Experimental verification of the law of gravity for laboratory distances

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The theory of extended supergravity predicts the existence of not only gravitons but also vector and scalar bosons that lead in the static limit to the appearance of additional terms in the expression for the gravitational potential. This effect can be interpreted as a dependence of an effective gravitational constant G on the distance between interacting masses. A different possibility of a dependence $G = G(r)$ in the form $G(r) = G_0[1 + \lambda \ln(r/1\ \text{cm})]$ derives from a proposed mechanism of polarization of the gravitational vacuum. The law of gravity has been tested in the range of distances 11–21 cm using data obtained in an experiment made to determine the absolute value of G at the P. K. Shternberg State Astronomical Institute of the Moscow State University. The result, $\lambda = (9 \pm 10)10^{-5}$, does not support a dependence $G = G(r)$. The possible values of some parameters of supergravity theories are estimated.

1. INTRODUCTION

Attempts to quantize the general theory of relativity in the form of a gauge theory with gravitons ($s = 2$) as gauge bosons come up against serious difficulties. One of them is the appearance of a large number of divergent integrals, which cannot be renormalized. The introduction of supersymmetry between fermions and bosons leads in a natural manner to supergravity as the best candidate for a renormalizable quantum theory of gravity. In the simplest realization of supergravity,[1] the graviton is associated in a multiplet with a new massless particle of half-integer spin ($s = 3/2$), called the gravitino. It was shown that the introduction of the gravitino eliminates the divergences, at least at the one-loop level.

To achieve a unification of gravity with the other forms of interaction, it is necessary to extend supersymmetry in such a way as to include particles with spin less than 3/2. Thus, attempts to create a unified field theory lead to the idea of extended supergravity, in which unification is achieved in the sense that all particles are related to one another by a supersymmetry transformation. Thus, the gravitational multiplet of the $N = 2$ extended supergravity model consists of a graviton ($s = 2$), two gravitinos ($s = 3/2$), and a vector particle ($s = 1$), which has been called a graviphoton.[1] The most recent and apparently most realistic supergravity model is the $N = 8$ model with spontaneous symmetry breaking. The particle spectrum of this model contains the graviton, eight gravitinos, 28 particles with spin 1, 56 particles with spin $\frac{1}{2}$, and 70 particles with spin 0.[2,3]

The presence in supergravity theories of new particles responsible for the gravitational interaction (the graviphoton, scalar bosons of Brans-Dicke type) leads to new phenomena predicted by these theories such as violation of the equivalence principle and deviation of the gravitational potential from the Newtonian form. For example, the graviphoton, a vector boson present in many models of extended supergravity (including the cases $N = 2$ and $N = 8$), corresponds to an intermediate interaction, in which two particles repel each other while a particle and antiparticle attract each other. In the static limit, this leads to antigravity, i.e., to mutual cancellation of the attractive and repulsive forces.[3]

If gravitation is due to the exchange of not only a graviton but also other particles with Compton wavelengths r_i^0 and dimensionless coupling constants α_i (which characterize the strength of the given interaction relative to the graviton interaction), then in the static limit the potential produced by a mass M will have besides the usual Newtonian part additional components of Yukawa type with finite range of order r_i^0, namely,

$$V(r) = -G_\infty \frac{M}{r}\left[1 + \sum_i \alpha_i \exp\left(-\frac{r}{r_i^0}\right)\right], \qquad (1)$$

where G_∞ is the gravitational constant at very large distances $r \gg \max r_i^0$.

Negative values of α_i correspond to repulsion due to exchange of graviphotons, and positive α_i correspond to attraction due to the exchange of scalar particles. We note that in extended supergravity the masses of both particles—the vector boson and the scalar—are zero. Nevertheless, these particles can acquire masses through the mechanism of spontaneous symmetry breaking.[3] Some variants of the scalar-tensor theory of gravitation also give an analogous form of the gravitational potential.[4,5]

