Anomalous spin magnetoresistance in the region of variable-range hopping conductivity

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The effect of many-particle spin interference on the electron tunneling probability is examined in the region of variable-range hopping conductivity. It is shown that, depending on the form of the wave functions of the impurities and the degree of doping, the many-particle interference can lead to either a positive or a negative giant magnetoresistance.

1. INTRODUCTION

In the region of variable-range hopping (VRH) conductivity the electrons execute tunneling hops over distances much larger than the average distance between impurities.[1]

The influence of impurity scattering of the tunneling electron on the asymptotic behavior of the electron wave function was studied in Refs. 2 and 3, where the single-particle problem was solved. In real semiconductor impurity bands, however, the population of the energy levels is always finite, and the many-particle nature of the quantum mechanical problem must be taken into account, particularly since it leads to qualitatively new effects.

The primary purpose of the present paper is to propose a new spin mechanism, which arises upon incorporation of many-particle effects, for the magnetoresistance in the VRH region in weak magnetic fields.

This mechanism can be understood qualitatively with the aid of Fig. 1, which shows three impurities. Suppose we are interested in the probability of an electron tunneling from impurity 1 to impurity 3. If impurity 1 is vacant (Fig. 1a) there are two tunneling paths: a direct hop from impurity 1 to impurity 3, with a probability amplitude of $A_{1\to3}$, and a compound hop involving an intermediate virtual state of the electron at impurity 2, with a probability amplitude of $A_{1\to2\to3}$. The total probability for the hop is

$$W_1=\frac{2\pi}{\hbar}\,|A_{1\to3}+A_{1\to2\to3}|^2.$$

Let us now consider the case in which impurity 2 is occupied (Fig. 1b); we shall assume that because of the Hubbard repulsion an impurity can be occupied by only a single electron. In addition, we shall for simplicity neglect the exchange interaction of the electrons at different impurities. The probability of the hop in question depends on the spin configuration of the two electrons in the initial state.

In fact, if the electron spins in the initial state are parallel, then the final spin states corresponding to the trajectories (1→3) and (2→3, 1→2) coincide (case 1 in Fig. 1b), and the total probability for the hop is again W_1. (Here the trajectories (2→3, 1→2) correspond to successive electron hops from impurity 2 to impurity 3 and then from 1 to 2.)

On the other hand, if the electron spins in the initial state are antiparallel (case 2 in Fig. 1b), the final spin configurations for trajectories (1→3) and (2→3, 1→2) are not the same—they are quantum-mechanically distinguishable—and the desired probability is therefore

$$W_2=\frac{2\pi}{\hbar}\,(|A_{1\to3}|^2+|A_{2\to3,1\to2}|^2).$$

The probabilities W_1 and W_2 thus differ by the interference term $2A_{1\to3}A^*_{2\to3,1\to2}$. A magnetic field alters the probability of the different spin configurations and thus affects the electron hopping probability, i.e., the conductivity.

The magnetoresistance mechanism outlined above is of a general nature for hopping conductivity, but it is manifested most clearly in the VRH region, where the conductivity is governed by tunneling hops over distances r which are much greater than the average distance between impurities, $N^{-1/3}$ (N is the impurity concentration). It was shown in Ref. 3 that for VRH the optimum number n of virtual electron hops and, hence, the number $\mathcal{N}$ of optimum tunneling trajectories are larger (see Fig. 2). In the absence of magnetic field the spins are disordered, and in an impurity band with finite compensation the different trajectories lead, with an overwhelming probability, to different final spin states, i.e., they do not interfere. The imposition of a magnetic field aligns the

