Electron fluctuations under phonon nonequilibrium conditions

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Current fluctuations in an electron-phonon system are investigated theoretically for the case in which the longwave phonons, with wave vectors considerably smaller than the electron wave vectors, are strongly nonequilibrium in character. It is shown that two contributions of different nature appear in the expressions for the spectral density of the current fluctuations. These depend on the phonon intensity. The first of them describes the direct effect of nonequilibrium phonons on the electron fluctuations. The second contribution is due to phonon fluctuations that appear in the current fluctuations as a result of electron-phonon coupling. Explicit expressions for the spectral density of the current correlators are presented for the case of weak nonlinearity in the hydrodynamic limit.

1. INTRODUCTION

There is a number of interesting physical problems associated with the interaction of electron and phonon systems under non-equilibrium conditions. Among these is the problem of phonon turbulence under conditions of acoustic instability in piezo-semiconductors, and also the problem of the fluctuations of the current under these conditions. The first of these has been solved in Ref. 1 under conditions of weak turbulence. The second problem has essentially not been solved to date, although for the diagnostics of phonon turbulence, the results of the measurement of current fluctuations (the so-called fluctuation spectroscopy) have also been used along with acoustical and optical methods (see, for example, Refs. 2 and 3). The absence of a developed theory of current fluctuations under conditions of strong nonequilibrium of the phonon system makes the interpretation of the experimental data difficult.[4–6] Moreover, this problem has a wider theoretical scope, since in it we encounter an example of a new specific non-equilibrium fluctuation phenomenon—correlation "through the medium." We shall see that, as a result, the analysis of the spectrum of the spatially homogeneous current fluctuations can become a method for the study of nonequilibrium, spatially inhomogeneous (for example, wave) processes in a medium. Such a form of fluctuation spectroscopy of a medium can have a number of advantages over the traditional.

In the present work, we consider the fluctuations in a gas of nondegenerate electrons interacting in a semiconductor with long wavelength ($\hbar q \ll p$, $\mathbf{q}$ is the wave vector of the phonon, $\mathbf{p}$ is the quasimomentum of the electron) strongly nonequilibrium phonons. In addition to this isolated group of phonons, the electrons interact (collide) with equilibrium, short wavelength ($\hbar q \sim p$) phonons and impurities, which play the role of a thermostat. The nonequilibrium nature of the electron and long wavelength phonon systems is caused by the external electric field $\mathbf{E}$, which creates a drift of the electrons (see, for example, Refs. 2 and 7).

Expressions are obtained for the spectrum of current fluctuations under conditions of weak turbulence, when we can limit ourselves to the first terms of the expansion in the nonequilibrium phonon intensity $N_\mathbf{q} \gg n_\mathbf{q}$ ($n_\mathbf{q} = T/\hbar\omega_\mathbf{q}$ is the equilibrium number of phonons, $\omega_\mathbf{q}$ is the frequency of the phonon, T is the temperature of the thermostat). Both the direct effect of the disequilibrium of the phonon distribution function, and the phonon fluctuations as such, are manifest in the electron fluctuations. The first contribution is connected with the effect of nonequilibrium phonons on the distribution function of the electrons and on their kinetics, while the second arises from the interelectron correlation brought about by the phonons.

In the case of weak turbulence, the first contribution is proportional to $N_\mathbf{q}$ and undergoes temporal dispersion within times of the order of the relaxation time of the spatially inhomogeneous excitations of the electron system γ_e. The second contribution is proportional to the square of the phonon intensity and, in contrast with the first contribution, has an additional dispersion over times of the order of the phonon damping time $1/\gamma_\mathbf{q}$. This contribution predominates at frequencies of the order of $\gamma_\mathbf{q}$; in addition, under certain conditions, it can exceed the level of intrinsic electron noise even under conditions of weak phonon turbulence. Moreover, a side peak is possible at the phonon frequency $\omega_\mathbf{q}$.

We note that the interelectron correlation via phonons is similar to the interelectron correlation as a consequence of electron-electron collisions[8,9]; however, in contrast to the latter, it does not occur instantaneously (within the time of the collision) and can be protracted over times up to times of the order of the phonon lifetime. In such a time scale, the electron system already acts as a simple indicator of fluctuations in the phonon system.

Correlations through the phonons and the appearance of phonon noise in the electron noise are a particular case of a more general phenomenon—the interelectron correlation "through the medium" and of "noise of the medium" in the current noise. Another example of this phenomenon is the correlation through immobile scatterers.

