Effect of texture on the propagation of collective modes in superfluid ^{3}He

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A phenomenological theory of collective modes in superfluid ^{3}He-*B*, similar to the Leggett theory for spin dynamics, is developed. The theory can be applied to study the dynamics of collective modes in an inhomogeneous texture of the order parameter. In the presence of a magnetic field, the quantization axis of the angular momentum J that characterizes the modes varies in space in this texture. This results in a Van Hove-type texture-dependent singularity in the absorption spectrum of the ultrasound that excites the collective modes. Thus, in the texture produced by a magnetic field between parallel plates and observed with the aid of ultrasound, a collective mode with $J = 2$ should be split not into five modes, as it should in open geometry, but into ten. The additional splitting of the mode spectrum on account of the textures can be used to investigate vortex structures in a rotating superfluid liquid.

1. INTRODUCTION

Cooper pairing in liquid ^{3}He leads to formation of a coherent superfluid state. One of the manifestations of the coherence of superfluid phases of ^{3}He is the existence in them of various collective modes. In contrast to zero-sound modes in a normal Fermi liquid, these collective modes exist in the limit of arbitrary long wavelengths and reflect the appearance of long-range order. At least 14 collective modes were observed in superfluid ^{3}He-*B*. They include four Goldstone modes, viz., sound and three spin-wave modes. By now these modes have been thoroughly studied; experiments with propagation of sound and of spin waves (NMR experiments) yield the basic information on the structure of the order parameter in superfluid ^{3}He phases (see, e.g., the review[1]) and on the various textures—spatially inhomogeneous states of the order parameter (see, e.g., Refs. 2 and 3). Experiments with ultrasound revealed ultrasound-excited collective modes with gaps (see the latest papers[4–6] and the references therein). In ^{3}He-*B* these are the fivefold degenerate squashing mode (SM), whose splitting has not yet been observed, and the real squashing mode (RSM), all five branches of which can be seen in a magnetic field. The existence of other collective modes is also indicated.[6]

The theory of collective modes with gaps was limited up to now to calculation of their spectrum and to their connection with ultrasound in a spatially homogeneous liquid. In ^{3}He-*B*, whose isotropic equilibrium state is an eigenstate with $J = 0$, the modes are classified in accord with the value of the total angular momentum J (Ref. 7) (see Ref. 8 for the quantum numbers that characterize the modes in ^{3}He-*A*). The mode spectrum was calculated by many reasonable methods: in the formalism of the two-particle Green's function,[9] by the method of matrix kinetic equation,[10] by the method of functional integration over the Fermi fields,[11] and others.

All these methods are quite complicated for use in the study of the propagation of collective modes in an inhomogeneous liquid. The latter, however, is essential, since the gap collective modes can serve as just a reliable source of texture information as the Goldstone modes, but in other frequency and wavelength bands. Thus, for example, it became clear recently[12] that texture effects are responsible for the sixfold[4] splitting of RSM with $J = 2$ observed in a magnetic field rather than the expected fivefold splitting. The additional splitting of the central line with $J_z = 0$ depends on the form of the texture, and this dependence can be used to identify the different textures, including those occurring in rotation of ^{3}He-*B* on account of formation of quantized vortices. The latter were investigated up to now only by the NMR method.[3] It becomes necessary therefore to develop a phenomenological theory that describes the dynamics of collective modes in the presence of a texture, similar to the macroscopic theory of spin dynamics developed by Leggett[13] (see also Refs. 14–16). The macroscopic theory should lead to all the qualitative results from the microtheory for a homogeneous medium, leaving for the latter the calculation of the phenomenological parameters.

Phenomenological equations for the order parameter, which can describe the propagation of gap collective modes in the presence of textures, are derived in Secs. 2 and 3 of the present article. These are generalizations of the Leggett equations[13–16] that describe the entire spin-wave dynamics in the presence of a magnetic field and of textures. There is one important difference between the equations for gap modes and the Leggett equations. Leggett's equations describe the dynamics of hydrodynamic variables and of soft Goldstone modes of the order parameter, and can be used at wavelengths exceeding the coherence length ($\lambda \gg \xi$) and at frequencies that are low compared with the gap Δ in the spectrum of the Fermi excitations ($\omega \ll \Delta$). Non-Goldstone modes, on the contrary, have a frequency comparable with Δ; thus, in the weak coupling approximation the RSM frequency as $\lambda \to \infty$ is equal to $\omega_0 = (8/5)^{1/2}\Delta$ (Refs. 7, 9–11). Therefore the region of applicability of the phenomenological equations for the order parameter, which describe the propagation of gap collective modes, is limited by other inequalities: $\lambda \gg \xi$ and $|\omega - \omega_0| \ll \Delta$. In all other respects the

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equations are similar and are derived from the same commutation relations as given for pair operators in Leggett's paper.[13] In both cases, the structure of the equations is determined by the symmetry of the "vacuum" phase of the superfluid phase of ^{3}He, i.e., by the type of broken symmetry (in ^{3}He-B it is the spin-orbit symmetry which is broken, and this leads to a nontrivial influence of the texture on the collective modes).

In Sec. 4 we consider the effect of the magnetic field on the mode spectrum, and in Sec. 5 we show how the joint influence of the magnetic field and of the texture leads to additional mode splitting. We determine the texture characteristics that can be studied with the aid of collective modes and point out the advantage of this method of studying the texture over NMR. In Sec. 6 we show how to study vortex states of ^{3}He-B in a rotating vessel with the aid of collective modes, and in the Conclusion we discuss the possibility of studying textures with the aid of collective modes in ^{3}He-A.