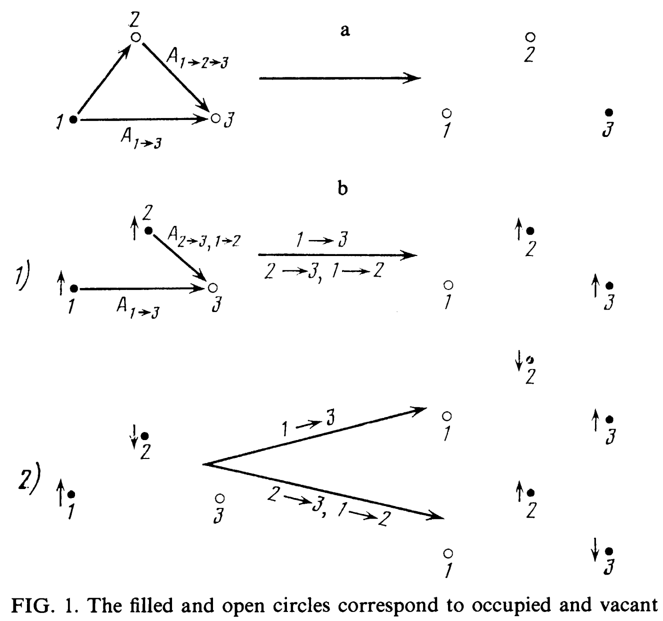

FIG. 1. The filled and open circles correspond to occupied and vacant donors.

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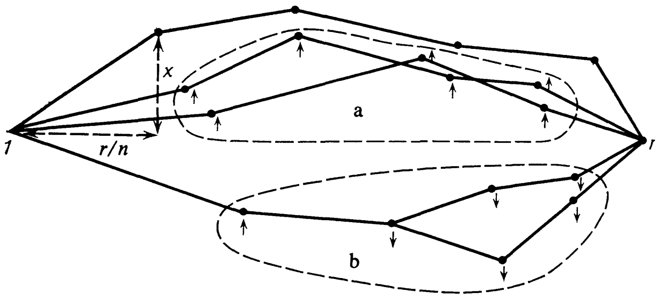

FIG. 2.

spins and increases the relative importance of the interfering paths. As a result, the hopping probability changes exponentially. In fact, in this region one speaks of a dependence of the localization radius on the magnetic field **H**.

Significantly, in the region of weak magnetic fields and low temperatures the magnetic-field dependence of the localization radius $\xi(H)$ is due entirely to spin effects, and the influence of the magnetic field on the orbital part of the wave functions can be neglected.

The material in this article is organized as follows. In Sec. 2 the case of light doping is considered, and it is shown that in the case when the impurities are described by a small-radius potential, the spin-interference correction to the localization radius has an H dependence which corresponds to a positive magnetoresistance. In Sec. 3 the case of intermediate doping is considered, and it is shown that in the critical region of the metal-insulator transition, the magnetoresistance due to spin interference changes sign and becomes negative.

We shall proceed from the impurity-band Hamiltonian of a doped semiconductor:

$$\hat{H}=\sum_{i\sigma}\varepsilon_i a_{i\sigma}a_{i\sigma}^{+}+\sum_{i\neq j}V_{ij}a_{i\sigma}^{+}a_{j\sigma}+U\sum_{i}a_{i\uparrow}a_{i\uparrow}^{+}a_{i\downarrow}a_{i\downarrow}^{+}. \quad (1)$$

Here $a_{i\sigma}^{+}$ and $a_{i\sigma}$ are the creation and annihilation operators for electrons at the ith impurity, V_{ij} is the matrix element for a transition between impurities i and j, U is the Hubbard repulsion energy at a single site, and ε_i is the energy of the single-particle state.

2. LIGHT DOPING CASE

For light doping, $Na_0^3 \ll 1$ (a_0 is the Bohr radius of an isolated impurity) the overlap of the wave functions of different impurities is small, and $|V_{ij}| \ll |\varepsilon_i - \varepsilon_j|$. We shall assume that the ε_i are random quantities distributed uniformly over the interval $(-\Delta, \Delta)$, where 2Δ is the width of the impurity band. In addition, we shall neglect spin exchange: $J \ll kT$; here $J \sim v_{ij}^2/U$ is the exchange integral and T is the temperature.

We shall show that the results depend rather strongly on the form of V_{ij}; let us begin with the case when the impurities are described by a small-radius potential, and

$$V_{ij}=-\frac{C}{r_{ij}}\exp\left(-\frac{r_{ij}}{a_0}\right). \quad (2)$$

Here $C \sim a_0 E_B$, where E_B is the Bohr energy, and $r_{ij}=|\mathbf{r}_i-\mathbf{r}_j|$, where $\mathbf{r}_i$ is the coordinate of the impurity. This case corresponds, for example, to a conductivity due to hopping via deep impurities in semiconductors.

