Fluctuation effects in macroscopic dynamics of two-dimensional ferromagnets

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Ferromagnets described by the Landau-Lifshitz equation with damping taken into account are considered. An effective action is found that permits the thermal fluctuations to be taken into account in the form of a perturbation-theory series in the nonlinearities of the dynamic equation. It is shown that in the two-dimensional situation the fluctuations lead to logarithmic corrections to the theory parameters, viz., to the static "charge" g and to the coefficients A and B that characterize the real and imaginary parts of the spin-wave spectrum. There are no corrections to A and B in the one-loop approximation. The renormalization-group equations are obtained in the two-loop approximation. The behavior of A and B is analyzed on the basis of these equations in the region where perturbation theory holds.

INTRODUCTION

Long-wave dynamics of ferromagnets is described by the Landau-Lifshitz equation.[1] In the macroscopic treatment we can take into account the oscillations of only the direction of the spontaneous magnetization, which we characterize by a single three-component vector n_μ in spin space (critical dynamics of ferromagnets is a separate problem). We neglect spin-orbit interaction, so that the spin and spatial indices can be separated. The free-energy density takes in this situation the following simple form:

$$F=(T/2g)\,\nabla n_\mu \nabla n_\mu. \tag{1}$$

Here T is the temperature. We shall have in mind hereafter the two-dimensional situation in layered Heisenberg magnets, in which the exchange integral inside the layer can exceed the interlayer integral by several orders.

In the two dimensional case the constant g in (1) is dimensionless and determines the force of the fluctuations. In the case $g \ll 1$ perturbation theory holds for the model considered. Analysis within the framework of this theory[2] shows that g becomes logarithmically renormalized by the thermal fluctuations and increases with increase of the scale. We shall assume that in the system considered there exists a large region of wave vectors k, in which perturbation theory is valid. This region is bounded from above by the cutoff Λ, and from below by the spin-orbit interaction or by interlayer exchange or else by the fact that g becomes of the order of unity. The last restriction is no burden, in view of the logarithmic character of the renormalization. The two-loop approximation was considered in this situation in Ref. 3, where the following renormalization-group equation was obtained for g:

$$\frac{\partial g}{\partial \ln \Lambda}=\frac{1}{2\pi}g^2+\frac{1}{4\pi^2}g^3. \tag{2}$$

Neglecting the second term of the right-hand side of (2), we get hence

$$g=g_0\left(1-\frac{g_0}{2\pi}\ln\frac{\Lambda}{k}\right)^{-1}.$$

The complete dynamic equation, corresponding to (1), for the vector $\mathbf{n}$ is of the form (A and B are constants)

$$\frac{\partial \mathbf{n}}{\partial t}=A[\mathbf{n}\times\nabla^2\mathbf{n}]+B(\nabla^2\mathbf{n}+\mathbf{n}(\nabla n_\mu)^2)+\mathbf{f}. \tag{3}$$

The first term in the right-hand side of (3) is reactive, the second is dissipative, and $\mathbf{f}$ denotes the random forces whose correlator is

$$\langle f_\mu(t_1,\ \mathbf{r}_1) f_\nu(t_2,\ \mathbf{r}_2)\rangle=2gB(\delta_{\mu\nu}-n_\mu n_\nu)\delta(t_1-t_2)\delta(\mathbf{r}_1-\mathbf{r}_2). \tag{4}$$

The structure of the second term in the right-hand side of (3) and the tensor structure of the right-hand side of (4) are connected by the condition $\mathbf{n}^2=1$. The fluctuation effects in the system described by Eq. (3) were considered for case $B\to 0$ in Ref. 4.

EFFECTIVE ACTION

Applying now to Eq. (3) the procedure described in Refs. 5 and 6 we can construct the effective action

$$I=\int dt\, d^2r\left\{\mathbf{p}\left(\frac{\partial \mathbf{n}}{\partial t}-A[\mathbf{n}\times\nabla^2\mathbf{n}]-B\nabla^2\mathbf{n}\right)+igB\mathbf{p}^2+\ldots\right\}. \tag{5}$$

Here $\mathbf{p}$ is an auxiliary Bose field that satisfies the condition $\mathbf{p}\cdot\mathbf{n}=0$. The dots in (5) denote the terms that contain the auxiliary Fermi fields ψ and $\bar\psi$ (Refs. 5 and 6) that ensure normalization of the distribution function $\exp(iI)$. To calculate these terms explicitly we must introduce some parametrization of the fields $\mathbf{p}$ and $\mathbf{n}$, which have two independent components each, and then use the procedure of Refs. 5 and 6. The fluctuation effects can now be taken into account in the model considered by perturbation theory, the series for which is constructed with the aid of (5) and is represented by Feynman diagrams whose vertices are determined by the nonlinear terms in Eq. (3).