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2. EFFECT OF NONEQUILIBRIUM PHONONS ON THE KINETICS OF ELECTRON FLUCTUATIONS

We assume that the usual conditions of applicability of the kinetic equation are satisfied for the electrons:

$$\hbar/\varepsilon_p\tau_p \ll 1, \quad \hbar eE/\varepsilon_p p \ll 1. \tag{1}$$

Here e is the charge, ε_p is the energy of the electron, τ_p is the relaxation time of the electrons in the thermostat. The stationary distribution function of the electrons with account of only the field and the thermostat satisfies the condition

$$I_p F_p \equiv e\mathbf{E}\frac{\partial F_p}{\partial \mathbf{p}} + I_p^{th} F_p = 0. \tag{2}$$

The function F_p is normalized to the total number of electrons $\Sigma F_p = N$. We shall take into account the interaction of the electrons with long-wavelength phonons by perturbation theory. In view of the fact that $N_q \gg n_q$, we can neglect the spontaneous emission of phonons, so that the problem reduces to the interaction of electrons with the classical wave field, the intensity of which it is nevertheless best characterized by the number of phonons N_q. We represent the perturbation-theory series in the form of diagrams of the kinetic classical diagram technique described in Refs. 10–12. At first we shall not take the Coulomb long-range interaction into account, for simplicity. This neglect corresponds to the case $q \gg \varkappa$, where $\varkappa$ is the reciprocal of the Debye radius. Figure 1 shows the first corrections to the electron distribution function in powers of the stationary phonon intensity N_q. The ratio of the second-approximation correction (Fig. 1c) to the first-approximation correction (Fig. 1b) is a measure of the development of the phonon turbulence.[1,10] In order of magnitude, it is equal to

$$K = \sum_q c_q^2 \frac{N_q}{B_q^2}\frac{q^2}{p^2}, \tag{3}$$

where c_q is the electron-phonon coupling constant, B_q is the modulus of the operator of the spatially inhomogeneous response of the electron system $\hat{B}_p(\mathbf{q},\omega)$ at the frequency $\omega = \omega_q$:

$$\hat{B}_p(\mathbf{q},\omega) \equiv -i\omega + i\mathbf{q}\mathbf{v} + I_p \tag{4}$$

($\mathbf{v}$ is the electron velocity). The nonlinearity parameter K we shall assume to be small and take into consideration only the zeroth and first terms of the perturbation-theory series, i.e., the diagrams a and b of Fig. 1.

The fluctuations of the electron distribution function are described by the diagram in Fig. 2. The break on the electron line corresponds to the Kronecker symbol δ_{pp_1}.[12] This diagram gives the usual Fourier half-transform for the spectrum of the distribution-function fluctuations

$$\langle \delta F_p(\tau)\delta F_{p_1}\rangle_\omega = \frac{1}{-i\omega + I_p} F_p \delta_{pp_1}. \tag{5}$$

The operator $(-i\omega + I_p)^{-1}$ describes the development of

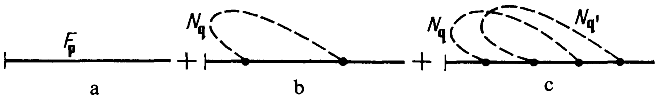

FIG. 1

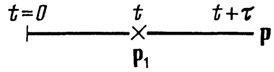

FIG. 2.

the fluctuations in time and corresponds to a vertical cut in the diagram of the segment between the observation points t and $t + \tau$.

By including the phonon line in this diagram, we describe (in first approximation in the phonon intensity) the effect of the phonon disequilibrium on the electron fluctuations. There are six diagrams in all, three of which are shown in Fig. 3 (the other three are obtained by reversing the direction of the phonon line). The diagram of Fig. 3a describes the change in the distribution function—see Fig. 1b. The spectrum of the fluctuations does not change here, only their level changes. The diagram of Fig. 3b describes the change, under the action of the phonons of the evolution, of the fluctuations that have already arisen, which changes their spectrum in the case of unchanged integrated intensity of the fluctuations. Finally, the diagram of Fig. 3c describes the appearance and evolution of fluctuations under conditions of spatial-temporal inhomogeneity produced by the phonon perturbation. This process also manifests itself in the level and in the spectrum of the fluctuations. We write down the analytic expression for this, most interesting, diagram

$$\sum_q c_q^2 N_q \frac{1}{-i\omega + I_p}\mathbf{q}\frac{\partial}{\partial \mathbf{p}} B_p^{-1}(\mathbf{q},\omega+\omega_q)\delta_{pp_1} B_p^{-1}(\mathbf{q},\omega_q)\mathbf{q}\frac{\partial F_p}{\partial \mathbf{p}}. \tag{6}$$

In this expression, the phonon intensity N_q corresponds to the phonon line itself, and the interaction constants to its entrance (ic_q) and exit ($-ic_q$). The gradients $\mathbf{q}\partial/\partial\mathbf{p}$ at these points reflect the classic nature of the forces acting on the electrons, while the propagators $(-i\omega + I_p)^{-1}$ and B_p^{-1} reflect the evolution of perturbations that are spatially homogeneous and spatially inhomogeneous, respectively. The spatially inhomogeneous perturbation splits up into two by the "act of observation"—at the break on the electron line, to which corresponds the Kronecker symbol δ_{pp_1}. Beyond (to the right of) this point, the frequency of observation ω is added to the frequency of the perturbation produced by the phonon. The tail of the diagram is the stationary unperturbed distribution function F_p. A diagram analogous to Fig. 3c, but with opposite direction of the phonon line, is obtained from (6) by taking the complex conjugate and also by making the substitution $\omega \to -\omega$. It is not difficult also to write out the analytical expression for the remaining diagrams (for details of the correspondence rules see Refs. 10 and 12).

