# Incommensurate structures with vortical configurations in dipole systems

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The possibility of formation of incommensurate vortex structures in systems with dipole interaction between the spins is considered. It is shown that the eigenvalue spectrum of the dipole-tensor Fourier components of compounds with hexagonal structure has a minimum inside the Brillouin zone; the wave vector corresponding to such a minimum is not symmetry-fixed. The incommensurate structure produced as a result at intermediate temperatures is therefore vortical because of the transverse polarization of the dipole-tensor eigenvector. The vortical structure becomes deformed in an external field and goes over into a sinusoidal (more accurately, a fan-shaped) structure at a certain critical value. The effect of volume forces as well as of the hexagonal anisotropy on the structure of the vortical configurations is considered.

## 1. INTRODUCTION

There exist by now a number of magnetic materials in which the periods of the magnetic structures are not commensurate with the crystal-lattice periods. The causes of the formation of incommensurate structures vary. In crystals without an inversion center, a phase transition between the commensurate and incommensurate phases is due to the possible presence, in the free energy, of the isotropic-interaction terms described by the Lifshitz invariant. The mechanism of such a transition with onset of helical structures (spin-density waves) was first considered by Dzyaloshinskiĭ.[1]

Recent experiments with antiferromagnets[21,3] have revealed the existence of structures of intermediate temperature and having an incommensurate magnetic phase. Shiba[4] (see also Ref. 5) has shown that these experimental data can be readily explained in terms of a conical-point instability[6] due to weak dipole-dipole interaction. It follows in particular from his calculations[4] that the temperature interval in which the incommensurate phase exists depends on the dipole interaction of the spins; this agrees well with the observed values.

At the same time there are many known substances in which the exchange interaction is substantially weakened. In these substances the dipole interaction predominates in the formation of the magnetic structure. Experimental investigations, more than a decade old, of the thermal and magnetic properties of certain dipole systems, namely rare-earth compounds (the exchange interaction is weakened by screening of the $4f$ electrons by the outer $5s$ and $5p$ electrons), have shown that the critical point at which ordering from the paraphase takes place lies somewhat higher than the point of transition into a state with a commensurate magnetic structure. It was indicated in this connection in Ref. 8 that for certain crystal lattices even pure dipole interaction can lead to a divergence in the susceptibility if the wave vectors do not correspond to the onset of some commensurate (ferro- or antiferromagnetic) structure, since the dipole interaction can be of either sign, depending on the direction. At the same time, a study of the ground and metastable states with a simple cubic lattice as the example has shown that dipole forces stimulate the onset of structures with vortical character.[9]

The present study deals with the possibilities of formation of intermediate-temperature vortical incommensurate structures due mainly to dipole-dipole interaction:

$$\mathscr{H}=-\frac{1}{2}\sum_{i,j;\alpha,\beta} D_{ij}^{\alpha\beta}S_i^\alpha S_j^\beta,\quad D_{ij}^{\alpha\beta}=(g\mu_B S)^2\left(\frac{\delta_{\alpha\beta}}{r_{ij}^3}-3\frac{r_{ij}^\alpha r_{ij}^\beta}{r_{ij}^5}\right),\quad (1)$$

where $r_{ij}$ is the distance between the spins $\mathbf{S}_i$ and $\mathbf{S}_j$ which are normalized to unity. We consider below compounds with hexagonal lattice, such as many salts of rare-earth elements.[10,11] Such lattices contain modulated structures because the Brillouin zone contains a "random" minimum and because the eigenvalues of the dipole tensor depend on the wave vector. On the other hand, the character of the polarization of the eigenvectors (of the mean values of the spins) at the minimum points leads to a vortical configuration of these structures. We examine therefore the possibility of the onset of intermediate phases if a magnetic field is present in the dipole systems. Finally, we consider the influence of weak exchange interaction, as well as of hexagonal anisotropy, on the structure of the vortical configurations.