The unrenormalized Gren's function $\langle\psi\bar\psi\rangle$ is determined by the structure of the linearized equation (3). In the Fourier representation it has singularities with respect to frequency only in the lower half-plane (as should be the case for a generalized susceptibility). The determinant that appears on integration of $\exp(iI)$ with respect to ψ and $\bar\psi$ is represented by a sum of diagrams with closed Fermi loops. The integrals with respect to frequency, which correspond to these loops, are equal to zero by virtue of the aforementioned

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analytic properties of the $\langle\psi\bar{\psi}\rangle$ functions, except for the diagrams that contain only closed single $\langle\psi\bar{\psi}\rangle$ lines. The expressions corresponding to these diagrams, however, vanish on regularization. Thus, at least within the framework of perturbation theory, the aforementioned determinant is equal to unity, so that the dependence of the effective action on the Fermi fields ψ and $\bar{\psi}$ can be omitted. The action (5) is quadratic in **p**, so that the distribution function can be explicitly integrated over this field. As a result we arrive at a distribution function $\exp(iI)$ with the following effective action:

$$I=i\int dt\, d^2r\,(4gB)^{-1}\{(\partial\mathbf{n}/\partial t)^2-2A[\mathbf{n}\times\nabla^2\mathbf{n}]\partial\mathbf{n}/\partial t + (A^2+B^2)[(\nabla^2\mathbf{n})^2-(\nabla\mathbf{n})^2(\nabla\mathbf{n})^2]\}. \quad (6)$$

We shall parametrize the components of the unit vector **n** in the following manner:

$$\mathbf{n}=((2-x^*x)^{1/2}(x+x^*)/2,\ -i(2-x^*x)^{1/2}(x-x^*)/2,\ 1-x^*x). \quad (7)$$

Here x and x^* are complex conjugate fields. The parametrization (7) is convenient because the Jacobian of the transition from x and x^* to **n** with the condition $\mathbf{n}^2 = 1$ is equal to unity. Expanding now the action (6) up to second order in x, we obtain the following expression:

$$I^{(2)}=i\int dt\, d^2r\,(2gB)^{-1}\left\{\frac{\partial x^*}{\partial t}\frac{\partial x}{\partial t} + iA\left(\frac{\partial x}{\partial t}\nabla^2x^*-\nabla^2x\frac{\partial x^*}{\partial t}\right)+(A^2+B^2)\nabla^2x^*\,\nabla^2x\right\}. \quad (8)$$

We obtain thus in the quadratic approximation the following expression for the paired mean value of the Fourier components:

$$D(\nu,\mathbf{q})=\langle x(\nu,\mathbf{q})x^*(-\nu,-\mathbf{q})\rangle=\frac{2gB}{\nu^2-2A\nu q^2+(A^2+B^2)q^4}. \quad (9)$$

Here ν is the frequency and **q** the wave vector. The proper spectrum of the system is determined by the poles of (9) and takes the form

$$\nu=Aq^2-iBq^2. \quad (10)$$

Thus, A determines the real part of the spectrum of the spin waves, and B their damping. For the equal-time correlator we find

$$\int\frac{d\nu}{2\pi}D(\nu,q)=\frac{g}{q^2}. \quad (11)$$

This expression agrees, as it should, with that calculated with the aid of the free energy (1).

To carry out the renormalization the variables must be divided into fast and slow parts. We do this via the change of notation

$$n_\mu\to R_{\mu\nu}n_\nu. \quad (12)$$

Here R is an orthogonal matrix that depends on the slow variables, and the unit vector n_ν depends now on the fast variables x. Substitution of (12) in the action (6) reduces to a replacement of the derivatives by "covariant" ones:

$$\partial/\partial t\to\partial/\partial t+\Phi,\quad \nabla\to\nabla+\mathbf{a},$$

where

$$\Phi=R^T\partial R/\partial t,\quad \mathbf{a}=R^T\nabla R \quad (13)$$

(the superscript T denotes the transpose). Putting now $x = 0$, we obtain for the slow variables an effective action equivalent to (6):

$$I_s=i\int dt\, d^2r\,(4gB)^{-1}\{\Phi_+\Phi_- -iA[\Phi_+(\nabla\mathbf{a}_-+\mathbf{a}_0\mathbf{a}_+)-\Phi_-(\nabla\mathbf{a}_+-\mathbf{a}_0\mathbf{a}_-)] +(A^2+B^2)(\nabla\mathbf{a}_-+\mathbf{a}_0\mathbf{a}_+)(\nabla\mathbf{a}_+-\mathbf{a}_0\mathbf{a}_-)\}. \quad (14)$$

