Theory of polarized radiation transfer occurring in cubic crystals located in a longitudinal magnetic field in the exciton resonance region

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The depolarization of resonant exciton radiation in a cubic crystal located in a longitudinal magnetic field is analyzed with allowance for the re-emission and multiple reflection of the light from the surface that occur under conditions of elastic exciton scattering by the impurities. An expression is found for the scattering matrix $\hat{P}_{\mathbf{H}}$ relating the Stokes parameters of the incident and scattered light for a single scattering event occurring in the magnetic field. The transfer equation is solved by the Chandrasekhar $\hat{S}$-matrix method, as generalized by E. L. Ivchenko, G. E. Pikus, N. Kh. Yuldashev [Sov. Phys. JETP **52**, 793 (1980)] by making allowance for the multiple reflection from the surface. The dependence of the back-scattered radiation polarization on the longitudinal magnetic field is computed for the case of excitation by linearly polarized light. It is shown that the radiation depolarization that occurs in a magnetic field under conditions of multiple scattering of the light by excitons is approximately given by the standard Hanle-effect formulas with two characteristic times: the effective lifetime ($\bar{\tau}_l$) and the effective alignment relaxation time ($\bar{\tau}_s$) of the exciton. A relation connecting $\bar{\tau}_l$ and $\bar{\tau}_s$ with the radiative and nonradiative exciton lifetimes is established.

§1. INTRODUCTION

In Ref. 1 Ivchenko, Pikus, and one of us construct a theory of polarized-resonance-radiation transfer in cubic crystals in the exciton region of the spectrum. There the case of weak exciton-photon interaction is considered, and, in particular, the polariton effects are neglected. The transfer of polarized radiation by polaritons is the subject of investigation in Ref. 2. It is shown in Ref. 1 that the multiple processes of reabsorption-re-emission of light and reflection from the inner crystal surface have an appreciable effect on the polarization and intensity of the scattered radiation, as well as on its spectral and angular distributions. In that case, as the quantity $\tilde{\omega}_0 = \tau_l/\tau_{\rm rad}$ (where $\tau_l^{-1} = \tau_0^{-1} + \tau_{\rm rad}^{-1}$, τ_0 and $\tau_{\rm rad}$ being the nonradiative and radiative exciton lifetimes), which determines the quantum yield of a single scattering event, increases, the degree of polarization of the radiation decreases, i.e., the re-emission and reflection processes constitute one of the mechanisms underlying effective spin relaxation.

As is well known, the Hanle effect, i.e., the change in polarization in a magnetic field, is widely used to determine the lifetimes and spin relaxation times under conditions of spin orientation.

The Hanle effect for excitons in the absence of re-emission is considered in Ref. 3 for cubic crystals and in Refs. 4 and 5 for uniaxial crystals (CdS and GaSe). In the simplest cases, e.g., for the $\Gamma_6 \times \Gamma_7$ exciton with angular momentum $J = 1$ in a cubic crystal, the longitudinal-magnetic-field ($\mathbf{H}_\parallel$) dependence of the linear polarization of the radiation in the coordinate systems X, Y and X', Y', rotated with respect to X, Y through an angle of $\pi/4$, is given by the formulas (Ref. 6, Table I):

$$\mathscr{P}'_{\rm lin}(H_\parallel) = \mathscr{P}'_{\rm lin}(0)\frac{1}{1+(\omega_\parallel\tau)^2}$$
$$\mathscr{P}''_{\rm lin}(H_\parallel) = -\mathscr{P}'_{\rm lin}(0)\frac{\omega_\parallel\tau}{1+(\omega_\parallel\tau)^2}. \tag{1}$$

Here $\hbar\omega_\parallel = g_\parallel \mu_0 H_\parallel$ is the Zeeman splitting of the exciton states with $m_z = \pm 1$ and τ is the half-life of the polarization ($\tau^{-1} = \tau_l^{-1} + \tau_s^{-1}$, τ_s being the spin-relaxation time). Here it is assumed that the exciting light is polarized along the X axis, and, consequently,

$$\mathscr{P}'_{\rm lin}(0) = \tau_s/(\tau_s+\tau_l) = \tau/\tau_l. \tag{1'}$$

In view of this the question arises whether it is possible to use the Hanle effect to determine τ_0 and $\tau_{\rm rad}$ under conditions of intense re-emission. Below we consider precisely the case of a longitudinal magnetic field, since the cylindrical symmetry $C_{\infty v}$ is then preserved, which enables us to obtain the exact solution with the aid of the Chandrasekhar $\hat{S}$-matrix method,[7] as generalized in Ref. 1 through the taking into account of the multiple reflection of the scattered light from the inner crystal surface.

We compute the linear polarization of radiation scattered without a change in its frequency under conditions of resonant excitation of the $J = 1$ triplet excitons in a cubic crystal by linearly polarized light. Here, as in Ref. 1, we neglect the polariton effect and the exciton diffusion (which is permissible when $\tau_r \ll \tau_l$, where τ_r is the momentum-relaxation time of the exciton), as well as the spin relaxation of the exciton ($\tau_l \ll \tau_s$). When the condition $\tau_r \ll \tau_l$ is fulfilled, we, unlike Silant'ev,[8] can neglect the Faraday effect, i.e., the rotation of the light's polarization plane during the propagation of the radiation in the crystal in the interval between two scattering events. Furthermore, we assume that the Zeeman splitting $\hbar\omega_\parallel$ of the $m_z = \pm 1$ exciton states in the magnetic

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field is small compared to the radiation-line width $\hbar\Gamma \approx \hbar/\tau_r$.

§2. THE BASIC EQUATIONS

The intensity and polarization of the radiation scattered in the crystal, and propagating in the direction of the unit vector $\mathbf{\Omega} = \mathbf{\Omega}(\theta,\varphi)$, where θ is the polar angle, determined by the directions of the vectors $\mathbf{n}^0$ and $\mathbf{\Omega}$ ($\mathbf{n}^0$ being the normal to the surface of the crystal), and φ is the azimuthal angle, will be represented by the Stokes matrix

$$\hat{I}(\Omega) = \begin{bmatrix} I_l \\ I_r \\ U \\ V \end{bmatrix}$$

(in the Chandrasekhar basis[7]). Here $I_l = d_{ll}$ and $I_r = d_{rr}$ are the intensities for the l and r polarizations, the unit basis vector $\mathbf{l}$ lies in the plane of the vectors $\mathbf{n}^0$ and $\mathbf{\Omega}$, the vector $\mathbf{r}$ is perpendicular to this plane, $U = 2\,\mathrm{Re}\, d_{lr}$, $V = 2\,\mathrm{Im}\, d_{lr}$, and $d_{ij} \propto \langle E_i E_j^* \rangle$. If the crystal is excited by monochromatic light that, after refraction at the surface, propagates in the direction $\mathbf{\Omega}_0$ (θ_0,φ_0), then the transfer equation for the radiation in the magnetic field can, when the above-indicated conditions are fulfilled, be written as:

$$\mu\frac{d\hat{I}(\Lambda,\mathbf{\Omega})}{d\Lambda} = \hat{I}(\Lambda,\mathbf{\Omega}) - \frac{\tilde{\omega}_0}{4\pi}\int_{-1}^{1} d\mu' \int_0^{2\pi} d\varphi' \hat{P}_H(\mathbf{\Omega},\mathbf{\Omega}')$$
$$\times \hat{I}(\Lambda,\mathbf{\Omega}') - \frac{\tilde{\omega}_0}{4}\exp\left(-\frac{\Lambda}{\mu_0}\right)\hat{P}_H(\mathbf{\Omega},\mathbf{\Omega}_0)\hat{F}(\mathbf{\Omega}_0),$$

where $\pi\hat{F}$ is the flux incident on a unit area normal to the direction of propagation of the light, $\mu = \cos\theta$, $\mu_0 = -\cos\theta_0 > 0$, and $\Lambda = \alpha z$, z being the distance from the crystal surface and α is the coefficient of absorption at the resonance frequency. The angular scattering matrix $\hat{P}_H(\mathbf{\Omega},\mathbf{\Omega}')$, being the kernel of the integral part of the equation, describes an elementary act of elastic scattering of the radiation in the magnetic field with excitation of an exciton in the intermediate state:

$$\hat{I}(\Lambda,\mathbf{\Omega}) = \hat{P}_H(\mathbf{\Omega},\mathbf{\Omega}')\hat{I}(\Lambda,\mathbf{\Omega}').$$

