Cosmological transitions with changes in the signature of the metric

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It is conjectured that there exist states of the physical continuum which include regions with different signatures of the metric and that the observed Universe and an infinite number of other Universes arose as a result of quantum transitions with a change in the signature of the metric. The Lagrangian in such a theory must satisfy conditions of non-negativity in the regions with even signature. Signature here means the number of time coordinates. The induced gravitational Lagrangian in a conformally invariant theory of Kaluza-Klein type evidently satisfies this requirement and leads to effective equations of the gravitational theory of macroscopic space identical to the equations of the general theory of relativity. It is suggested that in our Universe there exist in addition to the observable (macroscopic) time dimension two or some other even number of compactified time dimensions. It is suggested that the formation of a Euclidean region in the center of a black hole or in the cosmological contraction of the Universe (if it is predetermined by the dynamics) is a possible outcome of gravitational collapse.

1. INTRODUCTION

It is generally assumed that the signature of the metric of the space-time continuum is an absolutely given physical property, i.e., at each space-time point the metric tensor g_{ik} has one principal value corresponding in its sign to time, and three principal values of opposite sign, corresponding to space. Here and below, signature means the number of time coordinates. In theories of Kaluza-Klein type, the number of time coordinates is, as before, generally assumed to be 1, and the compactified factor space is assumed to be purely spatial. The exclusion from the theory of transitions with change in the signature of the metric is equivalent to nonvanishing of the determinant of g_{ik}, $g \neq 0$; at the same time, we assume the components of the metric tensor are continuous functions of the coordinates.

In this paper, we forego the assumption of invariance of the signature of the metric and consider states with different signatures. During work on this paper, I became acquainted with Vilenkin's paper,[1] in which he considers creation of a de Sitter Universe from a closed inflationary Universe as a result of a quantum transition with change in the signature of the sphere S_4, i.e., from a state with definite metric; he also gives references to earlier publications of similar ideas.

Notation. Q is the number of dimensions of the physical space-time continuum. We assume $Q > 4$, taking a theory of Kaluza-Klein type. The number of time coordinates in the given region of the space-time continuum (the signature) is σ. We take the signs of the principal values of the metric tensor corresponding to time to be negative, and the signs of the spatial directions to be positive. In the observed Universe we apparently have $\sigma = 1$ (see however below), i.e., the signs of the principal values are $(-, +, +, +)$. We shall denote regions of the space-time continuum with $\sigma = 1$ by the letter U, from the word Universe. We shall denote purely spatial regions with $\sigma = 0$ by P, from the name of the Greek philosopher Parmenides, who argued for a world without motion (Pushkin has the line: "There is no motion," said the bearded sage . . .). In quantum mechanics, the word *state* is used in two senses: 1) Usually as the set of values of the physical quantities at a given instant of time; 2) but sometimes as a set of values of physical quantities in space and time. In the present paper, following the majority of authors, the word state will be used in the first sense—as a set of values of quantities on a hypersurface of dimension $Q - 1$ (of codimension 1). For a set of values of quantities in the space of dimension Q we shall use the word *trajectory*.

In this paper, some consequences of the hypothesis allowing values of the signature σ not equal to 1 are discussed in connection with the so-called anthropic principle. During the period 1950–1970, several authors independently suggested that besides the observed Universe there are infinitely many "other" Universes, many of them having characteristics and properties entirely different from "our" Universe; our Universe and others like it are characterized by parameters which make possible the occurrence of structures (atoms, molecules, stars, planetary systems, etc.) capable of sustaining the development of life and intelligence. This hypothesis eliminates many questions of the type: Why is the world constructed precisely as it is and not otherwise?—by assuming that there are worlds constructed otherwise but they are not accessible to observation, at least not at the present. Some authors regard the anthropic principle as unfruitful and even not in accord with scientific method. I do not agree with this. I remark, in particular, that the requirement of validity of the fundamental laws of nature under conditions quite different from those in our Universe could have empirical value for the finding of these laws. P. Ehrenfest noted[2] as long ago as 1917 that the number[3] of dimensions of observed space may be explained by the fact that for a different number of dimensions the exponent in Coulomb's law is different and the existence of atoms impossible; this, of course, is an argument in the spirit of the anthropic principle. One of the earliest such papers known to me is due to Dicke[3]; Zel'dovich mentions an even earlier paper of Idlis (1959); see also Ref. 4. In 1980, Zel'dovich put forward the