a) The simplest case to consider is that of low compensation $K=N_A/N_D$ (N_A and N_D are the acceptor and donor concentrations), when the majority of the impurities are occupied by a single electron and the conductivity is governed by the hopping of the rare holes (by "hole" we mean a vacant impurity). If $\beta H \gg kT$ (β is the Bohr magneton) and all the electron spins are parallel, the problem of evaluating Δa (by definition, $\xi = a_0 + \Delta a$) reduces to the single-particle problem. Introducing the hole creation and annihilation operators $b_i^{+}=a_i$ and $b_i=a_i^{+}$ and using the commutation relations for a_i^{+} and a_i from (1), we obtain an iteration series for the wave function of a hole localized at impurity 1:[1)]

$$\Psi_1(\mathbf{r})=\Psi_1^{0}(\mathbf{r})+\sum_i\frac{\tilde{V}_{1i}\Psi_i^{0}(\mathbf{r})}{E_1-E_i}+\ldots, \quad (3)$$

where the Ψ_i^0 are the unperturbed wave functions of the holes at impurity i, E_i is the energy of the system with a hole at impurity i, and $\tilde{V}_{ij}=-V_{ij}$.

The scattering amplitude for a hole at impurity i is thus $\tilde{\mu}_i = C/(E_1-E_i)<0$ (because $E_i > E_1$ in the VRH region for the majority of the intermediate states). Each term in series (3) corresponds to a trajectory for the motion of a hole via impurities (see Fig. 2). It was shown in Ref. 2 (see also Ref. 3) that the result depends strongly on the value of the parameter $B=\tilde{\mu}^2 a_0 N$.

Let us begin with the case $B \ll 1$, for which[2]

$$\Delta a(H=\infty)=\tilde{\mu}a_0^3N. \quad (4)$$

The quantity $\Delta a=(H=\infty)$ corresponds to a complete alignment of the spins along the magnetic field.

It was shown in Ref. 3 that for $B \ll 1$ the main contribution to (3) is from trajectories with $n \sim r|\tilde{\mu}|a_0N$ scattering events ($\tilde{\mu} \sim -C/\Delta$) and with a deflection of the trajectory of the order of $x \sim (|\tilde{\mu}|N)^{-1/2}$ at each hop (see Fig. 2). In this case the number of scattering impurities in the "scattering cylinder" with base radius x and height r/n is $B^{-1} \gg 1$. This inequality ensures that the fluctuations of the wavefunction are relatively small (the continuum regime[3]) and makes it inconsequential whether it is the wavefunction itself or its logarithm that is averaged. (To evaluate the hopping conductivity it is necessary to know $\langle \ln|\Psi|^2\rangle$, where $\langle\ldots\rangle$ denotes an average over impurity configurations.[6])

For $\beta H \lesssim kT$ the electron spins are disordered, and different trajectories correspond to different final spin configurations. Here the total hopping probability is

$$W=\frac{2\pi}{\hbar}\sum_{\alpha}\Big|\sum_{\beta}A_{\beta\alpha}\Big|^2. \quad (5)$$

All the trajectories can be divided in groups, identified by index α, such that within each group all the trajectories correspond to the same final spin configuration. The index β enumerates the trajectories within a group. Examples of such groups are illustrated in Fig. 2; $A_{\beta\alpha}$ is the probability amplitude corresponding to the given trajectory.

It is important that the leading contribution to (5) is from groups of paths over impurities having the same spin direction (group a in Fig. 2). In fact, the probability amplitude corresponding to a single trajectory with n scattering events is of order[3]

$$A_0|\tilde{\mu}|^n\left(\frac{n}{r}\right)^n\exp\left(-\frac{x^2n^2}{ra_0}\right),$$

where x is the characteristic deflection of the trajectory in one hop (see Fig. 2): $A_0\sim e^{-r/a_0}$ is the probability amplitude of a forward hop, x^2n^2/r is the elongation (in comparison with a straight trajectory) of a trajectory having n scattering events with an average deflection of x.