2. SYMMETRY OF GROUND STATE AND DYNAMIC VARIABLES FOR COLLECTIVE MODES IN ^{3}He-B

The classification of collective modes in superfluid ^{3}He is determined by the symmetry of the vaccum, which is the equilibrium homogeneous state of the liquid. The vacuum in ^{3}He-B is characterized by the following symmetry elements. First is continuous symmetry, specified by the generator

$$\hat{J}_i=\hat{L}_i+R_{\alpha i}\hat{S}_\alpha, \tag{2.1}$$

where $\hat{\mathbf{L}}$ and $\hat{\mathbf{S}}$ are respectively the generators of the orbital and spin rotations, and $R_{\alpha i}$ is an orthogonal 3×3 matrix that characterizes the given degenerate state of the vacuum. The degeneracy is connected with the leeway in the choice of the orientation of the spin coordinate frame relative to the orbital frame neglecting the weak spin–orbit (so-called dipole) interaction. The appearance in ^{3}He-B of the connection (2.1) between the spin and orbital rotations, which is dictated by the state of the vacuum, means breaking of the spin–orbit symmetry in ^{3}He-B. In addition, there are two discrete vacuum symmetry elements: symmetry with respect to the time reversal operation $\hat{T}$, and combined parity $\hat{P}_{\rm comb}=\hat{P}\hat{U}_{\pi/2}$, where $\hat{P}$ is the operation of spatial inversion $\mathbf{r}\to-\mathbf{r}$, and $\hat{U}_{\pi/}$ is a gauge transformation with a phase parameter equal to $\pi/$ 2.

This symmetry determines uniquely the form of the amplitude of the Cooper pairing, on the Fermi surface, of particles with opposite momenta:

$$T_{ab}(\mathbf{p})=\langle a_{-\mathbf{p}a}a_{\mathbf{p}b}\rangle. \tag{2.2}$$

In the equilibrium state this amplitude should satisfy the vacuum symmetry conditions:

$$\hat{J}_iT_{ab}(\mathbf{p})=0,\qquad \hat{T}T_{ab}(\mathbf{p})=\hat{P}_{\rm comb}T_{ab}(\mathbf{p})=T_{ab}(\mathbf{p}) \tag{2.3}$$

with the following actions of the symmetry-transformation operators[5]:

$$\hat{L}_iT_{ab}(\mathbf{p})=\frac{1}{i}e_{ikl}p_k\frac{\partial}{\partial p_l}T_{ab}(\mathbf{p}),$$

$$\hat{S}_\alpha a_{\mathbf{p}a}={}^1/_2(\sigma_\alpha)_{ab}a_{\mathbf{p}b},\qquad \hat{U}_{\pi/2}a_{\mathbf{p}a}=e^{\pi i/2}a_{\mathbf{p}a},$$

$$\hat{P}T_{ab}(\mathbf{p})=T_{ab}(-\mathbf{p}),\qquad \hat{T}a_{\mathbf{p}a}=g_{ab}a_{-\mathbf{p}b}^{+},\qquad g=i\sigma_2,\qquad \hat{T}T_{ab}(\mathbf{p})=g_{ac}T_{cd}^{+}(\mathbf{p})g_{db}. \tag{2.4}$$

Here σ_a are Pauli matrices.

The general solution of Eqs. (2.3) is of the form

$$T_{ab}^0(\mathbf{p})=(\sigma_\alpha g)_{ab}R_{\alpha i}n_if(p),\qquad \mathbf{n}=\mathbf{p}/p, \tag{2.5}$$

where $f(p)$ is a real function that depends only on the modulus p. It describes the pairing of Fermi particles with a common spin $S=1$ and with a relative angular momentum $L=1$ of the pair particles. By virtue of its symmetry, T_{ab}^0 is a solution of the Gor'kov equations and effects an absolute minimum of the energy at certain parameters of the liquid.

The collective modes in ^{3}He-B, including also the spin waves considered by Leggett, are oscillations of the amplitude T_{ab} about its equilibrium value T_{ab}^0, coupled with the oscillations of the particle distribution function:

$$n_{ab}(\mathbf{p})=\langle a_{\mathbf{p}a}^{+}a_{\mathbf{p}b}\rangle. \tag{2.6}$$

Oscillations against the background of the vacuum (2.5) are characterized by quantum numbers that are determined by the symmetry of the vacuum. These are the total angular momentum J, which takes on arbitrary integer values, and the parities T and $P_{\rm comb}$ with values ± 1. To find the dynamic variables for which we shall construct the equations that describe the dynamics of collective modes with a given set of quantum numbers J, T, and $P_{\rm comb}$, we must separate the appropriate components from the amplitude T_{ab} ($\mathbf{p}$) and the distribution function n_{ab} ($\mathbf{p}$). These components must satisfy the equations

$$\begin{gathered}\hat{J}^2T_{ab}=J(J+1)T_{ab},\qquad \hat{J}^2n_{ab}=J(J+1)n_{ab},\\ \hat{P}_{\rm comb}T_{ab}=P_{\rm comb}T_{ab},\qquad \hat{P}_{\rm comb}n_{ab}=P_{\rm comb}n_{ab},\\ \hat{T}T_{ab}=TT_{ab},\qquad \hat{T}n_{ab}=-Tn_{ab}.\end{gathered} \tag{2.7}$$

Since, as we shall see below, T_{ab} and n_{ab} are canonical conjugates in the corresponding Hamilton equations, their components in this mode should have different time parity T.

We shall be interested hereafter only in the dynamics of experimentally observable collective modes with gaps: SM and RMS. The dynamics of the remaining modes with other quantum numbers is developed in similar fashion.

The SM has the quantum numbers $J=2$, $T=-1$, and $P_{\rm comb}=+1$. The solution of Eqs. (2.7) with these quantum numbers yields the following components of the pairing amplitude and of the distribution function, which oscillate in this mode:

$$n_{ab}(\mathbf{p})\propto\delta_{ab}\left(n_in_k-\frac{1}{3}\delta_{ik}\right)Q_{ik}(p),$$

$$T_{ab}(\mathbf{p})\propto i(g\sigma_\alpha)_{ab}\Big\{(n_iR_{\alpha k}+n_kR_{\alpha i}-{}^2/_3\delta_{ik}n_lR_{\alpha l})\times[v_{ik}(p)-{}^1/_5\tilde{v}_{ik}(p)]+\left(n_in_k-\frac{1}{3}\delta_{ik}\right)n_lR_{\alpha l}\tilde{v}_{ik}(p)\Big\}. \tag{2.8}$$

