Oscillation phenomena during the decay of a metastable state to the quasicontinuous spectrum

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The decay of a metastable state to the quasicontinuous spectrum, which may occur in a quasistationary system in an external field, is considered. For times $t < T$, where T is the period of the motion in the external field, this analysis predicts the Gamov exponential α-decay law for a finite system. New decay harmonics appear for $t = mT$ $(m = 1, 2,...)$. They correspond to the interaction of the decayed primary flux with the flux transmitted by the potential well produced by the external field. These interference harmonics oscillate as functions of the external field because discrete levels produced by the field are found to cross periodically the quasistationary level as the field is varied. For sufficiently long times $t \gg T$, the decay process corresponds to the distribution of the quasistationary state over the discrete levels. Possible systems for the observation of these effects are suggested.

1. FORMULATION OF THE PROBLEM. INTERFERENCE PHENOMENA DURING THE DECAY OF A QUASISTATIONARY STATE IN AN EXTERNAL FIELD

Consider two potential wells separated by a barrier (see the Figure). We shall suppose that the left well (I) contains a stationary state of energy E_0 when the interaction with the right well (II) is neglected. The size L of the right well will be assumed to be much greater than the size of the left well, and we shall suppose that the former contains a large number of energy states. At the initial time, the state under consideration is localized in well I. We shall examine the decay of the state in the course of time. In the limit as $L \to \infty$, we shall thus obtain the decay law for a quasistationary state that was considered by Gamov[1,2] as far back as 1928. It will be clear later that the derivation of this law relies heavily on the exponentially small level splitting due to the interaction between the two wells through the potential barrier. On the other hand, when L is finite, we find that, in addition to the usual decay law for the quasistationary state, there are also new exponentially falling harmonics that correspond to the decay of a metastable state in the first well to discrete states in the second well. The new harmonics appear for $t > T$, where T is the period of classical motion in well I with energy E_0, and are the result of interaction between the decaying state in well I and the wave that had passed through well II and has returned to well I.

Our problem reduces to the solution of the Schrödinger equation

$$i\frac{\partial\psi}{\partial t} = \left[-\frac{1}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x,t), \quad (1.1)$$

where $V(x)$ is the two-well potential shown in Fig. 1. The initial state at time $t = 0$ corresponds to the state in well I with wave function $\psi_0(x)$ and the energy E_0:

$$\psi(x,t)\,|_{t=0} = \psi_0(x), \quad (1.1a)$$

where $\psi_0(x)$ is the wave function for the isolated well I with energy E_0. In the absence of well II, the system would continue in this state in well I, and the corresponding wave function would be

$$\psi_0(x,\ t) = \psi_0(x)\exp(-iE_0 t). \quad (1.2)$$

We are interested in the probability amplitude for a transition from this state to a state $\psi(x,t)$ that is a solution of (1.1) with initial condition (1.1a):

$$p(t) = \int dx\, \psi_0(x,t)\,\psi^*(x,t). \quad (1.2a)$$

Let us expand $\psi(x,t)$ into a series in terms of the normalized wave functions $\psi_E(x)$ of (1.1):

$$\psi(x,t) = \sum_E C_E \psi_E(x)\exp(-iEt). \quad (1.3)$$

The coefficients C_E are determined by the initial condition (1.1a):

$$C_E = \int dx\, \psi_0(x)\,\psi_E^*(x). \quad (1.3a)$$

Substituting (1.3), (1.3a), and (1.2) and (1.2a), we obtain

$$p(t) = \sum_E |C_E|^2 \exp\{i(E-E_0)t\}. \quad (1.2b)$$

Before we can evaluate the transition amplitude $p(t)$ in (1.2b), we must know the coefficients C_E given by (1.3a), which are expressed in terms of the wave functions $\psi_0(x)$ and $\psi_E(x)$. The quasiclassical approximation[3] will suffice for the evaluation of the wave functions. Simple calculations yield the expression for $p(t)$, and if we expand all the quantities

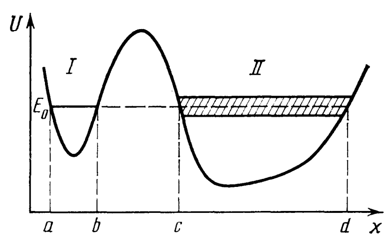

FIG. 1.