The contribution to (5) from paths with n scattering evenst on impurities having parallel spins is

$$\mathcal{N}_1^2A_0^2\exp\left\{-2\frac{x^2n^2}{ra_0}\right\}|\tilde{\mu}|^{2n}\left(\frac{n}{r}\right)^{2n},$$

where $\mathcal{N}_1=B_1^n$ is the number of paths which pass over identical spins and have n scattering events with an average deflection of x; here $B_1^{-1}=(r/n)x^2N_\uparrow\gg 1$ is the number of impurities with parallel spins in the scattering cylinder, and $N_\uparrow$ is the concentration of electrons with spins up, for example. This expression has a maximum of the order

$$A_0^2\exp(2r\Delta a_\uparrow/a_0^2)\text{ with }\Delta a_\uparrow=N_\uparrow\tilde{\mu}a_0^3$$

at

$$n=r|\tilde{\mu}|a_0N_\uparrow,\quad x\sim(|\tilde{\mu}|N_\uparrow)^{-1/2}.$$

Here $B_1=\tilde{\mu}^2N_\uparrow a_0$. If $N_\uparrow=N$, we arrive at (4).

Paths over completely different spins do not (with an overwhelming probability) interfere, and their contribution to (5) is of the order of

$$\mathcal{N}A_0^2\left(\frac{|\tilde{\mu}|n}{r}\right)^{2n}\exp\left\{-2\frac{x^2n^2}{ra_0}\right\},$$

where

$$\mathcal{N}=B^{-n}=\left(\frac{r}{n}x^2N\right)^n.$$

Thus, the contribution from such trajectories is small, characterized by the parameter[2)]

$$\mathcal{N}/\mathcal{N}_1^2=(B_1^2/B)^n\ll 1.$$

One can, of course, have intermediate situations in which, for example, the first m scattering events occur at impurities with the same spins and all the rest at impurities with different spins (Fig. 2, group b). However, the contribution to (5) from such trajectories is also small.

Considering only trajectories over identical spins, we again arrive at the single-particle problem, and in analogy with Refs. 2 and 3 we obtain

$$\Delta a(H)=\frac{a_0^2}{r}\ln\left[\exp\left(\frac{r}{a_0^2}\Delta a_\uparrow\right)+\exp\left(\frac{r}{a_0^2}\Delta a_\downarrow\right)\right],$$

$$\Delta a_{\uparrow\downarrow}=\tilde{\mu}a_0^3N_{\uparrow\downarrow}. \tag{6}$$

Here

$$N_{\uparrow\downarrow}=N[1+\exp(\pm 2\beta H/kT)]^{-1} \tag{7}$$

is the concentration of up or down spins.

The large number of impurities in the scattering cylinder ensures[3] that the relative fluctuations of the wavefunction will be small at large values of r: $\delta\Psi/\Psi\sim B_1\ll 1$. At $H\to\infty$ formula (6) goes over to (4).

As we have said, at low compensations we have $\tilde{\mu}<0$ and $\Delta a(H)<0$. Aligning the spins increases the modulus of Δa, thereby leading to a positive magnetoresistance. This result is a consequence of the competition of two factors: first, aligning the spins leads to interference of the amplitudes corresponding to paths with the same n, and, second, because $\tilde{\mu}$ is negative, paths with different n begin to cancel one another.

Let us now turn to the case $B\gg 1$. For $\beta H\gg kT$ the problem again reduces to the single-particle problem considered in Ref. 3. In this case it is necessary that a typical scattering cylinder contain at least one or a few impurities, i.e.,

$$x^2(r/n)N\sim 1, \tag{8}$$

and that the number of trajectories giving the main contribution to (5) be of the order of z^n, where $z\gtrsim 1$. The contribution to (5) from all trajectories with n scattering events is of the order of

$$A_0^2\tilde{\mu}^{2n}z^{2n}\exp\left(-2\frac{x^2n^2}{ra_0}\right)\left(\frac{n}{r}\right)^{2n}. \tag{9}$$

Seeking the maximum of this expression under condition (8), we find that it is reached at $n\sim r(zNa_0)^{1/2}\ln^{1/2}B$ and

$$\Delta a(H=\infty)\sim a_0(zNa_0^3)^{1/2}\ln^{3/2}B. \tag{10}$$

Significantly, allowance for the large number of optimal paths gives only a factor $z\gtrsim 1$ in (10), and the fluctuations of the wavefunction at large r turns out to be large: $\delta\Psi/\Psi\sim B\gg 1$ (the fluctuation regime[3]).

For $\beta H\sim kT$ the spins are disordered, and the outcome of the competition between the two factors discussed above remains unclear, i.e., for $B\gg 1$ neither the magnitude nor even the sign of the magnetoresistance is known. From the physical standpoint this is due to the strong fluctuations of the wavefunction at large r, while from the formal standpoint it is due to the difficulties in averaging the logarithm of the wavefunction.

b) The case of arbitrary compensation. Below the Fermi energy all the impurity states are singly occupied, while above it they are all vacant. Now a trajectory can contain both an electron part (when an electron travels along vacant impurities) and a hole part (when a hole travels along occupied impurities). An example of such a trajectory is illustrated in Fig. 3.