## 2. DIPOLE TENSOR

To find the ordered states that can be produced from an unstable paramagnetic state it is necessary to know the eigenvalues and the eigenvectors (see the next section) of the Fourier components of the dipole tensor

$$D_{\alpha\beta}(\mathbf{q})=\sum_{i-j} D_{ij}^{\alpha\beta}\exp(i\mathbf{q}\mathbf{r}_{ij}).$$

Using Ewald's method (see, e.g., Ref. 12) one can express the elements of the tensor $D_{\alpha\beta}(\mathbf{q})$ in the form of rapidly converging sums.

A symmetric dipole tensor in $q$-space has three eigenvalues $\lambda(\mathbf{q})$ determined from the condition for vanishing of the determinant, viz.,

$$\det(D_{\alpha\beta}(\mathbf{q})-\lambda\delta_{\alpha\beta})=0. \quad (2)$$

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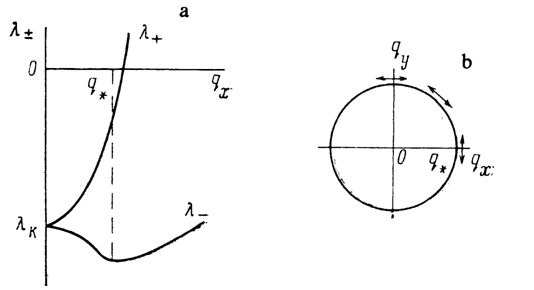


FIG. 1. a) Dependence of the eigenvalues $\lambda_\pm$ on $qx$ at $q_y = 0$ and $q_z = \pi/c$; b) contour with radius $q = q_*$, along which the smallest eigenvalue $\lambda_-(\mathbf{q})$ is continuously degenerate; the arrows along an equipotential line show the direction of the polarization.

Figure 1(a) shows the two lower branches of the function $\lambda(q_x)$ at $q_y = 0$ and $q_z = \pi/c$ (the $z$ axis is along the $c$ axis of the hexagonal lattice). The lowest branch $\lambda_-(q_x)$ has a minimum at $q_x = q_*$, and crosses the $\lambda_+(q_x)$ curve at the point $q_x = q_y = 0$ (conical point $K$). The smallest eigenvalue $\lambda_-$ is continuously degenerate along a contour of radius $q = q_*$ [Fig. 1(b)]. The value of $q_*$, on the other hand, is in the intermediate region and corresponds to neither ferromagnetic nor antiferromagnetic ordering of the spin in the basal plane. The presence of a minimum inside the Brillouin zone is due to the difference in the character of the dipole interaction of the spins in different coordinate spheres: the interaction tends to ferromagnetic ordering of the spins for some spheres and to antiferromagnetic ordering for the others. It is the competition between these contributions that causes the sag of the function $\lambda_-(\mathbf{q})$; the magnitude and location of the sag depend on the ratio of the parameters of the hexagonal lattice.[1]

Numerical calculations of $\lambda(\mathbf{q})$ show that at an hexagonal-lattice parameter ratio $c/a \gtrsim 1.5$ the eigenvectors corresponding to the smallest $\lambda(\mathbf{q})$ lie in triangular planes (perpendicular to the $c$ axis); in the neighboring layers, however, they are antiferromagnetically ordered ($q_z = \pi/c$). Thus, the smallest eigenvalue $\lambda_-$ corresponds to a wave vector $\mathbf{q}'_* = \mathbf{q}_* + \mathbf{k}\pi/c$, where $\mathbf{k}$ is a unit vector along the $c$ axis; the period of a mode with such a wave vector $\mathbf{q}'_*$ is not commensurate with the period of the lattice in the basal plane. The eigenvectors of the tensor $D_{\alpha\beta}(\mathbf{q})$ in the basal plane, however, are everywhere polarized along the tangent to the equipotential line $\lambda_-(\mathbf{q}'_*)$ [Fig. 1(b)]. It will be shown below that just this circumstance leads to the existence of the vortical structure.