Here

$$\Phi_\pm=\Phi_{31}\mp i\Phi_{32},\quad \Phi_0=i\Phi_{12} \quad (15)$$

and analogously for **a**.

RENORMALIZATION GROUP

The renormalization procedure consists of integrating the distribution function $\exp(iI)$ with respect to the fast variables and representing the result in the form $\exp(iI_{ren})$, where I_{ren} differs from (14) because of the fluctuation terms due to the interaction of the fast and slow variables. The integration is by perturbation theory, in which the averaging is over a distribution function with action (8). Expanding the action (6) (with "covariant" derivatives) ion powers of x, we obtain numerous non-Gaussian terms, both those containing Φ and **a** and those containing only x and x^*. It turns out that the fluctuation corrections to I which stem from these expansion terms lead to a logarithmic renormalization of the parameters of the model, but no structurally new logarithmic terms appear, so that the action (6) is found to be renormalizable.

To obtain the renormalization-group equations in the one-loop approximation it suffices to expand the action I to second order in x. This integration gives rise to fluctuation terms represented by the diagram of Fig. 1, where the solid line corresponds to the D function (9), and the black dot denotes an interaction vertex with slow variables. In some of the terms corresponding to this diagram, the integral with respect to ν and q diverges at the upper limit; these terms should be omitted in the regularization. As a result we arrive at purely logarithmic integrals (see the Appendix) which recombine the coefficients (14). It turns out that all three terms in (14) are renormalized in the same manner, so that A and B remain constant in the one-loop approximation.

This makes it necessary to consider the two-loop approximation. To obtain the renormalization group in this approximation the action must be expanded up to fourth order in x. In the integration with respect to x these expansion terms generate in the action a fluctuation contribution

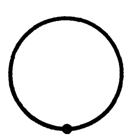

FIG. 1.

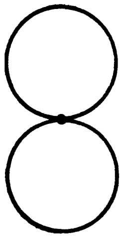

FIG. 2.

represented by the diagrams of Figs. 2 and 3. We note that the action fluctuation contribution represented by Fig. 4 (where the light circle denotes the vertex of the x self-action vertex) vanishes upon regularization, this being the consequence of the parametrization (7) assumed by us. The expression corresponding to the diagram of Fig. 2 is carried out in analogy with the regularization of the aforementioned terms. The regularization of the expression corresponding to Fig. 3 is somewhat more complicated and is considered in the Appendix. After the regularization, all the above contributions lead to a purely logarithmic renormalization of the parameters of the action (14).

Two-loop diagrams generate mutually canceling doubly logarithmic terms (this cancellation is an expression of the normalizationability theory). From these double logarithms one must separate the singly logarithmic term that causes the renormalization in the two-loop approximation; this is not a trivial manner. We carry out the calculations by the dimensional regularization method (see, e.g., Ref. 7) in the variant proposed in Ref. 8. In this method the (logarithmic) integrals are calculated in a space with dimensionality $d = 2 - \varepsilon$, thereby ensuring their convergence on the upper limit. Convergence on the lower limit is ensured by the pole-term substitution $q^2 \to q^2 + m^2$, where m has the dimensionality of the wave vector and determines the boundary between the fast and slow variables. It is also necessary to add in (6), (8), and (14) a factor Λ^{ε} to ensure their nondimensionality. After calculation in d-dimensional space, the limit as $\varepsilon \to 0$ must be taken.