It is natural to consider which of the new theories are in agreement with experiment. It is expected[6] that the typical supergravity effects will be manifested only at energies of the order of the Planck energy ($E_P \approx 10^{19}$ GeV). In modern accelerators, gravitational interaction of two particles can be realized at only a much lower energy level, and therefore macroscopic gravitational experiments are, at least at the present time, the most direct verification of supergravity theories. If virtual particles with mass $m_i < 10^{-4}$ eV are exchanged, the characteristic range is $\hbar/m_i c > 1$ cm, and in principle this can be observed in a laboratory. One such experiment permitting a testing of the consequences of supergravity in the static limit is a Cavendish type experiment for determining the gravitational constant G. The effective gravitational constant corresponding to a potential in the form

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(1) is

$$G(r)=G_\infty\left[1+\sum_i \alpha_i\left(1+\frac{r}{r_i^0}\right)\exp\left(-\frac{r}{r_i^0}\right)\right]. \quad (2)$$

Thus, G becomes a function of the distance between the interacting masses.

There is also another approach that also leads to a functional dependence $G = G(r)$. It is well known that in quantum electrodynamics there is a vacuum polarization effect; at short distances $r \ll \hbar/m_e c$, this adds to Coulomb's law an additional logarithmic term.[7] Long suggested[8] that the mechanism of polarization of the gravitational field vacuum consists of changing the "vacuum mass" density near some localized mass source, and that it leads to an effective gravitational constant having at laboratory distances an analogous additional logarithmic term

$$G(r)=G_0[1+\lambda\ln(r/r_0)], \quad (3)$$

where $G_0 = G(r \to 0)$, and $r_0 \approx 1$ cm. In the gravitational field, in contrast to the Coulomb field, $\lambda > 0$ must hold.

In his earlier experimental paper,[9] Long reported a violation of Newton's law at the laboratory scale. His result was $\lambda = (2 \pm 0.4)\cdot 10^{-3}$ for distances $r_1 = 4.5$ and $r_2 = 30$ cm. (It should be noted that a direct quantum-gravitation calculation of the effects due to vacuum polarization does not lead to a behavior of G in the form (3); in addition, such effects are extremely small.[10])

At the P. K. Shternberg State Astronomical Institute of the Moscow State University, an experiment was made some years ago to determine the absolute value of the gravitational constant G. The value of G found in the experiment is[11]

$$G=(6.6745\pm0.0008)\,10^{-8}\ \text{cm}^3/\text{g}\cdot\text{sec}^{-2}.$$

In the present paper, we analyze the data obtained in this experiment in order to see if there is any dependence of G on the distance between interacting bodies.

2. METHOD AND MAIN PARAMETERS OF THE EXPERIMENT

In principle, the experiment at the Shternberg Institute repeated the method of Heil and Chrzanowski for determining G by the torsion-balance method in the dynamic regime.[12] Assuming the hypothetical dependence $G = G(r)$ (3), the moment of the forces of the gravitational attraction between test masses m fixed at the ends of the horizontal balance arm and the source masses M can be written in the form

$$P=G_0\hat{P}+\lambda G_0\int \ln r\, d\hat{P}, \quad (4)$$

where $G_0\hat{P}$ is the usual Newtonian moment of the gravitational forces, r is the distance between the elements of the interacting masses, and $d\hat{P}$ is the "infinitesimal" moment accurate to the gravitational constant factor; the integration is over the volumes of the test and source masses.

The square of the frequency of the torsional vibrations of the balance in the gravitational field of the two source masses is determined in this case by the expression

$$\omega^2=\frac{1}{J}\left\{D+G_0\left[\frac{\partial}{\partial\varphi}\hat{P}+\lambda\frac{\partial}{\partial\varphi}\int\ln r\, d\hat{P}\right]_{\varphi=0}\right\}, \quad (5)$$

where D is the torsional rigidity of the balance, J is the moment of inertia about the suspension wire, and φ is the angle of the deviation of the torsion balance from the equilibrium position. Measuring the frequency of the torsional vibrations for two different positions of the source masses M, we can calculate the coefficient λ from the expression

$$\lambda=\frac{J(\omega_1^2-\omega_2^2)/G_0-(\mathfrak{M}_1^0-\mathfrak{M}_2^0)}{\mathfrak{M}_1(\ln r)-\mathfrak{M}_2(\ln r)}, \quad (6)$$

where

$$\mathfrak{M}_i^0=\left(\frac{\partial}{\partial\varphi}\hat{P}_i\right)_{\varphi=0},\quad \mathfrak{M}_i(\ln r)=\left(\frac{\partial}{\partial\varphi}\int\ln r\, d\hat{P}_i\right)_{\varphi=0},$$

and $i = 1, 2$, respectively, for the first and second positions of the source masses.