Here $Q_{ik}(p)$, $v_{ik}(p)$, and $\tilde{v}_{ik}(p)$ are real zero-trace symmetric matrices that depend only on the modulus p of the momentum. This dependence is inessential, since the oscillations take place only near the Fermi surface. We shall therefore consider as the dynamic variables integrals of Q, v, and $\tilde{v}$ with respect to the momenta, in analogy with the procedure used for spin waves.[13] Thus, the dynamic variables that describe the SM are Q_{ik}, v_{ik} and $\tilde{v}_{ik}$ are expressed in the following manner in terms of the distribution function and the pairing amplitude:

$$Q_{ik}=\sum_{\mathbf{p}} Q_{ik}(p)=\frac{5}{4}\sum_{\mathbf{p}}\left(n_i n_k-\frac{1}{3}\delta_{ik}\right)n_{aa}(\mathbf{p}),$$

$$v_{ik}=\sum_{\mathbf{p}} v_{ik}(p)$$

$$=\mathrm{Im}\left\{(g\sigma_a)_{ab}\sum_{\mathbf{p}}\left(n_i R_{ak}+n_k R_{ai}-\frac{2}{3}\delta_{ik}n_l R_{al}\right)T_{ab}(\mathbf{p})\right\}.$$

$$\tilde{v}_{ik}=\sum_{\mathbf{p}}\tilde{v}_{ik}(p) \tag{2.9}$$

$$=\mathrm{Im}\left\{(g\sigma_a)_{ab}\sum_{\mathbf{p}}\left(n_i n_k-\frac{1}{3}\delta_{ik}\right)n_l R_{al}T_{ab}(\mathbf{p})\right\}-\frac{1}{5}v_{ik}.$$

We note that v_{ik} describes the amplitude of pairing into a state with $S=1$, $L=1$, and $J=2$, whereas the corresponding state for $\tilde{v}_{ik}$ has $S=1$, $L=3$ and $J=2$; Q_{ik} is the quadrupole moment of the distribution function of the particles in the liquid.

Similarly, an RSM with quantum numbers $J=2$, $T=+1$ and $P_{\text{comb}}=+1$ is described by real zero-trace symmetric matrices M_{ik}, u_{ik}, and $\tilde{u}_{ik}$ expressed in terms of the distribution function and the pairing amplitude as follows:

$$M_{ik}=\frac{5}{12}(\sigma_a)_{ab}R_{aj}\sum_{\mathbf{p}}(e_{jli}n_l n_k+e_{jlk}n_l n_i)n_{ab}(\mathbf{p}),$$

$$u_{ik}=\mathrm{Re}\left\{(g\sigma_a)_{ab}\sum_{\mathbf{p}}\left(n_i R_{ak}+n_k R_{ai}-\frac{2}{3}\delta_{ik}n_l R_{al}\right)T_{ab}(\mathbf{p})\right\}, \tag{2.10}$$

$$\tilde{u}_{ik}=\mathrm{Re}\left\{(g\sigma_a)_{ab}\sum_{\mathbf{p}}\left(n_i n_k-\frac{1}{3}\delta_{ik}\right)n_l R_{al}T_{ab}(\mathbf{p})\right\}-\frac{1}{5}u_{ik}.$$

We note that unlike the variable Q_{ik} in the SM, the variable M_{ik} in the RSM reflects a change in the particle-spin distribution in the wave. Therefore, whereas an SM is intensely excited by ultrasound, an RSM should be excited by an rf magnetic field, i.e., in NMR experiments, and is substantially less connected with ultrasound (see, e.g., Ref. 4). We note also that all the variables of (2.9) and (2.10) vanish at equilibrium, when the only nonzero quantities are those T_{ab} and n_{ab} components that correspond to the quantum numbers $J=0$, $T=+1$, and $P_{\text{comb}}=+1$. One of these components is the order parameter at equilibrium:

$$A_{ai}^0=g_1\sum_{\mathbf{p}} n_i(g\sigma_a)_{ab}T_{ab}^0(\mathbf{p})=\Delta R_{ai}, \tag{2.11}$$

where g_1 is the first harmonic in the pairing interaction potential

$$g(\mathbf{nn}')=\sum_l g_l P_l(\mathbf{nn}'),$$

and the gap Δ in the excitation spectrum is expressed in terms of the function $f(p)$ from (2.5) as follows:

$$\Delta=-\frac{2}{3}g_1\sum_{\mathbf{p}} f(p). \tag{2.12}$$

The second component is the particle density

$$\rho=\sum_{\mathbf{p}} n_{aa}(\mathbf{p}). \tag{2.13}$$

3. POISSON BRACKETS AND DYNAMIC EQUATIONS FOR COLLECTIVE MODES

We proceed now to construct dynamic equations for the variables obtained. It is most convenient to derive these equations in the Hamiltonian formalism. Using the known commutation properties, written out in Ref. 13, for the operators $T_{ab}(\mathbf{p})$ and $n_{ab}(\mathbf{p})$ in second quantization, it is easy to obtain the Poisson brackets (PB) between the dynamic variables. Since we are interested in linear dynamics, we retain in the right-hand parts of the brackets only the equilibrium values of the operators, i.e., the variables (2.11) and (2.13). Calculation of the commutators yields the following PB for the variables (2.9) and (2.10):

$$\{Q_{ij},\ v_{mn}\}=(\Delta/g_1)\,(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm}-{}^2/{}_3\delta_{ij}\delta_{mn}), \tag{3.1}$$

$$\{M_{ij},\ u_{mn}\}=(\Delta/g_1)\,(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm}-{}^2/{}_3\delta_{ij}\delta_{mn}). \tag{3.2}$$

The remaining PB are zero. Since the variables $\tilde{v}_{ik}$ and $\tilde{u}_{ik}$ commute with all the variables, they are integrals of the motion and if they are zero at the initial instant, they remain so also at succeeding instants. We shall therefore disregard these variables.

We have thus ten pairs of canonically conjugate variables: five pairs of Q and v, which give five SM branches, and five pairs of M and u, which give five RSM branches.