The diagrammatic equations mentioned above are investigated in the Appendix. Calculation (in d-dimensional space) yields the following fluctuation corrections to the parameters of the action (14):

$$\Delta\frac{1}{gB} = -\frac{1}{2\pi\varepsilon B}\left(\frac{\Lambda}{m}\right)^{\varepsilon} + \frac{g}{8\pi^2\varepsilon B}\left(\frac{\Lambda}{m}\right)^{2\varepsilon}\operatorname{Re}\frac{B+iA}{B-iA}L, \tag{16}$$

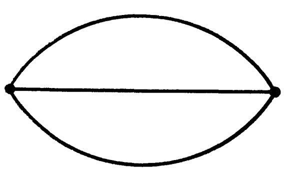

FIG. 3.

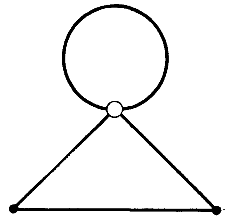

FIG. 4.

$$\Delta\frac{A}{gB} = -\frac{A}{2\pi\varepsilon B}\left(\frac{\Lambda}{m}\right)^{\varepsilon} + \frac{g}{8\pi^2\varepsilon B}\left(\frac{\Lambda}{m}\right)^{2\varepsilon}\operatorname{Re}(iB-A)L - \frac{gA}{8\pi^2\varepsilon B}\left(\frac{\Lambda}{m}\right)^{2\varepsilon}, \tag{17}$$

$$\Delta\frac{A^2+B^2}{gB} = -\frac{A^2+B^2}{2\pi\varepsilon B}\left(\frac{\Lambda}{m}\right)^{\varepsilon} - \frac{g(A^2+B^2)}{8\pi^2\varepsilon B}\left(\frac{\Lambda}{m}\right)^{2\varepsilon}L_1 - \frac{g(A^2+B^2)}{4\pi^2\varepsilon B}\left(\frac{\Lambda}{m}\right)^{2\varepsilon}. \tag{18}$$

Here

$$L = L_1 + iL_2 = \ln[1 - {}^1/_4(1 - iA/B)^2]. \tag{19}$$

As $\varepsilon \to 0$ the expressions in the right-hand sides of (16)–(18) diverge formally, this being due to the character of the assumed upper cutoff. We, however, are interested in the changes of g, A, and B following a shift of the real cutoff Λ, and these expressions remain finite as $\varepsilon \to 0$. The latter is ensured by the absence of terms $\propto \varepsilon^{-2}$ from the right-hand sides of (16)–(18) (by virtue of the renormalizability of the action).

RENORMALIZATION EQUATIONS

Differentiating (16)–(18) with respect to $\ln\Lambda$ and taking the limit as $\varepsilon \to 0$, we obtain the renormalization-group equations in the two-loop approximation. The resultant equation for g agrees with (2), as it should; the equations for A and B are:

$$\frac{\partial \ln A}{\partial \ln \Lambda} = -\left(\frac{g}{2\pi}\right)^2\left\{1 + \frac{b}{b^2+1}[2bL_1 + (b^2-1)L_2]\right\}, \tag{20}$$

$$\frac{\partial \ln B}{\partial \ln \Lambda} = -\left(\frac{g}{2\pi}\right)^2\left\{1 + \frac{1}{b^2+1}[(b^2-1)L_1 - 2bL_2]\right\}. \tag{21}$$

We introduced in these equations the dimensionless parameter $b = B/A$, the ratio of the imaginary and real parts of the spin-wave spectrum. The components of the logarithm L are expressed in terms of b as follows:

$$L_1 = \frac{1}{2}\ln\frac{9b^4 + 10b^2 + 1}{16b^4}, \quad L_2 = \operatorname{arctg}\frac{2b}{1+3b^2}. \tag{22}$$

From (20) and (21) we can obtain for b the equation

$$\frac{\partial \ln b}{\partial \ln \Lambda} = \left(\frac{g}{2\pi}\right)^2 (L_1 + bL_2). \tag{23}$$

We consider the case of weak damping, $b \ll 1$. In this case we obtain from (20) and (21)

$$\frac{\partial \ln A}{\partial \ln \Lambda} = -\left(\frac{g}{2\pi}\right)^2, \tag{24}$$

$$\frac{\partial \ln b}{\partial \ln \Lambda}=-\frac{g^2}{2\pi^2}\ln\frac{1}{b}. \tag{25}$$

Equation (24) agrees with the one in Ref. 4, although the statement made in that reference that the renormalization of $\ln A$ and $\ln H$ have the same character is incorrect ($\partial \ln H/\partial \ln A$ and (24) are of opposite sign). Equations (24) and (25) can be integrated if the second term in the right-hand side of (2) is neglected. In this case we get

$$A=A_0\exp(-g/2\pi), \tag{26}$$

$$\ln b^{-1}=\ln b_0^{-1}\exp(-g/\pi). \tag{27}$$

At small b_0 the value of b can thus increase considerably with increasing g. As already noted, however, too small values of b_0 cannot be expected; the lower limit is here the value $b_0\sim g_0^2$ obtained in Ref. 4.