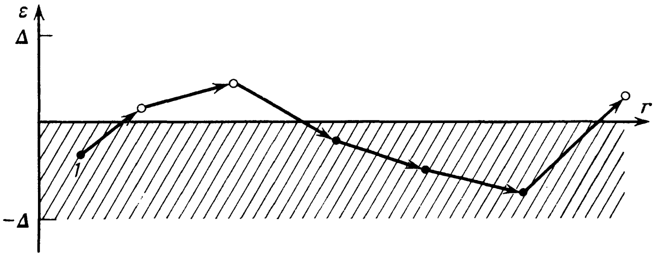

FIG. 3.

The conductivity in the VRH region is governed by hops of electrons in a narrow energy band $\delta\varepsilon \ll \Delta$, whereas the corrections to the localization radius are determined by all the states of the impurity band, and therefore one can assume that ε_1 is near the Fermi level and E_i is practically always greater than E_1 (E_i is the energy of the intermediate state).

By analogy with the above, one can see that each virtual hop of an electron to a vacant impurity having an energy higher than the Fermi level corresponds to a positive factor $V_{ij}(E_1 - E_i)$ in the amplitude, while each virtual hop of a hole to an initially occupied impurity corresponds to a negative factor $V_{ij}/(E_1 - E_j)$. As we have said, the additional factor of -1 stems from the anticommutation of Fermi operators.[4,5]

If $B \ll 1$, then the governing contribution to (5) is from trajectories containing hole parts moving via impurities with parallel spins, and the expression for $\Delta a(H)$ in this case is obtained from (6) by the substitution $N_{\uparrow\downarrow}\tilde{\mu} \rightarrow \nu_{\uparrow\downarrow}$, where

$$\nu_{\uparrow\downarrow} \sim N_D \frac{C}{\Delta} \left\{ K - (1-K) \left[1 + \exp\left(\pm 2 \frac{\beta H}{kT} \right) \right]^{-1} \right\} \qquad (11)$$

is the amplitude, averaged over the scattering cylinder, when the scattering on either only up spins or only down spins is taken into account.

Formula (11) is suitable for arbitrary degree of compensation and corresponds to a positive magnetoresistance.

The influence of a magnetic field on the orbital part of the wavefunction becomes important if $H^2(e^2/c^2\hbar^2)r^3a_0 > 1$ (Ref. 6), where $r \propto T^{-1/4}$ is the length of a hop in the VRH region. This effect can be neglected in comparison with the effect considered above at temperatures sufficiently low that $Na_0^3(a/r)^2(E_B/kT)^2 > 1$. If the opposite inequality holds, the spin effect in the magnetoresistance can nevertheless be distinguished with the aid of electron paramagnetic resonance.

3. INTERMEDIATE DOPING $Na_0^3 \sim 1$ AND THE CRITICAL REGION

1) The majority of experiments in the VRH regime have been done in the region $Na_0^3 \sim 1$, where the resistance of the sample is not too large. The most important thing is that in this region the spin exchange J is greater than kT and the spins are ordered.

For a qualitative understanding of this case it is useful to consider another case: $Na_0^3 \ll 1$, $kT \ll J$. The characteristic time for the tunneling of an electron a distance r is in order of magnitude $\tau_t \sim r(m/E_b)^{1/2}$, while the spin-flip time for the neighboring spins on account of exchange is $\tau_l \sim \hbar/J$.

If $\tau_t \ll \tau_l$, then during the tunneling time the spin will not flip on account of exchange, and expression (6) is valid. However, $N_\uparrow$ and $N_\downarrow$ must now be defined in terms of the total magnetic moment per unit volume, $\mathbf{M}(\mathbf{H})$, which is determined by the magnetic order of the spin subsystem at low temperatures and depends on the sign of J. For $Na_0^3 \lesssim 1$, it is most likely that $J < 0$ and the system forms a spin glass. Increasing $\mathbf{H}$ leads to an increase in $\mathbf{M}$, and we thus conclude that allowance for exchange in the case $B < 1$ will not alter the sign of the magnetoresistance—it will remain positive.

