Existence of superconducting domain walls in ferromagnets

A. I. Buzdin, L. N. Bulaevskiĭ, and S. V. Panyukov

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR
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A BCS model incorporating the orbital effect of the magnetic induction and the paramagnetic effect of the exchange field is used to investigate the conditions for the formation of an infinitesimal superconducting nucleus localized at domain walls in ferromagnets. It is shown that a solution of this type always exists in reentrant ferromagnetic superconductors, at least in the metastability region of the FN (ferromagnetic normal) phase. The existence of superconducting domain walls in the stable FN phase of ideal ferromagnets remains an open question, since the transition to this state is apparently of first order and the transition parameters are unknown. From the standpoint of realizing superconducting domain walls it seems more promising to consider reentrant superconductors possessing an irregular magnetic subsystem without a coexistence phase and having a small exchange contribution to the magnetic energy, i.e., compounds such as $Ho_{1-x}Y_xMo_6S_8$.

1. INTRODUCTION

The question of whether superconductivity exists in ferromagnets was first posed in 1956 by Ginzburg,[1] who treated the problem with allowance for the suppression of Cooper pairing by the magnetic field of localized magnetic moments in the ferromagnetic state, i.e., an electromagnetic mechanism of interaction between the superconducting and ferromagnetic ordering. Analysis of the ferromagnets known at that time showed that Cooper pairing can arise in a magnetically ordered phase only in exceptional situations where the effect of the magnetic induction (the orbital effect) is for some reason suppressed (for example, in a thin ferromagnet with magnetization lying in the plane of the slab or in a ferromagnet in the metastable state with the external field directed counter to the magnetic moment and compensating its effect).

Shortly thereafter, Matthias, Suhl, and Corenzwit[2] noted the existence of another mechanism working to destroy the superconductivity of localized (magnetic) electrons. This mechanism is due to the exchange interaction of the localized moments with the conduction electrons. Abrikosov and Gor'kov[3] showed that the presence of localized moments in the paramagnetic state gives rise to exchange (magnetic) scattering of the electrons by the localized moments, thus suppressing the Cooper pairing. In the ferromagnetic state an additional contribution to the exchange scattering arises from the effect of the exchange field, which destroys the singlet Cooper pairing of the electrons by causing electrons with opposite spins to become separated in energy (the paramagnetic effect[4]). In the end, the electromagnetic and exchange mechanisms of depairing practically rule out the possibility that superconductivity can coexist with homogeneous ferromagnetic ordering.

Matthias and Suhl[5] first noted that the conditions for the occurrence of superconductivity in a ferromagnet are more favorable near a domain wall than inside a domain. This circumstance stems from the fact that the magnetization here is inhomogeneous, and the electrons move in a field which varies in direction. Therefore, it is possible in principle for superconductivity of a localized type to occur near domain walls under conditions where Cooper pairing is suppressed in the interior of the domains.

The first attempt to describe this effect quantitatively was undertaken by Kopaev.[6] Specifically, he considered a ferromagnet having linear domain walls[7] within which the magnetization vector varies in magnitude and sign but remains parallel to the easy axis. At the center of such a wall the magnetization goes to zero, and Kopaev hypothesized[6] that a superconducting state can exist in the region of the wall where the magnetization is small. Kopaev assumed[6] that the localization length of the superconductivity is small compared to the superconducting correlation length and, working under this assumption, calculated the conditions for the occurrence of localized superconductivity. Such an assumption, however, ignores the proximity effect, which makes it impossible for superconducting regions with a localization length shorter than the superconducting length to exist in a metal.

The possibility that superconducting domain walls can exist in a normal ferromagnetic (FN) phase was raised anew[8,9] in connection with experimental studies of the reentrant ferromagnetic superconductors $ErRh_4B_4$ and $HoMo_6S_8$.[10–14] These compounds, which have a regular lattice of localized moments of the rare earth elements, undergo a transition to a superconducting state at a temperature T_{c1} (8.7 K and 1.8 K, respectively); below the point T_M (1 K and 0.7 K) they exhibit an inhomogeneous magnetic ordering in the superconducting state, and at a temperature T_{c2} (0.8 K and 0.65 K) they return by a first-order transition to a normal state with ferromagnetic ordering of the localized moments. Fertig *et al.*[10] discovered that below T_{c2}, down to the very lowest temperatures, the resistivity of $ErRh_4B_4$ is approximately 40% smaller than at temperatures above T_c, and that it reaches its normal level below T_{c2} only in magnetic fields exceeding 5 kOe. Analogous behavior is also ob-

