Effect of magnetic field on the photoproduction of electron-positron pairs

A. E. Lobanov and A. R. Muratov

Moscow State University
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The cross section for electron-positron production by two photons in a dc uniform magnetic field is obtained. It is shown that the magnetic field induces in the cross section oscillations having an amplitude considerably larger than the corrections determined from the perturbation-theory series.

We investigate in this article the production of electron-positron pairs by circularly polarized photons propagating counter to one another along a uniform dc magnetic field. This problem was already considered by many workers.[1–4] Their results, however, are valid in a limited range, so that no definite conclusions can be drawn concerning the effect of the magnetic field on this process.

The spectrum of the transverse motion of an electron in a magnetic field comprises a sequence of Landau levels. A change in the energy of the initial photons or of the magnetic field intensity alters jumpwise the phase space of the produced particles. This introduces into the cross section oscillating terms that cannot be obtained by perturbation theory with respect to the external field. In this paper we represent the reaction cross section as a sum of a monotonic part and an oscillating one. This yields specific estimates of the magnitudes of the effects and enables us to determine the conditions under which they can be observed.

The integral representation of the cross section σ for pair production can be obtained in the lowest order of perturbation theory in the quantum field by using the results of Ref. 5. In that article was calculated the polarization operator of a photon propagating in a superposition of a dc magnetic field and a plane electromagnetic wave. If the polarization operator is expanded in powers of the external-wave field intensity, the expansion term proportional to the square of the field intensity and taken on the mass shell is, apart from a coefficient, the amplitude for forward scattering of two photons in the magnetic field. In accordance with the optical theorem, the imaginary part of the amplitude determines the desired cross section. The result is (+ and − pertain to like and unlike photon polarizations, respectively)

$$\sigma^{\pm}=\pm r_0^2\frac{\eta}{\mu}\int\limits_{-1}^{1}d\beta\int\limits_{-\infty}^{\infty}\frac{dx}{\sin^2 x}\exp\left[-i\left(\beta\pm\frac{1}{\mu}\right)x\right]$$
$$\times\left\{\frac{\beta_+\beta_-}{\nu_+^2}e^{ix}\sin^2(x\beta_+\nu_+)\right.$$
$$-\frac{\beta_+\beta_-}{\nu_-^2}e^{-ix}\sin^2(x\beta_-\nu_-)$$
$$\left.-\left(\frac{\beta}{\nu_-\nu_+}+\frac{\beta\pm 1/\mu}{\nu_-^2\nu_+^2}\right)\sin(x\beta_+\nu_+)\sin(x\beta_-\nu_-)\right\}. \qquad (1)$$

Here m and e are the mass and charge of the electron, $r_0=e^2/4\pi m$ is its classical radius, ω amd ω' are the photon frequencies, $\eta=2\omega\omega'/eH$, $\mu=eH'm^2$, $\beta\pm=(1\pm\beta)/2$, $\nu_{\pm}=1\pm\beta\mp\eta^1$ The integrand in (1) has poles at the points $x=\pi n$, where n is an integer; these poles are understood to be bypassed from below.

The integration with respect to the variable x is by the method described in Refs. 6 and 7, so that $\sigma^{\pm}$ can be represented directly in the form of a sum of partial cross sections for the production of electrons and positrons on fixed Landau levels. Since the projection of the angular momentum on the magnetic-field direction is conserved, only a single summation is needed:

$$\sigma^-=\pi r_0^2 t\sum_{n=1}^{N_-}f_-(n),$$
$$f_-(x)=8\mu^2x(1-t\mu x)(1+2\mu x-t\mu^2)^{-2}(1-t+t^2\mu^2-2t\mu x)^{-1/2}, \qquad (2)$$
$$\sigma^+=\pi r_0^2 t\left\{\mu\frac{2-t+2t\mu+t^2\mu^2}{(1+2\mu+t\mu^2)^2(1-t)^{1/2}}+\sum_{n=1}^{N_+}f_+(n)\right\},$$

$$f_+(x)=\left\{\frac{2-t+t^2\mu^2(1+2\mu x)-2t\mu[1+2t\mu x(1+t\mu^2)]}{[1+2\mu(x-1)+t\mu^2]^2}\right.$$
$$+\frac{2-t+t^2\mu^2(1+2\mu x)+2t\mu[1+2t\mu x(1+t\mu^2)]}{[1+2\mu(x+1)+t\mu^2]^2}$$
$$\left.+\frac{16t^3\mu^4(1+t\mu^2)2\mu x}{[1+2\mu(x-1)+t\mu^2]^2[1+2\mu(x+1)+t\mu^2]^2}\right\}\frac{\mu}{(1-t-2t\mu x)^{1/2}}. \qquad (3)$$

We have used here the notation

$$a_+=\frac{1-t}{2t\mu},\quad a_-=\frac{1-t+t^2\mu^2}{2t\mu},$$

where $t=m^2/\omega\omega'$, $N_{\pm}=[a_{\pm}]$ is the number of states in which a pair is produced at a given energy of the initial photons, ([y] is the integer part of the number y). We note that pair production on the level $n=0$ takes place only if the photon polarizations coincide.

