Magnetic symmetry of the domain walls in magnetically ordered crystals

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A study is made of the relationship between the specific crystallomagnetic structure of magnetically ordered crystals and the properties of their domain walls. The magnetic symmetry classes are constructed for plane 180-degree domain walls in ferromagnets and antiferromagnets, and the coordinate dependence of the magnetization density and antiferromagnetism vector in the domain wall is characterized qualitatively for domain walls belonging to each of the classes. It is shown that there are 42 magnetic classes of domain walls in ferromagnets and 134 magnetic classes of domain walls in antiferromagnets. It is predicted that phase transitions associated with changes in the domain-wall symmetry can occur. A study is made of the change in the symmetry and spatial structure of the domain walls due to their motion.

The static and dynamical properties of the boundaries between magnetic domains in magnetically ordered crystals have been actively studied of late (see, e.g., Ref. 1). In the overwhelming majority of cases the studies have considered some model problem without taking into account the specific magnetic symmetry of the crystal. At the same time, however, individual calculations of the static[2,3] and dynamic[4,5] properties of domain walls in orthoferrites, for example, suggest that there are a number of important features which arise upon the systematic incorporation of the actual structure of the particular magnet.

In this paper we construct a complete symmetry classification of stationary and moving 180-degree domain walls in ferromagnetic and antiferromagnetic crystals. We consider only plane domain walls whose widths are much greater than the interatomic spacings. On the basis of our analysis we make a number of general statements about the properties of domain walls and predict the possibility of phase transitions associated with a change in the domain-wall symmetry.

The existence of domain structure in ferromagnets and ferrimagnets is mainly due to the magnetic dipole interaction.[6] A different situation is found in antiferromagnets, in which there is, in addition to the thermodynamically stable domain structure (antiferromagnetic, with a weak ferromagnetism), a nonequilibrium domain structure associated with the kinetics of the phase transition to the magnetically ordered state.[7] There is also active study of the thermodynamic-equilibrium domain structure that arises at first-order phase transitions in a magnetic field—the intermediate state.[8–15]

The theory elaborated below applies to the walls of all domain structures in which the ferromagnetism vectors (in ferromagnetic and ferrimagnetic crystals) or antiferromagnetism vectors (in two-sublattice antiferromagnetic crystals) of adjacent domains are antiparallel (i.e., it applies to 180-degree domain walls). The generalization to other domain walls can be done in an analogous way.

1. MAGNETIC SYMMETRY CLASSES OF PLANE 180-DEGREE DOMAIN WALLS IN MAGNETICALLY ORDERED CRYSTALS

If the radius of curvature r_0 of the domain walls is much larger than their thickness, segments with dimensions much smaller than r_0 can be considered approximately planar. It is thus meaningful to study the symmetry of plane domain walls. It is convenient to introduce an orthonormal basis tied to the plane wall. Unit vector $\mathbf{n}$ is perpendicular to the plane of the wall, while unit vectors $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ lie in the plane of the wall. The coordinate measured along the direction of $\mathbf{n}$ will be denoted ξ.

The magnetic symmetry group of a domain wall in a crystal is the set of all operations which do not change the spatial arrangement of the magnetic moments of the atoms in the magnet with the domain wall. This group is a subgroup of the magnetic (Shubnikov) group of the paramagnetic phase of the crystal.[1)] If all translations t are made identical to the unit operation, we arrive at the concept of the magnetic symmetry class of the crystal with the domain wall.

The properties of domain walls whose width ξ_0 is much larger than the dimensions a of the magnetic unit cell can be described by macroscopic quantities [e.g., the magnetization density $\mathbf{M}(\xi)$ and the density of the antiferromagnetism vector, $\mathbf{L}(\xi)$]. By introducing macroscopic quantities one is averaging over dimensions $a \ll x \ll \xi_0$. Consequently, the symmetries of wide domain walls can be characterized by magnetic classes.

We shall show that the symmetry of plane domain walls depends importantly on how the vector normal $\mathbf{n}$ and the ferromagnetism (antiferromagnetism) vectors in the domains are oriented with respect to the crystallographic axes.

Let us first enumerate all the possible magnetic symmetry classes G_k of plane 180-degree domain walls in ferromagnets.[2)] In this case the magnetic moments far from the wall are antiparallel, i.e., $\mathbf{M}(\xi=\infty) = -\mathbf{M}(\xi=-\infty)$. The magnetic class of the domain walls in the crystal can only

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contain symmetry elements which belong to the magnetic class of the paramagnetic phase of the crystal, and of them, only those which do not change the conditions at $\xi = \pm\infty$. These operations can be of two types. Operations of the first type ($g^{(1)}$) do not change the direction of either the T-odd axial vectors $\mathbf{M}(\pm\infty)$ or of the normal $\mathbf{n}$. Operations of the second type reverse both these directions: $g^{(2)}\mathbf{M}(\infty) = \mathbf{M}(-\infty)$, $g^{(2)}\mathbf{m} = -\mathbf{n}$. The set of operations $g^{(1)}$ and $g^{(2)}$ forms a group. This group corresponds to the maximum possible domain-wall symmetry in the given crystal for the chosen direction of vectors $\mathbf{n}$ and $\mathbf{M}(\pm\infty)$. All the remaining classes for a given crystal at fixed $\mathbf{n}$ and $\mathbf{M}(\pm\infty)$ can be obtained by a simple listing of its subgroups.

Three distinct orientations of the vectors $\mathbf{M}(\pm\infty)$ with respect to the normal to the wall are possible:

1. $\mathbf{M}(\pm\infty)\|\boldsymbol{\tau}_1$. The magnetic symmetry classes of the domain wall can consist of the following elements: $1, 2_2, \bar{2}_1, \bar{2}_n, \bar{1}', \bar{2}_2', 2_1', 2_n'$. In our notation $1_1, 2_2, 2_n$ are twofold rotations about the axes $\boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \mathbf{n}$, respectively, $\bar{1}$ is inversion, and $\bar{2}_1 = 2_1\cdot\bar{1}$, $\bar{2}_2 = 2_2\cdot\bar{1}$, and $\bar{2}_n = 2_n\cdot\bar{1}$ are reflection planes perpendicular to the axes $\boldsymbol{\tau}_1$, $\boldsymbol{\tau}_2$, and $\mathbf{n}$, respectively. As usual, a prime on a symmetry element means that a time-reversal operation is done simultaneously. The primed rotations are known as antirotations, and $\bar{1}'$ is known as anti-inversion.