At large r, when $\tau_t > \tau_l$, many spin flips due to exchange will occur in the course of the tunneling. The magnetoresistance in this case evidently remains positive, but its magnitude is an open question.

2) A completely different situation arises in the critical region of the metal-insulator transition. The perturbation series (3) in this case is hard to sum. Nevertheless, one can make certain qualitative arguments concerning the nature of the wavefunction near the transition.

We shall assume that $K \ll 1$, $J < 0$ and the ground state of the spin subsystem is a spin glass.

The metal-insulator transition, generally speaking, occurs in different ways depending on the relationship between V_{ij} and U in the critical region, and we will be discussing mainly the case $V_{ij} \ll U$, i.e., $J \sim V_{ij}^2/U \ll V_{ij}$ (the case $V_{ij} \sim U$ is discussed in the Conclusion).

The disorder of the spin subsystem leads to additional scattering for holes, and at $H = 0$ even in the ordered Hubbard model the hole wavefunction is localized and falls off at large distances as $\exp(rN^{-1/3})$.[7] This assertion is particularly true for a system of randomly distributed impurities.

It should be kept in mind that at $H = 0$ and at small distances (but larger than $N^{-1/3}$) the motion of a hole gives rise to a spin-disordered region of radius $R \gg N^{-1/3}$. This phenomenon was studied in connection with the magnetic properties of antiferromagnetic semiconductors[8,9] and crystalline ^{3}He.[7]

In a disordered system of impurities the formation of such a spin polaron is possible either in the critical region $|(N - N_c)/N_c| \ll 1$ or in the metallic region $N \gg N_c$. Here N_c is the critical concentration of impurities for the metal-insulator transition in the case when the spins are completely polarized.

The energy of formation of a spin polaron in a disordered system is

$$\mathscr{E} = D\hbar/R^2 + N_c|J|R^3, \qquad (12)$$

where D is the coefficient of diffusion. The minimum of (12) is reached at

$$R \sim (\hbar D_0 a_0/N_c|J|)^{1/6}; \quad D_0 N_c^{-2/3} V_{ij}/\hbar. \qquad (13)$$

Expressions (12) and (13) are meaningful if $N^{-1/3} \ll R \ll \xi_0$, where[10]

$$\xi_0 \sim N^{-1/3} |(N - N_c)/N_c|^{\nu}$$

is the correlation radius of the hole wavefunction in the critical region under the condition that the spin subsystem is completely polarized.

In this case at distances of the order of R the metallic state does not differ from the insulating state, and the diffusion coefficient at scales of the order of $D \sim D_0 a_0/R$ (Ref. 10), where $\bar{V}_{ij}$ is the characteristic overlap integral in the critical region. The first term in (12) is the energy needed to localize a hole in a region of dimension R. Following Thouless,[11] we can estimate this energy as $\hbar/\tau_R$, where $\tau_R \sim R^2/D$ is the characteristic time for a hole to diffuse a distance R.[3)] The second term in (12) is the energy loss in the spin polarization of a volume R^3.[4)]

The imposition of a magnetic field polarizes the spins and decreases the first term in (12) by an amount of order $N_c R^3 (\beta H)^2/|J|$, and as a result

$$R(H) \sim \left[\frac{\hbar D}{N_c|J|(1-(\beta H/J))^2}\right]^{1/5}. \qquad (14)$$

Expression (14) has meaning for $\beta H \ll |J|$.

Thus $R(H)$ grows with increasing H, and at relatively high temperatures this dependence dictates a negative magnetoresistance. However, at low temperatures in the VRH region a hole undergoes hops over distances $r \gg R$. At such distances the spins are completely disordered, and therefore, as we have said, the exponential decay of the hole wavefunction at such distances is governed by the localization length $\xi \sim N^{-1/3} \ll R$.

Increasing the magnetic field leads to polarization of the spins and decreases the degree of disorder of the spin subsystem, while the mean free path of a hole increases. This should lead to growth of the localization radius:

$$\xi(H) - \xi(H=0) \sim \xi(\beta H/J)^2 \quad \text{for} \quad \beta H \ll |J|.$$

The situation here is analogous to the single-particle Anderson transition in a disordered system. In particular, there is a critical magnetic field $H_c \sim |J|/\beta$ such that for $|(H_c - H)/H_c| \ll 1$ one has $\xi(H) \sim N^{-1/3}|(H-H_c)/H_c)H_c|^\gamma$, where γ is the critical exponent, as long as $\xi(H) \ll \xi_0$. If $N > N_c, |(N-N_c)/N_c| \ll 1$, then for $H > H_c$ there is a metal-insulator transition, the detailed description of which is beyond the scope of this paper.