Allowance for the weak exchange interaction

$$\mathscr{H} = -J\sum_i \mathbf{S}_i\mathbf{S}_{i+1} \tag{3}$$

lifts the continuous degeneracy along the circle $q_x^2 + q_y^2 = q_*^2$ on the $q_x$, $q_y$ plane. We shall discuss this case in greater detail in Sec. 4.

## 3. VORTEX STRUCTURE. EXTERNAL FIELD

The existence of a minimum of $\lambda_-(\mathbf{q})$ inside the Brillouin zone leads to the appearance of an intermediate phase in a certain temperature interval. To determine the structure of this phase and the temperature interval in which it is stable, let us calculate the free energy $F$ within the framework of the molecular field approximation. In this approximation, the dipole Hamiltonian takes the form

$$\mathscr{H}_{ef} = -\sum_i \mathbf{H}_i\mathbf{S}_i,$$

where the components of the effective field $\mathbf{H}_i$ are connected with the components of $\mathbf{h} = g\mu_B S\,\mathbf{h}_0$ ($\mathbf{h}_0$ is the uniform external field) and with the tensor $D_{ij}^{\alpha\beta}$ by the relation

$$H_i^\alpha = h_\alpha - \frac{1}{2}\sum_{j,\beta} D_{ij}^{\alpha\beta}\langle S_j^\beta\rangle. \tag{3'}$$

The mean value of the spin at the site $i$ is determined in terms of the partition function $Z = \mathrm{Sp}\exp(-\mathscr{H}_{\mathrm{ef}}/T)$ by the expression

$$\langle S_i^\alpha\rangle = \frac{\partial \ln Z}{\partial (H_i^\alpha/T)} = B_S\left(\frac{H_i}{T}\right)\frac{H_i^\alpha}{H_i}. \tag{4}$$

Here $B_S(x)$ is the Brillouin function:

$$B_S(x) = \frac{2S+1}{2S}\,\mathrm{cth}\left(\frac{2S+1}{2S}x\right) - \frac{1}{2S}\,\mathrm{cth}\,\frac{x}{2S}.$$

The expression for the free energy $F = -T\ln Z$ is represented as follows:

$$F = \frac{1}{2}\sum_{i,j;\alpha,\beta} D_{ij}^{\alpha\beta}\langle S_i^\alpha\rangle\langle S_j^\beta\rangle - \sum_i \mathbf{h}\langle \mathbf{S}_i\rangle + T\sum_i\int_0^{\langle S_i\rangle} B_S^{-1}(x)\,dx, \tag{5}$$

where $B_S^{-1}$ is the inverse of the Brillouin function. To obtain $F$ we used the relation (4). The equilibrium condition $\partial F/\partial\langle S_i^\alpha\rangle = 0$ leads, naturally, to expression (3′) for the effective field.

At low values of the order parameter $\langle S_i^\alpha\rangle$ the free energy can be written in the form of a Landau expansion. Retaining terms up to sixth order in $\langle S_i^\alpha\rangle$ we have

$$F = \Bigg(\sum_i \frac{1}{2}\sum_{j;\alpha,\beta} D_{ij}^{\alpha\beta}\langle S_i^\alpha\rangle\langle S_j^\beta\rangle + A\langle S_i\rangle^2 + B\langle S_i\rangle^4 + C\langle S_i\rangle^6 + \mathbf{h}\langle \mathbf{S}_i\rangle\Bigg), \tag{6}$$

where

$$A = 1.5\alpha T,\quad B = 0.45\alpha^4\beta T,\quad C = 0.3\alpha^6(9\alpha\beta^2/5 - 6\gamma/7)T,$$

$$\alpha = \frac{S}{S+1},\quad \beta = \frac{(2S+1)^4 - 1}{(2S)^4},\quad \gamma = \frac{(2S+1)^6 - 1}{(2S)^6}$$