The matrix P_H is most easily determined by treating the exciton in the magnetic field as a classical oscillating dipole. In the case of resonant excitation by linearly polarized light, the dipole moment of the exciton is initially oriented along the polarization vector $\mathbf{e} = \mathbf{E}/|\mathbf{E}|$ of the light. Upon the application of a magnetic field, this dipole begins to rotate about the direction of the vector $\mathbf{H}_\parallel$ with angular frequency $\omega_\parallel/2$. In the absence of a magnetic field, the matrix $\hat{P}(\mathbf{\Omega},\mathbf{\Omega}')$ can, according to the formula (11) in Ref. 2, be written in the form

$$\hat{P}(\mathbf{\Omega},\mathbf{\Omega}') = {}^3/_2\hat{U}\hat{\Theta}(\mu,\varphi)\hat{\Theta}^+(\mu',\varphi')\hat{U}^{-1}, \quad (2)$$

where

$$\hat{U} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and $\hat{\Theta}(\mu,\varphi)$ is a 4×9 matrix with elements $\Theta_{\alpha\beta,\eta\xi}(\mathbf{\Omega}) = e_{\alpha\eta}(\mathbf{\Omega})e_{\beta\xi}(\mathbf{\Omega})$, the $e_{\alpha\eta}(\mathbf{\Omega})$ being the Cartesian components $(\eta = x,y,z)$ of the polarization vector $\mathbf{e}_\alpha$ $(\alpha = l,r)$ of the light propagating in the direction of $\mathbf{\Omega}$. In a longitudinal magnetic field the azimuthal angle φ' acquires in the time period t an increment equal[1)] to $\omega_\parallel t/2$. Taking account of the fact that the probability for radiative recombination of the exciton at the instant t is determined by the lifetime τ_l:

$$f(t) = \frac{1}{\tau_l}\exp(-t/\tau_l),$$

we can write the angular matrix describing the scattering of light by excitons in a longitudinal magnetic field in form

$$\hat{P}_H(\mathbf{\Omega},\mathbf{\Omega}') = \frac{1}{\tau_l}\int_0^\infty \hat{P}\left(\mu,\varphi;\mu',\varphi'+\frac{1}{2}\omega_\parallel t\right)\exp\left(-\frac{t}{\tau_l}\right)dt.$$

The matrix $\hat{P}_H$, like the matrix $\hat{P}$ in the formula (6) in Ref. 1, can be expanded in terms of matrices $\hat{P}^{(m)}$ that respectively depend only on $m\psi$, where $\psi = \varphi' - \varphi$:

$$\hat{P}_H(\mathbf{\Omega},\mathbf{\Omega}') = \hat{Q}\{{}^3/_4\hat{P}^{(0)}(\mu,\mu') + [(1-\mu^2)(1-\mu'^2)]^{1/2}$$
$$\times[\mathcal{P}_x'\hat{P}^{(1)}(\mathbf{\Omega},\mathbf{\Omega}')$$
$$+\rho_x'\hat{P}_H^{(1)}(\mathbf{\Omega},\mathbf{\Omega}')] + \mathcal{P}_x\hat{P}^{(2)}(\mathbf{\Omega},\mathbf{\Omega}') + \rho_x\hat{P}_H^{(2)}(\mathbf{\Omega},\mathbf{\Omega}')\}. \quad (3)$$

Here the matrices $\hat{Q}$, $\hat{P}^{(0)}$, $\hat{P}^{(1)}$, and $\hat{P}^{(2)}$ are given by the expressions in the formula (6) of Ref. 1, and the additional matrices $\hat{P}_H^{(1)}$ and $\hat{P}_H^{(2)}$ arising in the magnetic field are given by the formulas

$$\hat{P}_H^{(1)}(\mathbf{\Omega},\mathbf{\Omega}') = \frac{3}{4}\begin{bmatrix} -4\mu\mu'\sin\psi & 0 & 2\mu\cos\psi & 0 \\ 0 & 0 & 0 & 0 \\ -2\mu'\cos\psi & 0 & -\sin\psi & 0 \\ 0 & 0 & 0 & -\sin\psi \end{bmatrix},$$
$$\psi = \varphi' - \varphi, \quad (4)$$

$$\hat{P}_H^{(2)}(\mathbf{\Omega},\mathbf{\Omega}') = \frac{3}{4}\begin{bmatrix} -\mu^2\mu'^2\sin 2\psi & \mu^2\sin 2\psi & \mu^2\mu'\cos 2\psi & 0 \\ \mu'^2\sin 2\psi & -\sin 2\psi & -\mu'\cos 2\psi & 0 \\ -\mu\mu'^2\cos 2\psi & \mu\cos 2\psi & -\mu\mu'\sin 2\psi & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In (4)

$$\mathcal{P}_x' = \frac{1}{1+{}^1/_4(\omega_\parallel\tau_l)^2}, \quad \rho_x' = \frac{1}{2}\omega_\parallel\tau_l\mathcal{P}_x',$$

and

$$\mathcal{P}_x = \frac{1}{1+(\omega_\parallel\tau_l)^2}; \quad \rho_x = \omega_\parallel\tau_l\mathcal{P}_x.$$

It is convenient to represent the scattering matrix $\hat{P}_H(\mathbf{\Omega},\mathbf{\Omega}')$, given by the expression (3), in a factorized form, similar to the formula (2) for the matrix $\hat{P}(\mathbf{\Omega},\mathbf{\Omega}')$:

$$\hat{P}_H(\mathbf{\Omega},\mathbf{\Omega}') = {}^3/_2\hat{U}\hat{\Theta}(\mathbf{\Omega})\hat{K}\hat{\mathcal{M}}(h)\hat{K}^+\hat{\Theta}(\mathbf{\Omega}')\hat{U}^{-1}, \quad (5)$$

where $h = \omega_\parallel\tau_*$,

$$\hat{\mathscr{M}}(h)=\begin{bmatrix}\hat{\mathscr{M}}_2(h) & 0 & 0 & 0\\ 0 & \hat{E}_3 & 0 & 0\\ 0 & 0 & \hat{\mathscr{M}}_1(h) & 0\\ 0 & 0 & 0 & \hat{\mathscr{M}}_1(h)\end{bmatrix},$$

$$\hat{\mathscr{M}}_1(h)=\begin{bmatrix}\mathscr{P}_x' & -\rho_x'\\ \rho_x' & \mathscr{P}_x'\end{bmatrix},\quad \hat{\mathscr{M}}_2(h)=\begin{bmatrix}\mathscr{P}_x & -\rho_x\\ \rho_x & \mathscr{P}_x\end{bmatrix}, \tag{6}$$

$$\hat{K}=\begin{bmatrix}\hat{\Sigma}_1 & 0\\ 0 & \hat{E}_5\end{bmatrix},\quad \hat{\Sigma}_1=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 1 & 0\\ 0 & 1 & -1 & 0\\ -1 & 0 & 0 & 1\end{bmatrix},$$

$\hat{E}_3$ and $\hat{E}_5$ being 3×3 and 5×5 unit matrices. The matrix $\hat{P}_{\rm H}$ given by the formulas (3) and (5) is normalized to unity (see the formula (7) in Ref. 2). Let us note that the scattering matrix $\hat{P}_{\rm H}(\mathbf{\Omega},\mathbf{\Omega}')$ in a longitudinal magnetic field in the form (3) and (5) can also be derived on the basis of the kinetic equation for the exciton density matrix.

