On the problem of the dynamics of the collapse of Langmuir waves

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(Submitted 23 January 1984)
Zh. Eksp. Teor. Fiz. **87**, 433–449 (August 1984)

We obtain a criterion for the instability of self-similar regimes of the supersonic collapse of Langmuir waves. We show that all self-similar regimes which at this moment are known satisfy this criterion. We discuss possible manifestations of the instability in numerical calculations. We consider the problems of the effect of a magnetic field and of electron non-linearities on Langmuir collapse dynamics.

§ 1. INTRODUCTION

In the ten years after Langmuir collapse was predicted[1] a large number of papers has been published devoted to the conditions for the occurrence, the dynamics, and the consequences of this phenomenon. Invariably the main attention has been paid to the problem of self-similar collapse regimes as they seem to be important and at the same time the most accessible to investigations. Study of this problem yielded a number of important results, in particular, a theorem that an infinite number of bound states exist in a self-similar caviton.[2] Recently the existence of self-similar regimes was confirmed by their direct computation in the cases of centrally symmetric scalar collapse and of Langmuir collapse of strongly oblate caviton (Ref. 3).[1)] Nonetheless, the level of understanding attained up to the present moment on collapse dynamics is not fully satisfactory, primarily due to the lack of clarity of the problem of the instability and of the possibility of the establishment of self-similar regimes. It is the aim of the present paper to study this and some other unclear problems of the collapse dynamics.

§ 2. NECESSARY INFORMATION ABOUT COLLAPSE

The equations describing the collapse of Langmuir waves have, in dimensionless variables, the following form:[1]

$$\nabla\,(i\partial/\partial t+\Delta-n)\,\nabla\varphi=0, \tag{1}$$

$$\partial^2 n/\partial t^2-\Delta\,(n+W)=0, \qquad W\equiv|\nabla\varphi|^2. \tag{2}$$

Here φ is the temporal envelope of the high-frequency electric potential, and n the perturbation of the ion density. Instead of (1), (2) one often studies the simpler set of equations:

$$(i\partial/\partial t+\Delta-n)\,E=0, \tag{1'}$$

$$\partial^2 n/\partial t^2-\Delta\,(n+W)=0, \qquad W\equiv|E|^2, \tag{2'}$$

usually called the "scalar model."

Equation (1) [or (1′)] allows us to establish a connection between the characteristic time ω^{-1} of the change in phase of the electric field envelope, the depth n and the spatial scale a of the collapsing caviton:

$$\omega\sim n\sim a^{-2}. \tag{3}$$

Using Eq. (2) we can estimate the characteristic time γ^{-1} of the deepening of the caviton. When there is no compensation of the pressure of the high-frequency field by the gas-kinetic plasma pressure we have $W+n\sim W$ and

$$\gamma\sim(W/na^2)^{1/2}\sim W^{1/2}. \tag{4}$$

As the deepening of the caviton is accompanied by its compression and by a growth of the energy density W of the waves trapped in it, γ^{-1} is also the time for W to double:

$$W_t\sim\gamma W. \tag{5}$$

Using (4) and (5) one easily finds the law for the growth of the energy density of the waves in the caviton as one approaches the time t_s when a singularity is formed:

$$W\propto(t_s-t)^{-2}. \tag{6}$$

One must emphasize that this result is obtained from rather rough considerations without assuming a self-similar evolution of the caviton. Just as rough is the conclusion about the "adiabaticity" of the collapse, i.e., about the satisfying of the condition

$$\omega\gg\gamma \quad (Wa^4\ll1). \tag{7}$$

When the size a of the caviton decreases this condition is satisfied better and better since the energy density of the waves in the caviton (when there are no external sources) cannot grow faster than a^{-3}:

$$W\lesssim W_0(a_0/a)^3. \tag{8}$$

It is relevant to remind ourselves here that the quantity W is bounded from below by the condition $W\gtrsim a^{-2}$ for mudulational instability which is necessary for the development of the collapse. If the more stringent limitation

$$W\gg a^{-2} \tag{9}$$

is satisfied, which enables us to neglect in Eq. (2) the term Δn, the collapse is called "supersonic."

To find the time-dependence of the size of the caviton one normally uses the assumption that the total energy of the waves trapped in it is constant. In that case the equal sign holds in (8) so that

$$W\propto a^{-3}, \tag{10}$$

and as one approaches the time t_s the condition (9) for supersonic compression is satisfied with an ever greater margin.

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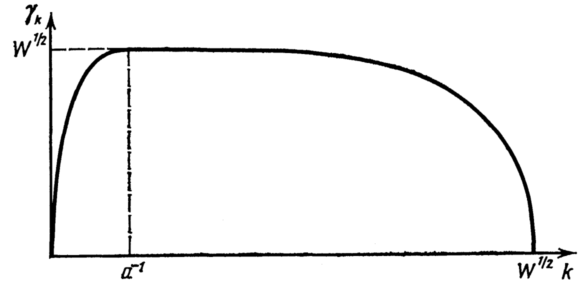

FIG. 1. Dependence of the modulational instability growth rate on the wave number of the perturbations when one is well above threshold ($W \gg a^{-2}$). In the region $k \gg a^{-1}$ the simple formula $\gamma_k \approx (W - k^2)^{1/2}$ is valid.

§ 3. THE POSSIBILITY OF A SMALL-SCALE INSTABILITY

A large excess of the energy density of the waves above the threshold for the modulational instability conceals any danger for the stability of the supersonic collapse regimes. The fact is that if condition (9) were satisfied in a stationary density well, not only perturbations with wavelengths $k^{-1} \sim a$ would be modulationally unstable, but also perturbations with appreciably smaller scales, down to $k^{-1} \sim W^{-1/2} \ll a$. Of course, an evolving caviton differs from stationary one: the fast change of the main scale a may in principle suppress the small-scale instability. However, the fact that the growth rate of the instability of a stationary caviton is practically independent of the wavelength k^{-1} of the perturbation in the whole range $a^{-1} \lesssim k \ll W^{1/2}$ (see Fig. 1) suggests a possible survival of a small-scale instability when we change to an evolving caviton. The answer to the problem of the stability of an evolving caviton is clearly determined by the numbers involved and can only be obtained through a quantitative analysis.