(in this section we need only an expansion of $F$ up to fourth order in $\langle S_i^\alpha\rangle$). Transforming to Fourier components

$$\langle \mathbf{S}_q\rangle = \sum_i \langle \mathbf{S}_i\rangle \exp(i\mathbf{q}\mathbf{r}_i)$$

in the expansion of the free energy (6) and diagonalizing it in the vicinity of the transition point, we find that the coefficient of $|\langle \mathbf{S}_q\rangle|^2$, which is the reciprocal susceptibility, as it

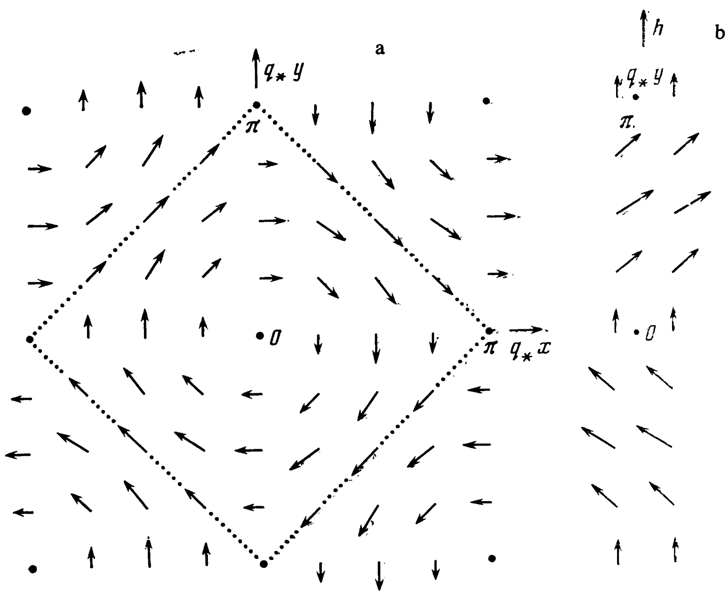


FIG. 2. Pattern of spins in incommensurate structures at $q_* a \ll 1$. a)—vortical structure; b—linear structure at finite value of the field $h$ (fan-shaped).

should, reverses sign at the point $T_* = -\lambda_-(\mathbf{q}_*')/3\alpha$ corresponding to the extremum (minimum) of the function $\lambda_-(\mathbf{q})$.

At $T = T_*$ the paramagnetic state is unstable to condensation of the transverse modes with $\mathbf{q} = \mathbf{q}_*'$, since the eigenvectors (the mean values of the spins) corresponding to the smallest eigenvalue $\lambda_-(\mathbf{q}_*')$, are orthogonal to the wave vectors $\mathbf{q}_*'$ viz. arrangement of the magnetic moments in Fig. 1(b) along a tangent to the circle $q_x^2 + q_y^2 = q_*^2$ and their antiferromagnetic arrangement in the neighboring layers of the hexagonal lattice. The average spin projections, expressed in terms of normal transverse modes, take therefore in the basal plane the form

$$\langle S_i^x \rangle = \Phi_x \exp(iq_* y_i) + \Phi_x^* \exp(-iq_* y_i),$$
$$\langle S_i^y \rangle = \Phi_y \exp(iq_* x_i) + \Phi_y^* \exp(-iq_* x_i) + m, \tag{7}$$

where $\Phi_x$ and $\Phi_y$ are complex quantities of like amplitude, and $m$ is the constant component of the spin along the magnetic field (the vector of the field $\mathbf{h}$ lies in the same basal plane and is oriented, to be specific, along the $y$ axis).