For a semifinite crystal the Stokes matrices of the radiation incident on, and the radiation scattered back in the opposite direction from, the inner crystal boundary ($z=+0$) are, just as in Ref. 1 [see the formula (15) there], connected by the relation

$$\hat{I}(+0,\mathbf{\Omega})=\frac{\tilde{\omega}_0}{4\mu}\hat{S}_H^{\hat{R}}(\mathbf{\Omega},\mathbf{\Omega}_0)\hat{F}(\mathbf{\Omega}_0).$$

The equation for the matrix $\hat{S}_{\rm H}^{\hat{R}}$ can be derived from the invariance principle formulated earlier [Ref. 1, formula (16)] for crystals, which remains valid in the presence of a magnetic field, since this field does not destroy the homogeneity of the medium. This equation differs from the formula (17) of Ref. 1 only in the replacement of $\hat{S}$ by $\hat{S}_{\rm H}$ and $\hat{S}^{\hat{R}}$ by $\hat{S}_{\rm H}^{\hat{R}}$:

$$\hat{S}_H^{\hat{R}}(\mathbf{\Omega},\mathbf{\Omega}_0)=\hat{S}_H(\mathbf{\Omega},\mathbf{\Omega}_0)+\frac{\tilde{\omega}_0}{4\pi}\int_0^1\frac{d\mu'}{\mu'}\int_0^{2\pi}d\varphi'\hat{S}_H(\mathbf{\Omega},\mathbf{\Omega}')\hat{R}(\mu')\hat{S}_H^{\hat{R}}(\mathbf{\Omega}',\mathbf{\Omega}_0). \tag{7}$$

Let us recall that the matrix $\hat{S}_{\rm H}$ relates the incident and scattered fluxes in a magnetic field without allowance for the light reflected from the inner surface. The equation determining the matrix $\hat{S}_{\rm H}$ also differs from the equation for $\hat{S}$ (Ref. 6) only in the replacement of $\hat{P}$ by $\hat{P}_{\rm H}$ and $\hat{S}$ by $\hat{S}_{\rm H}$:

$$\left(\frac{1}{\mu}+\frac{1}{\mu_0}\right)\hat{S}_{\rm H}(\mathbf{\Omega},\mathbf{\Omega}_0)=\hat{G}_{\rm H}(\mathbf{\Omega},\mathbf{\Omega}_0)+\frac{\tilde{\omega}_0}{4\pi}\int_0^1\frac{d\mu'}{\mu'}\int_0^{2\pi}d\varphi'\hat{S}_{\rm H}(\mathbf{\Omega},\mathbf{\Omega}')\hat{G}_{\rm H}(\mathbf{\Omega}',\mathbf{\Omega}_0), \tag{8}$$

where

$$\hat{G}_{\rm H}(\mathbf{\Omega},\mathbf{\Omega}')=\hat{P}_{\rm H}(\mathbf{\Omega},\overline{\mathbf{\Omega}}')+\frac{\tilde{\omega}_0}{4\pi}\int_0^1\frac{d\mu''}{\mu''}\int_0^{2\pi}d\varphi''\hat{P}_{\rm H}(\mathbf{\Omega},\mathbf{\Omega}'')\hat{S}_{\rm H}(\mathbf{\Omega}'',\mathbf{\Omega}'),$$

$\overline{\mathbf{\Omega}}=\overline{\mathbf{\Omega}}(-\mu,\varphi)$.

As indicated above, in a longitudinal magnetic field the matrices $\hat{S}_{\rm H}$ and $\hat{S}_{\rm H}^{\hat{R}}$, like $\hat{S}$ and $\hat{S}^{\hat{R}}$, are invariant under the operations of the symmetry group $C_{\infty v}$, while the invariance under time reversal leads to the relation

$$\hat{P}_{\rm H}(\mathbf{\Omega},\mathbf{\Omega}')=\hat{G}\hat{P}_{-{\rm H}}(\mathbf{\Omega}',\mathbf{\Omega})\hat{G}^{-1}$$

and similar relations for the matrices $\hat{S}_{\rm H}$ and $\hat{S}_{\rm H}^{\hat{R}}$. The form of the $\hat{G}$ matrix depends on the choice of the basis. For the Chandrasekhar basis it is given by the formula (23) of Ref. 1.

The changes that occur in the direction of propagation of the light and in the Stokes matrix upon the passage of the incident and scattered light through the crystal surface do not depend on the magnetic field, and are taken into account in the same way as is done in Ref. 1.

§3. SOLUTION OF THE EQUATION FOR THE MATRIX $S_{\rm H}$

Let us represent the matrix $\hat{S}_{\rm H}$ similarly to $\hat{P}_{\rm H}$ in the following factorized form:

$$\left(\frac{1}{\mu}+\frac{1}{\mu_0}\right)\hat{S}_{\rm H}(\mathbf{\Omega},\mathbf{\Omega}')={}^3/_2\hat{U}\hat{\Theta}(\mathbf{\Omega})\hat{\mathscr{H}}(\mu)\hat{K}\hat{\mathscr{M}}(h)\hat{K}^+\hat{\mathscr{H}}'^+(\mu')\hat{\Theta}^+(\mathbf{\Omega}')\hat{U}^{-1}. \tag{9}$$

Then the variables in Eq. (8) are separable, and for the new unknown, 9×9 matrices $\hat{\mathscr{H}}$ and $\hat{\mathscr{H}}'$, we obtain the integral equations:

$$\hat{\mathscr{H}}(\mu)=\hat{1}+\tilde{\omega}_0\mu\hat{\mathscr{H}}(\mu)\hat{\mathscr{M}}'(h)\int_0^1\frac{d\mu'}{\mu'+\mu}\hat{\mathscr{H}}'^+(\mu')\hat{\Psi}'(\mu'),$$

$$\hat{\mathscr{H}}'^+(\mu)=\hat{1}+\tilde{\omega}_0\mu\int_0^1\frac{d\mu'}{\mu'+\mu}\hat{\Psi}'(\mu')\hat{\mathscr{H}}(\mu')\hat{\mathscr{M}}'(h)\hat{\mathscr{H}}'^+(\mu), \tag{10}$$

$$\hat{\Psi}'(\mu)=\frac{3}{8}\int_0^{2\pi}d\varphi\hat{\Theta}^+(\mathbf{\Omega})\hat{\Theta}(\mathbf{\Omega}),\quad \hat{\mathscr{M}}'=\hat{K}\hat{\mathscr{M}}\hat{K}^+.$$