From these elements one can form sixteen groups, which are written out in the second column in Table I (and are numbered $k = 1$–16 in the first column). It is seen that groups G_2–G_{16} are subgroups of G_1. Only the generating elements of the classes are shown for $k = 24$–42). The third column gives the nonzero components of the vectors $\mathbf{M}(\pm\infty)$, and the fourth column gives the qualitative form of the coordinate dependence of the components of the magnetization vector $\mathbf{M}(\xi)$ (ξ is the coordinate along the normal to the wall, and the symbols S and A denote the presence of symmetric and antisymmetric parts, respectively, with respect to the substitution $\xi\to-\xi$). The last column gives the abbreviated international notation for the classes.

2. $\mathbf{M}(\pm\infty)\|\mathbf{n}$. In this case the magnetic classes of the domain walls can include the following operations:

$$1, 2_\perp, 2_n, 3_n, 4_n, 6_n, \bar{1}', \bar{2}_\perp', \bar{2}_n', \bar{3}_n', \bar{4}_n', \bar{6}_n',$$

where 3_n, 4_n, 6_n are threefold, fourfold, and sixfold rotations about the normal $\mathbf{n}$, while $2_\perp$ denotes twofold rotations about axes perpendicular to $\mathbf{n}$. From these elements one can make 31 groups which do not reduce to one another by a

TABLE I.

1	2	3	4			5
k	Symmetry elements	$\mathbf{M}(\infty)$	M_{τ_1}	M_{τ_2}	M_n	International symbols
1	$(1, \bar{2}_1, 2_2, \bar{2}_n)\times(1, \bar{1}')$	τ_1	A	O	O	mmm'
2	$1, \bar{2}_1, \bar{2}_2', 2_n'$	τ_1	A, S	O	O	mm'
3	$1, \bar{2}_1, 2_2, \bar{2}_n$	τ_1	A	O	O	mm
4	$1, \bar{1}', 2_1', \bar{2}_1$	τ_1	A	O	O	$2'/m$
5	$1, \bar{1}', 2_n', \bar{2}_n$	τ_1	A	A	O	$2'/m$
6	$1, \bar{2}_1$	τ_1	A, S	O	O	m
7	$1, 2_1', 2_2, 2_n'$	τ_1	A	S	O	$22'2'$
8	$1, 2_n'$	τ_1	A, S	A, S	O	$2'$
9	$1, 2_1', \bar{2}_2', \bar{2}_n$	τ_1	A	O	S	mm'
10	$1, 2_1'$	τ_1	A	S	S	$2'$
11	$1, \bar{2}_n$	τ_1	A	A	S	m
12	$1, \bar{2}_2'$	τ_1, n	A, S	O	A, S	m'
13	$1, 2_2$	τ_1, n	A	S	A	2
14	$1, \bar{1}', 2_2, \bar{2}_2'$	τ_1, n	A	O	A	$2/m'$
15	$1, \bar{1}'$	τ_1, τ_2, n	A	A	A	$\bar{1}'$
16	1	τ_1, τ_2, n	A, S	A, S	A, S	1
17	$1, \bar{2}_1', 2_2, \bar{2}_n'$	n	O	S	A	$m'm'$
18	$1, \bar{2}_n'$	n	S	S	A	m'
19	$1, 2_n$	n	O	O	A, S	2
20	$1, \bar{1}', 2_n, \bar{2}_n'$	n	O	O	A	$2/m'$
21	$1, 2_1, 2_2, 2_n$	n	O	O	A	222
22	$1, \bar{2}_1', \bar{2}_2', 2_n$	n	O	O	A, S	$m'm'$
23	$(1, 2_1, 2_2, 2_n)\times(1, \bar{1}')$	n	O	O	A	$m'm'm'$
24	3_n	n	O	O	A, S	3
25	$\bar{6}_n'$	n	O	O	A	$\bar{6}'$
26	$3_n, \bar{2}_1'$	n	O	O	A, S	$3m'$
27	$3_n, 2_1$	n	O	O	A	32
28	$\bar{6}_n', 2_1$	n	O	O	A	$\bar{6}'m'2$
29	$\bar{3}_n', \bar{2}_1'$	n	O	O	A	$\bar{3}'m'$
30	4_n	n	O	O	A, S	4
31	$4_n, \bar{2}_n'$	n	O	O	A	$4/m'$
32	$4_n, \bar{2}_1'$	n	O	O	A, S	$4m'm'$
33	$4_n, 2_1$	n	O	O	A	422
34	$4_n, \bar{2}_1', \bar{2}_n'$	n	O	O	A	$4/m'm'm'$
35	$\bar{4}_n'$	n	O	O	A	$\bar{4}'$
36	$\bar{4}_n', 2_1$	n	O	O	A	$42m'$
37	6_n	n	O	O	A, S	6
38	$6_n, \bar{2}_n'$	n	O	O	A	$6/m'$
39	$6_n, \bar{2}_1'$	n	O	O	A, S	$6m'm'$
40	$6_n, 2_1$	n	O	O	A	622
41	$6_n, \bar{2}_1', \bar{2}_n'$	n	O	O	A	$6/m'm'm'$
42	$\bar{3}_n'$	n	O	O	A	$\bar{3}'$

relabeling of the axes τ_1 and τ_2. These groups are given in Table I in the rows numbered $k = 12, 13,...,42$.

3. $\mathbf{M}(\pm\infty) = M_{\tau_1}(\pm\infty)\boldsymbol{\tau}_1 + M_n(\pm\infty)\mathbf{n}$. The symmetry of the crystal with the domain wall is described by one of the classes G_k with $k = 12, 13, 14$.

Table I thus gives all possible magnetic symmetry classes for a domain wall in a ferromagnet that are not mutually reducible by a relabeling of the unit vectors $\tau_1 \leftrightarrow \tau_2$.

To find out which types of domain walls can, in principle, occur in each particular crystal, one must sort out all possible crystallographically different directions of the normal $\mathbf{n}$ and for each such direction list all the classes G_k which, at fixed values of $\mathbf{M}(\pm\infty)$ determined from the stability conditions of the spatially homogeneous magnetically ordered state, are subgroups of the magnetic class of the paraphase of the crystal.

In ferrimagnets the domain-wall symmetry is also described by the classes G_k, since the transformation properties of the magnetization vectors of the sublattices in a ferrimagnet are the same as for the ferromagnetism vector in a ferromagnet. There are no additional symmetry operations, since the sublattices in a ferrimagnet are inequivalent.