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around the point $E = E_0$, we obtain

$$p(t)=\sum_{\delta E}\exp(i\delta Et)\left[1+\Delta^2(1+(\tau\delta E)^2)\frac{\omega_0(E_0)}{\omega(E_0)}\right]^{-1}, \quad (1.4)$$

where

$$\delta E=E-E_0, \quad \Delta(E)=\frac{1}{2}e^{-D(E)},$$

$$\tau^{-1}=\frac{\omega_0(E_0)}{4\pi}e^{-2D}, \quad D=\int_b^c|p|dx,$$

and $\omega_0(E)$ and $\omega(E)$ are the frequencies of classical motion with energy E in the first and second wells, respectively. The sum in (1.4) is evaluated over all the discrete states of the system that are defined by the following dispersion relation:

$$\operatorname{tg} S(E_0+\delta E)=-\tau\delta E \quad (1.5)$$

or

$$S(E_0)+\pi\delta E_n/\omega(E_0)=\pi n-\operatorname{arctg}[\tau\delta E_n], \quad (1.5a)$$

where

$$S(E)=\int_c^d p\,dx$$

is the classical action in well II.

The sum in (1.4) over the discrete levels δE_n, given by (1.5a), will be evaluated with the aid of the Poisson formula:

$$p(t)=\sum_{k=-\infty}^{+\infty}J_k, \quad (1.6)$$

$$J_k=\int_{-\infty}^{+\infty}dn\exp(2\pi ikn+i\delta E_nt)\times\left[1+\Delta^2(1+(\tau\delta E_n)^2)\frac{\omega_0(E_0)}{\omega(E_0)}\right]^{-1}. \quad (1.6a)$$

We now transform in (1.6a) from the integration variable n to the variable δE. The derivative $dn/d\delta E$ can be found from (1.5a), and the result is that (1.6a) assumes the form

$$J_k=\Delta^{-2}\omega_0^{-1}(E_0)\int_{-\infty}^{+\infty}dx\exp(2\pi ink+ixt)[1+(\tau x)^2]^{-1}. \quad (1.7)$$

We note that the factor in the numerator of (1.6a) cancels with the corresponding factor in the density of states. As a result, the main contribution to J_k given by (1.7) is due to the neighborhood of the point $x = i/\tau$. Substituting for n from (1.5a) into (1.7), we obtain

$$J_k=\pi^{-1}\exp[2ikS(E_0)]\int_{-\infty}^{+\infty}\exp(ix\gamma_k)dx(1+ix)^{2k}/(1+x^2)^{k+1},$$
$$\gamma_k=(t+kT)/\tau, \quad (1.8)$$

where $T = 2\pi/\omega(E_0)$ is the period of classical motion with energy E_0 is well II. The quantities J_k vanish identically for $k\geqslant 1$ since the contour of integration in (1.8) in the complex plane of the variable x can be closed in the upper half-plane $\gamma_k > 0$, and there are no poles in the upper half-plane in this case. In precisely the same way, $J_k = 0$ for $k < -t/T$. For the other values of k, the integral in (1.8) is readily evaluated, and (1.6) can be written in the form

$$p(t)=e^{-\gamma_0}+\sum_{1\leqslant m\leqslant m_0}\exp[-2imS(E_0)]\sum_{k=1}^{m-1}C_{m-1}^k\frac{(-1)^{m-k-1}}{(k+1)!}(2\gamma_{-m})^{k+1}e^{-\gamma_m},$$
$$\gamma_{-m}=(t-mT)/\tau, \quad m_0=t/T. \quad (1.9)$$

The first term in (1.9) corresponds to the usual exponential decay of a quasistationary state[1,2]

$$p(t)=e^{-\gamma_0}, \quad \gamma_0=t\frac{\omega_0}{4\pi}e^{-2D(E_0)}, \quad 0<t<T. \quad (1.9a)$$