§ 4. BASIC EQUATIONS

The well known estimates which for the convenience of the reader we collected in §2 show that at least in the rather late stages of the collapse the starting equations contain two small parameters—the ratio of the non-linear growth rate of the modulational instability γ to the oscillation frequency ω of the temporal envelope of the electric field in the caviton and the ratio of the sound speed to the collapse speed of the caviton:

$$g = \gamma/\omega \sim W^{1/2} a^2 \propto (t_s - t)^{1/3},$$
$$\varepsilon \sim \left(\frac{a}{t_s - t}\right)^{-1} \propto (t_s - t)^{1/3}. \tag{11}$$

The solution of the starting equations can be expanded independently in each of the parameters g and ε but it is convenient to write the two expansions as a single one—in powers of $(t_s - t)^{1/3}$. In the simplest case one must to obtain that expansion put

$$\varphi = \varphi_p \exp\left\{ i \int^t \omega_p(t')\, dt' \right\}, \tag{12}$$

where φ_p and ω_p are slowly varying functions, and by successive approximations solve the equations

$$\nabla(-\omega_p - n + \Delta)\nabla\varphi_p + i\Delta \frac{\partial \varphi_p}{\partial t} = 0,$$
$$\frac{\partial^2 n}{\partial t^2} - \Delta(n + |\nabla\varphi|^2) = 0. \tag{13}$$

If in (13) we change to new variables and introduce new functions by using the formulae

$$\tilde{t} = t_s - t, \quad \boldsymbol{\xi} = \mathbf{r}\tilde{t}^{-2/3},$$
$$\varphi_p(\mathbf{r}, t) = \tilde{t}^{-1/3}\psi_p(\xi, \tilde{t}),$$
$$n(\mathbf{r}, t) = \tilde{t}^{-4/3} u(\xi, t), \quad \omega_p(t) = \tilde{t}^{-4/3}\Omega_p(\tilde{t}),$$

we get the equations

$$\nabla(-\Omega_p - u + \Delta)\nabla\psi_p = -i\tilde{t}^{1/3}\Delta(\hat{L} - 3/2)\psi_p,$$
$$(\hat{L}^2 - 1/4)u - \Delta|\nabla\psi_p|^2 = \tilde{t}^{2/3}\Delta u. \tag{14}$$

Here ∇ indicates already differentiation with respect to ξ,

$$\hat{L} = \frac{1}{2} + \frac{4}{3} + \frac{2}{3}\xi\frac{\partial}{\partial \xi} - \tilde{t}\frac{\partial}{\partial \tilde{t}}.$$

The solution of Eqs. (14) as arbitrary functions of the time can be written in the form

$$f(\tilde{t}) = \sum_{j=0}^{\infty} (i\tilde{t}^{1/3})^j f^{(j)}(\tilde{t}). \tag{15}$$

To make the representation (15) unique one can arrange to leave in the coefficients $f^{(j)}(\tilde{t})$ only such a time dependence which cannot be expanded in a Taylor series in powers of $\tilde{t}^{1/3}$. Substitution of the expansion (15) of the functions u, ψ_p, Ω_p into (14) leads to the following equation chain:

$$\nabla(-\Omega_p^{(0)} - u^{(0)} + \Delta)\nabla\psi_p^{(0)} = 0, \quad (\hat{L}^2 - 1/4)u^{(0)} = \Delta|\nabla\psi_p^{(0)}|^2; \tag{16}$$

$$\nabla(-\Omega_p^{(0)} - u^{(0)} + \Delta)\nabla\psi_p^{(j)} = -\Delta\left(\hat{L} - \frac{3}{2} - \frac{j-1}{3}\right)\psi_p^{(j-1)}$$
$$+ \sum_{j_1=1}^{j} \nabla(\Omega_p^{(j_1)} + u^{(j_1)})\nabla\psi_p^{(j-j_1)},$$
$$\left[\left(\hat{L} - \frac{j}{3}\right)^2 - \frac{1}{4}\right]u^{(j)} = -\Delta u^{(j-2)}$$
$$+ \sum_{j_1=0}^{j} (-1)^{j_1}\Delta(\nabla\psi_p^{(j_1)*}\nabla\psi_p^{(j-j_1)}). \tag{17}$$

We assume in what follows that the eigenvalue $\Omega_p^{(0)}$ determined from the first of Eqs. (16) is non-degenerate. Without loss of generality one can then assume the functions $\psi_p^{(j)}$ to be real; the functions $u^{(j)}$ and $\Omega_p^{(j)}$ turn out to be automatically real. As, on the other hand, the odd coefficients in the expansions (15) of the functions u and Ω_p necessarily are imaginary, these coefficients are identically equal to zero.

The chain of equations which we have obtained enables us to look for the solution of the set (14) by successive approximations. In particular, the closed set of equations of the zeroth approximation consists of Eqs. (16) supplemented by the condition that the equations of the first approximation can be solved. This condition has the form

$$\frac{d}{d\tilde{t}} \int d^3\xi\, |\nabla\psi_p^{(0)}|^2 = 0 \tag{18}$$

and has the meaning of the conservation of number of waves trapped in the caviton.

The change from the initial set of equations to (16), (17) enables us in a natural way to pose the problem of stability of the solutions which are (asymptotically as $\tilde{t}\to 0$) self-similar, since they correspond to stationary solutions of Eqs. (16), (17). If they are linearized against the background of some stationary state these equations contain the time explicitly only in the combination $\tilde{t}(\partial/\partial\tilde{t})$ so that for them we have solutions in powers of $\tilde{t}$:

$$\delta u^{(j)}(\xi,\tilde{t})=u_\alpha^{(j)}(\xi)\tilde{t}^\alpha+\text{c.c.},\qquad \delta\Omega_p^{(j)}(\tilde{t})=\Omega_{p\alpha}^{(j)}\tilde{t}^\alpha+\text{c.c.},$$
$$\delta\psi_p^{(j)}(\xi,\tilde{t})=\psi_{p\alpha}^{(j)}(\xi)\tilde{t}^\alpha+\text{c.c.} \tag{19}$$

The representation of the temporal envelope of the potential φ in the form (12) which we use here refers to the case when only one of all possible adiabatically changing bound states in the caviton is populated. In the general case we must write on the right-hand side of (12) a sum over the numbers p of the states. The generalization of the zeroth approximation equations can also be reduced to a simple substitution

$$|\nabla\psi_p|^2\to\sum_p|\nabla\psi_p|^2$$

on the right-hand side of the second of Eqs. (16). The interference terms in the pressure of the high-frequency waves which we omitted in the zeroth approximation lead to the appearance of a fast oscillating correction, which is small of the order of the adiabaticity parameter, to the slowly changing density perturbation. This correction, in turn, gives a non-linear shift of the eigenfrequencies and, moreover, generates small corrections to the potential φ which oscillate with frequencies which differ from the eigenfrequencies. The new terms in φ lead to the appearance of new interference terms, and so on. All corrections mentioned can be expressed in terms of the slowly changing functions u, ψ_p, Ω_p for which one can ultimately obtain a chain of equations similar to (16), (17). The most important difference from the equations considered above of the single-mode regime consists in that resonances which inevitably arise in sufficiently high orders of perturbation theory lead to a weak (but no longer exponential) change in the occupation numbers of the bound states. This difference has no importance for what follows and only the rather rough structural properties of Eqs. (16), (17) which are equally characteristic for the equations of single-mode regimes are important.