We consider first the solution for $\langle \mathbf{S}_i \rangle$ in a zero field. Substituting (7) with $m = 0$ in the expression (6) for the free energy, we find from the equilibrium conditions $\partial F/\partial \Phi_x^* = 0$, $\partial F/\partial \Phi_y^* = 0$ that

$$|\Phi_x|^2 = |\Phi_y|^2 = |\Phi_0|^2, \tag{8}$$

where $|\Phi_0|^2 = 3\alpha(T_* - T)/20B$. In a zero field at $T < T_*$ the thermodynamically stable state corresponds not to a one-mode state, but to a state with simultaneous condensation of two transverse modes of equal amplitude. The structure of the produced state has a vortical configuration in the basal plane. The arrangement of the spins of such a structure is shown in Fig. 2(a).

$$\langle S_i^x \rangle = |\Phi_0| \sin q_* y_i, \quad \langle S_i^y \rangle = -|\Phi_0| \sin q_* x_i,$$

from which it can be seen that the orientation of the spins in the neighboring vortices is opposite. We emphasize here that the vortical character of the incommensurate structure is due to transverse polarization of the eigenvectors of the dipole-dipole tensor $D_{\alpha\beta}(q)$ along the contour on which the equipotential line has a minimum [Fig. 1(b)].

With decreasing temperature, the vortex configuration becomes unstable: a transition takes place into a homogeneous state with $q_x = q_y = 0$ (but, as before, the spins are antiferromagnetically ordered in neighboring planes, $q_z = \pi/c$). An estimate of the temperature region $\Delta T$ where the vortex structure exists can be obtained from the condition that the free energies of the incommensurate and homogeneous phases be equal. As a result we find, as we should, that this region is proportional to the depth of the potential well relative to $\lambda_K = \lambda_-(\mathbf{q}_K)$ of the conical point $\mathbf{q}_K = (0, 0, \pi/c)$: $\Delta T \sim \lambda_K - \lambda_-(\mathbf{q}_*')$.

Application of a magnetic field deforms the vortical incommensurate structure; this is reflected in inequality of the oscillation amplitudes in (7) and (8). Minimizing now the free energy (6) not only with respect to $\Phi_x$ and $\Phi_y$, but also with respect to $m$, we obtain at small $h$

$$|\Phi_x|^2 = |\Phi_y|^2 + 2m^2, \quad |\Phi_y|^2 = |\Phi_0|^2 - {}^7/{}_5 m^2, \tag{9}$$

where $m = h/(\lambda_K + 3\alpha T)$. The free energy takes in this state the form

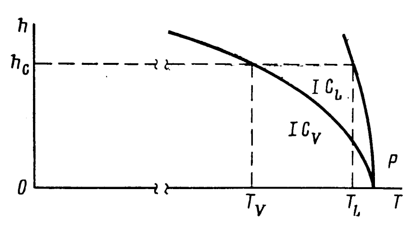


FIG. 3. Phase diagram: $IC_L$—region of existence of linear (fan-shaped) incommensurate phase, $IC_V$— region of vortical incommensurate phase, $P$—paraphase region.

$$F_V=-\frac{A_-^2+16A_-m^2B+144m^4B^2}{20B}+m^4B-\frac{1}{2}hm. \quad (10)$$

Here $A_-=3\alpha(T-T_*)$. As can be seen from (9), with increasing field $h$ along the $y$ axis the difference between the amplitudes $|\Phi_x|$ and $|\Phi_y|$ also increases, with the amplitude $|\Phi_x|$ transverse to the field increasing and the longitudinal $|\Phi_y|$ decreasing. Finally, when the field reaches the point $h=h_c$ the amplitude $|\Phi_y|$ vanishes: the vortex structure collapse and becomes sinusoidal.