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conjecture of multiple production of closed Universes from a primordial empty Minkowski world by a process of "sprouting" [Ref. 5, see also Ref. 6).

The present paper is in the spirit of the anthropic principle; as in Ref. 5, the creation of closed Universes is assumed, but from a different primordial substrate—a space with definite metric.

It is suggested in the paper that our Universe may possibly have a structure different from that usually assumed, namely, that in it compactified time dimensions exist in addition to the observed macroscopic time dimension. This hypothesis is discussed at the end of Sec. 2.

2. DYNAMICAL PRINCIPLE. THE PROBLEM OF INTERPRETATION. HYPOTHESIS ABOUT THE SIGNATURE OF THE OBSERVED UNIVERSE

The clarification of the fundamental questions considered in this section was helped by my becoming acquainted during work on this paper with the preprint of Ref. 7 of Hartle and Hawking, which was recommended by A. D. Linde.

A possible interpretation (not the only one) of quantum theory as applied to the Universe as a whole is to compare the probabilities of different states $B_1, B_2, \ldots$ on some distinguished hypersurface B in the presence of a measurement of state A_0 on some other hypersurface A. The probability amplitude of the states B_i is determined by quantum superposition (by functional integration) of the amplitudes of the trajectories "spanned" over the states B_i and A_0, i.e., satisfying on B and A the boundary conditions for the $(Q-1)$-dimensional metric tensor and the matter fields.

We assume that the trajectories are continuous but that they can have different topologies and different signature structures.

As an illustration, Fig. 1 shows two-dimensional trajectories spanned over two one-dimensional rings A and B. Trajectories 1 and 2 differ in their topology, the different one-dimensional sections of one and the same trajectory 2 having different one-dimensional topologies (ring, two rings, a figure of eight). The trajectories 3, 4, and 5 differ from 1 and 2 by the signature structure; the different sections have different signature structures. It is evident that the boundary of a U region with a P region must be spacelike on the side of the U region, and the boundary with a $\sigma = 2$ region must be timelike.

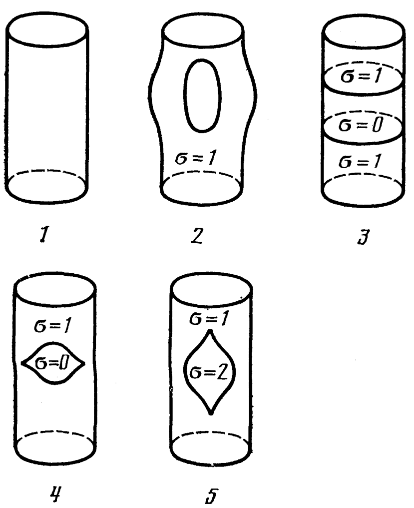

FIG. 1.

Differences in the signature structure of a trajectory appear just as natural as differences in the topological structure. It is less clear whether it is necessary to consider states with $\sigma \neq 1$ in interpreting the theory, i.e., to assume their realization in the preparation of the state on the hypersurface A and measurement on the hypersurface B. It would appear that this is also necessary.

Following Ref. 7, we note that the separation in time between the hypersurfaces is not given and in quantum theory does not have a rigorous meaning. However, for fixed states A_0 and B_i one can speak of the most probable separation in time.