Equations (2) and (3) are convenient for the analysis of the cross section for the process in very strong magnetic fields ($\mu\gtrsim 1$), when the number of the terms in the sum is relatively small. If, however, $\mu\ll 1$, it is advantageous to transform these expressions by using the Abel-Plana summation formula (see, e.g., Ref. 8). This permits the cross section to be written naturally as a sum of a monotonic and an oscillating part:

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$$\sigma^{\pm}=\sigma^{\pm}_{\text{mon}}+\sigma^{\pm}_{\text{osc}}. \tag{4}$$

The use of the summation formula is valid by virtue of the analyticity of the functions $f_{\pm}(x)$ in the region $b_{\pm}<\text{Re}\,x<a_{\pm}$, where $b_{+}=1-(1+t\mu^2)/2\mu$, $b_{-}=-(1-t\mu^2)/2\mu$. For the sums in Eqs. (2) and (3) to contain at least one term, the condition $t<(1+2\mu)^{-1}$ (for σ^{+}) or $t<(1+2\mu-(1+4\mu)^{1/2})/2\mu^2$ (for σ^{-}) must be satisfied. Therefore $b_{-}<0$ and $\mu<2$ and $b_{+}<0$ at $\mu<1/2$, If these conditions are satisfied, then[2]

$$\sigma^{\pm}_{\text{osc}}=i\pi r_0^2 t\int_0^{\infty}\left[\frac{f_{\pm}(a_{\pm}-iy)}{\exp(2\pi(y+ia_{\pm}))-1}-\frac{f_{\pm}(a_{\pm}+iy)}{\exp(2\pi(y-ia_{\pm}))-1}\right]dy, \tag{5}$$

$$\sigma^{-}_{\text{mon}}=\pi r_0^2 t\left\{\left(2+ts_{-}-\frac{1}{2}t^2s_{-}^2\right)\ln\frac{1+(1-ts_{-})^{1/2}}{1-(1-ts_{-})^{1/2}}\right.$$
$$\left.-(4+ts_{-})(1-ts_{-})^{1/2}+i\int_0^{\infty}\frac{f_{-}(iy)-f_{-}(-iy)}{e^{2\pi y}-1}dy\right\}, \tag{6}$$

$$\sigma^{+}_{\text{mon}}=\pi r_0^2 t\left\{\frac{2\mu^2[t(s_{+}^2+4\mu^2)-2s_{+}(2-t+t^2\mu^2)]}{v(s_{+}^2-4\mu^2)^2}\right.$$
$$+\frac{vs_{+}(2+4t\mu^2-ts_{+}^2)}{s_{+}^2-4\mu^2}$$
$$\left.+\frac{ts_{+}}{4}(2-ts_{+})\ln\frac{(1+v)^2-t^2\mu^2}{(1-v)^2-t^2\mu^2}+i\int_0^{\infty}\frac{f_{+}(iy)-f_{+}(-iy)}{e^{2\pi y}-1}dy\right\}, \tag{7}$$

where $v=(1-t)^{1/2}$, $s_{\pm}\ 1\pm t\mu^2$.

We investigate first the monotonic part of the cross section. The asymptotic expansion of $\sigma^{\pm}_{\text{mon}}$ in powers of μ coincides with the perturbation-theory series. In relatively weak fields it is quite sufficient, for both a qualitative and a quantitative description of the process, to retain the first two terms of the series:

$$\sigma^{-}_{\text{mon}}=\pi r_0^2 t\left\{\left(2+t-\frac{t^2}{2}\right)\ln\frac{1+v}{1-v}\right.$$
$$\left.-(4+t)v-\mu^2\left[t^2v^2\ln\frac{1+v}{1-v}+\frac{4-12t+12t^2}{6v}\right]\right\},$$

$$\sigma^{+}_{\text{mon}}=\pi r_0^2 t\left\{t\left(1-\frac{t}{2}\right)\ln\frac{1+v}{1-v}+(2-t)v+\mu^2\left[t^2v^2\ln\frac{1+v}{1-v}\right.\right.$$
$$\left.\left.+\frac{8-26t+5t^2+24t^3-12t^4}{6v^3}\right]\right\}. \tag{8}$$