As also in spin dynamics, the PB (3.1) and (3.2) are easily generalized to include the case of coordinate-dependent variables:

$$\{Q_{ij}(\mathbf{r}_1),\ v_{mn}(\mathbf{r}_2)\}=(\Delta/g_1)\,(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm}-{}^2/{}_3\delta_{ij}\delta_{mn})\,\delta(\mathbf{r}_1-\mathbf{r}_2), \tag{3.3}$$

$$\{M_{ij}(\mathbf{r}_1),\ u_{mn}(\mathbf{r}_2)\}=(\Delta/g_1)\,(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm}-{}^2/{}_3\delta_{ij}\delta_{mn})\,\delta(\mathbf{r}_1-\mathbf{r}_2). \tag{3.4}$$

The equations of motion are now easily obtained if the form of the Hamiltonian in known. Since we are interested in the linear equations that describe the wave propagation, it suffices to retain in the energy the terms quadratic in Q, M, v, and u. The quadratic energy expression invariant to transformations of $\hat{J}$, $\hat{T}$, and $\hat{P}_{\text{comb}}$ takes the following general form:

$$H=H_v+H_u+H_{uv}, \tag{3.5}$$

where

$$H_v=\frac{1}{2\chi_v}Q_{ik}^2+\frac{1}{2}a_v v_{ik}^2+\frac{1}{2}b_{1v}(\nabla_l v_{ik})^2+\frac{1}{2}b_{2v}(\nabla_k v_{ik})^2,$$

$$H_u=\frac{1}{2\chi_u}M_{ik}^2+\frac{1}{2}a_u u_{ik}^2+\frac{1}{2}b_{1u}(\nabla_l u_{ik})^2+\frac{1}{2}b_{2u}(\nabla_k u_{ik})^2, \tag{3.6}$$

and H_{uv} describes the interaction between the modes:

$$H_{uv}=\beta_u u_{ik}Q_{ik}+\beta_v v_{ik}M_{ik}. \tag{3.7}$$

This interaction is absent in the weak-interaction limit, for in this approximation there is symmetry between the quasiparticles and holes near the Fermi surface. Owing to this symmetry, the $\hat{T}$ operation breaks up into two independent operations, viz. complex conjugation and replacement of t by $-t$. Each of the symmetries forbids separately the appearance of terms of the type (3.7) in the energy. The interaction (3.7) is thus weak to the extent that Δ/ε_F is small, and we shall neglect it.

The Liouville equations $\dot{X}=\{H, X\}$ yield in this case the wave equations for the collective modes:

$$\ddot{u}_{ik}=-\omega_{0u}^2u_{ik}+c_{1u}^2\nabla^2u_{ik}+c_{2u}^2(\nabla_i\nabla_l u_{kl}-\nabla_k\nabla_l u_{il}-{}^2/_3\delta_{ik}\nabla_m\nabla_n u_{mn}), \tag{3.8}$$

$$\ddot{v}_{ik}=-\omega_{0v}^2v_{ik}+c_{1v}^2\nabla^2v_{ik}+c_{2v}^2(\nabla_i\nabla_l v_{kl}+\nabla_k\nabla_l v_{il}-{}^2/_3\delta_{ik}\nabla_m\nabla_n u_{mn}), \tag{3.9}$$

where

$$\omega_{0u,v}^2=\frac{\Delta^2a_{u,v}}{g_1^2\chi_{u,v}},\quad c_{1u,v}^2=\frac{\Delta^2b_{1u,v}}{g_1^2\chi_{u,v}},\quad c_{2u,v}^2=\frac{\Delta^2b_{2u,v}}{2g_1^2\chi_{u,v}}. \tag{3.10}$$

The spectrum of the waves $\omega(q)$ depends on the projection of the angular momentum J_z on the wave propagation direction. Expanding u_{ik} over states with different J_z, we obtain the RSM spectrum (the SM spectrum is similar in form). The variable for the collective mode with $J_z=0$ and the spectrum of this RSM are given by

$$u_{ik}=u^0(\delta_{ik}-3\hat{z}_i\hat{z}_k)/\sqrt{3}, \tag{3.11}$$

$$\omega_{J_z=0}^2=\omega_{0u}^2+(c_{1u}^2+{}^4/_3c_{2u}^2)q^2, \tag{3.12}$$

where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the unit vectors of the Cartesian coordinate frame. The modes with $|J_z|=1$ are described by two independent variables:

$$u_{ik}=u_x^1(\hat{x}_i\hat{z}_k+\hat{z}_i\hat{x}_k)+u_y^1(\hat{y}_i\hat{z}_k+\hat{y}_k\hat{z}_i), \tag{3.13}$$

and the doubly degenerate spectrum is of the form

$$\omega_{J_z=\pm1}^2=\omega_{0u}^2+(c_{1u}^2+c_{2u}^2)q^2. \tag{3.14}$$

Finally, two modes with $|J_z|=2$ are described by two variables:

$$u_{ik}=u_x^2(\hat{x}_i\hat{x}_k-\hat{y}_i\hat{y}_k)+u_y^2(\hat{x}_i\hat{y}_k+\hat{x}_k\hat{y}_i), \tag{3.15}$$

and their spectrum is

$$\omega_{J_z=\pm2}^2=\omega_{0u}^2+c_{1u}^2q^2. \tag{3.16}$$

4. COLLECTIVE MODES IN A MAGNETIC FIELD

RSM Zeeman splitting in a magnetic in which it is linear was calculated in Ref. 17 and experimentally observed in Ref. 18. Here we obtain this splitting in the phenomenological theory.

The main effect of the magnetic field is not the appearance of additional magnetic energy in the Hamiltonian (3.5), for this would lead only to effects quadratic in the field, but the change of the Poisson brackets. In fact, the liquid acquires in the magnetic field one other nonzero equilibrium property, besides the order parameter (2.11) and the density (2.13), viz., the spin density of the liquid

$$S_\alpha=\frac{1}{2}(\sigma_\alpha)_{ab}\sum_{\mathbf{p}}n_{ab}(\mathbf{p})=\frac{\chi}{\gamma}H_\alpha, \tag{4.1}$$

where χ is the magnetic susceptibility and γ is the gyromagnetic ratio. Therefore some of the PB, previously zero at equilibrium, acquire on account of $\mathbf{S}$ nonzero equilibrium magnetic values. For the SM and RSM these are

$$\{M_{ik}, M_{jm}\}=-{}^5/_{108}S_\alpha R_{\alpha l}(\delta_{ij}e_{kml}+\delta_{im}e_{kjl}+\delta_{kj}e_{iml}+\delta_{km}e_{ijl}),$$
$$\{u_{ik}, u_{jm}\}=-{}^4/_3S_\alpha R_{\alpha l}(\delta_{ij}e_{kml}+\delta_{im}e_{kjl}+\delta_{kj}e_{iml}+\delta_{km}e_{ijl}), \tag{4.2}$$
$$\{v_{ik}, v_{jm}\}=-{}^4/_3S_\alpha R_{\alpha l}(\delta_{ij}e_{kml}+\delta_{im}e_{kjl}+\delta_{kj}e_{iml}+\delta_{km}e_{ijl}).$$

We note that by virtue of the zero-spin character of the variable Q [Eq. (2.9)] the PB of the Q components with one another remain zero even in the presence of a magnetic field. As a result the SM reacts more weakly to a magnetic field than the RSM.