The functional integration must be made with respect to the matter fields (denoted by φ) and the components of the metric tensor and include summation over the discrete characteristics of the trajectories. The functional integral (in a somewhat nominal form, which ignores the gauge, constraint, and measure problems) has the form

$$\int \delta\varphi\, \delta g_{ik} \exp\left(-\int dx_0 \ldots dx_{Q-1} \sqrt{g}\, L\right) \ldots . \qquad (1)$$

Here and throughout, g is the determinant of g_{ik}. The argument of the exponential is purely imaginary for $g < 0$ ($\sigma = 1, 3 \ldots$) and real for $g > 0$ ($\sigma = 0, 2, 4, \ldots$). This is evidently a necessary consequence of the analytic structure of a theory with variable signature.

For convergence of the integral in regions in which $g/$ ((σ even), it is necessary to require in them $L \geqslant 0$. This is a nontrivial restriction, which may have empirical value for constructing the theory. In particular, the standard expression for the Einstein-Hilbert Lagrangian of the gravitational field, $L_g \propto R$, which is linear in the scalar curvature, does not satisfy this requirement. A theory with quadratic (or higher even power) Lagrangian defined in four-dimensional space-time (i.e., without compactified dimensions) contradicts experiment; not even Newton's law of gravity is reproduced. Thus, for $L_g \propto R^2$, the force of the gravitational interaction of two bodies does not depend on the distance between them. This difficulty is absent in theories of Kaluza-Klein type, and this is a further argument in favor of them. In these theories, it is assumed that at the present stage in the development of the Universe the characteristic compactification radii are small compared with the characteristic scales of the macrospace t, x_1, x_2, x_3. Integrating the Lagrangian of the gravitational field over the coordinates of the compactified factor space, we find an effective Lagrangian at the given point of the macrospace; in the first approximation, it contains only a constant term (cosmological constant) and a term linear in the curvature scalar of the macrospace:

$$L_g = \Lambda + \frac{1}{16\pi G} R.$$

We shall not consider here the mechanism that leads to compactification. We merely mention that the compactification radii must, if they are constant in the macrospace, be determined by dimensional parameters of the Lagrangian of the type of bare masses; the theory does not then possess conformal invariance and there are evidently difficulties with the indefiniteness of the Lagrangian and divergences. (It could be that for certain special values of the parameters the difficulties are absent.) It is of interest to consider alternative variants of the theory in which the Lagrangian is conformally invariant and the compactification radii depend on the macroscopic coordinates, keeping, however, constant ratios. Then sufficiently smooth variations of the compactification radii will not be observable, since all the dimensional characteristics of the effective Lagrangian are determined by the compactification scale; in particular, the Brans-Dicke theory reduces to Einstein's theory, and the equivalence principle is satisfied (cf. Ref. 8). As an example, we consider the induced (i.e., generated by quantum fluctuations of the matter fields, cf. Ref. 9) gravitational Lagrangian in a space with number of dimensions $Q = 4q$ that is a multiple of 4; the matter fields are massless and satisfy conformally invariant equations. On the basis of dimensional considerations and conformal invariance it must be assumed that the induced Lagrangian is described by an expression of the form $L_g \propto I^q$, where I is the quadratic invariant of the Weyl tensor (it is possible that a more accurate expression will include other invariants); the corresponding coefficient is dimensionless and there is hope that in supersymmetric theories it is finite. Denoting by ρ the compactification radius, we obtain for the effective Lagrangian of the macrospace (M is the scale of the effective particle masses m^i)

$$m_i \infty M \infty 1/\rho, \qquad \Lambda \infty 1/\rho^4, \qquad G \infty \rho^2.$$

So far in our exposition we have generally assumed (and will continue to do so, in particular in Sec. 3) that the signature of our Universe is $\sigma = 1$. However, it is of interest to consider variants of the structure of the Universe (and their consequences for the theory of elementary particles!) in which $\sigma > 1$. Compactification with respect to all the time coordinates except one is assumed in such a case.