The diagrams of Fig. 3 (as also the initial diagram of Fig. 2) describe the autocorrelation and are essentially one-particle diagrams. They have analogs in the response, for example, in the response to a weak external spatially inhomogeneous alternating field. The diagrams for the response are obtained by replacing the break point by the vertex of the interaction with the field. We note, however, that the contri-

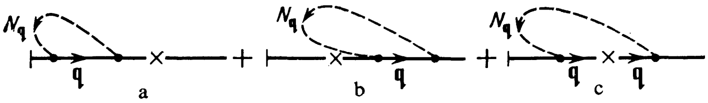

FIG. 3.

bution of the diagrams of Fig. 3 to the correlator of the currents is not expressed in terms of the conductivity corrections due to the phonon disequilibrium. They are expressed (as $\omega \to 0$) in terms of the corrections to the diffusion tensor of the electrons, just as the contribution of the diagram of Fig. 2 to the current fluctuations is expressed in terms of the diffusion tensor itself. If the electron system itself were at equilibrium, so that the distribution function F_p would be Maxwellian, then, in spite of the equality $\partial F_p/\partial \mathbf{p} = -\mathbf{v}F_p/T$, leading to the Einstein relation, the corrections from the phonon disequilibrium to the conductivity and to the diffusion would still be different.

3. ELECTRON CORRELATIONS THROUGH PHONONS

At first glance, it may seem that the diagrams of Fig. 3 cover exhaustively the effect of phonons on the electron fluctuations under conditions of weak turbulence (and at $q \gg \varkappa$). Actually, the addition of one more phonon line to the diagrams of Fig. 3 yields diagrams that are small in the nonlinearity parameter K—a parameter characterizing the smallness of the diagram of Fig. 1c relative to the diagram of Fig. 1b in the series for the distribution function. However, we can construct diagrams of a completely different topology which, even though they contain a higher power of the parameter of nonlinearity $K \ll 1$, nevertheless, can be not only comparable in magnitude with the diagrams of Fig. 3, but even exceed them. One of such diagrams is shown in Fig. 4. In contrast to the diagrams of Fig. 3, this diagram is essentially two-particle. It contains two electron lines coupled with one another only by the phonon lines. In the spatially homogeneous case, such a coupling requires no less than two phonons. Thus, it is quadratic both in the density of the electrons and in the phonon intensity. The physical meaning of this diagram is the correlation between the occupation number of the electron states through the phonon system. It is similar in many ways to the correlation that arise between the occupation numbers of the electron states in the nonequilibrium state because of collisions of the electrons with one another.[8,9,13] As in that case, the kinetic correlation through the phonons vanishes at equilibrium.[1)]

However, if at least one system—electron or phonon—is not in equilibrium, a correlation appears.

We now compare the orders of magnitude of the diagrams in Figs. 2–4. The ratio of the diagrams of Fig. 3 and Fig. 2 (we arbitrarily call the latter "thermal noise") is of the order of the coefficient of nonlinearity K. However, the ratio of the correlation diagram of Fig. 4 to the thermal noise is not K^2 as might be expected, but

$$\eta = K^2 (N/\Sigma^q), \tag{7}$$

where we denote by Σ^q the number of degrees of freedom in this segment of the phonon spectrum, where the phonons are in strong disequilibrium. Inasmuch as the number of electrons is not in any way connected with the number of nonequilibrium excited vibrational modes, the ratio N/Σ^q can be quite large. For example, if $KN/\Sigma^q \gtrsim 1$ the contribution of the correlation diagrams will be the principal one. Moreover, in the case $K^2N/\Sigma^q \gtrsim 1$ this contribution exceeds the level of thermal noise. Thus, as a consequence of the interelectron correlations through the phonons, the effect of the nonequilibrium phonons on the kinetics of the electrons and on their fluctuations is controlled by different parameters and can be completely different. For example, even in the case of weak effect of phonons on the kinetics of electrons ($K \ll 1$) the fluctuations of the latter can be determined precisely by the phonon contribution.

The correlation through the phonon has one important singularity. Whereas in pairwise collision the correlation arises instantaneously (within the time of the collision), in correlation through the phonons the process is protracted. Therefore, in contrast with the correlation as a consequence of pair collisions, which lowers only the level of the fluctuations, the correlation through the phonons can also affect the fluctuation spectrum—see Fig. 5. The diagram in Fig. 5a describes the change in the fluctuation spectrum at frequencies of the order of B_q (with corresponding change of their level), i.e., at the same frequencies as in diagrams of Figs. 3b and 3c. The diagram in Fig. 5b describes the change in the spectrum at much lower frequencies—at frequencies of the order of the phonon damping time γ_q. In this diagram, the perturbations on different electron lines are separated in time by τ' and thus the perturbed sections do not overlap: by the instant of time t some of the relaxation processes in the electron system have already been completed, while others begin only after the perturbation, at the time $t + \tau'$. In the interval between these instants of time, the only relaxation process is the slow change in the phonon amplitudes. The factor