Thus, a new phase exists in a field $h>h_c$ and corresponds to one transverse mode. To find the projections of the average spin for this incommensurate structure, we substitute again the expression (7) for $\langle \mathbf{S}_i\rangle$ in the expansion (6) for the free energy, but assume now that $\Phi_y=0$. As a result we obtain for the stable equilibrium

$$|\Phi_x|^2=\alpha(T_*-T)/4B-{}^1/_3m^2,$$
$$F_L=-(A_-+4m^2B)^2/24B+m^4B-{}^1/_2hm \quad (11)$$

(at small $h$ the expression for $m$ coincides with the analogous expression in the vortical phase). The structure induced by the field is fan-shaped at finite values of $h$ and is shown in Fig. 2(b).

The amplitude $|\Phi_x|$ in (11) becomes different from zero at the point $T_L=T_*-4m^2B/3\alpha$, where the transition from the paraphase takes place in a field $h\neq 0$. On the other hand, the transition point from the linear to the vortical structure, determined from the condition that the free energies $F_L$ and $F_V$ be equal in (10) and (11), is located at $T_V=T_*-28m^2B/3\alpha$. Figure 3 shows the region of existence of the phases: with decreasing field, the temperature interval of the linear incommensurate phase $(T_L-T_V\sim h^2)$ also decreases, and vanishes at $h=0$. In weaker fields the transition from one incommensurate phase to another is of second order in the molecular-field approximation.

## 4. ROLE OF EXCHANGE FORCES

Consider now the influence of the weak exchange interaction (3) on the form of the vortical configurations produced at finite values of $J$. The perturbation of the initial eigenvalues upsets the isotropy $\lambda_-(\mathbf{q}_*)$, but a discrete degeneracy still remains on a contour with radius

$$q_1=q_*-\sqrt{3}\frac{Ja}{\lambda_-''(\mathbf{q}_*)}\sin\frac{\sqrt{3}}{2}q_*a, \quad \lambda_-''(\mathbf{q}_*)=\frac{\partial^2\lambda_-(q_*)}{\partial q_*^2}. \quad (12)$$

Since $\lambda_-''(\mathbf{q}_*)>0$, the sign of the correction to the radius of the perturbed orbit is determined by the sign of the exchange interaction. Figure 4 shows the form of the eigenvalue $\Lambda_-(\mathbf{q})$ with account taken of the exchange interaction at $J>0$. The minima of the potential surface $\Lambda_-(\mathbf{q})$ at $q_z=\pi/c$ are located at six equivalent points $\mathbf{q}=\pm\mathbf{q}_s$, $s=1,2,3$, corresponding to degenerate values of the moduli of the wave vectors $(q_1=q_2=q_3)$. The $\Lambda_-(\mathbf{q})$ minimum points are in turn separated by a barrier whose maximum is reached at the saddle points $\mathbf{q}=\pm\mathbf{q}_a$, $\pm\mathbf{q}_b$, $\pm\mathbf{q}_c$ $(q_a=q_b=q_c)$. If the exchange interaction is antiferromagnetic $(J<0)$, the positions of the characteristic points of the function $\Lambda_-(\mathbf{q})$ on Fig. 4 change places: the minimum points correspond to saddle points and vice versa.

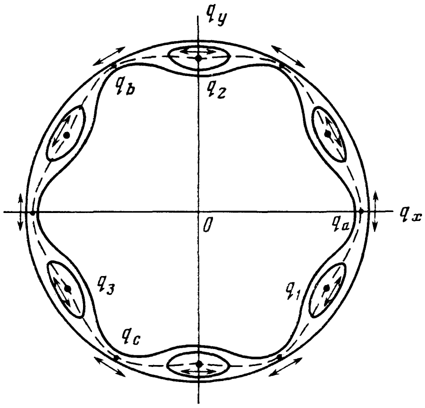


FIG. 4. Equipotential lines $\Lambda_-(\mathbf{q})$ with allowance for weak exchange (ferromagnetic) interaction. The minimum points are located at the vectors $\pm\mathbf{q}_1$, $\pm\mathbf{q}_2$, $\pm\mathbf{q}_3$; the saddle points are $\pm\mathbf{q}_a$, $\pm\mathbf{q}_b$, $\pm\mathbf{q}_c$.