In our Universe, the action of a trajectory determines the phase of its complex amplitude. Therefore, in accordance with (1) σ is an odd number, and the number of compactified coordinates is even. We note that the sign of the determinant g in the square root in (1) cannot be changed arbitrarily (for example, in connection with the indefiniteness of the Lagrangian of some particles in spaces with odd signature).

An important question of principle is the connection between the hypothesis discussed here and the causality principle. By the causality principle in relativistic dynamic theories (without allowance for the effects of quantum gravity) one understands the following assertion (which is one of the possible formulations): The state in some spatial region is maximally determined by the state on a spatial section of the exterior envelope of the light cones with vertices on the boundary of the region pointing into the past *or* the light cones pointing into the future *but not two such states at once* ("maximally determined" means here that the state outside the envelope does not influence the state in the region). In quantum gravity, the causality principle is to a large degree rendered nugatory, since the metric and, hence, the envelope of the light cones are different for the different trajectories whose amplitudes are superposed to determine the state.

A feature of a signature $\sigma > 1$, in contrast to the signature $\sigma = 1$ usually assumed, is that then already in the classical theory (and for individual trajectories in the quantum theory) the light cone does not have two different directions, i.e., *locally* there is no separation of past and future. For $\sigma > 1$, the light cone divides the space of ray directions only into two regions—spacelike and timelike—and not into three regions, as in the case $\sigma = 1$. The topology of ray directions of the light cone (the topology of the intersection of the cone with the unit sphere S_{Q-1} in the Q space) in the case $\sigma > 1$ is the direct product $S_{Q-\sigma-1} \times S_{\sigma p 1}$, a simply connected space. For $\sigma = 1$, the sphere $S_{\sigma-1}$ degenerates into two points, the cone is doubly connected and intersects the sphere S_{Q-1} in two spheres S_{Q-2}, distinguishing three regions of directions—into the future, into the past, and spatial directions.

However, the property of global ordering of the hypersurfaces of dimension $Q - 1$ with respect to the macrotime for the assumed compactification with respect to all time coordinates except one is preserved for all values of the signature. Therefore, it can be assumed that the absence of local ordering for $\sigma > 1$ does not affect macroscopic processes with the participation of particles with energies much less than the reciprocal radii of the time compactification (in the corresponding units). If the radii of time compactification are of the order of or less than the Planck length, then the effects of quantum gravity will be manifested at such or shorter distances.

3. P–U TRANSITION

We consider the geometry of Q space near the boundary of a P region ($\sigma = 0$) and a U region ($\sigma = 1$). We choose coordinates $x_0, \ldots, x_{Q-1}$ such that the value $x_0 = a$ corresponds to the boundary of the P and U regions. We assume that for the classical solution near the boundary of the regions

$$g_{00} = l/(x_0 - a). \tag{2}$$

For $x_0 > a$ we have the P region, for $x_0 < a$ the U region. In general, l depends on the coordinates $x_1, \ldots, x_{Q-1}$ but does not change in order of magnitude. In accordance with our assumption, the boundary N of the P and U regions is a closed nonsingular hypersurface with $Q - 1$ dimensions. In the case of our Universe and ones like it, N has the topological structure of a direct product:

$$N = M \otimes K.$$

In the case of our Universe, $M = 3$, and we have either a sphere S_3, or a torus $T_3 = S_1 \otimes S_1 \otimes S_1$, or $S_2 \otimes S_1$. The dimension of K is $Q - 1 - M$; the topological structure of K is that of the direct product of spheres of different dimensions and, possibly, closed topological spaces of more general type (for example, in the two-dimensional case spheres with p "arms," $2 \leqslant p < \infty$).