The characteristic matrix $\hat{\Psi}'(\mu)$, like the matrix $\hat{\mathscr{M}}'$, is real, and is an even function of μ. With the aid of the unitary transformation $\hat{D}^+\hat{\Psi}'\hat{D}=\hat{\Psi}$, with

$$\hat{D}=\begin{bmatrix}\hat{\Sigma}_1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \hat{\Sigma}_2\end{bmatrix},\ \text{where}\ \hat{\Sigma}_2=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & -1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix},$$

and $\hat{\Sigma}_1$ is given by the formula (6), we can reduce the matrix $\hat{\Psi}'$ to a quasidiagonal matrix $\hat{\Psi}$ (see Ref. 2). Performing the $\hat{D}$ transformation in (10), we find

$$\hat{H}(\mu)=1+\tilde{\omega}_0\mu\hat{H}(\mu)\hat{M}(h)\int_0^1\frac{d\mu'}{\mu'+\mu}\hat{H}'^+(\mu')\hat{\Psi}(\mu'), \tag{11}$$

where the matrix $\hat{M}=\hat{D}^+\hat{\mathscr{M}}'\hat{D}$ is obtained from the matrix $\hat{\mathscr{M}}$ given by the formula (6) by replacing the last $\hat{\mathscr{M}}_1(h)$ block by $\mathscr{M}_1(h)$. The solution to Eq. (11) has the form

$$\hat{H}(\mu)=\begin{bmatrix}\hat{H}^{(2)} & 0 & 0 & 0 & 0\\ 0 & H_v^{(0)} & 0 & 0 & 0\\ 0 & 0 & \hat{H}^0 & 0 & 0\\ 0 & 0 & 0 & \hat{H}^{(1)} & 0\\ 0 & 0 & 0 & 0 & \tilde{\hat{H}}^{(r)}\end{bmatrix}. \tag{12}$$

Here the matrices $\hat{H}^{(0)}$ and $H_v^{(0)}$ do not depend on the magnetic field, and the matrix

$$\hat{H}^0=\begin{bmatrix}\hat{H}^{(0)} & 0\\ 0 & H_v^{(0)}\end{bmatrix}$$

coincides with the $\hat{H}^0$ matrix given by the formula (31) of Ref. 1; $\hat{H}^{(1)}$, $\hat{H}^{(r)}$, and $\hat{H}^{(2)}$ are 2×2 matrices with components:

$$\hat{H}^{(i)}=\begin{bmatrix}H_1^{(i)} & -H_2^{(i)}\\ H_2^{(i)} & H_2^{(i)}\end{bmatrix}.$$

The matrix $\hat{H}'^{+}$, the equation for which is easily obtained from (10) in much the same way as (11) is derived, also has the form (12), with $\hat{H}'^{0+}(\mu)=\hat{H}^0(\mu)$ and a magnetic-field dependent part satisfying the condition $\hat{H}'^{+}(\mu)\equiv\hat{H}(\mu)$. Consequently, the solution of the system of equations (10) for $\mathscr{H}$ and $\mathscr{H}'$ reduces to the solution of three independent systems of two equations for the $\hat{H}^{(i)}(\mu)$-matrix elements.

The thus obtained solution to Eq. (8) can be written in the form

$$\hat{S}_H(\Omega,\Omega_0)=\hat{Q}\{{}^3/_4\hat{S}^{(0)}(\mu,\mu_0)+[(1-\mu^2)(1-\mu_0^2)]^{1/2}\times[\hat{S}_1^{(1)}(\mu,\mu_0)\hat{P}^{(1)}(\Omega,\bar{\Omega}_0)+\hat{S}_2^{(1)}(\mu,\mu_0)\hat{P}_H^{(1)}(\Omega,\bar{\Omega}_0)]+\hat{S}_1^{(2)}(\mu,\mu_0)\hat{P}^{(2)}(\Omega,\bar{\Omega}_0)+\hat{S}_2^{(2)}(\mu,\mu_0)\hat{P}_H^{(2)}(\Omega,\bar{\Omega}_0)\}. \quad (13)$$

This expression is equivalent to (9). For $H_{\parallel}=0$ it goes over into the formula (29) in Ref. 1. In (13), $\hat{S}^{(0)}(\mu,\mu_0)$ is the matrix (20) in Ref. 1, while

$$\hat{S}_1^{(1)}=\begin{bmatrix}\hat{S}_1^{(1)}\hat{E}_3 & 0\\ 0 & S_1^{(r)}\end{bmatrix},\quad \hat{S}_2^{(1)}=\begin{bmatrix}\hat{S}_2^{(1)}\hat{E}_3 & 0\\ 0 & S_2^{(r)}\end{bmatrix}. \quad (14)$$

The functions $S_j^{(i)}(h,\mu,\mu_0)$ (where $i=1,2$; r and $j=1,2$) in (13) and (14) combine into matrices of the form

$$\hat{S}^{(i)}=\begin{bmatrix}S_1^{(i)} & -S_2^{(i)}\\ S_2^{(i)} & S_1^{(i)}\end{bmatrix}, \quad (15)$$

which can be expressed in terms of the corresponding $\hat{H}^{(i)}$ matrices by the relations

$$\hat{S}^{(i)}(h,\mu,\mu_0)=\frac{\mu\mu_0}{\mu+\mu_0}\hat{H}^{(i)}(\mu)\hat{\mathscr{M}}^{(i)}(h)\hat{H}^{(i)}(\mu_0), \quad (16)$$

where $\hat{\mathscr{M}}^{(1)}=\hat{\mathscr{M}}^{(r)}=\hat{\mathscr{M}}_1$, $\hat{\mathscr{M}}^{(2)}=\hat{\mathscr{M}}_2$, and $\hat{\mathscr{M}}_1$ and $\hat{\mathscr{M}}_2$ are given by the formulas (6). In the absence of a magnetic field (i.e., for $h=0$) $S_2^{(2)}=S_2^{(1)}=S_2^{(r)}=0$ and $S_1^{(2)}, S_1^{(1)}$, and $S_1^{(r)}$ go over into the functions $S^{(2)}$, $S^{(1)}$, and $S_{44}^{(1)}$ of Ref. 1.

§4. SOLUTION OF THE EQUATION FOR THE MATRIX S_H^R

With the aid of a number of transformations, we can factorize the matrix integral equation (7), and reduce it to simpler equations more convenient for iteration. For this purpose we represent the solution to (7) in a form similar to (13):

$$\hat{S}_H{}^{\hat{R}}(\Omega,\Omega_0)=\hat{Q}\left\{\frac{3}{4}\hat{S}_R^{(0)}(\mu,\mu_0)+[(1-\mu^2)(1-\mu_0^2)]^{1/2}\hat{S}_R^{(1)}(\Omega.\ \Omega_0)+\hat{S}_R^{(2)}(\Omega,\Omega_0)\right\}. \quad (17)$$