An essentially different situation occurs in an antiferromagnet. We shall consider only those antiferromagnetic structures, including weakly ferromagnetic ones, which can be treated in the two-sublattice model[16] (a rigorous symmetry-based definition is found in Ref. 17).

As in the ferromagnetic case, the domain-wall symmetry classes in antiferromagnets can be obtained by listing the symmetry classes of the conditions at infinity and all their subgroups. The total number of domain-wall symmetry classes in this case is rather large, and we shall therefore give only a simple prescription for constructing them.

Following Ref. 16, let us for convenience separate all the symmetry operations included in the magnetic symmetry class of the paramagnetic phase of an antiferromagnetic crystal into odd and even operations (which do and do not, respectively, interchange the sublattices) with respect to the given antiferromagnetic structure. The even operations are denoted by a plus $(+)$ sign, the odd by a minus $(-)$ sign. Since the magnetic symmetry classes of the domain walls in an antiferromagnetic crystal are subgroups of the magnetic class of its paramagnetic phase, the operations which they contain can also be divided into even and odd operations with respect to the given antiferromagnetic structure.

The magnetic symmetry of an antiferromagnet is specified by giving the transformation properties of the antiferromagnetism vector $\mathbf{L}$ and ferromagnetism vector $\mathbf{M}$. Under even operations $\mathbf{M}$ and $\mathbf{L}$ transform alike, as T-odd axial vectors, while under odd operations the vector $\mathbf{L}$ is multiplied by an additional coefficient (-1). Since we are considering only antiferromagnets whose structures in the exchange approximation are collinear and specified by only a single antiferromagnetism vector, in constructing the domain-wall symmetry classes it is sufficient to consider only the transformation properties of the vector $\mathbf{L}$.

Let us first consider antiferromagnetic structures which are even with respect to all translations. In this case the magnetic symmetry classes of the domain walls can be constructed (using Table I) on the basis of the same elements as for the domain walls in ferromagnets.

To construct all the magnetic symmetry classes of domain walls in some particular antiferromagnet, one must proceed as follows:

a) determine which of the symmetry operations in the classes G_k in Table I are even for the given antiferromagnet and which ones are odd;

b) subject the odd elements in all classes G_k to an additional time-reversal operation.

Let us introduce a practical system of notation for the magnetic classes of domain walls in antiferromagnets, making use of the fact that the number of possible combinations of the parity of the elements in a given class G_k coincides with the number of real one-dimensional irreducible representations of the group $\widetilde{G}_k$ obtained from G_k by replacing the time-reversal operation with the unit operation. Here the characters of the irreducible representations of the group $\widetilde{G}_k$ coincide with the parity of the corresponding elements. It is therefore convenient to denote the magnetic symmetry class of the domain walls in an antiferromagnet by the symbol $G_k(\Gamma)$, where Γ is the symbol of the irreducible representation of the group $\widetilde{G}_k$ for the given antiferromagnetic structure.

By counting up the number of irreducible representations Γ one can show that there are 134 magnetic classes of domain walls in an antiferromagnet with the $t(+)$ structure.

Let us not consider antiferromagnetic structures which are odd with respect to some translation $t(-)$. In this case the magnetic class of the homogeneous antiferromagnet contains the odd time-reversal operation $1'(-)$. This operation remains present in the magnetic class of the inhomogeneous state if the dimensions of the inhomogeneity are substantially greater than the dimensions of the unit cell. It follows that the magnetic classes of the domain walls in an antiferromagnet with an odd translation are the direct product of the classes $G_k(\Gamma)$ with the group consisting of the unit element and the odd time-reversal operation: $G_k(\Gamma-) = G_k(\Gamma)\times(1,1'(-))$.

2. STRUCTURE OF 180-DEGREE DOMAIN WALLS

To describe the symmetry of the microscopic properties of crystals with plane 180-degree domain walls one must construct the magnetic groups which appear in the classes found above. It is necessary to know these groups in order to study walls of thickness comparable to the dimensions of the magnetic unit cell. Here, however, we shall consider only domain walls whose thickness is much greater than the dimensions of the magnetic unit cell of the crystal. Such domain walls are described by introducing the macroscopic magnetization density $\mathbf{M}(\xi)$ and the antiferromagnetism vector $\mathbf{L}(\xi)$. Here we are averaging the spatial distribution of the atomic magnetic moments over distances much greater than the interatomic distance but much smaller than the thickness of the domain walls. In order to find out which types of spatial distributions of $\mathbf{M}(\xi)$ and $\mathbf{L}(\xi)$ can in principle occur in each particular crystal, it is necessary to indicate which of the classes $G_k(\Gamma)$ are contained as subgroups in the magnetic class of the paramagnetic phase of the crystal. This is because each magnetic symmetry class $G_k(\Gamma)$ of the crystal with the domain walls corresponds to a definite type of coordinate dependence of the components of the magnetization density $\mathbf{M}_\alpha(\xi)$ and antiferromagnetism-vector density $L_\alpha(\xi)$. To identify this type one must study how the operations which appear in the class $G_k(\Gamma)$ act on the components of the T-odd axial vector **M** and the vector **L** remembering that the vector **L** has an additional sign change upon interchange of the sublattices.

As in the preceding section, it is reasonable to begin with the simplest case—the classes G_k of the domain walls in a ferromagnet. Separating all the operations in G_k into two types ($g^{(1)}$ and $g^{(2)}$, see Sec. 1) and seeing what restrictions on the form of the functions $M_\alpha(\xi)$ and $L_\alpha(\xi)$ $(\alpha = \tau_1, \tau_2, n)$ are imposed by operations of each type, we obtain the following rules.

If $g^{(1)}M_\alpha(\xi) = -M_\alpha(\xi)$, then $M_\alpha(\xi) = 0$; if $g^{(1)}M_\alpha(\xi) = M_\alpha(\xi)$, then $g^{(1)}$ does not impose any restrictions on the form of this components of the magnetization.

If $g^{(2)}M_\alpha(\xi) = -M_\alpha(-\xi)$, then $M_\alpha(\xi)$ is an antisymmetric (A) function of the coordinate ξ; if $g^{(2)}M_\alpha(\xi) = M_\alpha(-\xi)$, then $M_\alpha(\xi)$ is a symmetric (S) function.