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served in $HoMo_6S_8$, but here the residual conductivity is lower (by approximately 10%) and is suppressed in fields above 0.7 kOe (Ref. 4).[1)] Tachiki *et al.*[8] conjectured that the increased conductivity in the ferromagnetic phase below T_{c2} is due to the persistence of superconductivity on the domain walls. However, here again the idea of a superconductivity localized on the domain walls was not realistically rendered, since the analysis of that paper[8] was based on the assumption that the width of the rotating Bloch domain walls is large compared to the superconducting correlation length.[2)] Actually, however, in the ferromagnetic superconductors under discussion the situation is apparently just the opposite—the width of the domain wall in these compounds (of the order of several angstroms) is small compared to the superconducting correlation length (around 250 Å in $HoMo_6S_8$ and 200 Å in $ErRh_4B_4$).

In fact, the width of a rotating Bloch wall is given in order of magnitude by the relation $l_w \sim a(T_M/D)^{1/2}$ with $D \ll T_M$, where a is the magnetic hardness, which is of the order of an atomic length, and D is the anisotropy energy parameter. In the compounds under discussion, the strong effect of the crystalline field causes the parameter D to be of about the same order of magnitude as T_M.[3)] For $D \gg T_M$ one has a linear domain wall of width $l_w \sim a(1 - T/\Theta_c)^{-1/2}$ (Ref. 7), where Θ_c is the Curie temperature in the absence of superconductivity, and $\Theta_c \approx T_M$. Therefore, the width of the domain wall is greater than an atomic length only in the case of very small anisotropy or very close to the Curie point, but at temperatures in the region $T < T_{c2} < T_M$ for a reasonable value of the anisotropy, the width of the domain wall is close to an atomic length.

Therefore, with real compounds in mind, we should determine the conditions for the occurrence of localized superconductivity at domain walls whose width is small compared to the superconducting correlation length ξ. Because of the proximity effect, the superconducting solution will itself be localized in a region of the order of ξ or larger. In the limiting case $\xi \gg l_w$ the final results will not depend on the specific type of wall (rotating or linear).

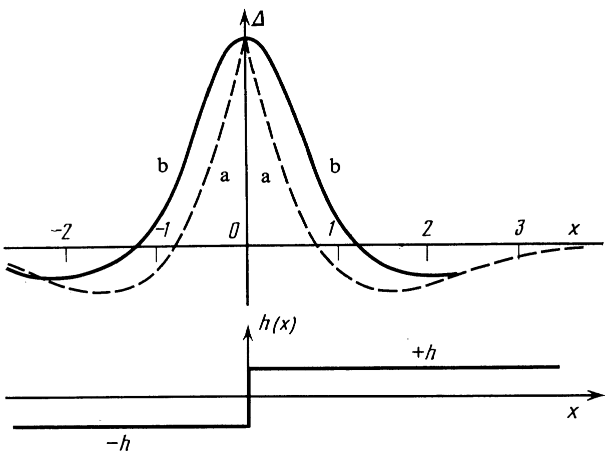

FIG. 1. Isolated domain wall. Shown below is the dependence of the exchange field on the coordinate in the x direction, perpendicular to the wall. Shown above is the shape of the superconducting order parameter: a) for a clean superconductor, on a scale of $3.76\xi_0$; b) for a dirty superconductor, on a scale of 1.66ξ.

In this paper we find the conditions for the formation of a critical superconducting nucleus of a superconducting domain wall with an infinitesimally small amplitude of the order parameter, under the assumption that in the absence of localized moments the superconductivity would arise at temperature T_{c0}. For such a formulation of the problem we should study the Cooper instability in the presence of a given magnetic order $\mathbf{m}(\mathbf{r},T)$ corresponding to the domain structure in the FN phase. In accordance with the condition $l_w \ll \xi$, we can approximate the magnetization $\mathbf{m}(\mathbf{r},T)$ and the magnetic induction $\mathbf{B}(\mathbf{r},T)$ near the domain wall by the step functions $m_z(x,T) = m_0(T)\mathrm{sgn}x$ and $B_z(x,T) = 4\pi m_0(T)\mathrm{sgn}x$ (the wall is located in the $x = 0$ plane and the easy axis is along z; see Fig. 1).