By substituting (17) into (7), we can verify that the first term $\hat{S}_R^{(0)}(\mu,\mu_0)$ of the solution, like the $\hat{S}^{(0)}(\mu,\mu_0)$ term in (13), does not depend on the magnetic field, and is given by the formula (35) in Ref. 1, while the matrices $\hat{S}_R^{(m)}$ $(m=1,2)$ satisfy the equations

$$\hat{S}_R^{(m)}(\Omega,\Omega_0)=\hat{S}_1^{(m)}(\mu,\mu_0)\hat{P}^{(m)}(\Omega,\bar{\Omega}_0)+\hat{S}_2^{(m)}(\mu,\mu_0)\hat{P}_H^{(m)}(\Omega,\bar{\Omega}_0)+\frac{\tilde{\omega}_0}{4\pi}\times\int_0^1\frac{d\mu'}{\mu'}\int_0^{2\pi}d\varphi'\Phi^{(m)}(\mu')[\hat{S}_1^{(m)}(\mu,\mu')\hat{P}^{(m)}(\Omega,\Omega')+\hat{S}_2^{(m)}(\mu,\mu')\hat{P}_H^{(m)}(\Omega,\Omega')]\hat{R}(\mu')\hat{Q}\hat{S}_R^{(m)}(\Omega,\Omega_0), \quad (18)$$

where $\Phi^{(1)}(\mu)=1-\mu^2$, $\Phi^{(2)}=1$. We can show that the solution to this equation for $\hat{S}_R^{(1)}$ has the following form:

$$\hat{S}_R^{(1)}(\Omega,\Omega_0)=\begin{bmatrix}B_1(\mu,\mu_0)\hat{E}_3 & 0\\ 0 & B_4(\mu,\mu_0)\end{bmatrix}\times\hat{P}^{(1)}(\Omega,\bar{\Omega}_0)-\begin{bmatrix}B_2(\mu,\mu_0)\hat{E}_3 & 0\\ 0 & B_3(\mu,\mu_0)\end{bmatrix}\times\hat{P}_{R1}^{(1)}(\Omega,\bar{\Omega}_0)+\begin{bmatrix}C_1(\mu,\mu_0)\hat{E}_3 & 0\\ 0 & C_4(\mu,\mu_0)\end{bmatrix}\hat{P}_H^{(1)}(\Omega,\bar{\Omega}_0)-\begin{bmatrix}C_2(\mu,\mu_0)\hat{E}_3 & 0\\ 0 & C_3(\mu,\mu_0)\end{bmatrix}\hat{P}_{R2}^{(1)}(\Omega,\bar{\Omega}_0). \quad (19)$$

Here the matrix $\hat{P}_{R1}^{(1)}$ is given by the formula (39) in Ref. 1, while the matrix

$$\hat{P}_{R2}^{(1)}(\Omega,\bar{\Omega}_0)=\frac{d}{d\psi}\hat{P}_{R1}^{(1)}(\Omega,\bar{\Omega}_0).$$

If we introduce the matrix $\hat{B}(\mu,\mu_0)$ composed of the unknown functions $B_k(\mu,\mu_0)$, $C_k(\mu,\mu_0)$ $(k=1-4)$:

$$\hat{B}=\begin{bmatrix}B_1 & -C_1 & B_2 & -C_2\\ C_1 & B_1 & C_2 & B_2\\ B_3 & -C_3 & B_4 & -C_4\\ C_3 & B_3 & C_4 & B_4\end{bmatrix}, \quad (20)$$

then, by substituting (19) into Eq. (18) and using (14)–(16) and (20), we can show that the matrix $\hat{B}$ satisfies the equation

$$\hat{B}(\mu,\mu_0)=\hat{S}^{(I)}(\mu,\mu_0)+\frac{3}{8}\tilde{\omega}_0\int_0^1\frac{d\mu'}{\mu'}\hat{S}^{(I)}(\mu,\mu')\hat{\chi}^{(I)}(\mu')\hat{B}(\mu',\mu_0), \quad (21)$$

where

$$\hat{S}^{(I)}(\mu,\mu')=\hat{H}^{(I)}(\mu)\hat{M}^{(I)}(h)\hat{H}^{(I)}(\mu'),$$

$$\hat{H}^{(I)}(\mu)=\begin{bmatrix}\hat{H}^{(1)}(\mu) & 0\\ 0 & \hat{H}^{(r)}(\mu)\end{bmatrix},\quad \hat{M}^{(I)}(h)=\begin{bmatrix}\hat{\mathscr{M}}_1(h) & 0\\ 0 & \hat{\mathscr{M}}_1(h)\end{bmatrix}, \quad (22)$$

$$\hat{\chi}^{(I)}(\mu)=(1-\mu^2)\begin{bmatrix}R_3-2\mu^2R_1 & 0 & -R_4 & 0\\ 0 & R_3-2\mu^2R_1 & 0 & -R_4\\ R_4 & 0 & R_3 & 0\\ 0 & R_4 & 0 & R_3\end{bmatrix},$$

the $R_k(\mu)$ being the elements of the matrix $\hat{R}(\mu)$ formed by the reflection coefficients. Now let us introduce a new matrix $\hat{b}(\mu,\mu0)$, which determines $\hat{B}(\mu,\mu_0)$ through the relation

$$\hat{B}(\mu,\mu_0)=\frac{\mu\mu_0}{\mu+\mu_0}\hat{H}^{(I)}(\mu)\hat{M}^{(I)}(h)\hat{b}(\mu,\mu_0)\hat{H}^{(I)}(\mu_0). \quad (23)$$

Substituting (23) into (21), we obtain an equation for $\hat{b}$:

$$\hat{b}(\mu,\mu_0)=1+\frac{3}{8}\tilde{\omega}_0(\mu+\mu_0)\times\int_0^1\frac{\mu'\,d\mu'}{(\mu+\mu')(\mu_0+\mu')}\hat{\Phi}^{(I)}(\mu')\hat{b}\ (\mu',\mu_0), \quad (24)$$

where

$$\hat{\Phi}^{(I)}(\mu)=\hat{H}^{(I)}(\mu)\hat{\chi}^{(I)}(\mu)\hat{H}^{(I)}(\mu)\hat{M}^{(I)}(h). \quad (25)$$

From this it is easy to observe [see (22) and (25)] that the solution to Eq. (24) has the form

$$\hat{b}=\begin{bmatrix}\hat{b}_1 & -\hat{b}_2\\ \hat{b}_2 & \hat{b}_3\end{bmatrix},\quad \hat{b}_i=\begin{bmatrix}b_{i1} & -b_{i2}\\ b_{i2} & b_{i1}\end{bmatrix},$$

and, consequently, it splits up into three independent systems of equations for the pairs of functions $b_{i1}(\mu,\mu_0)$ and $b_{i2}(\mu,\mu_0)$ $(i=1,2,3)$.