Let us illustrate this with an example. The operation $\bar{2}_n$ is of the type $g^{(2)}$, since $\bar{2}_n \mathbf{n} = -\mathbf{n}$. We note further that since $\bar{2}_n M_n(\xi) = M_n(-\xi)$, the presence of this operation in the magnetic class of the domain walls requires that $M_n(\xi)$ be a symmetric function of the coordinate ξ. The functions $M_{\tau_1}(\xi)$ and $M_{\tau_2}(\xi)$, on the other hand, should be antisymmetric, since $\bar{2}_n M_{\tau_1}(\xi) = -M_{\tau_1}(\xi)$, $\bar{2}_n M_{\tau_2}(\xi) = -M_{\tau_2}(\xi)$.

The types of functions $M_\alpha(\xi)$ found by this rule for all classes G_k are given in the fourth column of Table I.

The data in Table I determine the qualitative behavior of the magnetization vector within a domain wall not only in ferromagnets but also in ferrimagnets and antiferromagnets if in these last there is an antiferromagnetic ordering whose vector **L** is even under all the operations in G_k. In this case the behavior of the vector **L** in such a domain wall is qualitatively no different from the behavior of the vector **M**.

If the magnetic symmetry class of the domain walls in an antiferromagnet contains odd elements, the distributions $\mathbf{L}(\xi)$ and $\mathbf{M}(\xi)$ are determined in the following way. As we have said, the classes $G_k(\Gamma)$ are obtained from G_k by replacing all the odd operations $g(-)$ by the operations $g'(-)$. Here the action of the operation $g'(-)$ on **L** coincides precisely with the action of the operation $g(+)$ on **M**. Consequently, all classes $G_k(\Gamma)$ with the same value of the index k and different values of Γ correspond to the same type of behavior of $\mathbf{L}(\xi)$, which is like that of $\mathbf{M}(\xi)$ in Table I for the same index k. As to the behavior of $\mathbf{M}(\xi)$, it depends importantly on Γ as well as on k.

As an example, Table II gives the symmetry classes and structure of the high-symmetry domain walls in antiferromagnets. These classes are constructed from the classes G_7, G_9, and G_{17} by the method indicated above. It is seen from Table II that each of the listed classes "multiplies" into four classes. To each of the four there corresponds a single type of function $\mathbf{L}(\xi)$ and four different distributions of the magnetization $\mathbf{M}(\xi)$. It is particularly interesting that a nonzero inhomogeneous magnetization arises in all the domain walls regardless of whether the initial phases are weakly ferromagnetic or not. For example, $\mathbf{M}(\xi)$ arises in all structures with both even and odd inversions. Since the odd inversion prohibits the appearance of terms in the free energy which are linear in **L** (the Dzyaloshinskiĭ term, in particular), the appearance of inhomogeneous magnetization in such structures should be due to invariants containing the spatial derivatives, e.g., the invariant $\mathbf{M} \cdot \nabla \times \mathbf{L}$.

Let us make a few general remarks concerning the structure and properties of the domain walls.

If the symmetry class of the domain walls contains elements which interchange the places of the magnetic domains, one can introduce the concept of the "center" of the domain wall. Elements of this kind might be the twofold axes $2_\perp$ and $2'_\perp$, the center of anti-inversion $\bar{1}'$, and any of the elements $\bar{2}'_n$, $\bar{3}'_n$, $\bar{4}'_n$, $\bar{6}'_n$. The center of the domain wall is the plane passing through the indicated symmetry elements and perpendicular to the inhomogeneity axis ξ. In walls which lack a center, every nonzero component of the magnetization has both a symmetric and an antisymmetric part.

It is seen from Table I that in domain walls corresponding to the classes with $k = 1$–6, 14,15, and 19–42 there exists a plane $\xi = \xi^*$ on which $\mathbf{L}(\xi) = 0$ (antiferromagnet) or

TABLE II. Magnetic symmetry classes of high-symmetry domain walls in antiferromagnets and the spatial structure of these walls.

k	Γ	Symmetry elements	M_{τ_1}	M_{τ_2}	M_n	L_{τ_1}	L_{τ_2}	L_n
7	A	$1, 2_2(+), 2_1'(+), 2_n'(+)$	A	S	O	A	S	O
	B_1	$1, 2_2'(-), 2_1(-), 2_n'(+)$	S	A	O	A	S	O
	B_2	$1, 2_2'(-), 2_1'(+), 2_n(-)$	O	O	S	A	S	O
	B_3	$1, 2_2(+), 2_1(-), 2_n(-)$	O	O	A	A	S	O
9	A_1	$1, \bar{2}_2'(+), 2_1'(+), \bar{2}_n(+)$	A	O	S	A	O	S
	B_1	$1, \bar{2}_2(-), 2_1(-), \bar{2}_n(+)$	O	A	O	A	O	S
	A_2	$1, \bar{2}_2(-), 2_1'(+), \bar{2}_n'(-)$	O	S	O	A	O	S
	B_2	$1, \bar{2}_2'(+), 2_1(-), \bar{2}_n'(-)$	S	O	A	A	O	S
17	A_1	$1, 2_2(+), \bar{2}_1'(+), \bar{2}_n'(+)$	O	S	A	O	S	A
	B_1	$1, 2_2'(-), \bar{2}_1(-), \bar{2}_n'(+)$	S	O	O	O	S	A
	B_2	$1, 2_2'(-), \bar{2}_1'(+), \bar{2}_n(-)$	O	A	S	O	S	A
	A_2	$1, 2_2(+), \bar{2}_1(-), \bar{2}_n(-)$	A	O	O	O	S	A

$\mathbf{M}(\xi) = 0$ (ferromagnet). Since the length of the antiferromagnetism vector in an antiferromagnet or of the magnetic moment in a ferromagnet is determined by exchange forces, these pulsating domain walls can exist only near the magnetic ordering temperature or else in magnets in which the relativistic interactions are comparable to or even stronger than the exchange interaction. The remaining domain walls are rotating or quasirotating walls [in the latter case there is a rotation of the vector $\mathbf{L}$ (in an antiferromagnet) or $\mathbf{M}$ (in a ferromagnet) together with a simultaneous change of its length].

The classes $G_k(\Gamma)$ describe, among other things, the symmetry of the Bloch and Néel domain walls. The Bloch walls correspond to the classes $G_7(\Gamma)$, the Néel walls to $G_9(\Gamma)$. Consequently, the Bloch domain walls can form only in those magnets whose crystallographic class contains the class $\mathbf{D}_2$ as a subgroup, while the formation of Néel domain walls requires that the class $\mathbf{C}_{2v}$ be included among the subgroups of the crystallographic class of the magnet. If these conditions are not satisfied, the domain walls will have other structures.