With decreasing temperature the paramagnetic state now becomes unstable at a point $T_1=-\Lambda_-(\mathbf{q}_1')/3\alpha$ in which the modes with wave vectors $\mathbf{q}_s'=\mathbf{q}_s+\mathbf{k}\pi/c$ condense. In the general case, besides the coexistence of one and two modes, three modes can coexist. The modulated structures produced by a linear combination of three modes are known and have been considered in connection with solutions of other physical problems.[13-15] Taking into account the polarization of the eigenvectors at points $\mathbf{q}_s'$, we represent the solution for the average spin projections $\langle S_i^\alpha\rangle$ in the form

$$\langle S_i^x\rangle=a_1\cos(\mathbf{q}_1'\mathbf{r}_i+\gamma_1)-2a_2\cos(\mathbf{q}_2'\mathbf{r}_i+\gamma_2)+a_3\cos(\mathbf{q}_3'\mathbf{r}_i+\gamma_3),$$
$$\langle S_i^y\rangle=\sqrt{3}[a_1\cos(\mathbf{q}_1'\mathbf{r}_i+\gamma_1)-a_3\cos(\mathbf{q}_3'\mathbf{r}_i+\gamma_3)], \quad (13)$$

where $a_s$ and $\gamma_s$ are the amplitudes and phases of the corresponding modes. The states with one or two $\mathbf{q}_s'$ are thus obtained if one or two out of the three amplitudes in (13) are respectively different from zero.

Let us compare the free energies of the different states. To this end we replace $D_{ij}^{\alpha\beta}$ in the Landau expansion (6) by $\tilde{D}_{ij}^{\alpha\beta}=D_{ij}^{\alpha\beta}-2J\delta_{i+1\,j}\delta_{\alpha\beta}$ (the coefficients $A$, $B$, and $C$ remain unchanged here) and substitute next (13) in the so transformed expansion (6), in which now we retain terms up to sixth order in $\langle S_i^\alpha\rangle$. As a result we find at $h=0$ that $F_1$ in a state with one mode $(a_1=a_0,\ a_2=a_3=0)$ and $F_2$ with coexistence of two modes $(a_1=a_2=a_0/\sqrt{2},\ a_3=0)$ are equal and are given by

$$F_{1,2}=A_1a_0^2+6Ba_0^4+20Ca_0^6, \quad (14)$$

where $A_1=3\alpha(T-T_1)$.

The state with simultaneous condensation of three modes depends on the sum of their initial phases. The free energy $F_3$ in this state $(a_1=a_2=a_3=a_0/\sqrt{3})$ assumes the smallest value at $\gamma_1+\gamma_2+\gamma_3=(n+\frac{1}{2})\pi$, where $n$ is an integer, and is given by

$$F_3=A_1a_0^2+6Ba_0^4+(20-{}^{10}/_3)Ca_0^6. \quad (15)$$

From a comparison of (14) and (15) it can be seen that the formation of an incommensurate structure of three

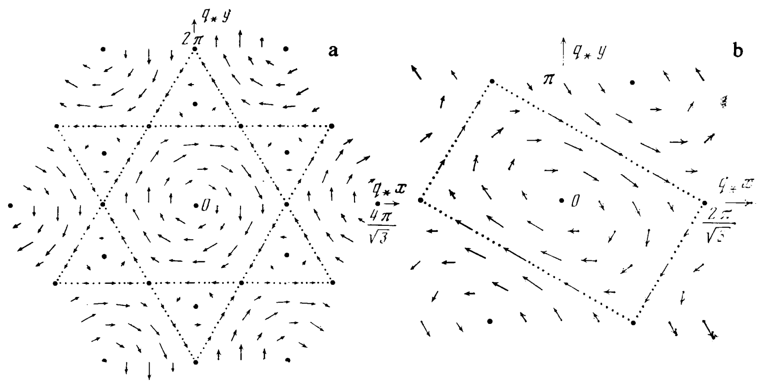


FIG. 5. Incommensurate structures with vortical configuration, due to the presence of weak exchange interaction: a—three-mode state; b—two-mode.