The term independent of the external field is the known Breit-Wheeler cross section. The correction to it, due to the presence of a magnetic field, is proportional to μ^2. When the photons have different polarizations, the coefficient of μ^2 is always negative; in the opposite case it reverses sign and becomes positive at $t\lesssim 0.42$. With decreasing photon energy, the value of the correction increases, and at $v\ll 1$ we have

$$\sigma^{-}_{\text{mon}}\approx\frac{8}{3}\pi r_0^2 v^3\left(1-\frac{\mu^2}{4v^4}\right),\quad \sigma^{+}_{\text{mon}}\approx 2\pi r_0^2 v\left(1-\frac{\mu^2}{12v^4}\right). \tag{9}$$

At low photon energies the effective parameters of the expansion is thus μ^2/v^4. To determine the behavior of the cross section at $v\sim\mu^{1/2}$ it is necessary again to use Eqs. (6) and (7), from which it follows that in this case

$$\sigma^{-}_{\text{mon}}=2\pi r_0^2(2\mu)^{3/2}\left\{\zeta\left(-\frac{1}{2},\frac{v^2}{2\mu}\right)-\frac{v^2}{2\mu}\zeta\left(\frac{1}{2},\frac{v^2}{2\mu}\right)\right\},$$
$$\sigma^{+}_{\text{mon}}=\pi r_0^2(2\mu)^{1/2}\left\{\frac{1}{2}\left(\frac{2\mu}{v^2}\right)^{1/2}-\zeta\left(\frac{1}{2},\frac{v^2}{2\mu}\right)\right\}, \tag{10}$$

where $\zeta(x,y)$ is the generalized zeta function (see e.g., Ref. 9). Substituting in (10) the value $v^2=2\mu$, we get

$$\sigma^{-}_{\text{mon}}\sim 2{,}5\pi r_0^2(2\mu)^{3/2},\quad \sigma^{+}_{\text{mon}}\sim 1{,}96\pi r_0^2(2\mu)^{1/2}. \tag{11}$$

Comparing (11) with (9) we easily verify that the approximate expression (9) works well even at the limit of applicability of Eqs. (6) and (7).

We proceed now to investigate the oscillating part of the cross section. At $\mu\ll 1$ it can be expanded in an asymptotic series of the form

$$\sigma^{\pm}_{\text{osc}}=\pi r_0^2 t\sum_{n=0}(2t\mu)^{n+1/2}C_n^{\pm}(t,\mu)\zeta\left(\frac{1}{2}-n,\{a_{\pm}\}\right). \tag{12}$$

Here $\{x\}=x-[x]$ is the fractional part of the number x. The expansion coefficients are given by

$$C_n^{-}(t,\mu)=2-t^2s_{-}^2+n[1-(1+ts_{-})^2]-\delta_{0,n}, \tag{13}$$
$$C_n^{+}(t,\mu)=\left\{[(n+1)(2-ts_{+}^2)+t\mu^2]\right.$$
$$\times[(1-t\mu)^{-2(n+1)}+(1+t\mu)^{-2(n+1)}]$$
$$\left.+t\mu s_{+}\left[2n+\frac{ts_{+}}{2}\right][(1-t\mu)^{-2(n+1)}-(1+t\mu)^{-2(n+1)}]\right\}\frac{t}{2}.$$

It follows from the properties of the zeta function that all but the first of the series (12) are regular, whereas the first term has a singularity at the point $\{a_{\pm}\}=0$. Taking this circumstance into account, we can write

$$\sigma^{-}_{\text{osc}}\approx\pi r_0^2 t(1-t^2)(2t\mu)^{1/2}R(a_{-}),$$
$$\sigma^{+}_{\text{osc}}\approx\pi r_0^2 t^2(2-t)(2t\mu)^{1/2}R(a_{+}), \tag{14}$$

where $R(a_{\pm})=\zeta(\frac{1}{2},\{a_{\pm}\}$. At small x

$$\zeta(1/2,x)\sim x^{-1/2}+\zeta(1/2)+O(x^{1/2})\quad(\zeta(1/2)\approx-1.46),$$

which yields the behavior of the cross section near the resonances, when the produced electron and positron have a zero c.m.s. momentum along the field. We note that the resonant behavior of the cross section for pair production in a magnetic field was first pointed out in Ref. 2.