$$(-i\omega + i\omega_q + \gamma_q/2 - i\omega_q + \gamma_q/2)^{-1} = (-i\omega + \gamma_q)^{-1}$$

corresponds to this process in the analytic expression for the diagram of Fig. 5b. (This factor is attributed to the vertical cut between the points t and $t + \tau'$.) This factor increases the contribution from the diagram at low frequencies, and its ratio to the thermal noise becomes of the order of

$$\eta_0 = \frac{\eta}{\gamma_q \tau_p} = K^2 \frac{N}{\Sigma^q} \frac{1}{\gamma_q \tau_p} \quad (\omega \ll \gamma_q). \tag{8}$$

In a time scale of the order of $1/\gamma_q$, all the processes in the

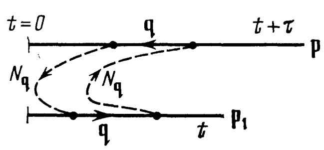

FIG. 4.

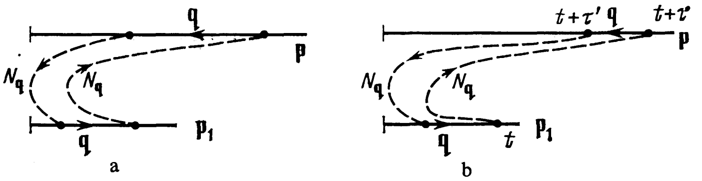

FIG. 5.

electron system can be regarded as instantaneous and the propagator segments of the electron lines can be contracted into points (cf. Ref. 14). The resultant diagram is shown in Fig. 6a (we do not show the nonrelaxing tails F_p). It is easy to see that this diagram describes the fluctuations of the phonon occupation numbers observable through the rapidly relaxing electron system. This becomes especially clear if we represent the change of the electron distribution function under the action of the phonons in similar fashion (see Fig. 6b and compare it with Fig. 1b). From these diagrams we find the contribution of the phonon fluctuations to the low-frequency electron fluctuations. We write it as follows:

$$(\delta j_\alpha \delta j_\beta)_\omega^{(ph)} = \sum_q j_\alpha{}^q j_\beta{}^q N_q{}^2 \frac{2\gamma_q}{\omega^2+\gamma_q{}^2}. \qquad (9)$$

Here

$$\mathbf{j}^q = 2\frac{e}{v_0} c_q{}^2 \operatorname{Re} \sum_p \mathbf{v} I_p{}^{-1} \mathbf{q} \frac{\partial}{\partial \mathbf{p}} B_p{}^{-1}(\mathbf{q}, \omega_q) \mathbf{q} \frac{\partial F_p}{\partial \mathbf{p}} \qquad (10)$$

(v_0 is the volume of the system). The current generated by the phonons is expressed in terms of this quantity (for acoustical phonons, it is customary to call this the electrosonic current):

$$\mathbf{j}^{(ph)} = \sum_q \mathbf{j}^q N_q. \qquad (11)$$

By setting the partial current $\mathbf{j}^q$ in correspondence with the vertices on the diagrams of Fig. 6, we can regard these diagrams as the diagrams for the acousto-electric current and its fluctuations. We see that the current fluctuations (9) are simply a reflection of the fluctuations of the phonon occupation numbers:

$$\overline{(\Delta N_q{}^2)}_\omega = N_q{}^2 \frac{2\gamma_q}{\omega^2+\gamma_q{}^2}. \qquad (12)$$

We note that the low-frequency current-noise peak caused by these fluctuations can appear even if the contribution of the phonons to the general level of fluctuations is small, i.e., even at $\eta = K^2 N/\Sigma^q \ll 1$ we can have $\eta_0 = \eta/\gamma_q \tau_p \gtrsim 1$. We wish to call attention to the fact that the appearance of the factor N/Σ^q is connected with the dual character of the discussed effect—the observation of the phonon fluctuations via the electron system: the general level of the fluctuations in the phonon system is inversely proportional to the number of (strongly) excited phonon modes Σ^q, while the general level of the electron fluctuations themselves is inversely proportional to N. The condition $\eta \gg 1$ means that the characteristic noise of the electron system is insignificant against the background of the very noisy phonon system. At $\eta \ll 1$, but $\eta_0 \gtrsim 1$, the noise of the phonon system appears, but only in the region of low frequencies.

The correlation diagrams, as we have already remarked, do not have an analog in the response, even though there is a low-frequency phonon contribution in the response (see Fig. 7a). However, this contribution, as also its fluctuation analog (Fig. 7b), is proportional to the first power of the phonon intensity[2)] N_q and is N_q/n_q times smaller than the contribution from the diagrams of Fig. 5b. The physical meaning of these diagrams is the effect of the change, under the action of the external force or thermal fluctuations, of the electron part of the damping coefficient of the phonons γ_q and the correction to their phase velocity.

