{d\omega}=\frac{16}{3}e^2\frac{\omega^3}{v^2}|\alpha_d(\omega)|^2\ln\frac{v\varepsilon}{m\omega R_{at}}.\tag{6}$$

We have discarded terms on the order of $\omega/\varepsilon\ll 1$ from (6). It is the integration over the region $q\sim q_{min}$ which is responsible for the logarithm of the large quantity $v\varepsilon/m\omega R_{at}\gg 1$ which appears here. For frequencies $\omega\gg I$ and in the Thomas-Fermi approximation for R_{at}, expression (6) becomes expression (2a) of Ref. 8.

Expression (6) is an important result of this study. It generalizes a result of the nonrelativistic theory: If we set $\varepsilon=m$ in (6), we find the expression derived in Refs. 4–6. Cross section (6) increases logarithmically with increasing energy of the incident particle. At high velocities of the incident particle, $v\lesssim 1$, the electron part of the bremsstrahlung is essentially independent of ε. The total cross section, $d\sigma^{(b)}/d\omega$, will therefore increase logarithmically with ε.

We derived cross section (6) in the logarithmic approximation. For this derivation it is sufficient to retain the terms which are linear in $\mathbf{q}$ in the expansion of the exponential function $e^{i\mathbf{q}\cdot\mathbf{r}_j}$. The corrections to the cross section $d\sigma^{(at)}/d\omega$ which come from terms containing higher powers of the expansion can be calculated for specific atoms, so that the arbitrariness resulting from the introduction of R_{at} can be eliminated. Here it is necessary to numerically calculate the matrix elements in (2) between wave functions of the atom.

The use of the Coulomb gauge makes it possible to explicitly determine the contributions to cross section (6) from the scalar (Coulomb) interaction between the incident electron and the atom and from the vector interaction. The logarithm in (6) would accordingly be written in the form

$$\ln (v\varepsilon/m\omega R_{at}) = \ln(\varepsilon/m) + \ln(v/\omega R_{at}).$$

The first term is the contribution of the transverse (vector) part of the electromagnetic interaction of the atomic electrons and the fast incident electron, while the second term is the contribution of the longitudinal (Coulomb) part of this interaction. With increasing energy, the first term becomes much larger than the second.

b) Angular distribution of the electromagnetic radiation

Integrating over the angular variables of the scattered electron in (5), we find

$$\frac{d\sigma^{at}}{d\omega\, d\Omega_{\mathbf{k}}} = \frac{e^2\omega^3}{\pi v^2} |\alpha_d(\omega)|^2 (1+\cos^2\theta) \ln \frac{v\varepsilon}{m\omega R_{at}}. \quad (7)$$

Here θ is the angle between the initial electron momentum and the photon emission direction. In deriving (7) we discarded terms of order ω/ε.

The θ dependence in (7) is the same as the angular distribution of the radiation from a rotating dipole. This distribution differs substantially from that in the case in which the incident electron radiates; this radiation is primarily into a cone whose axis is along $\mathbf{p}$ and whose vertex angle decreases rapidly with increasing electron velocity. In the limit $v \to 1$, all the electron bremsstrahlung occurs in an angular interval m/ε. The atomic bremsstrahlung in contrast, which is determined by the dynamic polarization of the target atom, is distributed in a generally isotropic way over all directions, and its shape $(1+\cos^2\theta)$ is independent of the electron velocity, with logarithmic accuracy. Incorporating the nonlogarithmic terms in (7) makes the coefficients of $\cos^2\theta$ and 1 functions of the energy ε. The complete angular distribution of the radiation can thus be characterized by a smooth angular dependence at $\theta > m/\varepsilon$ and a sharp anisotropy at $\theta \lesssim m/\varepsilon$. A similar photon angular distribution was first found by Hubbard and Ros[9] for the case of the bremsstrahlung of an electron colliding with an atomic nucleus having a dynamic polarization.