§ 5. SYMMETRY PROPERTIES OF THE BASIC EQUATIONS

Equations (13) are invariant under space-time shifts and spatial rotations; the chain of Eqs. (16), (17) allows time stretching:

$$\tilde{t}\to\beta\tilde{t}, \tag{20}$$

and the zeroth approximation Eqs. (16), (18) possess yet another symmetry under coordinate stretching:

$$\xi\to\lambda\xi,\qquad \psi_p^{(0)}\to\lambda\psi_p^{(0)},$$
$$u^{(0)}\to\lambda^{-2}u^{(0)},\qquad \Omega_p^{(0)}\to\lambda^{-2}\Omega_p^{(0)}. \tag{21}$$

An infinitesimally small spatial shift $\mathbf{r}\to\mathbf{r}+\boldsymbol{\delta}$ generates the transformation

$$\xi\to\xi+\frac{\delta}{\tilde{t}^{1/3}},\qquad u^{(j)}(\xi)\to u^{(j)}(\xi)+\frac{\delta}{\tilde{t}^{1/3}}\nabla u^{(j)}(\xi),$$
$$\psi^{(j)}(\xi)\to\psi^{(j)}(\xi)+\frac{\delta}{\tilde{t}^{1/3}}\nabla\psi^{(j)}(\xi), \tag{22}$$

i.e., three growing eigenmodes (19) with $\alpha=-2/3$.

The infinitesimal time shift $\tilde{t}\to\tilde{t}+\delta$ generates the transformation

$$\xi\to\xi-\frac{2}{3}\frac{\delta}{\tilde{t}}\xi,$$
$$u^{(j)}(\xi)\to u^{(j)}(\xi)+\frac{\delta}{\tilde{t}}\left[\frac{-4+j}{3}u^{(j)}(\xi)-\frac{2}{3}\xi\nabla u^{(j)}(\xi)\right],$$
$$\psi_p^{(j)}(\xi)\to\psi_p^{(j)}(\xi)+\frac{\delta}{\tilde{t}}\left[\frac{-1+j}{3}\psi_p^{(j)}(\xi)-\frac{2}{3}\xi\nabla\psi_p^{(j)}(\xi)\right], \tag{23}$$
$$\Omega_p^{(j)}\to\Omega_p^{(j)}+\frac{\delta}{\tilde{t}}\left(\frac{-4+j}{3}\right)\Omega_p^{(j)},$$

i.e., a growing eigenmode (19) with $\alpha=-1$.

Spatial rotations change one stationary solution of the chain of Eqs. (16), (17) into another and this generate three indifferently stable eigenmodes[2)] ($\alpha=0$).

As we noted above, the symmetry (20) allows us to look for solutions of Eqs. (16), (17) linearized with respect to the background of some stationary state in the form (19).

The additional symmetry (21) of the zeroth approximation equations lead, as will become clear in what follows, to a specific degeneracy of their solutions: it turns out that each stationary solution of the chain of Eqs. (16), (17) corresponds, generally speaking, to a one-parameter family of stationary solutions of the zeroth approximation equations. The reason why not all stationary solutions of Eqs. (16), (17) can be used as the zeroth approximation for constructing a stationary solution of the chain (16), (17) is that the expansion in the vicinity of the point $\xi=0$ of the solution of the set (16), (17) contains, generally speaking, non-integer powers of ξ while the solution of the set (16), (17) which is regular at zero can not contain them as in the opposite case due to the term $\Delta u^{(j-2)}$ in the second of Eqs. (17) for sufficiently large j in the expansions of the functions $u^{(j)}$, $\psi_p^{(j)}$ necessarily negative powers of ξ would appear. A similar difference occurs also between the linearized sets of Eqs. (16), (17) and (16), (18): for the first of them the eigenvalue spectrum α turns out to be discrete while for the second one it is, in general, continuous. The expansions in the vicinity of the point $\xi=0$ of the eigenfunctions of the continuous spectrum of the linearized set of Eqs. (16), (18) contain non-integer powers of ξ due to which it is impossible to establish for those zeroth-approximation functions a correspondence with any eigenfunctions of the exact problem which would be close to them: the corrections of the subsequent approximations turn out to be not small in the range

$$\xi\lesssim\tilde{t}^{1/3}.$$

One can, however, replace the zeroth approximation eigenfunctions by analytical functions (which do not contain fractional powers of ξ) which are close to them such that the corrections from the higher approximations remain small everywhere while the violation of the equations for the zeroth approximation eigenfunctions is inappreciable. This violation produces a weak temporal dependence of the eigenfunctions which are smooth at the origin which compensates it. One can thus arrange a correspondence between the eigenmodes of the continuous spectrum of the linearized set (16), (18) and the solutions of the exact linear problem which remain close to them for a long time—the "quasi-modes." It is clear that the quasi-modes are the closer to the eigenmodes of the zeroth approximation the smaller is the main nonanalytical term of the expansion of the latter as $\xi \to 0$. In view of the fact that the collapse time is finite and also that the cavitons can be destroyed by sufficiently large perturbations, the growing quasi-modes are practically as much a threat to the realizability of supersonic self-similar regimes as the true eigenmodes with $\mathrm{Re}\,\alpha < 0$. The eigenmodes given at the start of this section with $\alpha = -2/3$ and $\alpha = -1$ corresponding to shifts in position and time of the emergence of the singularity are, of course, not dangerous in this respect. However, the existence of only one eigenmode or quasi-mode, different from them, with $\mathrm{Re}\,\alpha < 0$ would mean a real instability.

§ 6. INSTABILITY CRITERION FOR SELF-SIMILAR REGIMES OF A "SCALAR" COLLAPSE

In the scalar collapse model the linearized zeroth approximation set of equations has the following form

$$\alpha \int d^3\xi\, E_p E_{p\alpha} = 0, \tag{24}$$

$$(-\Omega_p - u + \Delta) E_{p\alpha} = (u_\alpha + \Omega_{p\alpha}) E_p, \tag{25}$$

$$(\hat{L}_\alpha^2 - {}^1/_4) u_\alpha = 2\Delta \sum_p E_p E_{p\alpha}. \tag{26}$$