As a conclusion to this section, we make several remarks of a general character. The phonon lines on the diagrams can be replaced by lines of other elementary excitations in the medium surrounding the electrons. Thus, we arrive at the general concept of correlation through a medium and of the manifestation of noise of the medium in the electron fluctuations. We note two important circumstances. First, the effect of the excitations existing in the medium on the electron fluctuations can turn out to be significantly stronger than the effect on the response.[3)] For the fluctuations, as we have seen, not only is the integrated intensity of the excitations important, but also the number of the corresponding degrees of freedom. Second, the correlation through the medium can turn out to be retarded, and furthermore not so much because of the small velocity of propagation of the excitations as because of the slowness of their damping. Thus, we can obtain a long-lived correlation not only through phonons, magnons, plasmons, etc., but even through photons. The delay of the correlation by times longer than the relaxation time alters the spectrum of the electron fluctuations. Peaks appear in it both at zero frequency and at the frequencies of the weakly damped mixed excitations of the medium and of the electron system. These

FIG. 6

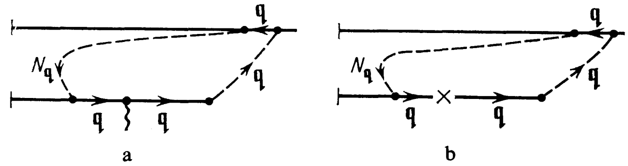

FIG.7.

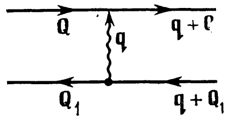

FIG. 8.

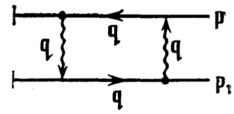

FIG. 10.

peaks can be clearly pronounced even if the contribution of the medium to the integrated intensity of the electron fluctuations is small.

Observation of spatially homogeneous fluctuations of the current is the usual methodology of noise experiments. We see that under nonequilibrium conditions, such experiments can serve as fluctuation spectroscopy of a medium by giving information not only on the spatially homogeneous but also on the spatially inhomogeneous excitations in it.

4. ACCOUNT OF COULOMB INTERACTION OF THE ELECTRONS

At $q \lesssim \varkappa$, it is necessary to take into account the long-range Coulomb forces that arise in the electron system in the case of spatially inhomogeneous perturbations. The action of these forces is described by the operator

$$U_{\mathbf{p}\mathbf{p}_1}^{\mathbf{q}} = i\frac{4\pi e^2}{\varepsilon q^2 v_0}\mathbf{q}\left(\frac{\partial}{\partial \mathbf{p}}-\frac{\partial}{\partial \mathbf{p}_1}\right), \tag{13}$$

where ε is the dielectric constant of the lattice. This operator is shown in the diagrams by a vertical (instantaneous electrostatic interaction!) wavy line connecting the two points on electron lines (see Fig. 8). The tip of the arrow corresponds to a derivative with a plus sign. Inclusion of this new element complicates the diagrams for the fluctuations. This leads to an added term with a self-consistent field in the operator of the evolution of the spatially inhomogeneous perturbation, i.e., to the replacement $\hat{B}_\mathbf{p}(\mathbf{q},\omega) \to \hat{\mathscr{B}}_\mathbf{p}(\mathbf{q},\omega)$, where

$$\hat{\mathscr{B}}_\mathbf{p}(\mathbf{q},\omega)x_\mathbf{p} \equiv \hat{B}_\mathbf{p}(\mathbf{q},\omega)x_\mathbf{p} - \sum_{\mathbf{p}_1} U_{\mathbf{p}\mathbf{p}_1}^{\mathbf{q}} F_\mathbf{p} x_{\mathbf{p}_1}. \tag{14}$$

Moreover, the constants of the electron-phonon interaction are renormalized: $ic_\mathbf{q} \to ic_\mathbf{q}/\varepsilon_{\mathbf{q},\omega\mathbf{q}}$ (entrance), $-ic_\mathbf{q} \to -ic_\mathbf{q}/\varepsilon_\mathbf{q}, \omega_\mathbf{q}$ (exit). The points nearest to the tails of $F_\mathbf{p}$ need not be specially renormalized, since for them processes leading to renormalization are automaticaltaken into account by the substitution (14) (see Ref. 10).

Coulomb interaction gives rise also to a new type of correlations diagrams (Fig. 9). They differ from the diagrams of Fig. 4 by the replacement of one of the phonon lines by a Coulomb-interaction line. The latter describes a mixed type of correlation—through the phonon and the electrostatic field. Upon replacement also of the second phonon by Coulomb interaction, we would have obtained a diagram (Fig. 10) which, at sufficiently large **q**, describes the correlation in terms of electron-electron collisions (see Ref. 11).

We note that correlations through the Coulomb field can be protracted in time (see Fig. 11) over times of the order of the damping time of the excitations in the electron system.