We can satisfy Eq. (18) for the matrix $\hat{S}_R^{(2)}(\mathbf{\Omega},\mathbf{\Omega}_0)$ by setting

$$\hat{S}_R^{(2)}(\Omega,\Omega_0)=\hat{S}_{R1}^{(2)}(\mu,\mu_0)\hat{P}^{(2)}(\Omega,\bar{\Omega}_0)+\hat{S}_{R2}^{(2)}(\mu,\mu_0)\hat{P}_H^{(2)}(\Omega,\bar{\Omega}_0). \quad (26)$$

If we now introduce, similarly to (15), the matrix

$$\hat{S}_R^{(2)}(\mu,\mu_0)=\begin{bmatrix}S_{R1}^{(2)} & -S_{R2}^{(2)}\\ S_{R2}^{(2)} & S_{R1}^{(2)}\end{bmatrix},$$

then, substituting (26) into Eq. (18) for $m=2$, we obtain the following integral equation:

$$\hat{S}_R^{(2)}(\mu,\mu_0)=\hat{S}^{(2)}(\mu,\mu_0)+\frac{3}{16}\tilde{\omega}_0\int_0^1\frac{d\mu'}{\mu'}\hat{S}^{(2)}(\mu,\mu')\chi^{(2)}(\mu')\hat{S}_R^{(2)}(\mu',\mu_0), \quad (27)$$

where

$$\chi^{(2)}(\mu)=R_1(\mu)\mu^4-2R_3(\mu)\mu^2+R_2(\mu),$$

and the matrix $\hat{S}^{(2)}(\mu,\mu_0)$ is defined by the relation (16). Let us simplify Eq. (27) by introducing the matrix $\hat{a}$ that determines $\hat{S}_R^{(2)}$ through the formula

$$\hat{S}_R^{(2)}(\mu,\mu_0)=\frac{\mu\mu_0}{\mu+\mu_0}\hat{H}^{(2)}(\mu)\hat{\mathscr{M}}_2(h)\hat{a}(\mu,\mu_0)\hat{H}^{(2)}(\mu_0). \quad (28)$$

The substitution of (28) into (27) yields an equation for $\hat{a}$:

$$\hat{a}(\mu,\mu_0)=1+\frac{3}{16}\tilde{\omega}_0(\mu+\mu_0)\times\int_0^1\frac{\mu'd\mu'}{(\mu+\mu')(\mu_0+\mu')}\hat{\Phi}^{(2)}(\mu')a(\mu',\mu_0), \quad (29)$$

where the characteristic matrix

$$\hat{\Phi}^{(2)}(\mu)=\chi^{(2)}(\mu)(\hat{H}^{(2)}(\mu))^2\hat{\mathscr{M}}_2(h).$$

The matrix $\hat{\Phi}^{(2)}$, like the matrices $\hat{H}^{(2)}$ and $\hat{\mathscr{M}}_2$, has two independent components:

$$\hat{\Phi}^{(2)}=\begin{bmatrix}\Phi_1^{(2)} & -\Phi_2^{(2)}\\ \Phi_2^{(2)} & \Phi_1^{(2)}\end{bmatrix},$$

and, consequently, the solution of the matrix equation (29) reduces to the solution of a system of two equations for the components a_1 and a_2 of the matrix

$$\hat{a}=\begin{bmatrix}a_1 & -a_2\\ a_2 & a_1\end{bmatrix}.$$

In the case of normal excitation of the crystal the intensity and degree of circular polarization of the radiation scattered back in the opposite direction are determined only by the matrix $\hat{S}_R^{(0)}$, and therefore do not undergo any changes upon the application of a longitudinal magnetic field. If in this case the incident light is linearly polarized, then the degree of linear polarization of the light scattered in the direction perpendicular to the surface decreases, according to the formulas (13), (17), and (26), with the field according to the law

$$\mathscr{P}'_{\text{lin}}(H_\parallel)=\frac{I_l-I_r}{I_l+I_r}=\frac{2S^{(2)}(h,n,\tilde{\omega}_0)}{\Sigma(n,\tilde{\omega}_0)}; \quad (30)$$

here n is the refractive index of the crystal, Σ is a quantity determining the scattered-light intensity $I=I_l+I_r$, which does not depend on the magnetic field, and is given by the formula (51) in Ref. 1; for $n=1$ we have $S^{(2)}=S_1^{(2)}$, while for $n\neq 1$ we have $S^{(2)}=S_R^{(2)}$, the quantities $S_1^{(2)}(h,\tilde{\omega}_0)$ and $S_{R1}^{(2)}(h,n,\tilde{\omega}_0)$ being given by the relations (16) and (28):

$$S_1^{(2)}=c_1\mathscr{P}_x-c_2\rho_x,$$
$$S_{R1}=(c_1a_1-c_2a_2)\mathscr{P}_x-(c_1a_2+c_2a_1)\rho_x, \quad (31)$$
$$c_1={}^1/_2[(H_1^{(2)}(1))^2-(H_2^{(2)}(1))^2],\quad c_2=H_1^{(2)}(1)H_2^{(2)}(1).$$

In the case when the excitons are excited by linearly polarized light in a longitudinal magnetic field, the rotation of their electric dipole moments in the scattered light results in the appearance of polarization $\mathscr{P}_{\text{lin}}^{\parallel}$ in the (l',r') coordinate system, rotated relative to the (l,r) system through an angle of $\pi/4$ about $-\mathbf{n}^0$:

$$\mathscr{P}''_{\text{lin}}(H_\parallel)=\frac{U}{I}=-\frac{2S_{\pi/4}^{(2)}(h,n,\tilde{\omega}_0)}{\Sigma(n,\tilde{\omega}_0)}. \quad (32)$$

Here $S_{\pi/4}^{(2)}=S_2^{(2)}$ for $n=1$ and $S_{\pi/4}^{(2)}=S_{R2}^{(2)}$ for $n\neq 1$. The functions $S_2^{(2)}(h,\tilde{\omega}_0)$ and $S_{R2}^{(2)}(h,n,\tilde{\omega}_0)$ are also determined from (16) and (28):

$$S_2^{(2)}=c_1\rho_x+c_2\mathscr{P}_x,\quad S_{R2}^{(2)}=(c_1a_1-c_2a_2)\rho_x+(c_1a_2+c_2a_1)\mathscr{P}_x. \quad (33)$$

In the case of small values of the quantum yield $\tilde{\omega}_0$ for a single scattering act, we can solve the integral equations for the matrices $\hat{S}_H$ and $\hat{S}_H^R$ by expanding them in series in powers of $\tilde{\omega}_0$. If we limit ourselves to the first approximation in $\tilde{\omega}_0$, i.e., if we consider only the first- and second-order contributions of the light scattering, then for $\mathscr{P}'_{\text{lin}}$ and $\mathscr{P}''_{\text{lin}}$ we obtain in the case when $\mu_0=\mu=1$ the following expressions: for $n^2=1$

$$\mathscr{P}'_{\text{lin}}=\mathscr{P}_x-[0{,}602\mathscr{P}_x-0{,}446(\mathscr{P}_x^2-\rho_x^2)]\tilde{\omega}_0,$$
$$\mathscr{P}''_{\text{lin}}=-[1-(0{,}602-0{,}892\mathscr{P}_x)\tilde{\omega}_0]\rho_x,$$

and for $n^2=10$

$$\mathscr{P}'_{\text{lin}}=\mathscr{P}_x-[0{,}766\mathscr{P}_x-0{,}518(\mathscr{P}_x^2-\rho_x^2)]\tilde{\omega}_0,$$
$$\mathscr{P}''_{\text{lin}}=-[1-(0{,}766-0{,}933\mathscr{P}_x)\tilde{\omega}_0]\rho_x.$$

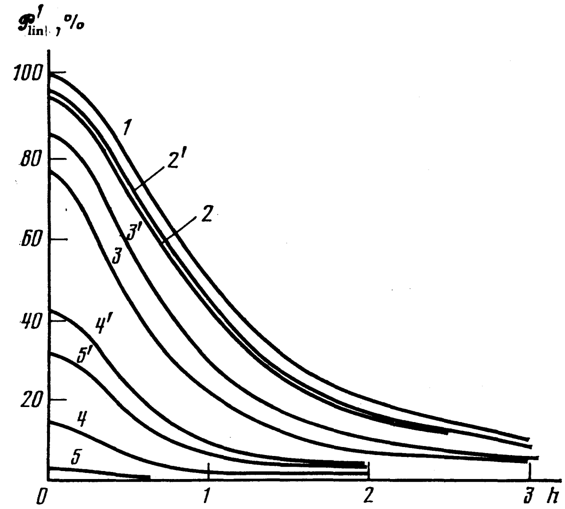

FIG. 1. Dependence of the degree of linear polarization $\mathcal{P}_{lin}$ of the exciton radiation on the longitudinal-magnetic-field strength $(h = \omega_{\parallel} \tau_l = g_{\parallel} \mu_0 H_{\parallel} \tau_l)$ in the case when the excitation is effected by linearly polarized light. The curves 2–5 were computed for $n^2 = 10$; the curves 2′ – 5′, for $n = 1$. The following $\tilde{\omega}_0$ values were used: for the curves 2 and 2′, $\tilde{\omega}_0 = 0.2$; for 3 and 3′, 0.6; for 4 and 4′, 0.99; for 5 and 5′, 1.0. The curve 1 depicts the Hanle contour $P_X = (1 + h^2)^{-1}$.