We give the main parameters of the experiment at the Shternberg Institute (Fig. 1). A more detailed description can be found in Refs. 11 and 13–15. The period of the characteristic torsional vibrations of the balance was $T_0 \approx 2310$ sec, the relaxation time $\tau^* \approx 10^6$ sec. The balance arm of length $2R \approx 35.5$ cm with cylindrical test masses at the ends was suspended by a tungsten wire of diameter 10^{-4} cm and length 100 cm. The copper masses were $m_1 = 29.899\,909 \pm 0.00\,003$ g and $m_2 = 29.94\,050 \pm 0.00\,003$ g. The torsion balance was placed in a copper chamber at pressure $p \approx 5\times 10^{-5}$ Torr. The source masses, $M_1 = 39.72\,791 \pm 0.00\,002$ kg and $M_2 = 39.79\,718 \pm 0.00\,002$ kg, were made of nonmagnetic steel in the form of right circular cylinders of diameter 18 cm and height 20 cm.

The vibrations of the balance were observed by means of two independent systems,[15] the main element of each of them being an indicator of the angular displacements based on the principle of an optical lever. The resolution of the indicators for quasistatic displacements was not worse than

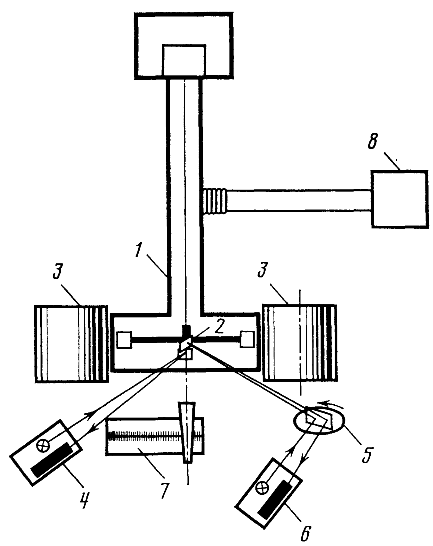

FIG. 1. Schematic arrangement of the experiment at the P. K. Shternberg State Astronomical Institute: 1) housing of the instrument, 2) torsion balance, 3) source masses, 4) and 6) indicators of small angular displacements, 5) rotating mirror, 7) device for linear measurements, 8) vacuum system.

TABLE I.

Source of error	Relative error of determination	Error in the value of the coefficient λ
Gravitational constant	$1{,}5\cdot 10^{-4}$	$6{,}2\cdot 10^{-5}$
Vibration frequency	$2\cdot 10^{-6}$	$5\cdot 10^{-5}$
Moment of inertia	$5\cdot 10^{-6}$	$3\cdot 10^{-6}$
Derivative of moment of the gravitational forces	$1\cdot 10^{-5}$	$2{,}7\cdot 10^{-5}$
Logarithmic term in the derivative of the moment of the gravitational forces	$1\cdot 10^{-3}$	$1\cdot 10^{-5}$

10^{-8} rad. The first detection system measured the period of the torsional vibrations as the interval of time between passages through the equilibrium position. The second detection system (a ranging system) contained a scanning mirror and made it possible to measure in digital form the instantaneous values of the angular coordinate of the oscillating balance arm with interval 20 sec.

By means of a special platform,[16] it was possible to place the source masses successively in four fixed positions. The masses were transferred from one position to another by remote control. The accuracy of the linear measurements was 2×10^{-4} cm.

To increase the interference immunity, the instrument was mounted on an antiseismic platform and placed in a passive thermostat. The copper housing of the instrument was grounded. Special measures were taken to improve the uniformity of the density and antimagnetic properties of the source masses. An experimental study was made of the effect on the stability of the operation of the torsion balance of microseismic disturbances, inhomogeneities of the gravitational field, and variation in the temperature. Helmholtz coils were used in special experiments to investigate the effect of a permanent magnetic field of the torsion balance. At the level of accuracy achieved in the measurement of the frequency ($\Delta\omega/\omega \approx 10^{-6}$), no such influence was observed.[13,16]

3. RESULTS OF MEASUREMENTS

The experiment to determine the gravitational constant consisted of a series of observations in each of which the frequencies of the torsional vibrations for four positions of the source masses and the distances between the torsion balance and the source masses were measured. On the basis of these measurements the coefficients $\mathfrak{M}_i^0$ were calculated by a sixfold numerical integration over the volumes of the interacting bodies. For the purposes of the present study, the logarithmic terms in the derivative of the moment of the gravitational forces $\mathfrak{M}_i(\ln r)$ (6) were also calculated.