The energy (3.5) remains the same as before, since we neglect here the effects quadratic in the field, although they can be easily taken into account if the quadratic Zeeman effect must be considered.[19] The influence of the magnetic field is thus characterized by the vector $\tilde{H}_i=R_{\alpha i}H_\alpha$, i.e., by a field rotated by the matrix $R_{\alpha i}$.

We consider first an RSM in a uniform effective field $\tilde{\mathbf{H}}$, i.e., both the external field $\mathbf{H}$ and the order parameter are homogeneous. We assume for simplicity that the splitting in the magnetic field is larger than the splitting due to finite q, considered in the preceding section. This corresponds to the typical experimental situation in fields $H>100$ G.[4] We regard therefore the gradient terms in the energy as a perturbation. We separate directly the components M_{ik} and u_{ik} corresponding to different projections of J_z on $\tilde{\mathbf{H}}$. They are given by Eqs. (3.11), (3.13), and (3.15) for u_{ik} and by similar formulas for M_{ik}. For these variables we have the following PB obtained from (4.2) and (3.2):

$$\{M^0, u^0\}=\Delta/g_1, \tag{4.3}$$

$$\{M_x^1, u_x^1\}=\frac{\Delta}{g_1},\quad \{M_y^1, u_y^1\}=\frac{\Delta}{g_1},$$
$$\{M_x^1, M_y^1\}=-\frac{5}{108}\frac{\chi}{\gamma}H,\quad \{u_x^1, u_y^1\}=-\frac{4}{3}\frac{\chi}{\gamma}H, \tag{4.4}$$

$$\{M_x^2, u_x^2\}=\frac{\Delta}{g_1},\quad \{M_y^2, u_y^2\}=\frac{\Delta}{g_1},$$
$$\{M_x^2, M_y^2\}=\frac{5}{54}\frac{\chi}{\gamma}H,\quad \{u_x^2, u_y^2\}=\frac{8}{3}\frac{\chi}{\gamma}H, \tag{4.5}$$

and the Hamiltonian

$$H=H^0+H^1+H^2, \tag{4.6}$$

$$H^0=\frac{(M^0)^2}{\chi_u}+a_u(u^0)^2+\left[\left(b_{1u}+\frac{1}{6}b_{2u}\right)q^2+\frac{1}{2}b_{2u}q_z^2\right](u^0)^2, \tag{4.7}$$

$$H^1=\frac{(M_x^1)^2+(M_y^1)^2}{\chi_u}+[(u_x^1)^2+(u_y^1)^2]\times\left(a_u+b_{1u}q^2+\frac{1}{2}b_{2u}q_z^2\right)+\frac{1}{2}b_{2u}(q_xu_x^1+q_yu_y^1)^2, \tag{4.8}$$

$$H^2=\frac{(M_x^2)^2+(M_y^2)^2}{\chi_u}+[(u_x^2)^2+(u_y^2)^2]\times\left[a_u+b_{1u}q^2+\frac{1}{2}b_{2u}(q^2-q_z^2)\right]. \quad (4.9)$$

The Hamiltonian (4.7) and the PB (4.3) lead, for the canonically conjugate variables M^0 and u^0 with projection $J_z=0$, to the following RSM spectrum:

$$\omega^2_{J_z=0}=\omega_{0u}^2+(c_{1u}^2+{}^1/_3c_{2u}^2)\,q^2+c_{2u}^2(\mathbf{q}\hat{\mathbf{h}})^2. \quad (4.10)$$

For future convenience we have introduced here the unit vector

$$\mathbf{h}=\tilde{\mathbf{H}}/H,\quad \tilde{H}_i=R_{\alpha i}H_\alpha, \quad (4.11)$$

which specifies the quantization axis in the magnetic field, such that $q_z{}^2=(\mathbf{q}\cdot\mathbf{h})^2$.

The Hamiltonian (4.8) with PB (4.4) leads to the following system of equations for the four dynamic variables of two RSM with $|J_z|=1$:

$$\begin{aligned}
\dot{u}_x^1&=\frac{2\Delta}{\chi_u g_1}M_x^1+\frac{4}{3}S[2a_u+2b_{1u}q^2+b_{2u}(q_y^2+q_z^2)]u_y^1,\\
\dot{u}_y^1&=\frac{2\Delta}{\chi_u g_1}M_y^1-\frac{4}{3}S[2a_u+2b_{1u}q^2+b_{2u}(q_x^2+q_z^2)]u_x^1,\\
\dot{M}_x^1&=\frac{5}{54}\frac{S}{\chi_u}M_y^1-\frac{\Delta}{g_1}[2a_u+2b_{1u}q^2+b_{2u}(q_x^2+q_z^2)]u_x^1,\\
\dot{M}_y^1&=-\frac{5}{54}\frac{S}{\chi_u}M_x^1-\frac{\Delta}{g_1}[2a_u+2b_{1u}q^2+b_{2u}(q_y^2+q_z^2)]u_y^1.
\end{aligned} \quad (4.12)$$

Assuming that the splitting due to the field exceeds the splitting due to the finite q, these equations lead to the spectrum

$$\omega^2_{J_z=\pm1}=\omega_{0u}^2+c_{1u}^2q^2+{}^1/_2c_{2u}^2[q^2+(\mathbf{q}\hat{\mathbf{h}})^2]+2\omega_{0u}J_zg_u\gamma H, \quad (4.13)$$

where

$$g_u=\frac{\chi}{\gamma^2\chi_u}\left(\frac{5}{108}+\frac{4}{3}a_u\chi_u\right) \quad (4.14)$$

is the Landé factor for the Zeeman splitting. The PB between the Q components in a SM are zero, therefore the g-factor for an SM differs from (4.14) in that there is no first term, and is equal to $g_v=4a_u\chi/3\gamma^2$. Since the second term of the SM g factor is less than the first in the weak-coupling limit, SM splitting in a magnetic field is substantially less than that of RSM. This is why SM splitting has not yet been observed.