We shall show that if the magnetic ordering influences the superconductivity only through the orbital effect of the magnetic induction (the electromagnetic mechanism), then the problem of the onset of Cooper pairing near the domain wall is completely equivalent to the problem of the nucleation of a superconducting center on the surface of the sample in a magnetic field parallel to the surface, i.e., the problem of determining H_{c3}.[15] Therefore, in regard to the electromagnetic mechanism the condition for the existence of superconducting domain walls is of the form $H^*_{c2} < 4\pi m_0(T) < H^*_{c3}$, where the upper critical orbital magnetic fields H^*_{c2} and H^*_{c3} depend on the superconducting correlation length $\xi(T)$; specifically, both fields H^*_{c2} and H^*_{c3} are proportional to $\Phi_0/\xi^2(T)$, where Φ_0 is the magnetic flux quantum, and differ only by a numerical factor.[4,15]

In real compounds we should take into account not only the electromagnetic mechanism but also the exchange interaction of the localized moments; these two interaction mechanisms together determine the magnetic ordering temperature and energy of the localized moments. In the compounds under discussion the contributions of the two mechanisms to the Curie temperature Θ_c in the absence of superconductivity and to the magnetic ordering temperature $T_M \approx \Theta_c$ in the superconducting phase are similar to order of magnitude, and in treating the problem of superconducting domain walls we should take the interaction of the localized moments and conduction electrons into account.

As we have already mentioned, the exchange interaction suppresses Cooper pairing in the ferromagnetic state through a scattering of the electrons by spin waves and through the paramagnetic effect of the exchange field. The first effect is characterized by a reciprocal magnetic-scattering time which is of the same order of magnitude as the contribution Θ_{ex} of the RKKY interaction to the temperature Θ_c. Because of this scattering, superconductivity of the ordinary or localized type can exist only under the condition $\Theta_{ex} \sim \Theta_c < T_{c0}$, which is satisfied in the reentrant superconductors $ErRh_4B_4$ and $HoMo_6S_8$. In compounds with $\Theta_{ex} \sim T_{c0}$ the suppression of the superconductivity is largely due to magnetic scattering, and the existence conditions for ordinary and localized superconductivity are not very different. One is therefore unlikely to find superconducting domain walls in ferromagnetic compounds which do not have a superconducting phase (such as $HoRh_4B_4$ or $GdMo_6S_8$). In what follows we shall consider only reentrant superconductors with $\Theta_{ex} \ll T_{c0}$ and determine the conditions for the formation of superconducting domain walls in their low-temperature phase.

In $HoMo_6S_8$ and $ErRh_4B_4$ the magnetic scattering by spin waves can be neglected by virtue of the condition $\Theta_{ex} \sim T_M \ll T_{c1}$, and only the paramagnetic effect of the exchange field $h(\mathbf{r}) = h_0 s(\mathbf{r},T)$ is important; here $s(\mathbf{r},T)$ is the normalized value of the magnetization, $s(\mathbf{r},T) = m(\mathbf{r},T)/\mu n$, μ is the magnitude of the magnetic moment at $T = 0$, and n is the concentration of localized moments. In order of magnitude one has $\Theta_{ex} \sim h_0^2 N(0)$, where $N(0)$ is the density of electron states per localized moment. The quantity h_0 is not small compared to T_{c0} even if $\Theta_{ex} \ll T_{c0}$, and in determining the conditions for the occurrence of localized superconductivity the effect of the exchange field should be taken into account along with the orbital effect of the magnetic induction $B_z(x) = B(0)s(x,T)$, where $B(0) = 4\pi\mu n$. Using the approximation $s(x,T) = s(T)\operatorname{sgn}x$, we shall determine the critical value $s(T_{DC}^{(w)})$ (as a function of h_0, B_0, and T_{c0}) for the nucleation of a superconducting center near a domain wall in the FN phase. The temperature $T_{DC}^{(w)}$ is clearly a superheating point of the FN phase with respect to localized superconductivity if the transition to the state with superconducting domain walls is in fact a first-order transition with a temperature $T_{DC} < T_{DC}^{(w)}$.

The above formulation of the problem corresponds to treating an isolated superconducting domain wall (see Fig. 1). This formulation is correct for the case in which the thickness of the domains is much greater than the superconducting correlation length ξ.

2. SYSTEM HAMILTONIAN AND THE EQUATIONS FOR THE SUPERCONDUCTING ORDER PARAMETER

We describe the system of localized moments and the Cooper pairing of electrons in the framework of the BCS model, taking into account the exchange field and magnetic field of the localized moments and the scattering of electrons by nonmagnetic impurities, but ignoring the magnetic scattering of electrons. The localized moments are located at lattice sites i and are described by the parameter s_i, which characterizes the normalized average value of the moment at site i (in the quantum-mechanical and statistical sense). The Hamiltonian of the system is of the form