On the basis of the errors in the determination of the quantities that occur in (6), one can estimate the expected accuracy in the determination of the coefficient λ. The data are given in Table I. The last column contains the errors in the value of the coefficient λ due to each source of error. The confidence interval for the coefficient λ at the level of one standard deviation, obtained as the square root of the sum of the squares of all the terms of the last column, is 9×10^{-5}. Therefore, it is to be expected that the experimental data permit the determination of the coefficient λ at this level of accuracy.

Each individual observation, including measurements of the frequencies of the torsional vibrations and the distances for four different positions of the source masses (the distances between the centers of the nearest interacting masses were, respectively, $r_1 = 11.25$ cm, $r_2 = 13.25$ cm, $r_3 = 16.25$ cm, $r_4 = 21.25$ cm), gives three independent values of λ. The results of the series of ten observations are given in Table II. Column I of the λ values was obtained for the interacting masses in positions r_1 and r_2; column II, for r_1 and r_3; column III, for r_1 and r_4.

Statistical analysis shows that all the experimental data constitute a sample belonging to one general set with a certain mean value λ. Thus, the estimate of the mean value of the coefficient λ at the 1σ level obtained in the experiment is $\lambda = (9 \pm 10)\cdot 10^{-5}$ in the range of distances $r \approx 11$–21 cm.

The ratio of the gravitational constants G corresponding to this result for masses interacting at the distances $r_1 = 11$ and $r_2 = 21$ cm is $G(r_2)/G(r_1) = 1.00062 \pm 0.00069$.

4. DISCUSSION OF THE RESULTS

Long's work[9] was followed by a series of experiments to test the Newtonian law of gravity in the range $r = (2–1)\times 10^4$ cm.[18–21] They all failed to confirm the result of Ref. 9. However, according to Long's estimate,[22] the accuracy of these experiments, except for that of Spero *et al.*,[19] was such as not to permit detection of the effect observed by Long (Table III). Thus, the only result that contradicts Long's is that of

TABLE II.

Number of observation	Experimental values $\lambda\cdot 10^4$			Number of observation	Experimental values $\lambda\cdot 10^4$		
	I	II	III		I	II	III
1	0.669	0.233	−5,270	6	4,816	3.891	0.540
2	7.730	−1,613	−3.693	7	−4,601	−0,149	−3.813
3	14,674	7.536	0,272	8	12,093	4,852	−1.496
4	4,282	−2.701	−6,930	9	10,020	−2,489	0.952
5	−2,184	−4,371	−7.474	10	4,690	5,407	−5,940

TABLE III.

Authors	Range of distances, cm	$(\lambda\pm\Delta\lambda)10^5$	Authors	Range of distances, cm	$(\lambda\pm\Delta\lambda)10^3$
Long[9]	4,5–30	200±40	Hirakawa *et al.*[20]	220–420	0±5300
Panov, Frontov[18]	40–300	150±300	Hirakawa *et al.*[21]	260–1070	−210±620
	40–1000	−60±410	Shternberg Institute	11–21	9±10
Spero *et al.*[19]	2–5	1±7			

Ref. 19. Nevertheless, if the mechanism of polarization of the gravitational vacuum proposed by Long is correct, then in a Null experiment of such Spero's this effect will not be observed. The point is that in null experiments one uses as a source mass either hollow spheres or long hollow cylinders. Within such bodies, the gravitational field is zero, so that according to Long there cannot be a change in the "vacuum mass" density, and a test body within the mass will not reveal the expected anomaly.[8]

In this connection, the result of the experiment at the Shternberg Institute, which was not a null experiment and in principle permits detection of the vacuum polarization effect, warrants attention. The estimate of our data shows that to within the accuracy of the experiment (which exceeds that of Long's by four times) there is no $G(r)$ dependence in the range of distances $r = 11$–21 cm.