Here $\hat{L}_\alpha$ is an operator obtained from $\hat{L}$ by replacing $\tilde{t}\,(\partial/\partial\tilde{t})$ by α; the upper index "0" indicating the zeroth approximation has been dropped for simplicity. Before we start with a study of the spectrum of the eigenvalues α of the set (24) to (26) in the general case it is useful to consider a simpler problem—that of the stability of a centrally symmetric caviton with a single occupied level under perturbations which do not destroy the symmetry. In this simplest particular case Eqs. (25), (26) are a set of linear ordinary fourth-order differential equations. Considering the term $\Omega_\alpha E$ (we drop for simplicity the index "p") on the right-hand side of Eq. (25) as a driving force one can write the general solution of the set (25), (26) in the form of a sum of a particular solution and four linearly independent solutions of the homogeneous set. The particular solution can, clearly, be chosen to be regular at the point $\xi = 0$. The asymptotic behavior of the solutions of the homogeneous set as $\xi \to 0$ are as follows:

$$E_\alpha \propto \xi^{-1}, \quad u_\alpha \propto \xi^{-1}; \tag{27}$$

$$E_\alpha \to \mathrm{const}, \quad u_\alpha \to \mathrm{const}; \tag{28}$$

$$E_\alpha \propto \xi^{c+2}, \quad u_\alpha \propto \xi^c,$$
$$c = {}^3/_2[\alpha - {}^1/_2 - {}^4/_3 \pm ({}^1/_4 + 2E_s^2)^{1/2}], \quad E_s \equiv E(0). \tag{29}$$

Depending on the value of $\mathrm{Re}\,\alpha$ the number of asymptotic solutions as $\xi \to 0$ can change from one to three. If

$$\mathrm{Re}\,\alpha < {}^4/_3 + {}^1/_2 - ({}^1/_4 + 2E_s^2)^{1/2}, \tag{30}$$

only the asymptotic form (28) is regular, and the general solution which is regular at the origin contains three free parameters: $E_\alpha(0), \Omega^\alpha$, and α. After reckoning solutions differing from one another solely by multiplication by a constant to be identical there remain two free parameters. One must choose the values of these two parameters such that the solution be regular as $\xi \to \infty$ and, moreover, satisfy condition (24).

The linearly independent asymptotic forms of the solutions of the homogeneous system as $\xi \to \infty$ have the form

$$E_\alpha \propto e^{\pm\xi} \tag{31}$$

[u_α is determined from Eq. (26)];

$$u_\alpha \propto \xi^{3(\alpha - 4/3 - 1/2 \pm 1/2)/2} \tag{32}$$

[E_α is determined from Eq. (25)].

In the range (30) of α values one of these asymptotic forms ($E \propto e^{\xi}$) is irregular. Hence the two parameters on which the solution which is regular at the origin depend must satisfy two equations and the spectrum of the eigenvalues α must thus be discrete.

If

$$\alpha_s \equiv {}^4/_3 + {}^1/_2 - ({}^1/_4 + 2E_s^2)^{1/2} < \mathrm{Re}\,\alpha < {}^4/_3, \tag{33}$$

there appears at the origin a second regular asymptotic form and, as before, there remains at infinity only one irregular one. This means that the eigenvalues α fill the whole band (33). If

$$E_s^2 > {}^{14}/_9 \tag{34}$$

the band (33) intersects the half-plane $\mathrm{Re}\,\alpha < 0$. Hence, (34) is a sufficient condition for instability.

The whole discussion given above can easily be transferred to the case of perturbations which violate the symmetry of the caviton. The angular dependence of such perturbations are given by spherical harmonics. For the lth harmonic Eqs. (27), (28) take the form

$$E_\alpha \propto \xi^{-l-1}, \quad u_\alpha \propto \xi^{-l-1}, \tag{27'}$$

$$E_\alpha \propto \xi^l, \quad u_\alpha \propto \xi^l, \tag{28'}$$

while Eqs. (29) to (34) are unchanged.

The results obtained remain valid also for cavitons which do not have a symmetry. This is already clear from the fact that when one changes from a symmetrical to an asymmetrical caviton there occurs merely an intermingling of the various spherical harmonics in the angular dependence of the eigenfunctions, but the number of asymptotic forms which are regular at the origin and irregular at infinity is not changed. The word "number" needs here an explanation: as there is a countable set of different asymptotic forms we must, strictly speaking, consider a finite-dimensional approximation of the starting equations (e.g., discard all harmonics with $l > N$), and afterwards take the limit as $N \to \infty$.

Finally, one can also generalize the results to the case where several levels are occupied in the caviton. In that case the quantity E_s^2 in the final formulae (33), (34) must be understood as follows:

$$E_s^2=\sum_p E_p^2(0). \tag{35}$$

§ 7. INSTABILITY CRITERION FOR SELF-SIMILAR REGIMES OF LANGMUIR COLLAPSE

The study of the linearized zeroth approximation Eqs. (16), (18)

$$\alpha\int d^3\xi\, \mathbf{E}_p\, \nabla\psi_{p\alpha}=0, \tag{36}$$

$$\nabla(-\Omega_p-u+\Delta)\nabla\psi_{p\alpha}=\nabla(\Omega_{p\alpha}+u_\alpha)\mathbf{E}_p, \tag{37}$$

$$(\hat{L}_\alpha^2-{}^1/_4)u_\alpha=2\Delta\sum_p \mathbf{E}_p\,\nabla\psi_{p\alpha}, \tag{38}$$

$$\mathbf{E}_p\equiv\nabla\psi_p$$

is in many ways similar to the one described above and can therefore immediately be pursued in the general case. To reduce the calculations it is useful to note that for a given regular function u_α Eqs. (36), (37) would be the same as the first-order equations of the stationary perturbation theory of the Schrödinger equation (with an appropriately chosen Hamiltonian). The regular functions $\psi_{p\alpha}$ would clearly be determined uniquely, i.e., the number of free parameters in the general solution of Eq. (37) would be equal to the total number of regularity conditions and of conditions[3] (36). As all solutions of linear equations which are proportional to one another are equivalent, for a given regular linear connection of the perturbation of the "potential" u_α with the functions $\psi_{p\alpha}$ one lacks just one parameter to satisfy the regularity conditions and the conditions (36). If the linear connection imposed upon Eq. (38) is, indeed, strictly specified one can use as the missing parameter only α and the spectrum of the eigenvalues α is discrete. If, however, the connection (38) leads in the general solution to the appearance of even one additional free parameter, different from α, one can choose the quantity α arbitrarily. The problem of the presence or absence of additional parameters may be solved by studying the specific (i.e., not peculiar to Eqs. (36), (37) with a given function u_α) asymptotic forms of the set of Eqs. (36) to (38). As $\xi\to\infty$ the additional asymptotic forms produced by the connection (38) are given by Eqs. (32) and in the range $\mathrm{Re}\,\alpha<4/3$ all are regular without exception. To liberate the parameter α in the range $\mathrm{Re}\,\alpha<4/3$ it is thus sufficient that there occur at least one specific asymptotic form which is regular at the origin. One can find the asymptotic forms of the solutions of the set (37), (38) as $\xi\to 0$ from the simplified set of equations:

$$\Delta\sigma_{p\alpha}=\nabla u_\alpha \mathbf{E}_p(0),\qquad \sigma_{p\alpha}\equiv\Delta\psi_{p\alpha},$$
$$(\hat{L}_\alpha^2-{}^1/_4)u_\alpha=2\sum_p \mathbf{E}_p(0)\,\nabla\sigma_{p\alpha}, \tag{39}$$

which is obtained by retaining in (37), (38) the most singular terms. Equations (39) have solutions in powers of ξ similar to (29):

$$\psi_{p\alpha}\propto\xi^{c+3},\qquad u_\alpha\propto\xi^c. \tag{40}$$

As the action of the operator $\hat{L}_\alpha$ on the power function ξ^c reduces to multiplying it by a number

$$L_{\alpha c}={}^1/_2+{}^4/_3+{}^2/_3c-\alpha, \tag{41}$$

one can easily eliminate perturbations of the charge density $\sigma_{p\alpha}$ from Eqs. (39) and the latter reduce to the equation for the density perturbation u_α:

$$(L_{\alpha c}^2-{}^1/_4)\Delta u_\alpha=2\sum_p(\mathbf{E}_p(0)\nabla)^2u_\alpha. \tag{42}$$

The symmetric second-order differential operator which occurs on the right-hand side of Eq. (42) can be diagonalized by an appropriate choice of the coordinate system $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$:

$$\sum_p(\mathbf{E}_p(0)\nabla)^2=\sum_{\beta=1}^{3}E_\beta^2\frac{\partial^2}{\partial\xi_\beta^2},\qquad E_\beta^2\equiv\sum_p(\mathbf{E}_p(0)\mathbf{e}_\beta)^2. \tag{43}$$

If the quantity $L_{\alpha c}^2-1/4$ (which one can easily show must be real) is larger than all quantities $2E_\beta^2$ ($\beta=1,2,3$):

$$L_{\alpha c}^2-{}^1/_4>2\max_\beta E_\beta^2=2\max_{|\mathbf{n}|=1}\sum_p(\mathbf{E}_p(0)\mathbf{n})^2, \tag{44}$$

by a suitable stretching of coordinates one can reduce Eq. (42) to a Laplace equation. A solution exists for integer c and can easily be found. However, the equations for the functions $\psi_{p\alpha}$ turn in this case to be insoluble. Indeed, in order that the equation

$$(L_{\alpha c}^2-{}^1/_4)u_\alpha=2\Delta\sum_p\mathbf{E}_p(0)\nabla\psi_{p\alpha}$$

can be solved one needs the orthogonality of the angular part of the function $u_\alpha\propto\xi^c$ to the harmonic with index $l=c+2$ and this condition is clearly not satisfied.

In the range of values of the parameters α and c complementary to (44):

$$L_{\alpha c}^2-{}^1/_4\leqslant 2\max_{|\mathbf{n}|=1}\sum_p(\mathbf{E}_p(0)\mathbf{n})^2, \tag{45}$$

one needs for the existence of a regular asymptotic form (40) the inequality

$$\mathrm{Re}\,\alpha\geqslant\frac{1}{2}+\frac{4}{3}-\left[\frac{1}{4}+2\max_{|\mathbf{n}|=1}\sum_p(\mathbf{E}_p(0)\mathbf{n})^2\right]^{1/2}. \tag{46}$$

The sufficiency of the condition (46) is proved by the following example of a solution of the Eqs. (39):

$$u_\alpha=(\xi\mathbf{n})^c,\qquad \psi_{p\alpha}=A_p(\xi\mathbf{n})^{c+3},$$
$$L_{\alpha c}^2-\frac{1}{4}=2\sum_p(\mathbf{E}_p(0)\mathbf{n})^2,\qquad |\mathbf{n}|=1. \tag{47}$$

The deduced presence of a whole band of eigenvalues α remains thus valid also for Langmuir waves.

Defining the quantity E_s^2 by the relation

$$E_s^2\equiv\max_{|\mathbf{n}|=1}\sum_p(\mathbf{E}_p(0)\mathbf{n})^2, \tag{48}$$

one can retain Eqs. (33), (34) for this band for the instability criterion.

One should note that both in the scalar and in the Langmuir case there are among the unstable perturbations necessarily small-scale ones. They correspond to eigenvalues α with $|\mathrm{Im}\,\alpha|\gg 1$ and are calculated explicitly by the WKB method. We have already mentioned in §3 why small-scale perturbations may be unstable. The absence of instabilities in the region of very large wavenumbers $k\gtrsim W^{1/2}\sim\tilde{t}^{-1}$ in the figure given in §3 is connected with the stabilizing effect of the term Δn in Eq. (2) which is dropped when we go over to the supersonic limit. This term is relatively small for perturbations with $k \ll W^{1/2} \sim \tilde{t}^{-1}$, i.e,

$$|\mathrm{Im}\,\alpha| \sim ka \ll \tilde{t}^{-1/3}. \qquad (49)$$

Condition (49) guarantees simultaneously the narrowness of the region $\xi \lesssim \tilde{t}^{1/3}|\mathrm{Im}\alpha|$ in which the zeroth approximation eigenmodes must be made smooth in order to get the quasimodes of the exact problem.

§ 8. THE CONDITIONS FOR THE EXISTENCE OF SELF-SIMILAR SOLUTIONS

It is also convenient to start the discussion of this problem with the simplest case—the centrally symmetric scalar collapse of a caviton with a single occupied level. In that case the self-similar solutions are determined from the equations

$$\frac{1}{\xi}\frac{d^2}{d\xi^2}\xi E = (\Omega+u)E, \quad \left(\hat{L}_0^2 - \frac{1}{4}\right)u = \frac{1}{\xi}\frac{d^2}{d\xi^2}\xi E^2. \qquad (50)$$

Because of the symmetry (21) the eigenfrequency Ω is here not a parameter to be determined but a given positive quantity which can without loss of generality be assumed to equal unity. The general form of the solution of Eqs. (50) which is regular at the origin for small values of ξ is the following:

$$E = \sum_{r,q} E_{r,q}\xi^{r+cq}, \quad u = \sum_{r,q} u_{r,q}\xi^{r+cq}. \qquad (51)$$