$$\mathscr{H} = \int d\mathbf{r}\left[\frac{1}{2m}\psi^+(\mathbf{r})\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2\psi(\mathbf{r})+\Delta(\mathbf{r})\psi^+(\mathbf{r})i\sigma_y\psi^+(\mathbf{r}) -\Delta^*(\mathbf{r})\psi(\mathbf{r})i\sigma_y\psi(\mathbf{r})+\sum_i J(\mathbf{r}-\mathbf{r}_i)\psi^+(\mathbf{r})\sigma_z(g-1)s_i\psi(\mathbf{r}) +N(0)\frac{|\Delta(\mathbf{r})|^2}{\lambda}\right]+\mathscr{H}_{sc},\qquad \mathbf{B}=\operatorname{rot}\mathbf{A}, \quad (1)$$

where $\Delta(\mathbf{r})$ is the superconducting parameter for the singlet pairing of electrons, $\psi(\mathbf{r})$ is a spinor, the components of $\boldsymbol{\sigma}$ are the Pauli matrices, $\mathbf{A}$ is the vector potential, $J(\mathbf{r})$ is the exchange integral, λ is the dimensionless electron-phonon interaction parameter, g is the g factor, and the term $\mathscr{H}_{sc}$ describes the scattering of electrons by nonmagnetic impurities. The exchange field $h(\mathbf{r})$ is given by the expression[1]

$$h(\mathbf{r})=\sum_i J(\mathbf{r}-\mathbf{r}_i)(g-1)s_i. \quad (2)$$

This field has both a part which is slowly varying on an atomic scale and a rapidly varying part with wave vectors which are of the order of the vectors of the reciprocal lattice of the localized moments. The influence of the rapidly varying part on the superconductivity can be neglected, and then[4] $h(x) = h_0 s(x)$, where $h_0 = n(g-1)\int d\mathbf{r}J(\mathbf{r})$.

We describe the superconducting system using the quasiclassical approximation for the anomalous Gor'kov functions integrated over the energy variable.[5] To first order in $\Delta(\mathbf{r})$ the Eilenberger equations for the anomalous function $f(\mathbf{v},\mathbf{r})$ are of the form[16]

$$\left[\tilde\omega+ih(x)+i\frac{e}{c}\mathbf{v}\mathbf{A}(\mathbf{r})+\frac{1}{2}(\mathbf{v}\nabla)\right]f(\mathbf{v},\mathbf{r})=\tilde\Delta\operatorname{sgn}\omega,$$
$$\tilde\omega=\omega+\frac{1}{2\tau}\operatorname{sgn}\omega,\qquad \tilde\Delta(\mathbf{r})=\Delta(\mathbf{r})+\frac{1}{2\tau}\bar f(\mathbf{r}),$$
$$\bar f(\mathbf{r})=\int\frac{d\Omega}{4\pi}f(\mathbf{v},\mathbf{r}),\qquad \Delta(\mathbf{r})=\pi T\lambda\sum_\omega \bar f(\mathbf{r}),\quad (3)$$
$$\omega=\pi T(2k+1),\qquad A_y=-B(0)\int_0^x s(x)\,dx,\qquad A_x=A_z=0,$$

where τ is the characteristic time for the scattering of electrons by nonmagnetic impurities, and the integration over Ω is over the direction of the electron velocity $\mathbf{v}$. The boundary conditions on equations (3) reduce to the continuity condition for the function $f(\mathbf{v},x)$ and the condition that $f(\mathbf{v},x)$ vanish at $|x| \to \infty$ for the localized solution.

In Ref. 17 we also found the free-energy functional of the superconducting system. By minimizing this functional with respect to f and g one obtains the Eilenberger equations, and the minimum value gives the free energy functional $\mathscr{F}\{\Delta(\mathbf{r})\}$. To lowest order in $\Delta(\mathbf{r})$ this functional is of the form

$$\mathscr{F}\{\Delta(\mathbf{r})\}=N(0)\int d\mathbf{r}\left[\frac{|\Delta(\mathbf{r})|^2}{\lambda}-\pi T\sum_\omega \bar f(\mathbf{r})\Delta^*(\mathbf{r})+\frac{1}{4\tau}\bar f^2\right], \quad (4)$$

where the function $\bar f(\mathbf{r})$ is the solution of equations (3).

The form of equations (3) for $h = 0$ implies that the conditions for the formation of a superconducting nucleus within a domain wall [$s(x) = s\operatorname{sgn}x$ and $f(v_x, x\to +0) = f(v_x, x \to -0)$] are equivalent to the corresponding conditions on the surface of the sample [$s(x) = s, x > 0$ and $f(v_x, x\to +0) = f(-v_x, x\to +0)$]. In fact, a change in the sign of the magnetic field in equations (3) is equivalent to a change in the direction of the electron velocity.