A comparison of the different correlation diagrams that are shown in Figs. 4, 5, 9–11, demonstrates both the similarity and the difference between "instantaneous" correlation created by collisions, and that protracted in time via waves and via relaxation processes.

5. CURRENT FLUCTUATIONS IN THE HYDRODYNAMIC APPROXIMATION

It remains to write down the explicit expressions for the current fluctuations that depend on the phonon intensities—for electroacoustic noise—under concrete conditions. We consider a situation in which strongly excited phonons with such small **q** that the inequalities of the hydrodynamic approximation are satisfied for the electrons:

$$\omega_\mathbf{q}\tau_\mathbf{p} \ll 1, \quad ql_\mathbf{p} \ll 1. \tag{15}$$

Correspondingly, we shall investigate the spectrum of fluctuations in the region of sufficiently low frequencies:

$$\omega\tau_\mathbf{p} \ll 1. \tag{16}$$

Here $l_\mathbf{p} = v\tau_\mathbf{p}$ is the free path length of the electron. Moreover, we shall assume that the inequalities

$$w_\mathbf{q}/v \ll 1, \quad V/v \ll 1 \tag{17}$$

are satisfied ($w_\mathbf{q}$ is the phase velocity of the sound, $\mathbf{V} = \Sigma \mathbf{v} F_\mathbf{p}/N$ is the drift velocity of electrons in the state unperturbed by phonons), as well as the inequality

$$1/qv\tau_M \ll 1, \tag{18}$$

where τ_M is the Maxwellian relaxation time of the electron system. Under the condition (18), only the diagrams of Figs. 3–5 survive. In the hydrodynamic approximation (15), we can neglect the diagrams of Figs. 3a and 3b in comparison with the diagram of Fig. 3c. In this approximation, the inverse of the evolution operator (14) acts in the following fashion (see Refs. 15, 10, 9):

$$\begin{aligned} x_\mathbf{p} &= \hat{\mathscr{B}}_\mathbf{p}^{-1}(\mathbf{q},\omega)y_\mathbf{p} \\ &= \frac{\Delta N}{N}\Big[F_\mathbf{p} - iqI_\mathbf{p}^{-1}(\mathbf{v}-\mathbf{V})F_\mathbf{p} + I_\mathbf{p}^{-1}\sum_{\mathbf{p}_1} U_{\mathbf{p}\mathbf{p}_1}^{\mathbf{q}}F_\mathbf{p}\Big] \\ &\quad + I_\mathbf{p}^{-1}(y_\mathbf{p} - F_\mathbf{p}y), \end{aligned} \tag{19}$$

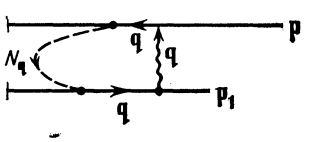

FIG. 9.

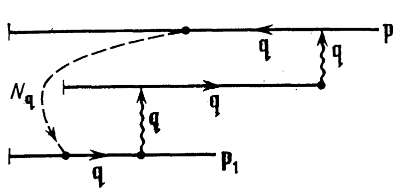

FIG. 11.

where

$$\Delta N=\left\{\sum_{\mathbf{p}}y_{\mathbf{p}}-i\mathbf{q}\sum_{\mathbf{p}}\mathbf{v}I_{\mathbf{p}}^{-1}(y_{\mathbf{p}}-F_{\mathbf{p}}y)\right\}[-i(\omega-\mathbf{qV})+\gamma_e]^{-1},$$

$$y=\sum_{\mathbf{p}}y_{\mathbf{p}}/N. \tag{20}$$

The damping coefficient of the densiton (more often called the electron-density wave in hydrodynamics)

$$\gamma_e=4\pi\sigma/\varepsilon+q^2D \tag{21}$$

is expressed in terms of the nonequilibrium diffusion tensor

$$D_{ik}=\frac{1}{N}\sum_{\mathbf{p}}v_iI_{\mathbf{p}}^{-1}(v_k-V_k)F_{\mathbf{p}} \tag{22}$$

and of the differential conductivity tensor

$$\sigma_{ik}=-\frac{e^2}{v_0}\sum_{\mathbf{p}}v_iI_{\mathbf{p}}^{-1}\frac{\partial F_{\mathbf{p}}}{\partial p_k}. \tag{23}$$

In (21) and subsequently, the projections of these tensors on the $\mathbf{q}$ direction are denoted by σ and D.

The partial current $\mathbf{j}^{\mathbf{q}}$ entering into the expression for the electrosonic current (11) is, in the hydrodynamic approx imation, equal to

$$\mathbf{j}^{\mathbf{q}}=\mathbf{q}\frac{T\sigma}{eNn_{\mathbf{q}}}\frac{\gamma_{\mathbf{q}}}{\omega_{\mathbf{q}}}, \tag{24}$$

where the damping coefficient of the phonons is given by the expression

$$\gamma_{\mathbf{q}}=\frac{2c_{\mathbf{q}}^2q^2\sigma v_0\Omega_{\mathbf{q}}}{\hbar e^2(\Omega_{\mathbf{q}}^2+\gamma_e^2)}$$

$$\Omega_{\mathbf{q}}=\omega_{\mathbf{q}}-\mathbf{qV}. \tag{25}$$

(We assume that the phonon damping is essentially determined by just this interaction with the electrons.)