c) Angular distribution of the scattered electrons

As the velocity of the incident electron increases, a structural feature appears in the angular distribution of scattered electrons. We begin by writing the differential cross section for the atomic bremsstrahlung as a function of the momentum transfer $q = |\mathbf{q}|$:

$$\frac{d\sigma^{(at)}}{d\omega\, dq} = \frac{16}{3} e^2 \frac{\omega^3}{v^2} |\alpha_d(\omega)|^2 \frac{q}{q^2-\omega^2}. \quad (8)$$

The denominator $(q^2-\omega^2)$ here leads to a definite shape of the angular distribution of the scattered electrons. As we have already noted, the minimum value of q is ω/v, so that at $v \lesssim 1$ the electrons will be scattered through extremely small angles. The contribution of the small-angle region to the total cross section increases with increasing ε. It is in this region that $d\sigma^{(at)}$ is especially large in comparison with $d\sigma^{(el)}$. An increase in the role of atomic bremsstrahlung at relativistic velocities of the incident electron was noted by Amus'ya.[6]

A limit is imposed on the growth of the bremsstrahlung cross section in (8) at extremely small scattering angles by the inequality $v < 1$ (which always holds) and thus the inequality $q_{min} > \omega$.

d) Polarization of the atomic bremsstrahlung

To find the angular distribution of the polarized photons in the scattering of unpolarized electrons by an atomic system, we choose the vectors $\mathbf{e}^{(1)} = [\mathbf{p}_1 \times \mathbf{k}]/|[\mathbf{p}\times\mathbf{k}]|$ and $\mathbf{e}^{(2)} = [\mathbf{k}/\omega, \mathbf{e}^{(1)}]$ as polarization unit vectors. We then find

$$\frac{d\sigma^{at}}{d\omega\, d\Omega_{\mathbf{k}}}(\xi_i) = \frac{e^2\omega^3}{2\pi v^2} |\alpha_d(\omega)|^2 \ln\left(\frac{\varepsilon v}{m\omega R_{at}}\right) [1+\cos^2\theta + \bar{\xi}_3 \sin^2\theta], \quad (9)$$

where $\bar{\xi}_3$ is the Stokes parameter averaged over the electron emission angle. The quantity $\bar{\xi}_3$ is a measure of the degree of linear polarization of the photon along the vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$: The polarization probability along $\mathbf{e}^{(1)}$ is $(1/2)(1+\bar{\xi}_3)$, while that along $\mathbf{e}^{(2)}$ is $(1/2)(1-\bar{\xi}_3)$. Consequently, the radiation which is independent of the polarization of the incident electron turns out to be linearly polarized. The electron bremsstrahlung has the same property (see Ref. 7, for example).

4. TOTAL BREMSSTRAHLUNG CROSS SECTION

Let us examine the complete spectrum of bremsstrahlung photons:

$$d\sigma^{(b)}/d\omega = d\sigma^{(at)}/d\omega + d\sigma^{(el)}/d\omega.$$

Since the expression $d\sigma^{el)}/d\omega$ for arbitrary electron velocities is quite complicated, we consider only the ultrarelativistic limit. We show that in this limit the cross section $d\sigma^{(b)}/d\omega$ is fundamentally different from that in the nonrelativistic case.

To find the expression for $d\sigma^{(at)}/d\omega$ in the ultrarelativistic limit, we set $v=1$ in (6):

$$\frac{d\sigma^{(at)}}{d\omega} = \frac{16}{3} e^2\omega^3 |\alpha_d(\omega)|^2 \ln\frac{\varepsilon}{m\omega R_{at}}. \quad (10)$$

The corresponding expression for the electron component has been found previously (see Ref. 7, for example):

$$\frac{d\sigma^{(el)}}{d\omega}=\frac{16}{3}Z^2e^2\frac{r_e^2}{\omega}\ln(mR_{at}). \quad (11)$$

The logarithm $\ln(mR_{at})$ appears here because we are taking the screening of the nucleus by the atomic electrons into account; r_e is the classical radius of the electron. Terms of order ω/ε have been discarded from (10) and (11).