In analogy with the derivation of (4.13), the spectrum of an RSM with $|J_z|=2$ is derived from a system of four equations and turns out to be

$$\omega^2_{J_z=\pm2}=\omega_{0u}^2+c_{1u}^2q^2+c_{2u}^2[q^2-(\mathbf{q}\mathbf{h})^2]+2\omega_{0u}J_zg_u\gamma H. \quad (4.15)$$

The condition for the applicability of the expressions (4.10), (4.13), and (4.15) obtained for the RSM spectrum is given by the inequalities

$$c_{2u}^2q^2/2\omega_{0u}\ll g_u\gamma H\ll\omega_{0u}. \quad (4.16)$$

We point out that the spectrum of all five RSM is determined in this approximation by only four parameters: ω_{0u}, c_{1u}, c_{2u}, and g_u, which can be obtained from experiment.

5. EFFECT OF TEXTURES ON COLLECTIVE MODES

The textures in ^{3}He-B, which are inhomogeneous states of the degeneracy parameter $R_{\alpha i}$, are the result of competition between different orienting actions exerted on $R_{\alpha i}$ by the walls, the magnetic field, the superfluid flow, and the quantized vortices produced in a rotating vessel (see, e.g., Ref. 3). In addition, owing to the nontrivial topological structure of the degeneracy space, there are metastable textures that have topological charge and long lifetime due to the preservation of this charge.[20] We point out among them the Maki solitons[21] observed in NMR spectra, solitons with a behavior similar to that of domain walls. The characteristic dimensions of typical structures in ^{3}He-B are quite large because of the high isotropy of this liquid. They are determined by the characteristic magnetic length ξ_H, which is inversely proportional to the magnetic field. In fields $H\sim100$ G this length, at low pressures and far from T_c, reaches several millimeters. In those Maki solitons whose topological charge is connected with the dipole interaction,[20] the dimension of the texture is of the order of the dipole length $\xi_D\sim10^{-3}$ cm. Finally, in objects whose topological charge is determined by the condensation energy, such as quantized vortices, the dimension of the texture is of the order of the coherence length $\sim10^{-5}$ cm.

The textures in ^{3}He-B were investigated up to now in NMR experiments. We shall here that experiments with ultrasound that excites collective modes can also yield information on textures.

The field of the degeneracy parameter $R_{\alpha i}(\mathbf{r})$ in a texture exerts two types of effects on the dynamics of the collective modes. First, even in a uniform magnetic field $\mathbf{H}$, the texture $R_{\alpha i}$ makes the quantization axis $\mathbf{h}$ [see (4.11)] inhomogeneous in space. As a result the RSM spectrum (4.10), (4.13), and (4.15) depends on the coordinates via $\mathbf{h}(\mathbf{r})$. Second, texture produces in the Hamiltonian terms of the dynamic variable u_{ik} that are linear in the gradients. A possible term, e.g., is

$$e_{ikl}B_j^lu_{im}\nabla_nu_{jk}, \quad (5.1)$$

where the tensor associated with the texture

$$B_j^l=e^{lik}R_{\alpha i}\nabla_jR_{\alpha k} \quad (5.2)$$

plays the role of a non-Abelian gauge field for the collective mode.

We consider here broad textures with dimension ξ_H or ξ_D substantially larger than the wavelength of an RSM excited by ultrasound (the wave vector q of the RSM-exciting ultrasound is obtained by comparing the ultrasound frequency $\omega=cq$ (c is the speed of sound) with the RSM frequency, i.e., $q\sim\Delta/c\sim10^4$–10^6 cm$^{-1}\gg\xi_D^{-1}\gg\xi_H^{-1}$). In these textures, the first mechanism changes the frequency of the collective mode by a measurable value $c_2^2q^2/\omega_0$, whereas the second changes it by the much smaller value $c_2^2q/\omega_0\xi_H$ or $c_2^2q/\omega_0\xi_D$. The second effect becomes inportant if an RSM is excited in NMR experiments by an rf magnetic field of large wavelength.

The dependence of the RSM spectrum on the coordinates leads to the following important consequence. Since the wavelength is much smaller than the texture dimension it can be assumed that ultrasound excites in a given point of the texture a collective mode independently of the remaining sections of the texture. The ultrasound absorption intensity is therefore proportional to the density of states, i.e., to the relative volume of the texture acted upon by the given frequency:

$$P(\omega)=\frac{1}{V}\int dV\delta(\omega-\omega(\mathbf{r})). \tag{5.3}$$

The main contribution to this spectral density is made by those texture sections in which the spectrum is stationary, i.e., $\nabla\omega = 0$. To frequencies equal to the value of the spectrum at the stationary points there corresponds a singularity in the absorption spectrum. We can thereby determine the topology of stationary points, lines, or surfaces in a texture, and hence the form of the texture. Particularly strongly pronounced in the absorption spectrum is the singularity in the case of a planar texture, e.g., in a Maki soliton or in a liquid placed between parallel plates. In this case $\nabla\omega$ can vanish on the entire plane, corresponding to infinite $P(\omega)$ at the frequency at which the spectrum is stationary. This means appearance of a line in the ultrasound-absorption spectrum.

We consider just such a case, which is typical for experiments with ultrasound propagating between parallel plates. We shall show that in this geometry the five RSM correspond not to one ultrasound-absorption line each, as should be the case in an unbounded liquid, but to two lines each if the angle β between the magnetic field and the normal to the plates differs from zero. The texture between parallel plates in a magnetic field is determined by the competition between the magnetic and surface energies F_M and F_s (see Ref. 2):

$$F_M=-a\int dV(\mathbf{nH})^2,\quad F_s=-d\int dS(s_iR_{\alpha i}H_\alpha)^2, \tag{5.4}$$

where s is the normal to the plates (the x axis), and the unit vector $\mathbf{n}$ specifies the rotation angle defined by the orthogonal matrix

$$R_{\alpha i}(n,\ \theta_0)=\delta_{\alpha i}+(1-\cos\theta_0)(n_\alpha n_i-\delta_{\alpha i})+e_{\alpha il}n_l\sin\theta_0. \tag{5.5}$$

The rotation angle is fixed by the spin–orbit interaction: $\cos\Theta_0 = -1/4$.