The summation is over all integer non-negative values of r and q, while c is in general an irrational positive number. Substituting (51) into (50) leads to the following chain of recurrence relations:

$$(r+cq+3)(r+cq+2)E_{r+2,q} = \Omega E_{r,q} + \sum_{r_1=0}^{r}\sum_{q_1=0}^{q} u_{r_1,q_1}E_{r-r_1,q-q_1},$$

$$\{[1/2+4/3+2/3(r+cq)]^2 - 1/4\}u_{r,q} \qquad (52)$$

$$= (r+cq+3)(r+cq+2)\sum_{r_1=0}^{r+2}\sum_{q_1=0}^{q} E_{r_1,q_1}E_{r+2-r_1,q-q_1},$$

$$E_{0,q}|_{q\geq 1} = 0, \qquad E_{1,q} = 0.$$

One sees easily that together with $E_{1,q}$ all coefficients $E_{r,q}$, $u_{r,q}$ for which r is odd vanish. When $q=0$ one obtains from (52) a closed set of equations for the quantities $E_{r,0}$ and $u_{r,0}$. [We assumed earlier that this just corresponds to the general solution of Eqs. (50) which is regular at the origin.] This set has been studied in rather much detail up to the present time. The main results are contained in Ref. 3 and consist of the following. All quantities $E_{r,0}$, $u_{r,0}$ can be expressed in terms of $E_{0,0} \equiv E(0) \equiv E_s$. In particular,

$$u(0) \equiv u_{0,0} = \frac{\Omega E_s^2}{14/9 - E_s^2}. \qquad (53)$$

For an arbitrary value of E_s the solution which is regular at the origin has in some point $\xi = \xi_s$ a singularity of the form

$$E(\xi)|_{\xi\approx\xi_s} \approx \pm\frac{2\sqrt{2}}{3}\frac{\xi_s}{\xi-\xi_s}, \quad u(\xi)|_{\xi\approx\xi_s} \approx \frac{2}{(\xi-\xi_s)^2}.$$

Only when

$$\xi_s = \infty, \qquad (54)$$

does the solution turn out to be bounded on the whole of the real axis. The condition (54) determines the discrete (countable) set of values E_s. By virtue of Eq. (53) and the obvious requirement $u(0) < 0$ (one can also check that this is necessary by formal means) all solutions satisfy the inequality

$$E_s^2 > 14/9. \qquad (55)$$

A more detailed analysis, given in the same paper[3] shows that for the solution which is the nth in order of growth of E_s the quantity E_s^2 is confined to the interval

$$2/3\,n(4/3\,n+1) < E_{sn}^2 < 2/3(n+1)[4/3(n+1)+1]. \qquad (56)$$

For given values of $E_{r,0}$, $u_{r,0}$ one obtains from (52) a closed set of equations for the quantities $E_{r,1}$, $u_{r,1}$. In the coordinate representation this set of equations is the same as the Eqs. (50) linearized against the background of the solution which is analytical at the origin. Using the results of §6 we can obtain the answer without calculations. The condition that the equations for the quantities $E_{2,1}$ and $u_{0,1}$ have a non-trivial solution reduces to Eq. (29) with $\alpha = 0$:

$$c = 3/2[(1/4 + 2E_s^2)^{1/2} - 1/2 - 4/3]. \qquad (57)$$

For such c one of these quantities, e.g., $u_{0,1}$ can be chosen arbitrarily, i.e., there occurs a free parameter in the solution. Higher-order terms in q are proportional to the qth power of $u_{0,1}$ as one can see easily from Eqs. (52); the quantities $u_{r,q}$ are non-zero for $r \geq 2(q-1)$ and $E_{r,q}$ for $r \geq 2q$. One can thus "hook on" to the solution of Eqs. (50) which is analytical at the origin a non-analytical addition with arbitrary weight $u_{0,1}$. The condition that this addition is regular at the origin ($c > 0$) is the same as the necessary condition (55) for the existence of self-similar solutions and is thus satisfied automatically.

One can also easily write down expansions similar to (51) for the non-stationary solutions which differ from the stationary ones by an addition which is non-analytical at the origin with a weight $u_{0,1}$ which depends on a power of the time. An estimate of the coefficients of the non-stationary expansions shows that the instability considered in §6 goes over into the non-linear stage when $u_{0,1} \sim 1$, i.e., when the shape of the caviton has been deformed considerably.

One must in the case of an asymmetric caviton replace in Eqs. (50) the radial parts of the Laplacians by the complete Δ operators. The expansions (51) remain valid, except that the coefficients are no longer numbers but functions of the angles or, what amounts to the same, of the unit vector $\mathbf{n} = \boldsymbol{\xi}/\xi$. Finally one must add in Eqs. (52) the angular part of the Laplace operator to each of the factors $(r+cq+3)$ $(r+cq+2)$. This fact leads to the result that the coefficient $E_{1,0}$ no longer necessarily vanishes, since the equation $\Delta\xi E_{1,0} = 0$ has the non-trivial solution:

$$E_{1,0} = \mathbf{An},$$

where $\mathbf{A} \equiv |\nabla E|_{\xi=0}$ is a vector which is independent of $\boldsymbol{\xi}$. The appearance of a term which is linear in $\boldsymbol{\xi}$ in the expansions (51) alters Eq. (53) somewhat:

$$u(0) = \frac{\Omega E_s^2 + A^2}{14/9 - E_s^2}. \qquad (53')$$

However, this change does not affect at all the condition for the existence of the self-similar solutions (55) and thus does not affect the results obtained above.

When there are several occupied bound states in the caviton there also do not occur significant changes: the relation (53′) and all consequences following from it remain valid if we define E_s and A by Eqs. (35) and

$$A^2 = \sum_p (\nabla E_p|_{\xi=0})^2.$$

All conclusions remain valid also in the case of the Langmuir collapse of a strongly flattened caviton as along its short axis the self-similar solutions satisfy the same equations as in the scalar model and then also condition (55). However, one can apparently not extend condition (55) to the general case of Langmuir collapse. The qualitative explanation of this lack of correspondence consists in the fact that as $r \to 0$ the field E in the first of Eqs. (1′) of the scalar model is the analog not of the vector field $\nabla\varphi$ but of the charge density $\Delta\varphi$ in Eq. (1); on the other hand, in the second equation [see (2) and (2′)] the scalar and the vector fields enter on a par. As the values of the vector field and of the charge density at the origin are not connected by a simple algebraic relation, there is in the Langmuir case no formula similar to (53′) and the exclusion of the existence of self-similar solutions which do not satisfy condition (55) which is connected with it is removed.