The correlation diagrams of Figs. 4 and 5 give in the hydrodynamic approximation the following contribution to the current fluctuations:

$$(\delta j_\alpha\delta j_\beta)_\omega^{(ph)}=\sum_{\mathbf{q}}j_\alpha^{\mathbf{q}}j_\beta^{\mathbf{q}}N_{\mathbf{q}}^2\frac{2\gamma_{\mathbf{q}}}{\omega^2+\gamma_{\mathbf{q}}^2}\mathscr{L}_{\mathbf{q}}^{(ph)}(\omega). \tag{26}$$

We see that along with the peak at zero frequency, of width $\gamma_{\mathbf{q}}$ and due to the phonon fluctuations [see (9)], there is also a smooth dispersion at frequencies, of the order of $\Omega_{\mathbf{q}}$ and (or γ_e), which characterize the densitons:

$$\mathscr{L}_{\mathbf{q}}^{(ph)}(\omega)$$
$$=\{4\Omega_{\mathbf{q}}^2[(\Omega_{\mathbf{q}}-\omega)^2+\gamma_e^2][(\Omega_{\mathbf{q}}+\omega)^2+\gamma_e^2]\}^{-1}\{\omega^4(\Omega_{\mathbf{q}}^2+\omega_D^2)$$
$$+\omega^2\Omega_{\mathbf{q}}^2(5\gamma_e^2-3\Omega_{\mathbf{q}}^2-8\gamma_e\omega_D+\omega_D^2)$$
$$+\omega^2\gamma_e^2\omega_D^2+4\Omega_{\mathbf{q}}^2(\Omega_{\mathbf{q}}^2+\gamma_e^2)^2\}, \tag{27}$$

where for brevity we have used the notation $q^2D\equiv\omega_D$.

We now write down the explicit expression for the contribution, which is linear in the phonon intensity, from the diagram of Fig. 3c. Since this contribution, as we have seen, originates from the perturbation of the electron fluctuations by the phonons, we arbitrarily label it by the symbol (e)

$$(\delta j_\alpha\delta j_\beta)_\omega^{(e)}$$
$$=\frac{T}{v_0N}\sum_{\mathbf{q}}\frac{N_{\mathbf{q}}}{n_{\mathbf{q}}}\frac{\gamma_{\mathbf{q}}}{\omega_{\mathbf{q}}}\frac{q_\varphi q_\delta}{q^2}[\sigma_{\alpha\varphi}\mathscr{L}_{\mathbf{q},\beta\delta}^{(e)}(\omega)+\sigma_{\beta\varphi}\mathscr{L}_{\mathbf{q},\alpha\delta}^{*(e)}(\omega)]. \tag{28}$$

Here $\mathscr{L}_{\mathbf{q}}^{(e)}(\omega)$ is the dimensionless factor that determines the spectral component of the contribution (e) to the fluctuations:

$$\mathscr{L}_{\mathbf{q},\beta\delta}^{(e)}(\omega)=\{[\Omega_{\mathbf{q}}^2-(-i\omega+\gamma_e)\omega_D](D_{\beta\delta}+D_{\delta\beta}-D\sigma_{\beta\delta}/\sigma)q^2$$
$$+(-i\omega+\gamma_e+\omega_D)(q^2V_\beta/q_\delta+\Omega_{\mathbf{q}}\sigma_{\beta\delta}/\sigma)\}$$
$$\times\{\Omega_{\mathbf{q}}(\Omega_{\mathbf{q}}^2+\gamma_e^2-\omega^2-2i\omega\gamma_e)\}^{-1}. \tag{29}$$

We see that the contribution (e) undergoes dispersion only at frequencies of order $\Omega_{\mathbf{q}}$ and (or) γ_e.

We make the formulas (28) and (29) explicit for the axially symmetric situation, in which $\alpha\|\beta$, $\mathbf{V}\|\alpha$ or $\mathbf{V}\perp\alpha$ and, moreover, $\sigma_{\alpha\beta}=D_{\alpha\beta}=0$ if $\alpha\neq\beta$:

$$(\delta j_\alpha^2)_\omega^{(e)}=\frac{2T\sigma_{\alpha\alpha}}{v_0N}\sum_{\mathbf{q}}\frac{N_{\mathbf{q}}}{n_{\mathbf{q}}}\frac{\gamma_{\mathbf{q}}}{\omega_{\mathbf{q}}}\frac{q_\alpha^2}{q^2}\operatorname{Re}\mathscr{L}_{\mathbf{q},\alpha\alpha}^{(e)}(\omega), \tag{30}$$