Let us now study the role of the electromagnetic and exchange interactions in determining the conditions for the nucleation of a superconducting domain wall. By comparing the orbital term evA/c for $A \sim B\xi$ with the exchange term h in equations (3), one sees that in a clean superconductor the relative role is determined by the parameter

$$\rho=(2eB(0)v_F\xi(T)/h_0c)^2\sim(B(0)\Delta(T)/H_{c2}^*(T)h_0)^2,$$

where $\Delta(T)$ is the order parameter at temperature T in the absence of localized moments, and $\xi(T)=v_F/\pi\Delta(T)$. One is readily convinced that this same parameter ρ describes the relative contribution of the electromagnetic and exchange mechanism in dirty superconductors as well if $\xi(T)$ is replaced by the corresponding value of the correlation length.[4] We show below that the parameter ρ is small in the temperature region $T\ll T_{c1}$ in the reentrant superconductors $ErRh_4B_4$ and $HoMo_6S_8$. Therefore, for these compounds (in which $T_M\ll T_{c1}$) the conditions for the formation of a superconducting nucleus near a domain wall can be found without taking the magnetic induction into account. In addition, in the temperature region $T\ll T_{c1}$ the superconducting characteristics can be calculated for the limiting case of zero temperature, i.e., the sums over ω in (3) and (4) can be converted to integrals over ω.

As in the case of a uniform exchange field, the form of the solution for a superconducting nucleus in a nonuniform exchange field depends in an important way on the purity of the crystal. According to the results of Larkin and Ovchinnikov[18] and Fulde and Ferrel,[19] in a clean superconductor the solution for a nucleus in a constant exchange field is spatially inhomogeneous, while in a dirty superconductor the inhomogeneity of $\Delta(\mathbf{r})$ over distances characterized by l/ξ_0 need not be taken into account (see Ref. 20).

For a clean crystal we thus seek a solution of the following form for a superconducting nucleus in a system with a domain wall:

$$\Delta(\mathbf{r})=\exp\left(i\frac{2h}{v_F}\mathbf{k}_\perp\mathbf{r}\right)\Delta(\mathbf{k}_\perp,x),\qquad k_\perp=(0,k_y,k_z),$$
$$\Delta(\mathbf{k}_\perp,x)=\frac{1}{2\pi}\int dk_x\,\Delta(\mathbf{k})\exp\left(i\frac{2h}{v_F}k_x x\right).\tag{5}$$

Substituting (5) into (3) and (4), we obtain the functional

$$\mathcal{F}\{\Delta(\mathbf{k})\}=\frac{v_F N(0)}{2h}\left\{\frac{1}{2\pi}\int dk_x\left[\frac{|\Delta(\mathbf{k})|^2}{\lambda}-\frac{1}{2\pi}\int dk_x'\,\Delta(\mathbf{k})\Delta^*(\mathbf{k}')K(k,k')\right]\right\},$$
$$K(\mathbf{k},\mathbf{k}')=\frac{1}{2}\int_0^\infty dx\int_0^\infty dx'\int_0^{2\pi}\frac{d\varphi}{2\pi}\int_0^1\frac{d\mu}{\mu}\int_0^{\omega_D/h}d\omega\exp[i(k_x x-k_x'x')-\Omega|x-x'|/\mu]\cos\left[\frac{1}{\mu h}\int_x^{x'}h(\xi)\,d\xi\right],$$
$$\Omega=\omega+ik_\perp\cos\varphi(1-\mu^2)^{1/2},\tag{6}$$

where ω_D is the Debye frequency. Let us separate out from the kernel $K(\mathbf{k},\mathbf{k}')$ in (6) that part $K_0(\mathbf{k},\mathbf{k}')$ which corresponds to a uniform exchange field $h(x)=h$:

$$K_0(\mathbf{k},\mathbf{k}')=2\pi\delta(k_x-k_x')\left[\ln\frac{\omega_D}{h}-\psi(k)\right],$$
$$\psi(\mathbf{k})=-\frac{1}{2}\int_0^1 d\mu\ln[1-\mu^2(k_x^2+k_\perp^2)].\tag{7}$$

In the remaining part $K_1(\mathbf{k},\mathbf{k}')$ of the kernel the integration over ω can be extended to infinity. The linear integral equation for $\Delta(\mathbf{r})$ has the form of an equation for determining the minimum eigenvalue E:

$$(\psi(\mathbf{k})-E)\Delta(k)=\int K_1(\mathbf{k},\mathbf{k}')\frac{dk_x'}{2\pi}\Delta(\mathbf{k}'),\qquad E=-\ln(2h/\Delta_0),\tag{8}$$

with $K_1=0$ for $h(x)=h$. The function $\psi(x)$ is minimum at a wave vector $k=k_0=1.2$, which gives the condition for the formation of a delocalized nucleus in a uniform exchange field, i.e., within the domains, as $|h|=h_{0c}^{(w)}=0.754\Delta_0$.[18,19] Let us now find the kernel $K_1(\mathbf{k},\mathbf{k}')$ for the domain wall and the corresponding value of the critical field.