We consider now the case of strong damping, $b\gg 1$. We then obtain from (20) and (21)

$$\frac{\partial \ln A}{\partial \ln \Lambda}=-\left(\frac{g}{2\pi}\right)^2\left(\frac{5}{3}-2\ln\frac{4}{3}\right), \tag{28}$$

$$\frac{\partial \ln B}{\partial \ln \Lambda}=-\left(\frac{g}{2\pi}\right)^2\left(1-\ln\frac{4}{3}\right). \tag{29}$$

In this limit, (3) is a pure diffusion equation, and (29) determines the law that governs the diffusion coefficient with increasing scale. This diffusion equation can be extensively generalized to a number of so-called nonlinear σ models, in which a renormalization equation similar to (29) is obtained for the diffusion coefficient (in the two-dimensional situation).[9] Equations (28) and (29) can be integrated if the second term in the right-hand side of (2) is neglected. The result is then similar to Eq. (26):

$$A=A_0\exp[-(5/3-2\ln 4/3)g/2\pi],\quad B=B_0\exp[-(1-\ln 4/3)g/2\pi]. \tag{30}$$

The technique developed has thus made it possible to investigate in detail, on the basis of the renormalization-group procedure, the behavior of the coefficients of the two-dimensional Landau-Lifshitz equation with damping in the region $g<1$ where perturbation theory is valid. The coefficient A, which determines the real part of the spin-wave spectrum, decreases with increasing scale, but this decrease cannot be appreciable in the region of validity of perturbation theory, by virtue of the condition $g<1$. The coefficient B, which determines the spin-wave damping, increases at small ratios B/A with increasing scale, although it does not manage to approach $\sim A$ in the region where perturbation theory is valid. At $B\gtrsim A$ the coefficient B decreases with increasing scale just as negligibly as A. We note that the ratio B/A always increases with increasing scale.

APPENDIX

The diagram shown in Fig. 1 gives rise to the following fluctuation contribution to the coefficient of the first term of (14):

$$\Delta\frac{1}{gB}=-\Lambda^\varepsilon\int\frac{d\nu\, d^dq}{(2\pi)^{d+1}}\frac{1}{gB}D(\nu,q). \tag{A.1}$$

Here $d=2-\varepsilon$. Calculating the integral with respect to the frequency ν, which reduces to a residue at a pole of (9), as well as integrating over the angles, we obtain, introducing the required cutoff,

$$\Delta\frac{1}{gB}=-\Lambda^\varepsilon\int_0^\infty\frac{S_dq^{d-1}\,dq}{(2\pi)^d(q^2+m^2)B}=\frac{1}{B}\left(\frac{\Lambda}{m}\right)^\varepsilon\frac{S_d}{(2\pi)^d}\frac{\pi/2}{\sin(\pi\varepsilon/2)}. \tag{A.2}$$

Here $S_d=2\pi^{d/2}/\Gamma(d/2)$ is the surface of a d-dimensional sphere. Retaining in this expression the leading term in the small ε, we obtain the first terms of the right-hand side of (16). In perfect analogy we obtain the first terms of the right-hand sides of (17) and (18). We consider now the fluctuation contribution to the coefficient of the first term of (14), a contribution due to diagram of Fig. 2:

$$\Delta\frac{1}{gB}=\Lambda^{2\varepsilon}\int\frac{d\nu_1\,d^dq_1\,d\nu_2\,d^dq_2}{(2\pi)^{2d+2}gB}D(\nu_1,\mathbf{q}_1)D(\nu_2,\mathbf{q}_2). \tag{A.3}$$

This integral is transformed in the same way as (A.1) and yields

$$\Delta\frac{1}{gB}=\frac{g}{B}\left(\frac{\Lambda}{m}\right)^{2\varepsilon}\frac{S_d^2}{(2\pi)^{2d}}\frac{(\pi/2)^2}{\sin^2(\pi\varepsilon/2)}. \tag{A.4}$$