where

$$\operatorname{Re}\mathscr{L}_{\mathbf{q},\alpha\alpha}^{(e)}(\omega)=\{\Omega_{\mathbf{q}}[(\Omega_{\mathbf{q}}-\omega)^2+\gamma_e^2]$$
$$\times[(\Omega_{\mathbf{q}}+\omega)^2+\gamma_e^2]\}^{-1}\{\omega^2[\Omega_{\mathbf{q}}^2(\gamma_e\sigma_{\alpha\alpha}\{\sigma$$
$$-2q^2D_{\alpha\alpha})-\gamma_e\omega_D^2(2D_{\alpha\alpha}/D-\sigma_{\alpha\alpha}/\sigma)+\Omega_{\mathbf{q}}V_\alpha4\pi\sigma q^2/\varepsilon q_\alpha]$$
$$+(\Omega_{\mathbf{q}}^2+\gamma_e^2)[\Omega_{\mathbf{q}}^2(\gamma_e\sigma_{\alpha\alpha}/\sigma$$
$$+2q^2D_{\alpha\alpha})-\gamma_e\omega_D^2(2D_{\alpha\alpha}/D-\sigma_{\alpha\alpha}/\sigma)+\Omega_{\mathbf{q}}(\gamma_e+\omega_D)q^2V_\alpha/q_\alpha]\}. \tag{31}$$

Both the contributions (26) and (30), under conditions of weak electron damping $\gamma_e\ll\Omega_{\mathbf{q}}$, contain peaks at the frequency $\Omega_{\mathbf{q}}$, as is seen from the denominators of (27) and (31). At frequencies of such order, the contributions (ph) and (e) can compete [we note that, as is seen from (31), the correction (e) to the thermal noise can also be negative]. In the region of the same low frequencies $\omega\lesssim\gamma_{\mathbf{q}}$ in hydrodynamics, the contribution (ph) exceeds the contribution (e) relative to the disequilibrium parameter $N_{\mathbf{q}}/n_{\mathbf{q}}\gg1$.

6. CONCLUSION

We now discuss the general form of the spectrum of current noise in the considered case. At high frequencies of the fluctuations ($\omega\sim1/\tau_{\mathbf{p}}$) we have the usual Lorentzian. It can be represented roughly as a plateau at $\omega\lesssim1/\tau_{\mathbf{p}}$ and a cut $1/\omega^2$ at $\omega\gtrsim1/\tau_{\mathbf{p}}$.

Owing to the effect of the nonequilibrium phonons at frequencies of the order of the hydrodynamic damping of the electron density wave (densiton) γ_e, a bump appears on the plateau. Several densitons participate in its formation and therefore it has a complicated form (in contrast with the simple Lorentzian typical to spatially inhomogeneous electron fluctuations,[9,15] where only a single densiton is involved). Nevertheless, this bump—we call it a quasi-Lorentzian—can also be represented roughly in the form of a plateau at $\omega\lesssim\gamma_e$ and a quadratic cut at $\omega\gtrsim\gamma_e$. The bump itself, under conditions of weak turbulence, will be small (of the order of the parameter of nonlinearity). With increase in the nonlinearity, its magnitude should, however, increase [under certain conditions we can have instead of a bump a depression—see (31)].

At very low frequencies of the order of the phonon damping $\gamma_{\mathbf{q}}$, we come into the region of phonon fluctuations, which manifest themselves in current noise. The form of the spectrum in this region is a superposition of Lorentzians with different $\gamma_{\mathbf{q}}$ (with weights that depend weakly on ω). Inasmuch as in a nonequilibrium phonon system, the phonons near a certain characteristic $\mathbf{q}_0$ predominate, the spectrum can be described by a single Lorentzian and approximated by means of a plateau at $\omega \lesssim \gamma_{\mathbf{q}0}$ and a quadratic cut at $\omega \gtrsim \gamma_{\mathbf{q}0}$. It should be emphasized that the magnitude of this low-frequency bump, even under conditions of weak phonon turbulence, can be very significant and can greatly exceed the level of thermal noise [see (8)]. Under conditions of developed turbulence, the contribution of the phonon noise to the current fluctuations should be the major one at low frequencies ($\omega \lesssim \gamma_{\mathbf{q}}$).

The effect of nonequilibrium phonons on the current fluctuations has been observed in a number of experiments.[4–6]

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[1)]The correlation vanishes in equilibrium when account is taken of spontaneous (in addition to stimulated) exchange of phonons between electrons, by virtue of the identity

$$\mathrm{Re}\, B_{\mathbf{p}}^{-1}(\mathbf{q}, \omega_{\mathbf{q}}) \left(\frac{T}{\omega_{\mathbf{q}}} \mathbf{q} \frac{\partial}{\partial \mathbf{p}} + 1 \right) \exp\left(-\frac{\varepsilon_{\mathbf{p}}}{T} \right) = 0.$$

[2)]The phonon line emerging from the end of the electron line, cannot carry intensity.[10,14]

[3)]To avoid misunderstanding, we note that the reverse is not the case: thus, in our case, at $\Sigma^{\mathbf{q}} \gg N$, the effect of the phonons on the fluctuations at the expense of the uncorrelated contributon is of the same order as the effect on the response.

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