3. CONDITION FOR THE FORMATION OF SUPERCONDUCTING DOMAIN WALLS IN CLEAN COMPOUNDS

For an isolated domain wall the exchange field is $h(x)=h\,\mathrm{sgn}x$, and the superconducting kernel is given by the expression

$$K_1(\mathbf{k},\mathbf{k}')=\frac{1}{(k_x+k_x')}\int_{-1}^{1}\frac{d\mu\,\mathrm{sgn}\,\mu}{2-\mu(k_x+k_x')}\times[L(\mu,\mathbf{k}_\perp,k_x,k_x')+L(\mu,\mathbf{k}_\perp,k_x',k_x)],\tag{9}$$
$$L(\mu,\mathbf{k}_\perp,k_x,k_x')=\arcsin[2(1-\mu k_x)/(|\alpha_+|+|\alpha_-|)],$$
$$\alpha_\pm=1-\mu k_x\pm k_\perp(1-\mu^2)^{1/2}.$$

In the problem of an isolated wall there is no explicit small parameter. For this reason equation (8) with kernel (9) was solved numerically. The form of the function $\Delta(\mathbf{k}_\perp,x)$ which minimizes E is shown in Fig. 1 (case a); here $h_{DC}^{(w)}=0.794\Delta_0$ and $k_\perp=1.1$. The values of $h_{DC}^{(w)}$ and $h_{0c}^{(w)}$ are nearly equal, since the wave function $\Delta(\mathbf{k}_\perp,x)$ is localized in a region of approximately $4\xi_0$.

4. SUPERCONDUCTIVITY AT DOMAIN WALLS IN DIRTY COMPOUNDS

For a dirty superconductor the function $\bar{f}(x)$ satisfies the Usadel equation[21]

$$\left[|\omega|+ih(x)\,\mathrm{sgn}\,x-\tfrac{1}{2}D\frac{\partial^2}{\partial x^2}\right]\bar{f}(x)=\Delta(x),\qquad D=v_F^2\tau/3,\tag{10}$$

with a continuity condition of $\bar{f}(x)$ and its derivative in the plane of the domain wall. For an isolated domain wall we find the kernel

$$\tilde{K}(\mathbf{k},\mathbf{k}')=\left[\frac{1}{2}\ln(1+k^4)-E\right]\delta(\mathbf{k}-\mathbf{k}')-4\int_0^\infty\frac{dx\,b(x)\,(1+x^2)^{1/2}}{[(x+k)^2+1][(x+k')^2+1]},$$
$$b^2(x)=[(1+x^2)^{1/2}-x]/2.\tag{11}$$

Numerical solution of the integral equation gives $h_{DC}^{(w)} = 0.717\Delta_0$, and the function $\Delta(x)$ is shown in Fig. 1 (case b). The value obtained for $h_{DC}^{(w)}$ differs quite substantially from the critical exchange field $h_{DC} = 0.5\Delta_0$ of the bulk solution, i.e., the localization length in a dirty compound is approximately equal to the superconducting correlation length $\xi \approx (\xi_0 l)^{1/2}$.

The asymptotic behavior of the function $\Delta(x)$ for $x \gg \xi_0$ can be found analytically. It is governed by the poles of the function $\Delta(k)$ which lie closest to the real axis. These poles are found from the equation

$$E - {}^1/_2 \ln(1+k^4) = 0, \qquad (12)$$

which gives $k^2 = \pm 2i\beta^2$ and $\beta = 0.6$. Transforming to coordinate space, we obtain for $x \gg \xi_0$

$$\Delta(x) = f[\beta x (2h/D)^{1/2}], \qquad (13)$$

where $f(x) = \exp(-|x|)\cos(|x| + \pi/4)$.