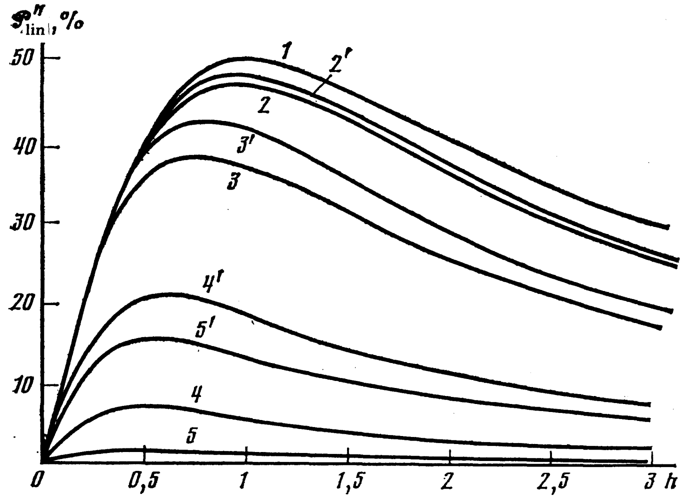

FIG. 2. Longitudinal magnetic field dependence of the degree of linear polarization $\mathcal{P}''_{lin}$ of the exciton radiation in the coordinate system X', Y' making an angle of 45° with the polarization plane of the exciting light. The curves 2–5 and 2′–5′ were computed with those same n^2 and $\tilde{\omega}_0$ values that were used to compute the corresponding curves in Fig. 1. The curve 1 depicts the Hanle contour $\rho_X = h/(1 + h^2)$.

§5. RESULTS OF THE NUMERICAL COMPUTATION

As follows from the preceding sections, to compute the angular distributions of the intensity and degree of polarization of the scattered light in a longitudinal magnetic field, we must numerically solve the integral equations (11) for $\hat{H}^{(1)}$, $\hat{H}^{(r)}$, and $\hat{H}^{(2)}$ and (24) and (29) for $\hat{b}$ and $\hat{a}$. The field-independent matrices $\hat{S}^{(0)}$ and $\hat{S}_R^{(0)}$ are computed in Ref. 1.

In the simplest case, when the crystal is illuminated normally and the light scattered back in the opposite direction is detected ($\mu = \mu_0 = 1$), it is sufficient to determine the four functions $H_1^{(2)}$, $H_2^{(2)}$, a_1, and a_2. The integral equations (11) for $\hat{H}^{(2)}$ and (29) for $\hat{a}$ were solved by the iteration method on a computer.[9] The convergence of this method for all h and $\tilde{\omega}_0$ values turned out to be sufficiently rapid. The iterative procedure was carried out with a relative error of 0.01% for the values of the sought quantities at all μ points in the interval [0,1].

Figure 1 shows the dependence $\mathcal{P}'_{lin}(H_{\parallel})$ of the degree of linear polarization of the exciton radiation on the strength of the longitudinal magnetic field in the case when the crystal is excited by linearly polarized light. The curves were computed from the formulas (30) and (31). For comparison we show in the same figure the Hanle curve, 1 (for $\mathcal{P}'_{lin}(0) = 1$), corresponding to $\tilde{\omega}_0 \to 0$, which has the classical Lorentz shape, and describes the single-scattering-induced depolarization of the radiation in the magnetic field. It can be seen that multiple scattering of the light does not lead to any qualitative change in the $\mathcal{P}'_{lin}(h)$ contour, the shape of which remains almost Lorentzian. But as $\tilde{\omega}_0$ and n increase, and the effects of the re-emission and specular reflection from the inner surface of the crystal become important, the value of $\mathcal{P}'_{lin}(h)$ decreases rapidly with increasing $\tilde{\omega}_0$, and the halfwidth $h_{1/2}$ of the $\mathcal{P}'_{lin}(h)$ curves decreases. Here $h_{1/2}$ corresponds to $\mathcal{P}'_{lin}(h_{1/2}) = \frac{1}{2}\mathcal{P}'_{lin}(0)$.

The behavior of the degree of linear polarization $\mathcal{P}''_{lin}(h)$ is depicted in Fig. 2; the curves were computed from the formulas (32) and (33) for $n^2 = 1$, 10 and $\tilde{\omega}_0 = 0.2$, 0.6, 0.99, 1.0. It can be seen that these curves also do not qualitatively differ in shape from the corresponding Hanle curve (the curve 1). As the values of n and $\tilde{\omega}_0$ increase, the peak of the $\mathcal{P}''_{lin}(h)$ curve shifts into the region of lower magnetic-field intensities, its halfwidth and the peak value decreasing monotonically in the process.

In order to find out to what extent the curves in Figs. 1 and 2 can be approximately described by the formulas (1) with τ_l, τ_s, and τ in them replaced by the effective times $\bar{\tau}_l$,

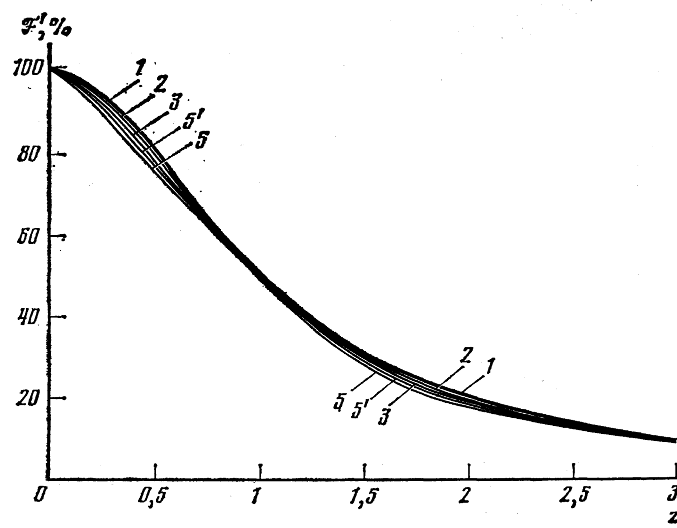

FIG. 3. Comparison of the results obtained for $\mathcal{P}'_{lin}$ in the exact and approximate computations carried out with the aid of the formulas (34) and (1). The curves 1–5 and 5′ correspond to the curves 1–5 and 5′ in Fig. 1.