In perfect analogy we obtain

$$\Delta\frac{A}{gB}=0,\quad \Delta\frac{A^2+B^2}{gB}=\frac{g(A^2+B^2)}{B}\left(\frac{\Lambda}{m}\right)^{2\varepsilon}\frac{S_d^2}{(2\pi)^{2d}}\frac{(\pi/2)^2}{\sin^2(\pi\varepsilon/2)}. \tag{A.5}$$

Much more difficult to analyze is the renormalization due to the diagram of Fig. 3. We consider first the contribution made to the coefficient of the first term of (14) and given by the integral

$$\Delta\frac{1}{gB}=-\Lambda^{2\varepsilon}\int\frac{d\nu_1\,d^dq_1\,d\nu_2\,d^dq_2}{(2\pi)^{2d+2}}\frac{(\nu_3-Aq_3)^2}{g^2B^2}\times D(\nu_1,\mathbf{q}_1)D(\nu_2,\mathbf{q}_2)D(\nu_3,\mathbf{q}_3). \tag{A.6}$$

Here $\nu_3=\nu_1+\nu_2-\omega$; $\mathbf{q}_3=\mathbf{q}_1+\mathbf{q}_2-\mathbf{k}$; ω and $\mathbf{k}$ are the external frequency and external wave vector. By virtue of the logarithmic character of the integral in (A.6) the dependence of ω and $\mathbf{k}$ can be left out of it. After integrating over the frequencies we obtain, introducing the required cutoff,

$$\Delta\frac{1}{gB}=-\Lambda^{2\varepsilon}\int\frac{d^dq_1\,d^dq_2}{(2\pi)^{2d}(q_1^2+m^2)(q_2^2+m^2)}\frac{g}{B}\times\mathrm{Re}\left[1+\frac{(1-iA/B)(Q^2-2q_3^2)}{Q^2-i(A/B)(Q^2-2q_3^2)}\right]. \tag{A.7}$$

Here $Q^2=q_1^2+q_2^2+q_3^2$. The first term in the integrand of (A.7) gives rise to a contribution that is exactly canceled out by (A.4). The second term in the integrand of (A.7) leads to a contribution $\propto\varepsilon^{-1}$, so that under our accuracy requirement the integration over the angle can be carried out in it in a space with $d=2$. The remaining integration with respect to

q_1 and q_2 can be easily carried out by changing to the variable q_1/q_2. As a result we obtain in the leading approximation in the small ε the second term of the right-hand side of (16).

The diagram of Fig. 3 causes also the following contribution to the integrand of (14):

$$-\frac{i}{2}\Lambda^{2\varepsilon}\Phi_+\int\frac{d\nu_1\,d^dq_1\,d\nu_2\,d^dq_2}{(2\pi)^{2d+2}(gB)^2}D(\nu_1,\mathbf{q}_1)D(\nu_2,\mathbf{q}_2)D(\nu_3,\mathbf{q}_3)$$
$$\times[\nu_3-A\mathbf{q}_3(\mathbf{q}_1+\mathbf{q}_2)][A(\nu_3\mathbf{q}_2+\nu_2\mathbf{q}_3)$$
$$-(A^2+B^2)(q_3^2\mathbf{q}_2+q_2^2\mathbf{q}_3)]\mathbf{a}_-. \quad (A.8)$$

The integral in (A.8) diverges formally at the upper limit, but this divergent part is eliminated by integrating over the angles. We must therefore separate in (A.8) that part of the integrand which is linear in $\mathbf{k}$ (we recall that $\mathbf{q}_3=\mathbf{q}_1+\mathbf{q}_2-\mathbf{k}$); the coefficient of $\mathbf{k}$ is now determined by a logarithmic integral. Recognizing that $\mathbf{k}$ is the wave vector of $\mathbf{a}_-$, i.e., $\mathbf{k}_i\cdot\mathbf{a}_-=-i\nabla_i\cdot\mathbf{a}_-$, we arrive at the conclusion that (A.8) renormalizes the second term of (14). It is technically simpler to integrate first over the frequencies and then separate that part of the result which is linear in $\mathbf{k}$. We then obtain, introducing a lower cutoff,

$$\Delta\frac{A}{gB}=-\Lambda^{2\varepsilon}\,\mathrm{Re}\int\frac{d^dq_1\,d^dq_2}{(2\pi)^{2d}}\frac{4g}{B}\left(1+i\frac{A}{B}\right)$$
$$\times\left[\frac{iB^2}{(q_1^2+m^2)(BQ^2+iA(Q^2-2q_3^2))}+\frac{Aq_2q_3}{d(q_1^2+m^2)(q_2^2+m^2)(q_3^2+m^2)}\right]. \quad (A.9)$$