Let us discuss the obtained experimental data in the framework of the hypothetical dependence (1) deduced from supergravity theories. Using the data given in Table III, we construct in the plane of the parameters (α_i, r_i^0) the region that precludes the dependence (1) at the confidence level 2σ. This region is shown in Fig. 2 together with the set of points of the possible combination (α_i, r_i^0) determined by Long's result.

The Brans-Dicke scalar boson present in the $N = 8$ extended supergravity model interacts in the static limit with matter with a force proportional to the mass, like the graviton, and therefore we have an expected $\alpha_\sigma \approx 1$ (Ref. 3). (Some scalar-tensor theories of gravity predict $\alpha_\sigma = 1/3$ for r_σ^0 satisfying either $10 < r_\sigma^0 < 10^3$ m or $r_\sigma^0 \lesssim 1$ cm.[4]) The result of

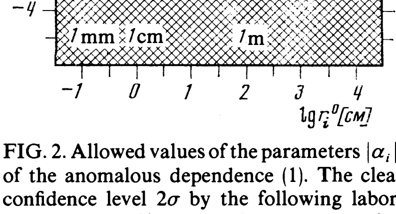

FIG. 2. Allowed values of the parameters $|\alpha_i|$ and r_i^0 under the assumption of the anomalous dependence (1). The clear region is ruled out at the confidence level 2σ by the following laboratory experiments: Ref. 19 (curve 2), P. K. Shternberg State Astronomical Institute (curve 3), Refs. 18 and 21 (curve 4). The upper hatched region (curves 1) represents the boundaries at the 2σ level for values of $|\alpha_i|$ and r_i^0 agreeing with Long's experiment.[9]

the Shternberg experiment rules out values $\alpha_\sigma \approx 1$ for interval 1 cm $< r_\sigma^0 < 10$ m. The complete set of experimental data of Fig. 2 extends this interval to 0.15 cm $< r_\sigma^0 < 50$ m. Taking into account some indirect astrophysical estimates,[23] it appears that one can assert that values $\alpha_\sigma \approx 1$ are possible only for $r_\sigma^0 \lesssim 0.15$ cm, which corresponds to a mass $m_\sigma \gtrsim 10^{-3}$ eV of the scalar boson.

In the $N = 2$ and $N = 8$ supergravity models, the force due to exchange of a graviphoton and leading to the phenomenon of antigravity is proportional in the static limit of the masses of the interacting particles if they can have different charge states (quarks, leptons) and is equal to zero for truly neutral particles (photons, gluons). In the gravitational interaction of composite particles such as nucleons, the graviphoton "sees" the quarks and "does not see" the gluons. Therefore, the coupling constant in the given case will be proportional to $-(3m_u/M_p)^2$, where m_u is the quark mass and M_p the proton mass. Taking $m_u \approx 10$ MeV, $M_p \approx 1$ GeV, we obtain the estimate $\alpha_\varphi \approx -10^{-3}$ (Ref. 3).

This method of combining the graviphoton with matter naturally leads to a violation of the equivalence principle. Now this has been tested experimentally[24] at the level $\sim 10^{-12}$, which gives an upper limit for the antigravity region: $r_\varphi^0 < 2.5$ m.[3] Further restrictions can be obtained from Fig. 2. The Shternberg experiment rules out values $|\alpha_\varphi| \approx 10^{-3}$ in the range $3 < r_\varphi^0 < 20$ cm. Together with the result of Ref. 19 and the estimate obtained from the equivalence principle, this gives for the possible values of r_φ^0 either $r_\varphi^0 < 1$ cm($m_\varphi > 10^{-4}$ eV) or 20 cm $< r_\varphi^0 < 2.5$ m ($10^{-6} < m_\varphi < 10^{-5}$ eV). It should be noted that if the quark mass m_u is somewhat lower than given above, then $|\alpha_\varphi| \lesssim 10^{-4}$, which is no longer ruled out by the laboratory experiments.

To strengthen the bounds on the new effects that follow from supergravity we require experiments to test the law of gravity in both the range 10–100 m and $r < 1$ cm. A raising of the accuracy of such experiments in the range 1 cm–10 m and of the accuracy of the experimental verification of the equivalence principle can also give new material for these purposes.

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