In the critical region of the metal-insulator transition the magnetoresistance is thus negative. The transition from a positive to a negative magnetoresistance occurs at $Na_0^3 \sim 1$.

The results presented above are valid at degrees of compensation all the way to order unity. The influence of magnetic field on the orbital part of the hole wavefunction can be neglected if $|J| \ll \bar{V}_{ij}$ in the critical region.

4. CONCLUSION

Allowance for spin interference effects thus leads to a dependence of the electron localization radius on the magnetic field and, hence, to a magnetoresistance. The magnitude and sign of the magnetoresistance in the VRH region is determined by the function $\xi(\mathbf{H})$,[1,6] i.e.,

$$\rho(T) = \rho_0 \exp\left(\frac{T_0}{T}\right)^{1/\eta}, \qquad T_0 \propto \xi^{1-\eta}. \qquad (15)$$

Here ρ is the resistivity, and η can be equal to two or four depending on whether or not the Coulomb gap is important.

In the case of deep impurities described by a short-range potential, the magnetoresistance turns out to be positive for light doping and $B \ll 1$. In the critical region $|(N-N_c)/N_c| \ll 1$ the magnetoresistance turns out to be negative. It is possible that the giant negative magnetoresistance observed in Ref. 12 is due to the mechanism considered in Sec. 3.

In conclusion we note that, strictly speaking, the results presented above are relevant to deep impurities having the wavefunction (2). One is readily convinced that the wavefunctions of shallow impurities for $Na_0^3 \ll 1$ are strongly fluctuating at large distances, in complete analogy with the case $B \gg 1$ considered in Sec. 2, and therefore neither the magnitude nor even the sign of the magnetoresistance for this case is known at the present time.

Satisfaction of the inequality $\bar{V}_{ij} \ll U$ in the critical region is also most realistic for a system of deep impurities.

The spin susceptibility of SiC$\langle N \rangle$ in the critical region near the metal-insulator transition was studied experimentally in Ref. 13. It had a Curie-Weiss character at all nitrogen concentrations clear up to the transition point. This circumstance indirectly confirms the possibility of such a metal-insulator transition regime.

In the case $|J| \sim \bar{V}_{ij}$ the influence of magnetic field on the orbital part of the wavefunction is apparently every bit as important as its influence on the spin part. For this reason the magnitude and sign of the magnetoresistance in this case remain unclear. Nevertheless, in this case the effects studied above can be distinguished from orbital effects by studying the hopping magnetoresistance under conditions of electron paramagnetic resonance. In this case the electron spins are disordered, and this changes the value of ξ, which, as we have said, depends on the magnetization $\mathbf{M}$. The contribution to the conductivity here turns out to be negative if the localization radius $\xi(H)$ calculated above increases with increasing H (corresponding to a negative magnetoresistance) and to be positive if $\xi(H)$ falls off with increasing H (corresponding to a positive magnetoresistance).

Apparently only positive changes in the conductivity have been observed under conditions of the electron paramagnetic resonance in doped semiconductors (see, e.g., Ref. 14). The detailed study of this phenomenon is beyond the scope of this paper.

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1) The change in the sign of $\bar{V}_{ij}$ in comparison with V_{ij} for a single particle is due to the anticommutation of Fermi operators. This circumstance is also extremely important in considering the ordered Hubbard model with infinite repulsion at a single site, and its leads to the situation that for certain types of lattices the ground state of a system having one vacancy, with the remaining sites singly occupied, corresponds to an unsaturated ferromagnet.[4,5]

2) It is easily verified that allowance for interfering paths of this type only will not change the localization radius from a_0 at all.

3) Interestingly, an estimate based on the assumption that the spin-polarized state contains a hole energy band with a width of order $\bar{V}_{ij}$ gives the same result (in order of magnitude).

4) Strictly speaking, in a disordered system the ground state of a spin-polarized region containing a hole is an incompletely saturated ferromagnet.[4,5] For making order-of-magnitude estimates, however, this circumstance is of no importance.

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