We are thus able to derive the decay law for a quasistationary state of a finite system, and remove the methodological difficulties[2] that are encountered for an infinite system (for example, the difficulties with the normalization of the wave functions). In a finite system, the primary wave that has decayed is found to return to well I for times $t > T$, and interacts with the state in this well. Interference between these two wave results in a decay law that is more complicated than the exponential law given by (1.9a). Thus, for $T < t < 2T$, we have

$$p(t)=e^{-\gamma_0}+2\gamma_{-1}\exp[-2iS(E_0)]e^{-\gamma_1},$$
$$\gamma_0=t/\tau, \quad \gamma_{-1}=(t-T)/\tau \quad (1.9b)$$

which, in addition to the usual exponential term (1.9a), contains a further term that describes interference between the primary flux during the decay of the state in well I and the waves reflected from the right-hand edge of well II when they interact near the barrier between the wells. The second term also contains an exponential, but its argument involves the time measured from the instant of interaction between the two waves. The second term also contains a factor that is linear in time, i.e., it falls more slowly with t than the exponential given by (1.9a). It is clear from (1.9b) that $p(t)$ is a continuous function, but the first derivative with respect to the time, $p'(t)$, has a discontinuity at T. For large times ($t > mT$, where m is an integer), the interference terms contain an exponential whose argument is proportional to $(t - mT)$, i.e., the time is measured from the instant when the decayed wave traverses the right well m times and interferes with the primary flux (1.9). The factor in front of the exponential contains a polynomial of degree m in the time t. The function $p(t)$ is continuous at the point mT, but all its derivatives from the first to the mth inclusive exhibit a discontinuity at this point. The above interference terms are hardly small, and their scale is determined by the ratio of the period of motion T in well I to the constant characterizing the decay time τ. Although ω_0T is always much greater than unity, the ratio T/τ [see (1.4)] can be either greater or smaller than unity because of the exponentially small penetration between the wells. The interference effects come into their own when the constant τ is less than the period of motion in well II, i.e., when $\tau\lesssim T$.

In addition to the nonstandard dependence of the interference term on time, the expression (1.9b) describes oscillations in $p(t)$ as a function of the classical action $S(E_0)$ (the factor $\exp[-2iS(E_0)]$ in the interference term). Since $S(E_0)$ depends on the strength of the external field, the probability $p(t)$ exhibits oscillations as a function of this field. As the external field is varied, the energy levels successively approach the quasistationary field E_0, and this periodic passage of a "ridge of levels" across E_0 is responsible for the oscillations in $p(t)$. This effect is completely analogous to the oscillations accompanying quantization in a solid, when a variation of the field is accompanied by the passage of the successive discrete states across the Fermi level. Naturally, all that we have said above applies not only to (1.9b) but also to all times $t > T$ in (1.9).

2. ASYMPTOTIC DECAY OF A METASTABLE STATE

The representation of $p(t)$ by the sum of harmonics (1.9), measured from the time of interaction between the decaying wave of the quasistationary state and the wave that has traversed the right well m times (see Fig. 1), is convenient provided m is not too large, since the terms in (1.9) grow rapidly for large m and tend to cancel each other. In the present section, we shall derive formulas that are convenient for large t: $t/T \gg 1$.

It follows from the results of Sec. 1 that $p(t)$ can be written in the form

$$p(t)=\sum_{0\leqslant m\leqslant m_0}\pi^{-1}\exp[-2imS(E_0)]\Phi_m,\qquad m_0=t/T, \tag{2.1}$$

$$\Phi_m=\int\limits_{-\pi/2}^{\pi/2}d\varphi\exp\{i\gamma_{-m}\,\mathrm{tg}\,(iR+\varphi)-2im\,(iR+\varphi)\},$$

$$\gamma_{-m}=(t-mT)/\tau>0, \tag{2.1a}$$

where R is an arbitrary number greater than zero. The representation given by (2.1a) for $T=0$ is obtained from (1.7) by substituting for n from (1.5a) and then replacing the variable so that $\delta E = \tan\varphi$. For $R>0$, the representation given by (2.1a) is obtained by transforming to the complex plane of the variable φ, and taking into account the fact that the integrand is periodic in φ with period π. We are interested in the asymptotic behavior of the integral in (2.1a) for $m \gg 1$ and, generally, $\gamma_{-m} \gg 1$. We shall use the saddle-point method. The principal contribution to the integral in (2.1a) is due to the neighborhood of the points φ at which the derivative of the argument of the integrand in (2.1a) vanishes:

$$\cos^2\varphi=\gamma_{-m}/2m. \tag{2.2}$$

Since $\gamma_{-m}>0$ and $m>0$, we find that $\cos\varphi$ must be real. We can then distinguish two cases:

$$(\gamma_{-m}/2m)^{1/2}<1, \tag{2.2a}$$

$$(\gamma_{-m}/2m)^{1/2}>1. \tag{2.2b}$$

In the former case, φ is real and equal to $\pm\varphi_m$, where $\varphi_m = \arccos(\gamma_{-m}/2m)^{1/2}$. In the latter case, φ is purely imaginary:

$$\varphi=\pm iR_m,\qquad R_m=\ln\{[\gamma_{-m}^{1/2}+(\gamma_{-m}-2m)^{1/2}]/(2m)^{1/2}\}. \tag{2.3}$$

The asymptotic behavior is essentially different for these two cases. For (2.2a), we use (2.1a) with $R=0$. This yields $\Phi_m = \chi_m$, where

$$\chi_m=\frac{2^{1/2}\Gamma(1/3)}{3^{1/6}m^{1/3}}\rho_m\cos[2m\varphi_m-\gamma_{-m}\,\mathrm{tg}\,\varphi_m-\vartheta_m],$$

$$\Gamma(1/3)=2{,}6789\ldots, \tag{2.4}$$

and

$$\rho_m=1,\quad \vartheta_m=0\quad \text{for}\quad m\,\mathrm{tg}\,\varphi_m\leqslant1, \tag{2.4a}$$

$$\rho_m=3^{1/6}\sqrt{\pi}/2\Gamma(1/3)\,m^{1/6}\,\mathrm{tg}^{1/2}\,\varphi_m,$$

$$\vartheta_m=-\pi/4\quad \text{for}\quad m\,\mathrm{tg}\,\varphi_m\gg1. \tag{2.4b}$$

Since $m \gg 1$, we have the narrow region $m\ \mathrm{tg}\,\varphi_m \gtrsim 1$ in which the law defined by (2.4) and (2.4a) is replaced with (2.4) and (2.4b). In the case defined by (2.2b), we can use (2.1a) with $R=R_m$ (2.3), and the result is $\Phi_m = \psi_m$, where

$$\psi_m=\frac{2^{1/2}\Gamma(1/3)}{3^{1/6}m^{1/3}}\tilde\rho_m\exp(-\gamma_{-m}\,\mathrm{th}\,R_m+2mR_m), \tag{2.5}$$

and

$$\tilde\rho_m=1\quad \text{for}\quad m\,\mathrm{th}\,R_m\leqslant1, \tag{2.5a}$$

$$\tilde\rho_m=(3/m)^{1/6}(\pi/\mathrm{th}\,R_m)^{1/2}/2\Gamma(1/3)\quad \text{for}\quad m\,\mathrm{th}\,R_m\gg1. \tag{2.5b}$$

The asymptotic behavior defined by (2.5) and (2.5a) is replaced by (2.5) and (2.5b) in the narrow region in which $m\ \mathrm{th}\,R_m \gtrsim 1$. The result of this is that (2.1) can be written as the sum of two terms:

$$p(t)=\sum_{0\leqslant m<m_c}\pi^{-1}\psi_m\exp(-2imS(E_0))+\sum_{m_c<m\leqslant m_0}\pi^{-1}\chi_m\exp(-2imS(E_0)). \tag{2.6}$$