§ 9. DISCUSSION OF NUMERICAL RESULTS

Condition (55) for the existence of self-similar regimes of the scalar collapse is the same as the instability criterion (34); many, though possibly not all, self-similar regimes of Langmuir wave collapse also satisfy this criterion so that it is natural to pose the problem of the appearance of the predicted instability in numerical calculations. Up to the present time the centrally symmetric scalar collapse of a caviton with a single occupied level has been calculated with great detail. In that case the self-similar regimes correspond to the following values of the field at the center of the caviton E_s and the growth index α_s of the least stable perturbations:[4)]

$$E_{s1}^2=1.926, \quad \alpha_{s1}=-0.193;$$
$$E_{s2}^2=5.276, \quad \alpha_{s2}=-1.454; \qquad (58)$$
$$E_{s3}^2=12.433, \quad \alpha_s=-3.178;$$
$$\cdots\cdots\cdots$$

By virtue of (56) the quantities α_{sn} satisfy the inequalities

$$^4/_3(n-1) < -\alpha_{sn} < ^4/_3 n. \qquad (59)$$

The numerical solution of the non-stationary centrally symmetric equations of the scalar model[4] has shown that after some time the first of the self-similar solutions (58) is established. This fact agrees qualitatively with the conclusion about the instability of the second and higher self-similar solutions. As regards the first solution its instability is very weak ($\alpha_s = -0.19$) and was probably suppressed by the sound term Δn which in the calculations discussed here did not manage to become sufficiently small:

$$\varepsilon \sim [n/E^2(0)]^{1/2} \geqslant 0.2-0.3.$$

For a more detailed discussion of this problem it would be useful to extend the calculation and also to repeat it without the term Δn giving analytical and non-analytical initial conditions at the center of the caviton.

In the case of an axially symmetric Langmuir wave collapse self-similar solutions are known only for a strongly flattened caviton (see footnote 1). In that model

$$E_{s1}^2=2.506, \quad \alpha_{s1}=-0.461;$$
$$E_{s2}^2=6.750, \quad \alpha_{s2}=-1.875; \qquad (60)$$
$$\cdots\cdots\cdots$$

The assumption that the self-similar caviton is strongly flattened is based upon the results of a numerical solution of the axially symmetric Cauchy solution for Eqs. (1), (2).[5,6] The confirmation of self-similarity in Refs. 5,6 was carried out using time-dependences $|n(0,t)|^{-3/4}$ and $|\nabla\varphi(0,t)|^{-1}$ which must be linear when self-similarity is present. It turned out that these time-dependences are, indeed, nearly linear during some time interval, but this interval is too small to reach reliable conclusions: the size of the caviton managed to change during that time only by a factor 2 to 3, and over small sections it is well known that any function is nearly linear. If nevertheless we assume self-similarity and using the slope of the linear section of the curve of $|\nabla\varphi(0,t)|^{-1}$ to calculate E_s it turns out that $E_s^2 \approx 6$. This is close to the second of the values (60) but the shape of the solution along the axis of the caviton (see Fig. 5 in Ref. 6) differs appreciably from the corresponding self-similar solution (see Fig. 1a in Ref. 3). The difference may be explained by the fact that the strongly flattened caviton model is non-adiabatic, by the small time interval of the calculation, or by the instability of the self-similar collapse regimes discussed above. The numerical calculations performed so far of the dynamics of axially symmetric Langmuir collapse are insufficient to reach unambiguous conclusions. Improvements of these calculations, also in the scalar model, would also be useful: increasing their duration and varying within wider limits the initial values of the various parameters, in particular, the sound parameter ε. One should note that the authors of Ref. 6 mention numerical calculations performed in the limit $\varepsilon = 0$, but their results are not given. Meanwhile the results of such calculations might give important information about the stability of self-similar collapse regimes even for a relatively small time of the calculation. Indeed, in the case $\varepsilon \ll 1$ modulational perturbations with strongly differing spatial scales (see the figure in §3) are equally unstable so that it is *a priori* unclear what cavitons are formed from an initial state with wavelength k_0^{-1}. If stable self-similar supersonic collapse regimes exist we may expect the appearance of cavitons of size k_0^{-1} as the perturbations of that size occur in the initial state with a large weight. In the opposite case one may expect the occurrence of cavitons of smaller size which, collapsing, will in turn be split into smaller cavitons. Such an evolution must lead to a natural selection of the most unstable cavitons. The example of the centrally symmetric collapse shows that for them the index α_s may be very small and the splitting process strongly suppressed. Apparently the splitting of the cavitons has in the numerical experiments so far not been fixed sufficiently reliably. The formation of several cavitons of smaller size at the position of the initial caviton described in Ref. 7 occurred in the case of a rather strong magnetic field. Besides, it was emphasized that such an effect occurs also when there is no magnetic field, but the results of the corresponding calculations were not given. It is difficult to understand without them whether the simultaneous formation of several cavitons was observed or whether the wave energy was sufficient for a secondary collapse event.

One must note that the splitting up of the cavitons is not the only possible variant of the collapse dynamics when there are no stable self-similar solutions of the usual form. Another possibility consists in the existence of stable (not necessarily self-similar) collapse regimes in which the caviton has two spatial scales which differ in their time dependence. One can easily extend the list of alternatives. In view of the complexity of the problem more refined numerical calculations are necessary for the clarification of the true nature of the collapse dynamics.

§ 10. THE ROLE OF ELECTRONIC NON-LINEARITIES IN THE COLLAPSE DYNAMICS

If we assume the existence of stable supersonic collapse regimes one must expect, as we noted already above, that if the energy density of the Langmuir turbulence is high enough cavitons will occur in which the wave energy density will from the start strongly exceed the threshold for the modulational instability:[5)]

$$W_0 \gg n_0 T (r_D/a_0)^2. \quad (61)$$

At the moment when the wave energy density in the collapsing caviton becomes equal to the thermal energy density $n_0 T$ of the plasma, the caviton size, which decreases as $a \propto W^{-1/3}$, reaches the value

$$a_e \sim a_0 (W_0/n_0 T)^{1/3}.$$

By virtue of (61) the size a_e may turn out to be so much larger than the Debye radius r_D that the absorption of waves with wavelengths of the order a_e will be negligibly weak. This fact raises the very urgent problem of the role of the electron non-linearities in the collapse dynamics, for at $W \gtrsim n_0 T$ the velocity of the electron oscillations in the field of the Langmuir waves exceeds the thermal velocity and the dispersion of these waves is non-linear. The non-linear dispersion terms were calculated in Ref. 8. It was shown there that the contribution to the Hamiltonian corresponding to them is positive in a number of cases and, based upon the assumption that this property is universal, a conclusion was reached that the collapse is halted by the electronic non-linearities. The authors of Ref. 9 also reached the same conclusion. Recently this conclusion was confirmed by numerical calculations in which the electronic non-linearities were taken into account using a model, by adding a positive term quadratic in the electric field to the plasma density perturbation.[10] Nonetheless there are reasons for doubting the reliability of this conclusion. First of all it is necessary to note that from the hypothesis that the contribution of the electron non-linearities to the Hamiltonian[8] is positive follows a conclusion which contradicts the one reached. Indeed, this positive contribution may be cancelled by the negative contribution from the term $n|E|^2$. The condition for the cancellation has the form