The classes $G_{17}(\Gamma)$ describe high-symmetry rotating domain walls of the Néel type, but with other conditions at infinity.

The domain walls with $k = 8$ are of the Bloch type in terms of rotation but do not have a center, while those with $k = 12$ are of the Néel type without a center.

The symmetry classes $G_k(\Gamma)$ which we have found describe the symmetry of the magnet only in the place where the domain wall is found, while far from domain walls the symmetry is that of the spatially homogeneous magnetically ordered state.

We also note that the number of qualitatively different behaviors of $\mathbf{M}(\xi)$ or $\mathbf{L}(\xi)$ is substantially smaller than the number of classes $G_k(\Gamma)$. Nevertheless, domain walls with the same behavior of $\mathbf{M}(\xi)$ or $\mathbf{L}(\xi)$ but belonging to different classes $G_k(\Gamma)$ will differ substantially in their other properties—the distribution of the electric charge density and electric polarization,[18] and their elastic, magnetoelastic, and optical properties, etc.

To conclude this section let us discuss the possibility of phase transitions associated with a change in the domain-wall symmetry upon changes in the thermodynamic parameters.

First, we note that by virtue of what was said above, not all of these are spin-reorientation transitions.

Second, the change in the domain-wall symmetry can be caused by a reorientation of the atomic magnetic moments in the interior of the domains.

Third, the symmetry of the domain wall can change without a change in the magnetic ordering inside the domains. In particular, such a change could be the transformation of a Bloch domain wall (G_7) into a wall with symmetry G_8, G_{10}, G_{13}, G_{16}, or the lowering of the symmetry of a Néel domain wall (G_9) to one of the groups G_{10}, G_{11}, G_{12}, G_{16}. It is just such symmetry transitions that are the most interesting from the standpoint of studying the domain wall properties. A spin reorientation of this kind has apparently been observed experimentally in Ref. 19 (a theoretical discussion of this question for domain walls in orthoferrites is given in Refs. 3 and 5).

3. KINEMATIC SYMMETRY OF DOMAIN WALLS

Let us now turn to the problem of steadily moving domain walls. We shall consider a plane domain wall moving uniformly with velocity $\mathbf{v} = v\mathbf{n}$. To describe the symmetry of the momentum distribution in such a domain wall at any given instant, we introduce the concept of the kinematic magnetic symmetry class. The kinematic magnetic class is obtained from the magnetic class of the stationary domain wall by removing from the latter all operations which change the direction of the T-odd true vector $\mathbf{v}$.

We use the term kinematic symmetry because we are not considering the causes of the domain-wall motion. Here, of course, we do not take into account the change in symmetry of the magnet due to applied external forces. Clearly, each of the kinematic classes of domain walls in ferromagnets coincides with one of the classes G_k given in Table I, while in antiferromagnets it coincides with one of the classes $G_k(\Gamma)$ constructed by the prescription given in Sec. 1. Therefore, the kinematic class is specified by giving the index k in

TABLE III. Change in the magnetic symmetry of domain walls in ferromagnets due to their motion (of the number k of the classes G_k corresponding to the same wall for $\mathbf{v} = 0$ and $\mathbf{v} \neq 0$).

Magnetic class	Kinematic class	Magnetic class	Kinematic class	Magnetic class	Kinematic class
1	4	14	15	28	25
2	6	15	15	29	42
3	6	16	16	30	30
4	4	17	18	31	31
5	15	18	18	32	30
6	6	19	19	33	30
7	10	20	20	34	31
8	16	21	19	35	35
9	10	22	19	36	35
10	10	23	20	37	37
11	16	24	24	38	38
12	16	25	25	39	37
13	16	26	24	40	37
		27	24	41	38
				42	42

TABLE IV. Symmetry and structure of moving domain walls in antiferromagnets.

$\mathbf{v}=0$		$\mathbf{v}\neq 0$								
k	Γ	k	Γ	Symmetry elements	M_{τ_1}	M_{τ_2}	M_n	L_{τ_1}	L_{τ_2}	L_n
7	A	10	A	$1,\ 2_1'(+)$	A	S	S	A	S	S
	B_1	13	B	$1,\ 2_2'(-)$	S	A	S	A	S	A
	B_2	7	B_2	$1,\ 2_1'(+),\ 2_2'(-),\ 2_n(-)$	O	O	S	A	S	O
	B_3	8	B	$1,\ 2_n(-)$	O	O	A, S	A, S	A, S	O
9	A_1	10	A	$1,\ 2_1'(+)$	A	S	S	A	S	S
	B_1	12	A''	$1,\ \bar{2}_2(-)$	O	A, S	O	A, S	O	A, S
	A_2	9	A_2	$1,\ 2_1'(+),\ \bar{2}_2(-),\ \bar{2}_n'(-)$	O	S	O	A	O	S
	B_2	11	A''	$1,\ \bar{2}_n'(-)$	S	S	A	A	A	S
17	A_1	18	A'	$1,\ \bar{2}_n'(+)$	S	S	A	S	S	A
	B_1	17	B_1	$1,\ \bar{2}_1(-),\ 2_2'(-),\ \bar{2}_n'(+)$	S	O	O	O	S	A
	B_2	13	B	$1,\ 2_2'(-)$	S	A	S	A	S	A
	A_2	12	A''	$1,\ \bar{2}_1(-)$	A, S	O	O	A, S	O	A, S

the case of ferromagnets and by giving the pair of indices k, Γ in the case of antiferromagnets. The change in symmetry of a domain wall due to its motion can be found in Table III (for ferromagnets). Knowing the index k of the kinematic class, one can use Tables I and III to find the qualitative form of the function $\mathbf{M}(\xi)$ in a moving domain wall for each particular case.

In the case of antiferromagnetic crystals we shall give only the kinematic symmetry tables for the high-symmetry rotating walls (see Table II). The kinematic classes of these domain walls and the corresponding forms of the functions $\mathbf{L}(\xi)$ and $\mathbf{M}(\xi)$ for these classes are given in Table IV.

It is seen in Table III that in the motion of Bloch (G_7) and Néel (G_9) domain walls in ferromagnets their symmetry is lowered to G_{10}, i.e., the distribution of the magnetization in the moving wall is a superposition of the distributions of walls G_7 and G_9. This familiar result was obtained by a direct calculation in Ref. 20.