Of course, the cross section always remains finite, and cutoff factors must be taken into account. At relatively high energies, the principal among these factors is the Landau-level broadening by synchrotron radiation. To estimate the cross-section oscillation amplitude we can use the radiation probability W_{synchr} obtained in the quasiclassical approximation.[10] If $\mu v\ll t^{1/2}$ the level width is $\Gamma=\frac{1}{2}W_{\text{synchr}}\approx\alpha v\mu m$, where α is the fine-structure constant. We have thus in order of magnitude

$$\max R(a_{\pm})\sim(\Gamma/t^{1/2}\mu m)^{-1/2}\sim[t/\alpha^2v^2]^{1/4}. \tag{15}$$

The estimate becomes unsatisfactory at $t\sim\alpha^2$, when the widths of the levels become comparable with their separations. This, however, is immaterial, for at such photon energies one cannot neglect the contribution of the diagrams that

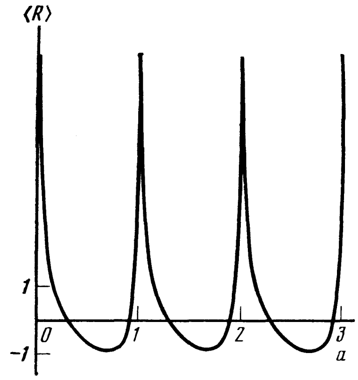

FIG. 1. Behavior of $\langle R(a)\rangle$ at $D\sim 10^{-3}$.

describe the emission from the produced particles, so that the equations derived no longer hold.[11]

At low energies of the initial photons, the spectrum of the produced pair is an assembly of series that result from the splitting of the Landau levels because of the Coulomb interaction of the electron and the positron. If $\alpha/v\ll 1$, which is the criterion for the validity of the results at low energies, we can estimate the amplitudes of the cross-section amplitudes to be equal to the width $\Gamma'\sim(\alpha/v)\mu m t$ of these series (see the review[12]). Then

$$\max R(a_\pm)\sim[v^2/t\alpha^2]^{1/4}. \tag{16}$$

We examine now the conditions needed for the observation of the effects of the magnetic field. Assume that the parameters $a_\pm$ have a distribution characterized by a probability density $\rho(a_0^\pm, D)$, where $a_0^\pm$ is the mean value and

$$D=(2t\mu)^{-2}\{(\Delta\Omega/\pi)^2+(\Delta\omega/\omega)^2+(\Delta\omega'/\omega')^2+(\Delta\mu/\mu)^2v^4\} \tag{17}$$

is the dispersion of $a_\pm$. In (17), $\Delta\omega$, $\Delta\omega'$, $\Delta\mu$ are respectively the mean squared deviations of the photon frequencies and of the magnetic field intensity, while $\Delta\Omega$ is the solid angle into which the photon beam is collimated. We shall assume that

$$D\ll(a_0^\pm)^2. \tag{18}$$

Let for the sake of argument the $a_\pm$ have a Gaussian distribution, i.e.,

$$\rho(a_0^\pm, D)=(2\pi D)^{-1/2}\exp[-(a_\pm-a_0^\pm)^2/2D]. \tag{19}$$

Taking the condition (18) into account, we obtain for the averaged value of the oscillating part of the cross section (see Fig. 1):

$$\langle R(a_\pm)\rangle=\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^{1/2}e^{-2D(\pi n)^2}\sin\left(2\pi n a_0^\pm+\frac{\pi}{4}\right). \tag{20}$$

It can be seen directly from (20) that at $D\gg 1$ the oscillation amplitude decreases exponentially with increasing dispersion. If, however $D\ll 1$, it can be readily shown that max $\langle R(a_\pm)\rangle\sim D^{-1/4}$. Thus, oscillations of the cross section can be observed if the condition $D\ll 1$ is satisfied.

Naturally, the mean value of the monotonic part of the correction is not zero also at $D\gtrsim 1$. A detailed analysis shows, however, that the measurement errors are in this case much larger than the monotonic correction, whose experimental observation is thus less likely.

Comparison of the results confirms that, depending on the photon energy and on the observation conditions, the monotonic and oscillatory terms are equally likely to predominate. We note that our results agree with the statement made in Ref. 13 that at nonrelativistic energies the oscillatory corrections predominate in a number of quantum-electrodynamic processes in an external field.

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[1] We use a system of units in which $c=\hbar=1$.
[2] Since $b_\pm<1$ for the indicated values of t and for arbitrary μ, it is easy to generalize Eqs. (5)–(7) to cover arbitrary values of the magnetic field intensity, but the resultant expressions are more unwieldy.

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