Since we are interested in the distribution of the quantization axis in the texture, we express the energies (5.4) in terms of $\mathbf{h}$, using (5.5):

$$F_M=-\frac{4}{5}aH\int dV\,\mathbf{Hh},\quad F_s=-dH^2\int dS(\hat{\mathbf{s}}\mathbf{h})^2. \tag{5.6}$$

In fields H that are strong enough, when the distance l between the plates exceeds the magnetic length ξ_H, the quantization axis is parallel to the magnetic field in practically the entire volume between the plates, and this ensures a minimum of the volume energy F_M. In the surface regions near plates of size ξ_H, the vector $\mathbf{h}$ is rotated in such a way that it becomes aligned with the normal on the surface itself: $\mathbf{h} = \pm\mathbf{s}$, since this is ensured by the minimum of the surface energy F_s (see Fig. 1, which shows the two textures of the field $\mathbf{h}$ that are possible in such a geometry).

We consider now excitation of RSM in such a texture by ultrasound propagating between the plates such that $\mathbf{q}\|\mathbf{s}$.

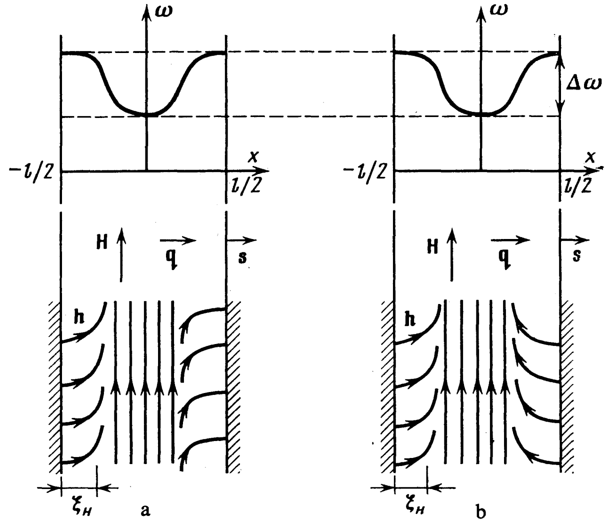

FIG. 1. Distribution of quantization axis $\mathbf{h}$ of the internal angular momentum J of collective modes in two possible textures between parallel plates in a strong magnetic field parallel to the plates. In the upper part of the figure is shown the dependence of the frequency ω of the spectrum of a collective mode with quantum numbers $J = 2$ and $J_z = 0$ on the coordinate x along the wave vector $\mathbf{q}$. The distance between the stationary points of the function $\omega(x)$ determines the observable splitting $\Delta\omega$ of this mode in the texture. Its splitting in strong fields does not depend on the field and on the forms of textures a and b.

We choose for the sake of argument an RSM with $J_z = 0$. Its spectrum has an absolute maximum $\mathbf{q}\|\mathbf{h}$ [(see (4.10)], i.e., on the surfaces of the plates, and an absolute minimum at those points of the texture where the deflection of $\mathbf{q}$ away from $\mathbf{h}$ is a maximum. The latter is realized on a line midway between the plates (see Fig. 1), where $\mathbf{h}\|\mathbf{H}$. Thus, the spectral density

$$P(\omega)=\frac{1}{l}\int_{-l/2}^{l/2}dx\,\delta(\omega-\omega(x))=\left(l\left|\frac{\partial\omega}{\partial x}\right|_{x=x(\omega)}\right)^{-1} \tag{5.7}$$

becomes infinite at two frequencies equal to the RSM frequencies at $x = 0$ and at $x = \pm l/2$:

$$\begin{aligned}\omega^2_{J_z=0}(x=\pm l/2)&=\omega_{0u}^2+(c_{1u}^2+{}^4/_3c_{2u}^2)q^2,\\ \omega^2_{J_z=0}(x=0)&=\omega_{0u}^2+(c_{1u}^2+{}^1/_3c_{2u}^2+c_{2u}^2\cos^2\beta)q^2.\end{aligned} \tag{5.8}$$

The RSM with $J_z = 0$ should therefore split into two lines with a distance between them

$$\Delta\omega_{J_z=0}\approx\frac{1}{2\omega_{0u}}c_{2u}^2q^2\sin^2\beta. \tag{5.9}$$

We point out that the splitting does not depend on the magnetic field if the latter is strong enough to satisfy the condition $\xi_H \ll l$. In weak fields, when $\xi_H \gtrsim l$, the vector $\mathbf{h}(0)$ is no longer parallel to $\mathbf{H}$ and depends on the texture. Figure 2 shows the textures obtained from the textures of Fig. 1 when the field is weakened. It can be seen that the resultant splittings are different:

$$\Delta\omega_{J_z=0}\approx(c_{2u}^2/2\omega_{0u})[\mathbf{qh}(0)]^2. \tag{5.10}$$

When the field tends to zero the magnetic length is $\xi_H \gg l$ and the texture with minimum energy becomes homogeneous, so that there is no splitting.

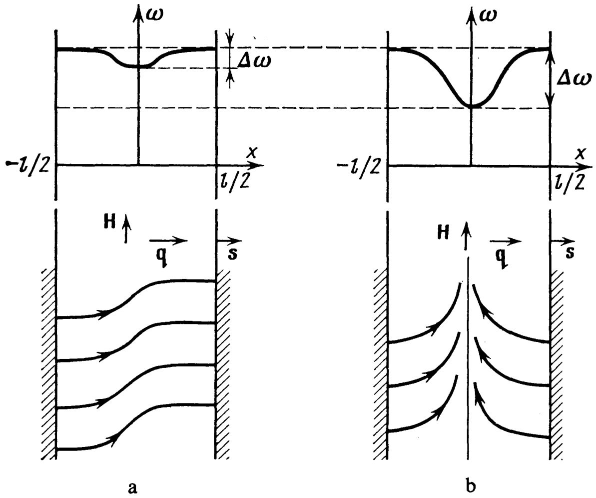

FIG. 2. The same as in Fig. 1, but in a weak field. The splitting of the collective-mode spectrum by the stable texture shown in Fig. 2a decreases to zero with decreasing field. The metastable structure in Fig. 2b leads to the same splitting as in a strong field. The observed hysteresis in the splitting is due to the metastable structure produced on going into the superfluid state in the presence of a field.