A substantially different situation obtains in antiferromagnets (see Tables II and IV). Here, if the domain wall belongs to the classes $G_7(B_2)$, $G_9(A_2)$, $G_{17}(B_1)$, its motion does not lead to a change of symmetry. In all the remaining cases the kinematic class of the high-symmetry domain wall consists of only two elements, and the structure of the moving wall is therefore qualitatively different from that of the stationary domain wall.

The change in the domain-wall structure due to motion affects the value of the effective mass and the limiting velocity of the wall.[21]

4. DOMAIN WALLS IN PARTICULAR FERROMAGNETS AND ANTIFERROMAGNETS

Let us consider some examples of domain walls in various magnetically ordered crystals of the monoclinic, rhombic, and tetragonal systems.

Depending on the relationship between the constants which determine the energy of the magnet, it can have a great diversity of magnetic configurations. We shall consider only those cases in which the ferromagnetism (or antiferromagnetism) vector occupies some preferred direction in the crystal.

Ferromagnets of class $C_{2h}(2_z, \bar{2}_z)$

If the vector $\mathbf{M}$ in the crystal is directed along the z axis, the possible domain-wall symmetries are described by the classes with $k = 4, 6, 10, 15, 16$ for $\mathbf{n}\perp z(\boldsymbol{\tau}_1\|z)$ and the classes with $k = 15, 16, 18, 19, 20$ *for* $\mathbf{n}\|z$.

If the vector $\mathbf{M}$ is perpendicular to the z axis, domain walls belonging to the classes $k = 5, 8, 11, 15, 16$ can exist for $\mathbf{n}\|z$.

The walls with $k = 4, 5, 6, 15, 19, 20$ are pulsating walls, while walls with $k = 8, 10, 11, 16, 18$ are rotating walls; of these, only the wall with $k = 8$ can be characterized by a single rotation angle $\theta(z)$ of the magnetic moment: $\mathbf{M} = M_0(\boldsymbol{\tau}_1 \cos\theta + \boldsymbol{\tau}_2 \sin\theta)$. This domain wall is of the Bloch type but without a center, i.e., the function $\theta(z)$ has a symmetric (with respect to $z \to -z$) part as well as an antisymmetric part. The symmetric part is due to the term

$$-\lambda_1 \frac{dM_y}{dz}\frac{dM_x}{dz} = \frac{1}{2} M_0^2 \theta'^2 \sin 2\theta; \quad \theta' = \frac{d\theta}{dz} \tag{1}$$

in the invariant expansion of the energy density w of the magnetic subsystem of the crystal in powers of the ferromagnetism vector and its derivatives. Here the variational equation

$$\frac{\partial w}{\partial \theta} - \frac{d}{dz}\frac{\partial w}{\partial \theta'} = 0 \tag{2}$$

contains a term $\lambda_1 M_0^2(\theta'^2 \cos 2\theta + \theta'' \sin 2\theta)$ which breaks the invariance of this equation with respect to inversion of the coordinate z.

Ferromagnets of class $C_{2v}(2_z, \bar{2}_y, \bar{2}_x)$

The domain walls in crystals of this class can have the following symmetry.

If $\mathbf{M}(\pm\infty)\|z$, then $k = 9$–$12, 16$ for $\mathbf{n}\perp z$ and $k = 12, 16, 19, 22$ for $\mathbf{n}\|z$.

If $\mathbf{M}(\pm\infty)\|y$, then $k = 2, 6, 8, 12, 16$ for $\mathbf{n}\|z$, $k = 3, 6, 11, 13, 16$ for $\mathbf{n}\|x$, and $k = 13, 14, 16, 17, 18$ for $\mathbf{n}\|y$.

It is seen that the Néel domain wall ($k = 9$) in the ferromagnetic class C_{2v} can exist only for $\mathbf{M}(\pm\infty)\|z$. For the other directions of the magnetization vector in the crystal only centerless rotating domain walls of the Néel ($k = 12$)

and Bloch ($k = 8$) types can exist. For $\mathbf{n}\|z$, $\mathbf{M}(\pm\infty)\|y$, for example, the antisymmetry of the function $\theta(z)$ in the domain wall with $k = 8$ is broken by the following term in the free energy density w:

$$w_A=\frac{\lambda_2}{4}\left(M_y\frac{d^3M_y}{dz^3}-M_x\frac{d^3M_x}{dz^3}+\frac{dM_x}{dz}\frac{d^2M_x}{dz^2}-\frac{dM_y}{dz}\frac{d^2M_y}{dz^2}\right). \quad (3)$$

The variational equation accurate to derivatives of third order is

$$\frac{\partial w}{\partial\theta}-\frac{d}{dz}\frac{\partial w}{\partial\theta'}+\frac{d^2}{dz^2}\frac{\partial w}{\partial\theta''}-\frac{d^3}{dz^3}\frac{\partial w}{\partial\theta'''}=0. \quad (4)$$

Owing to the presence of invariant (3), viz., $w_A = \lambda_2 Q'''\sin 2\theta$, this equation acquires a term

$$\lambda_2(2\theta'^3\cos 2\theta+3\theta'\theta''\sin 2\theta)M_0^2, \quad (5)$$

with the result that inversion disappears from the symmetry group of Eq. (4). An analogous result for domain walls with $k = 12$ follows from the invariant obtained from (3) by the substitution $M_x \to M_z$.

Ferromagnets of class D_{2h}

Let the magnetization in the crystal be parallel to one of the twofold axes. In this case domain walls with the following types of magnetic symmetry can occur: $k = 1$–16 for $\mathbf{n}\perp\mathbf{M}(\pm\infty)$ and $k = 12$–23 for $\mathbf{n}\|\mathbf{M}(\pm\infty)$. Thus in the ferromagnetic class D_{2h} both the Bloch ($k = 7$) and Néel ($k = 9$) domain walls can be stable.

In antiferromagnetic crystals we consider only the high-symmetry rotating domain walls. As we have noted, the symmetry of these domain walls is determined by the particular magnetic structure of the antiferromagnet and the position of the domain walls with respect to the symmetry elements of the crystal.