$$n \sim n_0 T_{\text{eff}} / m \omega_p^2 a^2. \quad (62)$$

The quantity $T_{\text{eff}} \equiv W/n_0 + T$ is approximately equal to the average electron energy and can be interpreted as the effective electron temperature of the plasma in the caviton. Such an obvious interpretation not only gives us a valid estimate of the non-linear dispersion terms but also to a larger extent than the model of Ref. 10 corresponds to their structure: these terms, like the linear ones, contain fourth spatial derivatives of the potential φ (except in some degenerate cases). When $W \gtrsim n_0 T$ substitution of (62) into the equation

$$\partial^2 n/\partial t^2 \sim W/Ma^2 \quad (63)$$

leads to the conclusion that the wave energy density increases exponentially with time:

$$\ln (W/n_0 T) \sim \omega_{pi} (t - t_e). \quad (64)$$

Here t_e is the moment that the electron non-linearities are switched on and ω_{pi} the ion plasma frequency. There is thus a basis for assuming that the electron non-linearities themselves do not stop the collapse but only somewhat decelerate it: the explosive growth of the wave energy density is changed to an exponential growth. The slowing down of the collapse, of course, facilitates the absorption of waves trapped in the caviton through Landau damping or the interaction of electron trajectories.

§ 11. COLLAPSE IN A MAGNETIC FIELD

In this section we discuss the formation of cavitons and collapse in a weak magnetic field which nevertheless appreciably changes the dispersion of the Langmuir waves:

$$k_0^2 r_D^2 \ll \omega_H^2/\omega_p^2 \ll 1. \quad (65)$$

Here ω_H is the electron cyclotron frequency and k_0 a characteristic wave number in the Langmuir spectrum; we assume that the phase velocity of the waves ω_p/k_0 is smaller than the velocity of light. To establish a correspondence with the results of Ref. 11 we must in what follows distinguish between the characteristic values of the wave number at right angles to and along the magnetic field, or introduce the angular width of the spectrum $\theta_0 \sim k_{\perp 0}/k_0$. In order that the "magnetic" dispersion correction to the frequency of the Langmuir waves be larger than the "thermal" one the following condition must be satisfied:

$$\theta_0 \gtrsim (\omega_p/\omega_H) k_0 r_D. \quad (66)$$

As for the parameters of the Langmuir spectrum, we shall assume again that the width of the spectrum is much larger than the growth rate of its modulational instability, i.e., that the adiabaticity parameter is small. It was shown in Ref. 12 that in that case even for a very insignificant level of the plasma density perturbations the fastest of the non-linear processes becomes the elastic scattering of Langmuir waves leading to the ergodization of their spectrum.[6)] Under the conditions (65), (66) the surfaces of constant frequency in **k**-space to a large extent are the same as cones with an axis directed along the magnetic field. Therefore, in the process of the ergodization of the spectrum preceding the formation of a caviton its angular width remains approximately unchanged. As to the characteristic wave number, it increases and by the time the ergodization of the spectrum is completed (when the longitudinal "thermal" correction $\omega_p k_{\parallel}^2 r_D^2$ to the Langmuir wave frequency becomes comparable to the "magnetic" correction) it reaches the value

$$k_0' \sim (\omega_H/\omega_p)\theta_0 r_D^{-1}.$$

The transverse wave number then reaches the value $k'_{10} \sim k'_0 \theta_0$. If the initial angular width of the spectrum is not small ($\theta_0 \sim 1$), not only the longitudinal, but also the transverse "thermal" correction to the wave frequency turns out after the ergodization to be of the order of the "magnetic" correction and the collapse proceeds right from the start practically in the same way as when there is no magnetic field.

If $\theta_0 \ll 1$ after the ergodization of the spectrum cavitons are formed[7)] with a longitudinal size $a_0 \sim 1/k'_0$ and a transverse size $b_0 \sim a_0/\theta_0$. The relation

$$b \sim a^2 r_D^{-1}(\omega_H/\omega_p)$$

between the longitudinal and the transverse sizes of the caviton is conserved in the collapse process until they become approximately equal. After that the effect of the magnetic field on the collapse dynamics stops being important. As the adiabaticity parameter is small the first of the self-similar regimes considered in Ref. 11 is realized at the start of the intermediate stage of the collapse; the volume of the caviton decreases as $ab^2 \propto a^5$, the adiabaticity parameter increases as[8)] $(Wa^4)^{1/2} \propto a^{-1/2}$ and in principle can reach unity. If initially this parameter was less than $\theta_0^{1/2}$ this does not take place and the condition of adiabaticity will be satisfied the whole time. In the opposite case the adiabaticity condition will be violated when the intermediate stage of the collapse comes to an end and the second self-similar regime considered in Ref. 11 may occur.

§ 12. CONCLUSION

The main result of the present paper is the instability criterion for self-similar regimes of the supersonic adiabatic collapse: $E_s^2 > 14/9$. In the scalar model this criterion is satisfied for all self-similar solutions. In the Langmuir case all solutions found up to the present also satisfy this criterion, but the theorem forbidding the existence of self-similar solutions with $E_s^2 < 14/9$ is unknown. Solutions with $E_s^2 < 14/9$, if they exist, are very attractive: they not only are not subject to the predicted instabilities but are also insensitive to the degeneracy of the equations of the supersonic approximation[9)] and are therefore well defined already in the framework of those equations. If there are no stable self-similar solutions of the usual form, the problem arises of the true nature of the collapse dynamics. One may propose rather many plausible answers to that problem and it is difficult to establish which of them is correct. An important help in this respect might turn out to be more refined numerical calculations than have been performed so far. Good numerical experiments are necessary also for a definite elucidation of the problems touched upon at the end of the paper about the effect of electronic non-linearities and magnetic fields on the collapse dynamics.

The author expresses his gratitude to V. V. Krasnosel'skikh, E. A. Kuznetsov, A. M. Rubenchik, D. D. Ryutov, G. M. Fraĭman, V. D. Shapiro, and V. V. Yan'kov for useful discussions about this paper and the papers mentioned in it.

1) One should note that for a strongly flattened caviton self-similar solutions are not found in the whole of space so that there may remain some doubt about the existence of complete solutions.

2) One should note that indifferently stable eigenmodes form a countable set, as the set of stationary solutions depends on a countable number of parameters.

3) We have in mind here again a finite-dimensional approximation with subsequent taking of the limit.

4) The values of the field are taken from Ref. 4. They differ somewhat from the results of earlier calculations.[3]

5) In this and the following sections we use dimensional quantities.

6) This conclusion was reached in Ref. 12 using the example of an isotropic plasma but it is also valid in the general case.

7) Starting at this point we use the usually implied assumption that there exist stable regimes of supersonic collapse.

8) The adiabatic self-similar regime of the intermediate stage of the collapse was rejected in Ref. 11 on this basis for no valid reason.

9) One should note that these properties are not independent: the instability is connected with the degeneracy.

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Translated by D.ter Haar