A similar splitting of an RSM with $J_z = 0$ was actually observed in an experiment[4] in a magnetic field perpendicular to the normal **s**, i.e., at $\beta = \pi/2$. It was seen that the splitting depended on the value of **H** in weak fields, but saturated at $\mathbf{H} \gtrsim 500$ G, i.e. precisely when $\xi_H < l$, where $l = 4$ mm. In weak fields, when $\xi_H < l$, hysteresis is observed and is due to formation of the two different structures in Figs. 1 and 2. The seemingly unusual splitting of RSM with $J_z = 0$ finds thus a simple explanation within the framework of texture effects.

The remaining four RSM with other J_z should be split by the texture in similar manner:

$$2\Delta\omega_{|J_z|=2} = -\Delta\omega_{|J_z|=2} = \frac{c_{2u}^2}{2\omega_{0u}}[\mathbf{q}\times\mathbf{h}(0)]^2. \tag{5.11}$$

No such splitting was observed so far. The reason is that RSM with $J_z \neq 0$ are much more weakly excited by ultrasound than the mode with $J_z = 0$, so that observation of the splitting of these modes is more difficult.

We note that the texture splitting amplitude $c_{2u}^2/2\omega_{0u}$ is connected with the RSM splitting, known from other experiments (see Ref. 4), in a zero magnetic field by the action of finite q [see (3.12), (3.14), and (3.16)]. Thus, for example, comparing (5.10) with (3.14) and (3.16) we get

$$\frac{\Delta\omega(J_z=0,\ H)}{\omega(|J_z|=1,\ H=0)-\omega(|J_z|=2,\ H=0)} = \frac{[\mathbf{q}\times\hat{\mathbf{h}}(0)]^2}{q^2}. \tag{5.12}$$

Thus, by measuring the left-hand side of (5.12) from two experiments (with and without a field), it is possible to determine directly the angle between the ultrasound propagation and the order parameter midway between the plates. It must only be recognized that if a field is used it must satisfy the conditions for the applicability of the expressions obtained for this case, i.e., (4.16).

6. HOW TO USE COLLECTIVE MODES TO INVESTIGATE TEXTURES IN ROTATING ^{3}He-*B*

Quantum vortices produced upon rotation change the texture,[3] and from this change one can obtain the vortex parameters of the structure of the vortex core. Consider ^{3}He-*B* contained between two plates and rotating about a normal to the plate (the rotation angular-velocity direction is $\hat{\Omega} \parallel \mathbf{s}$). Added to the orienting actions (5.4) and (5.6) of the magnetic field on the degeneracy parameter is the orienting action of the vortices, which is expressed in terms of dimensionless vortex parameter proportional to the density of the vortices.[22] The vortex action corresponds to a contribution to the free energy

$$F_V = \frac{2}{5}a\lambda\int dV(\hat{\Omega}_i R_{\alpha i} H_\alpha)^2 = \frac{2}{5}a\lambda H^2\int dV(\hat{\Omega}\mathbf{h})^2. \tag{6.1}$$

If the distance l between the plates exceeds the magnetic length ξ_H, the splitting of the spectrum of an RSM with $J_z = 0$ is given by (5.10), where the direction of the quantization axis $\mathbf{h}(0)$ in the volume is now determined by the joint action of the magnetic field and the vortices. This direction is obtained by minimizing $F_M + F_V$ with respect to **h**. As a result we arrive at the following connection between the RSM splitting and the vortex parameter λ:

$$\lambda = |\hat{\Omega}\mathbf{h}(0)|^{-1}\cos\beta - |[\hat{\Omega}\times\mathbf{h}(0)]|^{-1}\sin\beta. \tag{6.2}$$

Thus, the parameter λ can be directly extracted from measurements of the spectrum of the collective RSM. This parameter was previously measured in NMR experiments,[3,23] and its determination from the experimental data on the NMR-signal absorption line shape entails certain difficulties not encountered in experiments with ultrasound. The point is that in NMR experiments on textures the spin waves are excited by a uniform rf field. Therefore the levels excited in the spin waves are mainly the lower ones, which correspond to localized states in the potential produced by the textures. As a result, resonance peaks corresponding to excitation of spin waves localized on the texture are superimposed on the NMR-signal absorption line. To determine the parameter λ from the experimental data it was necessary either to find the energy levels by solving the Schrödinger equation (in which case some of the parameters are not known with sufficient accuracy), or to neglect the effects of the resonance peaks and investigate the shift of the envelope of the line shape, a procedure of equally low accuracy.

On the contrary, the wavelengths of collective modes excited by ultrasound are exceedingly small compared with the texture dimension, so that the quasiclassical approximation (or the geometric-optics limit) is applicable here, and expression (5.3) used by us for the spectral density of the ultrasound absorption is valid with good accuracy.

CONCLUSION

In the texture of superfluid ^{3}He-*B* the quantization axis of the internal angular momentum J that characterizes the collective modes $R_{\alpha i}(\mathbf{r})H_\alpha$ varies in space. This leads to an observable spectrum splitting not governed by additional quantum numbers and determined by the topology of the set of the stationary points of the spectrum in the texture. This permits new information on the textures in ^{3}He-B to be extracted.

The textures can lead to a similar splitting of the mode spectrum also in ^{3}He-A, and no magnetic field is needed for this purpose. In anisotropic ^{3}He-A the quantization axis for the collective modes is the anisotropy vector **l**. Its change in the texture should cause a texture-induced splitting of the spectrum. In contrast to the integral texture characteristics **l**, obtained from data on the damping of ultrasound, measurement of the splitting yields a local characteristic. This is particularly important for textures with dimension substantially larger than the dipole length ξ_D, since such textures are difficult to observe in NMR experiments.

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