Antiferromagnets with the orthoferrite structure ($2_z(+)$, $2_x(-)$, $\bar{1}(+)$, $t(+)$)

The energy density of the magnetic subsystem of these antiferromagnets can be approximated[3)] by the standard expression[16]

$$w=\frac{1}{2}\delta\mathbf{M}^2-\frac{1}{2}\beta_xL_x^2-\frac{1}{2}\beta_yL_y^2-\frac{1}{2}\beta_zL_z^2-d_yM_xL_y$$
$$-d_xM_yL_x-\frac{1}{2}\alpha_x\left(\frac{\partial\mathbf{M}}{\partial x}\right)^2-\frac{1}{2}\alpha_y\left(\frac{\partial\mathbf{M}}{\partial y}\right)^2-\frac{1}{2}\alpha_z\left(\frac{\partial\mathbf{M}}{\partial z}\right)^2. \quad (6)$$

Depending on the relationship between the phenomenological parameters appearing in this expression, three symmetric homogeneous phases are possible in zero magnetic field[16]:

$$\begin{aligned}&\text{I.}\quad \mathbf{L}=\pm L_0\mathbf{e}_z,\quad \mathbf{M}=0.\\&\text{II.}\quad \mathbf{L}=\pm L_0\mathbf{e}_x,\quad \mathbf{M}=\pm d_xL_0\mathbf{e}_y/\delta,\\&\text{III.}\quad \mathbf{L}=\pm L_0\mathbf{e}_y,\quad \mathbf{M}=\pm d_yL_0\mathbf{e}_x/\delta.\end{aligned} \quad (7)$$

The signs $\pm$ refer to different domains of the same phase. Phase I is antiferromagnetic, while phases II and III are weakly ferromagnetic.

The domain wall in orthoferrites have been well studied.[2,3] In each of the phases a domain wall can be situated in three different crystallographic planes, and each fixed position of the wall can correspond to two rotation planes of the vector $\mathbf{L}$. Thus eighteen different domain walls can exist. Let us list them. For all domain walls the rotation of the vector $\mathbf{L}$ within the domain wall is given in the leading approximation in the ratio of the relativistic forces to the exchange force by the equation

$$\sin\theta(\xi)=\pm\left[\mathrm{ch}\left(\frac{\xi}{\xi_0}\right)\right]^{-1}. \quad (8)$$

Let us first consider the domain walls in phase I:

a_I $\quad\mathbf{n}\|x,\quad \xi_0^2=\alpha_x/(\beta_z-\beta_y^*),$ (9)
$$\mathbf{L}=L_0(\mathbf{e}_z\cos\theta+\mathbf{e}_y\sin\theta),\quad \mathbf{M}=\frac{d_y}{\delta}L_0\mathbf{e}_x\sin\theta.$$

b_I $\quad\mathbf{n}\|x,\quad \xi_0^2=\alpha_x/(\beta_z-\beta_x^*),$ (10)
$$\mathbf{L}=L_0(\mathbf{e}_z\cos\theta+\mathbf{e}_x\sin\theta),\quad \mathbf{M}=\frac{d_x}{\delta}L_0\mathbf{e}_y\sin\theta.$$

c_I $\quad\mathbf{n}\|z,\quad \xi_0^2=\alpha_z/(\beta_z-\beta_x^*),$ (11)
$$\mathbf{L}=L_0(\mathbf{e}_z\cos\theta+\mathbf{e}_x\sin\theta),\quad \mathbf{M}=\frac{d_x}{\delta}L_0\mathbf{e}_y\sin\theta.$$

In addition, $a_\mathrm{I}'\mathbf{n}\|y$ is obtained from a_I by the substitution $x\leftrightarrow y$ in formula (9); $b_\mathrm{I}'\mathbf{n}\|y$ is obtained from b_I by the substitution $x\leftrightarrow y$ in (10); $c_\mathrm{I}'\mathbf{n}\|z$ is obtained from c_I by the substitution $x\leftrightarrow y$ in (11).

Phase II:

a_II $\quad\mathbf{n}\|z,\quad \xi_0^2=\alpha_z/(\beta_x^*-\beta_y^*),\quad \mathbf{L}=L_0(\mathbf{e}_x\cos\theta+\mathbf{e}_y\sin\theta),$ (12)
$$\mathbf{M}=L_0(d_x\mathbf{e}_y\cos\theta+d_y\mathbf{e}_x\sin\theta)/\delta.$$

b_II $\quad\mathbf{n}\|z,\quad \xi_0^2=\alpha_z/(\beta_x^*-\beta_z),$ (13)
$$\mathbf{L}=L_0(\mathbf{e}_x\cos\theta+\mathbf{e}_z\sin\theta),\quad \mathbf{M}=L_0\frac{d_x}{\delta}\mathbf{e}_y\cos\theta.$$

a_II' $\quad\mathbf{n}\|y,\quad \xi_0^2=\alpha_y/(\beta_x^*-\beta_z),$ (14)
$$\mathbf{L}=L_0(\mathbf{e}_x\cos\theta+\mathbf{e}_z\sin\theta),\quad \mathbf{M}=L_0\frac{d_x}{\delta}\mathbf{e}_y\cos\theta,$$

b_II' $\quad\mathbf{n}\|y,\quad \xi_0^2=\alpha_y/(\beta_x^*-\beta_y^*),$ (15)
$$\mathbf{L}=L_0(\mathbf{e}_x\cos\theta+\mathbf{e}_y\sin\theta),$$
$$\mathbf{M}=L_0(d_x\mathbf{e}_y\cos\theta+d_y\mathbf{e}_x\sin\theta)/\delta.$$

c_II $\quad\mathbf{n}\|x,\quad \xi_0^2=\alpha_x/(\beta_x^*-\beta_z),$ (16)
$$\mathbf{L}=L_0(\mathbf{e}_x\cos\theta+\mathbf{e}_z\sin\theta),\quad \mathbf{M}=\frac{d_x}{\delta}L_0\mathbf{e}_y\cos\theta.$$

c_II' $\quad\mathbf{n}\|x,\quad \xi_0^2=\alpha_x/(\beta_x^*-\beta_y^*),$ (17)
$$\mathbf{L}=L_0(\mathbf{e}_x\cos\theta+\mathbf{e}_y\sin\theta),$$
$$\mathbf{M}=L_0(d_x\mathbf{e}_y\cos\theta+d_y\mathbf{e}_x\sin\theta)/\delta.$$

The domain walls in phase III are obtained from expressions (12)–(17) by the substitution $x\leftrightarrow y$. In formulas (9)–(17) we have used the notation

$$\beta_x^*=\beta_x+d_x^2/\delta,\qquad \beta_y^*=\beta_y+d_y^2/\delta. \quad (18)$$

In walls of type a or a' the rotation of the vector $\mathbf{L}$ occurs in the plane of the domain wall (Bloch walls), while in walls of type b, b', c, c' the rotation occurs in a plane perpendicular to the plane of the domain wall (Néel and head-to-head walls).