5. ORDER OF TRANSITION AND THE EXISTENCE REGION OF SUPERCONDUCTING DOMAIN WALLS

We have found the superheating field $h_{DC}^{(w)}$ of the normal phase with respect to the onset of localized superconductivity around a domain wall in dirty $(0.717\Delta_0)$ and clean $(0.791\Delta_0)$ superconductors. However, there is reason to believe that the transition to the superconducting-domain-wall state should be of first order. Let us consider the sign of the coefficient in front of the fourth-order term in $\Delta(k)$ in the expansion of the superconducting functional at the point $T = T_{DC}^{(w)}$. In evaluating this coefficient we can regard the exchange field $h(x)$ as given (and equal to $h_0 s \operatorname{sgn} x$), and we should also take into account the additional contribution due to the change in the magnetization $s(x)$ under the influence of the superconductivity. This additional contribution is always negative in sign and is equal in order of magnitude to $(\partial\mathscr{F}/\partial s)^2/(\partial^2\mathscr{F}_M/\partial s)^2$, where $\mathscr{F}$ is given by formulas (9) and (11) for clean and dirty compounds, respectively, and $\mathscr{F}_M \approx \Theta s^2$ near the point T_M. As a result, at small $s \approx \Delta_0/h_0$ we obtain a contribution of the order of $\Theta_{ex} N(0)\Delta^4\xi/\Theta_c \Delta_0^2 s^2$. At the same time, the fourth-order term in $\Delta(k)$ at a given exchange field h is $N(0)\Delta^4\xi/\Delta_0^2$ (and is of unknown sign). Thus, for $\Delta_0/h_0 \ll 1$ the term which we know to be negative is the dominant one, and the transition to the superconducting-domain-wall state should be a first-order transition at the point h_{DC}, which can be substantially higher than $h_{DC}^{(w)}$ in a system with $h_0 \gg \Delta_0$. In compounds with approximately equal values of h_0 and Δ_0 it is possible to predict the order of the transition to the superconducting-domain-wall state only on the basis of a quantitative treatment. It is clear, however, that in this case h_{DC} can differ from $h_{DC}^{(w)}$ only by a numerical factor of order one.

A solution of the superconducting-domain-wall type thus exists for $h = h_0 s(T) < h_{DC}$. However, for a superconducting domain wall to exist it is also necessary that the FN phase itself be stable within the domains.

The stability region of this phase is determined by the condition $h > h_{c2}$, where $h_{c2} = h_0 s(T_{c2})$ is the critical value of the exchange field for the first-order transition from the superconducting state to the FN phase. Thus a superconducting domain wall exists only in superconducting ferromagnets for which $h_{DC} > h_{c2}$.

In a regular compound with an ideal magnetic subsystem, T_{c2} is the point of a transition from the FN phase to a superconducting phase DS with an inhomogeneous magnetic ordering of the domain-structure type,[17,22,23] and

$$h_{c2} = 0.65\Delta_0(\xi_0/\tilde{a})^{1/4}, \quad (\tilde{a}\xi_0)^{1/2} \ll l \ll v_F/h_0,$$
$$h_{c2} = 0.44\Delta_0(\xi_0/\tilde{a})^{1/4}, \quad l \gg \xi_0, \qquad (14)$$

where $\tilde{a}$ is a parameter which determines the domain-wall surface energy $\eta = \Theta_{ex} s^2 \tilde{a} n$ and is of the order of an atomic length. It is seen from (14) that h_{c2} is higher than $h_{DC}^{(w)}$, but the relationships between h_{c2} and h_{DC} are unknown (see Fig. 2).

In the compound $HoMo_6S_8$, estimates[17] based on the experimental data of Refs. 24 and 25 give $h_0 \approx 16$ K, $h_{c2} \approx 10$ K, $s_{c2} \approx 0.6$, and $\Delta_0 \approx 3$ K. The data of Ishikawa and Fisher[26] give $H_{c2}^*(0) \approx 3$ kOe for the upper critical field. Substituting these estimates and the value of $B(T_{c2}) = B(0)s_{c2}$ at $B(0) = 4.8$ kOe into the expression for ρ, we obtain $\rho \approx 0.04$. We see that the existence region of the superconducting domain walls is actually determined mainly by the paramagnetic effect of the exchange field, and we can use the results obtained above. The matter of whether superconducting domain walls exist in $HoMo_6S_8$ remains an open question, however, since $h_{c2} > h_{DC}^{(w)}$, and the value of h_{DC} is unknown.

In a ferromagnetic superconductor with an irregular magnetic subsystem (e.g., in the pseudoternary compounds $Ho_{1-x}Y_xRh_4B_4$ or $Ho_{1-x}Y_xMo_6S_8$) the coexistence phase DS may be absent, and such a compound on cooling will exhibit a first-order transition $S \rightarrow$ FN at a temperature T_{c2}. The S-FN transition point is determined by the condition that the magnetic and superconducting energies be equal, $\Theta_c s_{c2}^4 \approx N(0)\Delta_2^0/2$ for $N(0)\Delta_0^2/2 \ll T_M$. From this we obtain, for $N(0)\Delta_0^2 \ll T_{c2}$,

$$h_{c2}^2 \approx \Delta_0(\Theta_{ex}/\Theta_c N(0)\Delta_0^2)^{1/4}$$

and $h_{c2} \lesssim \Delta_0$ if $\Theta_{ex} \lesssim T_M \Delta_0^2/h_0^2$. This last condition can be satisfied for $h_0 \gg \Delta_0$ when the ratio Θ_{ex}/T_{c2} is small, i.e.,