modes corresponds to the lowest state and is due to the correlation between their initial phases. The structure of this state, which has vortices of two scales, is shown in Fig. 5a ($\gamma_1 = \gamma_2 = \gamma_3 = \pi/2$); the spin directions in the neighboring vortices is opposite, just as in Fig. 2a. The temperature region, in which the new vortical structure exists, is proportional to depth of the potential wells relative to the saddle points, which depends in turn on the exchange interaction $J$ between the nearest spins[2)]; as $J \to 0$ the region of existence of the new phase vanishes.

It is easy to show that states with one and two $\mathbf{q}_s'$ appear when additional account is taken, in the expansion for $F$, of the energy of the hexagonal anisotropy in the basal plane

$$F_a = K \sum_i (\langle S_i^+\rangle^6 + \langle S_i^-\rangle^6)/2,$$

where $K$ is the sixth-order anisotropy constant. If the easy-magnetization axes coincide with the directions of the eigenvector polarizations—the easy axes are oriented along the vectors $\pm \mathbf{q}_a$, $\pm \mathbf{q}_b$, $\pm \mathbf{q}_c$, corresponding to saddle points ($K < 0$), a single mode state is realized at $|K| \gtrsim 0.06C$. On the other hand, a vortical configuration with two $\mathbf{q}_s$ (at the same values $K \gtrsim 0.06$ C) is produced when the easy magnetization axes are directed along the vectors $\mathbf{q}_s$ ($K > 0$), on which the minima of $\Lambda_-(\mathbf{q})$ are located. The structure of this state is shown in Fig. 5b ($\gamma_1 = \gamma_2 = -\pi/2$); it can be seen from the arrangement of the spins in the $x$, $y$ plane that the force lines making up the vortex are oblate.

## 5. CONCLUSION

We have, thus, shown that incommensurate structures with vortical configuration can be produced in hexagonal-lattice compounds in which the dipole interaction plays the principal role. The existence of intermediate-temperature vortical states is due to the spectral properties of dipole tensor. The presence of a minimum inside the Brillouin zone leads to instability of the symmetry point $\mathbf{q}_K = (0, 0, \pi/c)$ at which the eigenvalues (the energy levels) intersect. On the other hand, at the minimum points the wave vector is not fixed by symmetry, and the modes produced have transverse polarization. When a magnetic field is applied, the vortical structure is deformed and becomes unstable in a definite temperature range in which it goes over into a fan-shaped structure.

Allowance for weak exchange interaction in dipole systems leads to the onset of a new incommensurate phase with two vortex scales, so that two intermediate-temperature modulated structures are produced with a vortical spin configuration. The two-scale vortical structure is due to the coexistence of three transverse modes whose wave vectors form a three-prong star. If the hexagonal-anisotropy field in the crystal is strong enough, this structure is not realized. Depending on the sign of the anisotropy constant, its place is taken either by a simple (but deformed) vortex structure ($K > 0$) or by a purely sinusoidal one ($K < 0$).

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[1)]Note the following: it was shown in a preceding paper[9] that continuous degeneracy takes place also in structures with cubic lattice. In contrast to the present case, however, where the spins are located at the sites of an hexagonal lattice, in Ref. 9 degenerate values of the wave vector **q** corresponded to states with antiferromagnetic ordering of the spins (states with two antiferromagnetic sublattices—two simple sublattices—with arbitrary angle between their antiferromagnetism vectors).

[2)]Allowance for the finite radius $\gamma^{-1}$ of the exchange interaction leads to expansion of this region, inasmuch as at small $\gamma^{-1}$ the exchange interval becomes renormalized (see the Appendix of Ref. 16).

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