It is assumed that in the early stage of the evolution of the Universe the space K contracts (is compactified), and M

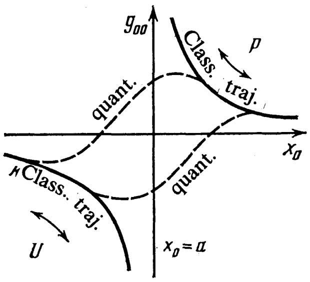

FIG. 2

expands, forming the observed macroscopic space. The space K is the factor space of Kaluza-Klein theory. In the conformally invariant variants of the theory the radii of the K space are formally, as I noted above, variable, and may increase, but at the present stage of evolution of our Universe they remain much less than the characteristic scales of the macrospace, and their smooth variations can be eliminated by conformal transformation.

The solution (2) has a discontinuity of g_{00}. The quantum trajectories must satisfy requirements of continuity of the dynamical variables, including the components of the metric tensor. The qualitative form of continuous trajectories of $g_{00}(x_0)$ with a change of signature is shown in Fig. 2 by the broken curves. In a region of width of order l a continuous trajectory cannot satisfy the classical equations, i.e., there is a quantum transition.

In each of the regions P and U the singularity of (2) can be eliminated by a coordinate transformation:

$$\text{In the } P \text{ region: } \quad x_0 - a = y^2/4l, \quad g_{00} \to g_{00}' = 1.$$
$$\text{In the } U \text{ region: } \quad a - x_0 = t^2/4l, \quad g_{00} \to g_{00}' = -1. \qquad (3)$$

The variables y and t take both positive and negative values. The classical solution (2) for the P region in the variables y can be extended to positive and negative values of y, and the classical solution for the U region can be extended to positive and negative values of t. Quantum transitions with a change in the signature of the Universe (like the topology) will occur with greatest probability at the minimal spatial scales of the Universe. If the described picture bears a relation to our Universe, then when extrapolating its observed state into the distant past we must assume a superposition of a U state with near-maximal density and a state with definite metric, i.e., a P state. To illustrate this situation, we can use analogy with the quantum-mechanical problem of one-dimensional motion of a wave packet in a space divided by a potential barrier. Suppose that at time $t_1 > 0$ the state is described by a wave packet, the group velocity being $v_1 > 0$ (motion from the barrier). Then for $t_2 < 0$, the solution of the Schrödinger equation extrapolated backward in time is a superposition of two coherent states—a wave packet to the right of the barrier and a wave packet to the left of the barrier. For $t < 0$, both packets move toward the barrier, and at $t = 0$ they merge into a single packet, which moves away to the right. The packet to the right of the barrier is the analog of the U state; the packet to the left, that of the P state. The analogy is not exact, since in the P region there is no time.

The state of the Universe with minimal spatial scales is possibly a state with "false vacuum." In accordance with the picture described above, the solution can be continued into the future and into the past (in the neighborhood of the "zero" point, but after compactification with expansion in accordance with the "catenary" law $\cosh(t/t_0)$ away from this point). A vacuum state, including a false vacuum state, has minimal entropy. Therefore, the entropy increases with increasing distance from the vacuum point into the past and into the future, i.e., there is a "reversal of the arrow of time." There are other possible realizations of a reversal of the arrow of time; see Ref. 10.

To conclude this section, we make a remark about black holes and cosmological collapse. It is possible that a U–P transition occurs when there is gravitational collapse and is its outcome (or one of the outcomes; an alternative is expansion into "another" space; a classical solution of this type is known for a charged black hole). We leave out of consideration the compactification. We suppose that at the center of the black hole (to be specific, assumed to be formed in a symmetric gravitational collapse and then undergo Hawking evaporation) there can exist a four-dimensional P region with a spacelike three-dimensional closed boundary. The P region is spherically symmetric and elongated along the spacelike axis of the T region of the Schwarzschild solution (which in our terminology is the U region). We assume that the formation of an analogous P region is possible in the cosmological collapse of a closed Universe (if its dynamics predetermines replacement of expansion by contraction) after one or several expansion-contraction cycles. Of course, within the P region there may again be inclusions with other signatures.