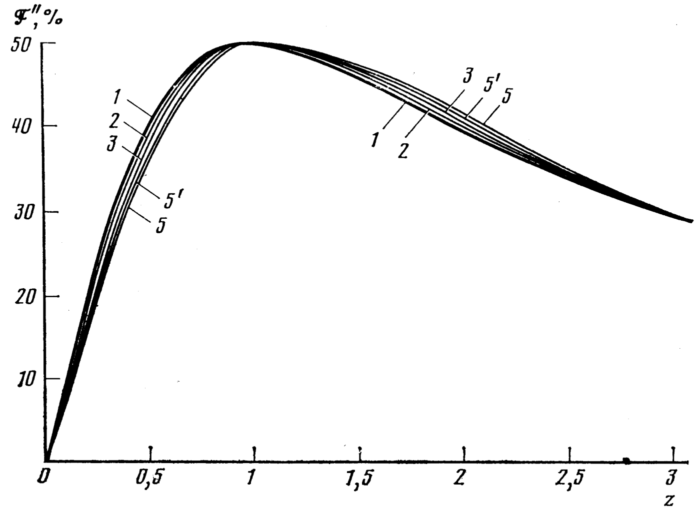

FIG. 4. Comparison of the results obtained for $\mathscr{P}''_{\rm lin}$ in the exact and approximate computations carried out with the aid of the formulas (34) and (1). The curves 1–5 and 5′ correspond to the curves 1–5 and 5′ in Fig. 1.

$\bar\tau_s$, and $\bar\tau$, where $\bar\tau^{-1}=\tau_l^{-1}+\tau_s^{-1}$, we constructed the functions

$$\mathscr{F}'(z)=\frac{\mathscr{P}'_{\rm lin}(z)}{\mathscr{P}'_{\rm lin}(0)},\quad \mathscr{F}''(z)=\frac{-\mathscr{P}''_{\rm lin}(z)}{\mathscr{P}'_{\rm lin}(0)},\quad z=\frac{h}{h_{1/2}}. \tag{34}$$

Figures 3 and 4 show plots (thin lines) of these functions for different values of n and $\tilde\omega_0$. The thick lines are plots of the functions $\mathscr{F}'_{(z)}=(1+z^2)^{-1}$ and $\mathscr{F}''_{(z)}=z(1+z^2)^{-1}$, which correspond to (1). It can be seen that the deviation of the exact curves (34) from (1) increases with increasing n and $\tilde\omega_0$, but does not exceed 5%.

Further, we determined $\bar\tau_l$ and $\bar\tau_s$ from the values of $\mathscr{P}'_{\rm lin}(0)$ and $h_{1/2}$, assuming in accordance with (1) and (1′) that

$$\bar\tau_l/\bar\tau_s=\mathscr{P}'^{-1}_{\rm lin}(0)-1,\quad \bar\tau_l^{-1}+\bar\tau_s^{-1}=h_{1/2}\tau_l^{-1}. \tag{35}$$

Figure 5 shows plots, obtained with the aid of the formulas (35) and the curves in Fig. 1, of $\tau_l/\bar\tau_l$, $\tau_l/\bar\tau_s$, and $\tau_l/\bar\tau$ as functions of the quantity $\tilde\omega_0$ for $n^2=1$ and 10. The dashed curves are $\tau_l/\bar\tau(\tilde\omega_0)$ curves computed by the method of iterations in powers of $\tilde\omega_0$ with allowance for only single and double scattering. It can be seen that these iteration curves already deviate from the exact curves at $\tilde\omega_0\geqslant 0.1$.

It follows from Fig. 5 that, as $\tilde\omega_0$ (i.e., the number of reemission events) increases, $\bar\tau_l$ increases, attaining at $n^2=10$ and $\tilde\omega_0=1$ the value $\bar\tau_l\approx 70\tau_l$. The quantity $\bar\tau_s$ decreases at the same time, but remains greater than τ_l, i.e., greater than $\tau_{\rm rad}$. Thus, when $\tilde\omega_0=1$, i.e., $\tau_l=\tau_{\rm rad}$, and $n^2=10$ we have $\bar\tau_s\approx 2.4\tau_l$. When $\tilde\omega_0$ is close to 1, $\bar\tau_s$ is smaller than $\bar\tau_l$. In this case, despite the decrease of $\bar\tau_s$, $\bar\tau$ increases monotonically with increasing $\tilde\omega_0$. For $n^2=10$ and $\tilde\omega_0=1$ we have $\bar\tau/\tau_l\approx 2.3$.

Let us emphasize that, although the shape of the $\mathscr{P}'_{\rm lin}(H_\parallel)$ and $\mathscr{P}''_{\rm lin}(H_\parallel)$ curves in the case under consideration is close to the shape given by the formulas (1) obtained in the simple theory, the computation of $\bar\tau_s$ and $\bar\tau_l$ even in the region $\tilde\omega_0\gtrsim 0.1$ or 0.2 requires exact allowance for the reemission process, i.e., the exact solution of the radiation-transfer problem. Let us also note that, in the case under consideration, the total quantum yield and the angular distribution of the intensity of the exciton radiation in the polarization plane of the exciting light do not depend on the longitudinal magnetic field.

In conclusion the authors consider it their pleasant duty to thank G. E. Pikus and E. L. Ivchenko for suggesting the theme and for fruitful discussions in the course of the performance of the computations and the preparation of the manuscript.

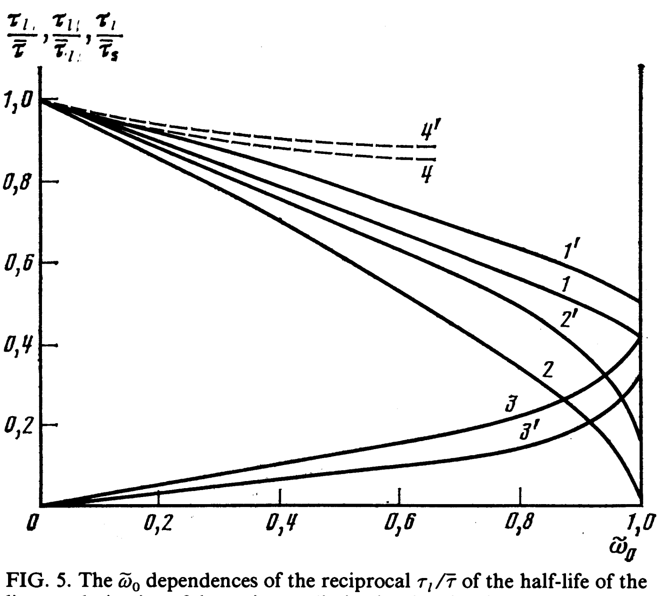

FIG. 5. The $\tilde\omega_0$ dependences of the reciprocal $\tau_l/\bar\tau$ of the half-life of the linear polarization of the exciton radiation in a longitudinal magnetic field (the curves 1, 1′ and 4, 4′), the reciprocal $\tau_l/\bar\tau_l$ of the effective lifetime (the curves 2 and 2′), and the reciprocal $\tau_l/\bar\tau_s$ of the effective spin relaxation time (the curves 3 and 3′) of the exciton. The curves 1–4 were computed with $n^2=10$; the curves 1′–4′, with $n^2=1$; and the dashed lines 4 and 4′, with allowance for only the single and double scattering.

[1)]If the magnetic field $\mathbf{H}_\parallel$ is oriented along the $0Z\parallel(-\mathbf{n}^0)$ axis of a right-handed Cartesian coordinate system, then the increment $\Delta\varphi'=\frac{1}{2}\omega_\parallel t>0$ when $g_\parallel>0$.

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