The distribution of these domain walls over magnetic classes is given in Table V.

TABLE V. Magnetic symmetry classes of domain walls in orthoferrites.

a_I, a_I'	b_I, b_I'	c_I, c_I'	a_{II}	b_{II}	a_{II}'	b_{II}'	c_{II}	c_{II}'
$G_7(B_2)$	$G_9(A_2)$	$G_{17}(B_1)$	$G_7(B_1)$	$G_9(B_1)$	$G_7(B_3)$	$G_9(B_2)$	$G_{17}(A_2)$	$G_{17}(B_2)$

For moving domain walls in antiferromagnets with the orthoferrite structure, the kinematic symmetry class and, hence, the distribution of the vectors **M** and **L**, are given in Tables V and IV.

It is seen from these tables that only the domain walls in the antiferromagnetic phase do not change their symmetry upon motion, and the distribution of **M** and **L** in them is determined, as before, by formulas (9)–(11) with the definite functional form $\theta = \theta(\xi - vt)$. In weakly ferromagnetic phases, there is a lowering of the symmetry of the domain walls upon their motion.

Although each of the six domain walls in a weakly ferromagnetic phase has its own symmetry, only two of the domain walls have qualitatively different distributions of **M** and **L**.

The first type includes the walls a_{II}, b_{II}', and c_{II}', in which the vector **M** rotates in the same plane as the vector **L** as one goes from one domain to another. Upon the motion of domain walls of this type the vectors **M** and **L** leave the plane, and the component of **M** which arises thereby is a symmetric function of the coordinate, while the component of **L** is antisymmetric. It is important to note that these domain walls retain their center upon motion.

The second type includes the walls b_{II}, a_{II}', and c_{II}, in which the transition from one domain to the next occurs by means of a plane rotation of the vector **L**, but the magnetization vector here has only a single nonzero component, which is perpendicular to the plane of rotation of the vector **L** and varies only in modulus. New components of **M** and **L** do not arise upon the motion of these domain walls, but the walls become less symmetric—they lose their center.

The features of the motion of domain walls in weak ferromagnets were studied in Ref. 5. The theory developed in that study is in basic agreement with the results of the above analysis. There are slight differences due to the model approximations used in that study[5] for the form of the equation of motion and the energy (6).

TABLE VI. Magnetic symmetry classes of domain walls in antiferromagnets with the structure of fluorides of transition elements. The z axis is directed along the principal crystallographic axis, the x axis along an even twofold axis.

$L(\pm\infty)$	n	Magnetic classes
$L_0\mathbf{e}_x$	$\mathbf{e}_z$	$G_7(A)$, $G_9(A_1)$
	$\mathbf{e}_y$	$G_7(A)$, $G_9(A_1)$
	$\mathbf{e}_x$	$G_{17}(A_1)$
$\frac{L_0}{\sqrt{2}}(\mathbf{e}_x+\mathbf{e}_y)$	$\mathbf{e}_z$	$G_7(B_1)$, $G_9(B_1)$
	$\frac{1}{\sqrt{2}}(\mathbf{e}_x-\mathbf{e}_y)$	$G_7(B_3)$, $G_9(B_2)$
	$\frac{1}{\sqrt{2}}(\mathbf{e}_x+\mathbf{e}_y)$	$G_{17}(A_2)$, $G_{17}(B_2)$
$L_0\mathbf{e}_z$	$\mathbf{e}_z$	$G_{17}(A_1)$, $G_{17}(B_1)$
	$\frac{1}{\sqrt{2}}(\mathbf{e}_x+\mathbf{e}_y)$	$G_7(B_2)$, $G_9(A_2)$
	$\mathbf{e}_x$	$G_7(A)$, $G_9(A_1)$

Antiferromagnets with the structure of transition-element fluorides ($4_z(-)$, $2_x(+)$, $2_{xy}(-)$, $\bar{1}(+)$, $t(+)$)

The magnetic classes of the high-symmetry rotated domain walls in antiferromagnets with this structure are given in Table VI, from which it is seen that all the domain walls listed in Table II can occur here. Using Tables VI and IV, we arrive at the conclusion that in the easy-plane phase ($\mathbf{L}\perp\mathbf{e}_z$) the motion of a domain wall always lowers the symmetry of the latter and, accordingly, changes its spatial structure. As to the domain walls in the easy-axis phase ($\mathbf{L}\parallel\mathbf{e}_z$), here there are two fundamentally different possibilities. If the rotation of the antiferromagnetism vector in the domain wall occurs in a plane passing through the principal axis and an even twofold axis, the structure of the moving domain wall is qualitatively different from its structure for $\mathbf{v}=0$ (here the domain walls of both the Bloch and Néel types are transformed for $\mathbf{v}\neq 0$ into a "superposition" of Bloch and Néel walls). If the antiferromagnetism vector in the domain wall rotates in a plane passing through the principal axis and an odd twofold axis, the structure of the moving wall is qualitatively similar to that of the stationary wall.

A calculation of the static and dynamic properties of domain walls in easy-axis antiferromagnets having the structure of fluorides of transition elements was done in Ref. 21. The conclusions of the symmetry analysis agree with the results of this study. The only difference is that by virtue of the approximations made in Ref. 21, the calculated theoretical structure of the domain walls turned out to depend importantly on the particular crystallographic plane in which **L** rotated but was completely insensitive to a change in the direction of the normal **n**. This result is a consequence of the neglect of the inhomogeneous relativistic interactions.

It is especially noteworthy that, as was shown in Ref. 21, the limiting domain-wall velocity depends strongly on whether the spatial structure of the domain wall changes upon motion or remains unchanged. This is a good indication of the importance of taking into account the particular symmetry of ferromagnets and antiferromagnets in studying their domain structure.

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[1)]This is always true except for those rare cases in which the magnetic symmetry group of the magnetically ordered phase is not a subgroup of the magnetic symmetry group of the paramagnetic phase.

[2)]Such domain walls can occur in crystals with any symmetry, since in the absence of field the energy of the crystal is time-reversal invariant and, consequently, has a ground state which is at least twofold degenerate.

[3)]Here we are not treating the problem of domain walls in the presence of spin reorientation, so invariants of higher order are not needed.

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