4. ANTHROPIC PRINCIPLE AND THE COSMOLOGICAL CONSTANT

The different regions of the Q space may differ in their discrete and continuous parameters. In the spirit of the anthropic principle we assume that the observed Universe is distinguished by a set of values of the parameters favorable for the development of life and intelligence. In particular, it is possible that the signature (equal to 1 or another odd number) is one such parameter.

For a Universe with given signature, as further discrete parameters we must consider the number of dimensions of the compactified factor space K and the macrospace $M = Q - \sigma - K$, which is not necessarily equal to 3. This possibility, which follows from the compactification hypothesis, is a natural realization of the idea[4] that Universes with different numbers of spatial dimensions M arise; evidently, Ehrenfest's arguments[2] for the reason why "our" case $M = 3$ is distinguished remain valid.

The topological characteristics of the boundaries of the P and U regions are also discrete parameters. The discrete parameters determine the effective Lagrangian of the macrospace.

The continuous parameters are the initial values of the characteristics of the matter fields and the initial deviations from symmetry of the transition boundaries. These parameters together with the discrete parameters determine the evolution of the Universe.

It is well known that the cosmological constant is zero, $\Lambda = 0$, or anomalously small and, moreover —and this is particularly remarkable—not in the internally symmetric state of the "false" vacuum but in the state of the "true" vacuum with broken symmetries. The smallness or vanishing of Λ is one of the main factors that ensures a prolonged existence of the Universe, sufficient for the development of life and intelligence. It is therefore natural to invoke the anthropic principle to solve the problem of the cosmological constant.

If the small value of the cosmological constant is determined by "anthropic selection," then it is due to the discrete parameters. At the same time, Λ is either exactly equal to zero in some variant, or exceptionally small. In the latter case, it must be assumed that the number of variants of the set of discrete parameters is sufficiently large to make the spectrum of Λ values in the neighborhood of the point $\Lambda = 0$ sufficiently "dense." This obviously requires a large value of the number of dimensions K of the compactified space or (and) the presence in some topological factors of a complicated topological structure (such as a large number of "arms").

We note in conclusion that in P space one must consider an infinite number of U inclusions (for the complete set of trajectories or even for one trajectory); at the same time, the parameters of an infinite number of them may be arbitrarily close to the parameters of the observed Universe. Therefore, it can be assumed that the number of Universes similar to ours, in which structures, life, and intelligence are possible, is infinite. This does not rule out the possibility that life and intelligence are also possible in an infinite number of very different Universes that form a finite or infinite number of classes of "similar" Universes, including Universes with signature different from ours.

5. CONCLUSIONS

In this paper, I have advanced and discussed the hypothesis of the existence of trajectories of the physical space-time continuum with different values of the signature of the metric. It is evident that in a theory that admits trajectories with even signature the Lagrangian in regions with such signature cannot be negative. This, together with the requirement of a correspondence with the general theory of relativity, greatly restricts the admissible class of theories. Induced nonlinear gravitation in a theory of Kaluza-Klein type with number of dimensions a multiple of 4 and matter fields with conformally invariant Lagrangian is considered as an example satisfying these requirements.

In accordance with the hypothesis, the prehistory of the observed Universe is a quantum superposition of a quasiclassical trajectory with reversed arrow of time and trajectories with quantum transitions, including regions with definite metric, and also various regions with ordinary signature $\sigma = 1$ and other signatures $\sigma = 2, 3 \ldots$, etc. A possible explanation for the anomalous smallness of the cosmological constant using the anthropic principle has been proposed. This explanation does not depend on the conjecture of transitions in which the signature of the metric changes.

It has been suggested that the signature of the observed Universe is in reality not equal to 1 and that in it there is an even number of additional compactified time dimensions. A possibility of reconciling this assumption with the causality principle has been discussed.

It is suggested that a P region is formed in gravitational collapse as its outcome or one of the possible outcomes.

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