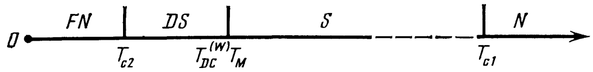

FIG. 2. Temperature intervals for the stability of the normal nonmagnetic phase N, the superconducting nonmagnetic phase S, the phase DS in which superconductivity coexists with inhomogeneous magnetic ordering, and the normal ferromagnetic phase FN in a compound with an ideal magnetic subsystem. The point $T_{DC}^{(w)}$ lies in the stability region of the DS phase; the position of the point T_{DC} is unknown. Superconducting domain walls can occur for $T_{DC} > T_{c2}$.

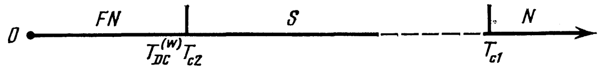

FIG. 3. The N, S, and FN phases in a compound with an irregular magnetic subsystem. Here the DS phase is absent at a sufficiently high degree of disorder. The point $T_{DC}^{(w)}$ can lie above T_{c2} if the exchange contribution to the magnetic energy is small. The position of the point T_{DC} is unknown.

when the dominant contribution to the magnetic energy of the system of localized moments is from the magnetic dipole interaction. In this case it is even possible to satisfy the inequality $h_{c2} < h_{DC}^{(w)}$ (see Fig. 3). For example, in $HoMo_6S_8$ we have $\Theta_{ex} \approx 0.08$ K, $T_M \approx 0.7$ K, and $(\Theta_{ex}^2/\Theta_c N(0)\Delta_0^2)^{1/4} \approx 1.4$. Therefore, in the pseudoternary compounds $Ho_{1-x}Y_xMo_6S_8$ one can expect superconducting domain walls to arise at concentrations x sufficient to destroy the DS phase.

The ferromagnetic superconductor $ErRh_4B_4$ apparently belongs to the class of asperomagnets which have a transition from a superconducting nonmagnetic phase to an asperomagnetic phase FN.[27] For this compound $h_0 \approx 42$ K, $s_{c2} \approx 0.88$, and $H_{c2}^* \approx 10$ kOe, according to the estimates made in Ref. 27 from the experimental data of Refs. 28–30. For $\Delta_0 \approx 15$ K and $B(0) = 6.5$ kOe we obtain $\rho \approx 0.05$ for this compound. The existence of superconducting domain walls here remains an open question, since $h_{c2} > h_{DC}^{(w)}$.

Thus the question of whether it is possible for systems having a regular magnetic subsystem to exhibit superconducting domain walls remain unresolved. To elucidate this matter it will be necessary to determine the critical value h_{DC} for a first-order transition to the superconducting-domain-wall state. Our results show that more promising compounds from the standpoint of realizing supeconducting domain walls are pseudoternary ferromagnetic superconductors having a first-order transition $S \rightarrow$ FN and a small exchange contribution to the magnetic energy of the system.

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[1)]The absence of complete short-circuiting of the normal metal here could be due to the polycrystalline nature of the samples, since in such a situation there is no infinite superconducting path.

[2)]In such an unrealistic model the conditions for the occurrence of superconductivity within the rotating walls are practically the same as within the domains, and the width of the existence region for localized superconductivity is negligibly small. The calculations of Ref. 8, being of an heuristic nature, did not determine the conditions under which superconductivity persists within the domain walls but is absent inside the domains.

[3)]For the ideal $ErRh_4B_4$ crystal the parameter D in the easy plane is ≈ 0.1 K (Ref. 15).

[4)]The Hamiltonian for the exchange interaction of the localized moments and electrons is usually written in the literature as $2(g-1)I\hat{s}\hat{J}$, where $\hat{J}$ is the operator for the localized moments and $\hat{s}$ is the operator for the spin density of the conduction electrons. The parameter I is related to the magnitude of the exchange field h_0 by $h_0 = (g-1)I\langle J_z\rangle_{T=0}/\nu$, where ν is the number of atoms in the chemical formula and $\langle J_z \rangle$ is the average value of the z component of the moment at $T=0$.

[5)]The Ginzburg-Landau approximation cannot be used to solve the problem posed in the present paper because we are interested in the temperature region $T < T_{c2} \ll T_{c1}$. In the low-temperature region the Ginzburg-Landau approximation is unsuitable for describing the paramagnetic effect of the exchange field.

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