The factor d^{-1} in the second term of (A.9) is due to the averaging over the angles in d-dimensional space. In (A.9) we can already leave out the dependence on ω and $\mathbf{k}$. The terms proportional to ε^{-2} in (A.9) cancel one another; the integration over the angles can therefore be carried out at our required accuracy in a space with $d=2$. The remaining integration with respect to q_1 and q_2 can be easily carried out by changing to the variable q_1/q_2. As a result we obtain the second and third terms of (17).

The calculation of the contribution due to the diagram of Fig. 3 leads to two terms. The first is determined by a logarithmic integral and yields

$$\Delta\frac{A^2+B^2}{gB}=\frac{2}{d}\Lambda^{2\varepsilon}\int\frac{d\nu_1\,d^dq_1\,d\nu_2\,d^dq_2}{(2\pi)^{2d+2}}\frac{A^2+B^2}{g^2B^2}q_3^2$$
$$\times[2A\nu_3-(A^2+B^2)Q^2]D(\nu_1,\mathbf{q}_1)D(\nu_2,\mathbf{q}_2)D(\nu_3,\mathbf{q}_3). \quad (A.10)$$

The factor d^{-1} in (A.10) is due to averaging over the angles in two-dimensional space. The integration over the frequencies reduces (A.10) to an expression similar to that obtained from (A.3), and leads to the result

$$\Delta\frac{A^2+B^2}{gB}=-\frac{4g(A^2+B^2)}{dB}\left(\frac{\Delta}{m}\right)^{2\varepsilon}\frac{S_d^2}{(2\pi)^d}\frac{(\pi/2)^2}{\sin^2(\pi\varepsilon/2)}. \quad (A.11)$$

The diagram of Fig. 3 leads also to the following contribution to the integrand of (14):

$$-\frac{i}{2}\int\frac{d\nu_1\,d^dq_1\,d\nu_2\,d^dq_2}{(2\pi)^{2d+2}g^2B^2}D(\nu_1,\mathbf{q}_1)D(\nu_2,\mathbf{q}_2)D(\nu_3,\mathbf{q}_3)$$
$$\times\mathbf{a}_+\{(A^2+B^2)[\mathbf{q}_3(q_1^2+q_2^2)+(\mathbf{q}_1+\mathbf{q}_2)q_3^2]-A\nu_3(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)\}$$
$$\times\{(A^2+B^2)(\mathbf{q}_3q_2^2+\mathbf{q}_2q_3^2)-A[\nu_1\mathbf{q}_2+\nu_2(\mathbf{q}_2+\mathbf{q}_3)]\}\mathbf{a}_-. \quad (A.12)$$

The integral in (A.12) diverges at the upper limit, and must therefore be regularized by subtracting from it expression (A.12) at $\omega=k=0$. The remainder must be expanded in terms of ω and k. The term linear in k is made to vanish by the integration over the angles, and the term linear in ω also vanishes, as can be verified after integrating it over the frequencies. We now must separate in (A.12) the term quadratic in k, whose coefficient is now determined by a logarithmic integral. Recognizing that $k_i\mathbf{a}_-=-i\nabla_i\mathbf{a}_-$, we arrive at the conclusion that the resultant term renormalizes the third term of (14). I is technically simpler to integrate first over the frequencies, and then separate that part of the expression which is quadratic in k. As a result we find, after introducing the required cutoff,

$$\Delta\frac{A^2+B^2}{gB}=4\Lambda^{2\varepsilon}\,\mathrm{Re}\int\frac{d^dq_1\,d^dq_2}{(2\pi)^{2d}}\frac{g(A^2+B^2)}{q_1^2+m^2}\frac{1}{BQ^2+iA(Q^2-2q_3^2)}. \quad (A.13)$$

It is necessary first to separate in this integral the part proportional to ε^{-2}, which turns out to be equal to (A.5). Subtracting (A.5) from (A.13) we can integrate over the angles in this difference, at our accuracy, in two-dimensional space and then evaluate the integral with respect to q_1 and q_2 by transforming to the variable q_1/q_2. The result is the second term of (18). The last term of (18) is obtained by adding (A.11) to (A.5) multiplied by two.

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