We now consider frequencies ω which are large in comparison with atomic frequencies. The dynamic polarizability of an atom of arbitrary atomic number Z is $\alpha_d(\omega)\simeq -Z/m\omega^2$ in this case. Substituing $\alpha_d(\omega)$ into (10), and combining (10) and (11), we find the following expression for the total bremsstrahlung spectrum at $\omega\gg\omega_{at}$:

$$\frac{d\sigma^{b}}{d\omega}=\frac{16}{3}Z^2e^2\frac{r_e^2}{\omega}\ln\frac{\varepsilon}{\omega}. \quad (12)$$

This expression differs only by a factor of Z^2 from the cross section for the bremsstrahlung of a recoil electron in the scattering of an ultrarelativistic electron by a slow free electron (see Ref. 7, for example). This agreement is not by chance. At $\omega\gg\omega_{at}$ the atomic electrons can be regarded as free. Their velocities are negligibly small in comparison with that of the incident electron. In this case the amplitude of the bremsstrahlung which arises in the collision of an ultrarelativistic electron with an atom consists of three components:

$$S^{(b)}=ZS_1+ZS_2+S_3,$$

where S_1 is the emission of a photon by the incident electron as it interacts with the atomic electron, S_2 is the emission by the atomic electron as it interacts with the incident electron, and S_3 is the emission by the incident electron as it interacts with the nucleus.

In the ultrarelativistic case the amplitude of the bremsstrahlung of a fast electron colliding with a free particle at rest does not depend on the mass of this particle; it is determined exclusively by its charge. Accordingly, of the three terms above the sum ZS_1+S_3 gives us zero and $S^{(b)}$ reduces to ZS_2. This result means that the total bremsstrahlung cross section in the ultrarelativistic case is due exclusively to the emission by atomic electrons.

For an ultrarelativistic electron, therefore, the result is in a sense the opposite of the result of the nonrelativistic case, where the bremsstrahlung spectrum at $\omega\gg\omega_{at}$ is described by

$$\frac{d\sigma^{(b)}}{d\omega}=\frac{16}{3}Z^2e^2\frac{r_e^2}{\omega v^2}\ln\frac{2mv^2}{\omega}, \quad (13)$$

i.e., is the same as the cross section for the bremsstrahlung emitted in a collision with a Coulomb center Ze (Ref. 3).

Some qualitative arguments show the reason for this difference. In the nonrelativistic limit, distances between the incident electron and the atom on the order of $R\sim 1/q_{min}$ are important for the atomic bremsstrahlung. In turn, we have $q_{min}\approx\omega/v$. The effective distances for the atomic bremsstrahlung are therefore much smaller than the wavelength of the emitted photon: $R\sim v/\omega\sim v/\lambda\ll\lambda$. For the composite system consisting of the incident electron and the atom, we can therefore use the dipole approximation. However, two free electrons cannot emit a dipole photon. Accordingly, at $\omega\gg\omega_{at}$ we are left with only the emission from the nucleus. In the ultrarelativistic case the effective distances are $R\sim 1/q_{min\perp}$, where $q_{min\perp}\sim\omega/v\gamma$ is the component of the vector $\mathbf{q}$ which is transverse with respect to $\mathbf{p}$, and $\gamma=(1-v^2)^{-1/2}$. Distances $R\sim\gamma v/\omega$ are much greater than the photon wavelength ($R\gg\lambda\sim 1/\omega$), so that we cannot restrict the analysis to the dipole radiation of an electron colliding with an electron. It is this circumstance which causes (12) to differ from its nonrelativistic analog, (13).

Returning to expressions (10) and (11), we note that the atomic bremsstrahlung is determined entirely by a dynamic characteristic of the atom, $\alpha_d(\omega)$, and does not depend on the mass of the incident particle. In contrast, $d\sigma^{(el)}$ is inversely proportional to the square of the mass M. For particles of large mass we can therefore completely ignore the bremsstrahlung in the static atomic potential, and we can observe the atomic bremsstrahlung in its pure form. By selecting atoms with a large dynamic polarizability we can therefore increase $d\sigma^{(at)}$ substantially.

To find the amplitude of the bremsstrahlung emitted in the collision of a relativistic positron with an atom, we reverse the sign of expression (4) (since the charge of the positron is opposite that of the electron) and add the result to expression (1). Expressions (6)–(10) and (12) for the cross sections for the atomic bremsstrahlung thus also apply to the case of a relativistic positron.

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