The critical value m_c that separates the two ranges of validity of the asymptotic quantities ψ_m and χ_m is defined by (2.2a) and (2.2b), and is given by

$$m_c=t/(T+2\tau). \tag{2.6a}$$

Since $m_0 = t/T$ (2.1), we have

$$m_c/m_0=(1+2\tau/T)^{-1}. \tag{2.6b}$$

The sums in (2.6) can readily be evaluated by using the above expressions for φ_m (2.5) and χ_m (2.4). Let us consider the first sum. We note first that, since $\mathrm{ch}^2 R_m = \gamma_{-m}/2m$, the argument of the exponential in ψ_m is always negative and equal to $-2m(\mathrm{sh}\,R_m\,\mathrm{ch}\,R_m - R_m) < 0$. Correspondingly, the maximum contribution to the first sum is due to the region near the point $m = m_c$ at which the argument vanishes. Expanding all quantities around this point, and recalling that the argument of the exponential ψ_m tends to zero near this point as $\sigma(m_c - m)^{3/2}$, where the small coefficient is given by $\sigma \sim m_0^{-1/2}$, we obtain the following expression for the first sum:

$$p_1(t)=\frac{\sqrt{2}}{\pi}\Gamma(1/3)\,(m_c\sqrt{3})^{-1/3}\frac{\exp[-2im_cS(E_0)]}{1-\exp[-2iS(E_0)]}. \tag{2.7}$$

We note that the right-hand side of (2.7) becomes infinite for $S(E_0) = 2\pi n$, where n is an arbitrary integer. To remove this divergence, we must allow for the fact that the coefficient σ in the expansion for ψ_m is finite. The second sum in (2.6) can be evaluated by analogy. In precisely the same way as in (2.7), but with a further factor of two, we now have the contribution due to the neighborhood of the point $m = m_c$, but there is also the contribution due to the neighborhood of the point m_0 that corresponds to the maximum harmonic in the sum. The final result can be written in the form

$$p(t)=\frac{3\sqrt{2}}{\pi}\Gamma(^1/_3)(m_c\sqrt{3})^{-1/3}\frac{\exp[-2im_cS(E_0)]}{1-\exp[-2iS(E_0)]}+p_{m_0}(t), \tag{2.8}$$

$$p_{m_0}(t)=(2\pi m_0\,\mathrm{tg}\,\varphi_{m_0})^{-1/2}\exp[-2im_0S(E_0)]$$
$$\times\left\{\frac{e^{i\alpha}}{1-\exp[2iS(E_0)-i\varkappa]}+\frac{e^{-i\alpha}}{1-\exp[2iS(E_0)+i\varkappa]}\right\}, \tag{2.8a}$$
$$\alpha=2m\varphi_m-\gamma_{-m}\,\mathrm{tg}\,\varphi_m,\qquad \varkappa=2\varphi_{m_0}+(T/\tau)\,\mathrm{tg}\,\varphi_{m_0}.$$

The structure of (2.8a) is somewhat different from that of (2.7) because, when all the quantities are expanded around the point m_0, the derivative within the cosine is not zero near the point m_0, as was the case in the summation leading to (2.7).

Thus, the asymptotic expression given by (2.8) has an oscillatory character for large t. Two harmonics are emphasized. The first contains $\exp\{-i(t/T)S(E_0)\}$, and the second

$$\exp\left\{-i\frac{t}{T}\left(1+\frac{2\tau}{T}\right)^{-1}S(E_0)\right\}.$$

The oscillatory character of the asymptotic expressions for large t corresponds to the Fock-Krylov theorem[4]: in a system with discrete states, the decay of quasistationary state is oscillatory and not exponential. The presence of two emphasized oscillatory harmonics is due to the fact that the system has two characteristic energy parameters, namely, the level separation T^{-1} in well I and the level broadening τ^{-1} in the individual wells due to the interaction between them. The close connection between the characteristic level separation in a system and the way it reaches its asymptotic behavior has been investigated in detail in connection with problems on the stochastization of motion in dynamic quantum-mechanical systems (see, for example, the review by Chirikov[5]).

The picture of the decay process that we have established can be described as follows. For times t such that $\tau<t<T$, we have the usual exponential decay of the quasistationary state [see (1.9a)], and the period T has no effect on the decay. For $t>T$, a new harmonic of the exponential decay $\exp[-(t-T)/\tau]$ (1.9b), due to the interaction between the primary flux and the flux that has traversed well II and has returned to well I. The higher harmonics of the exponential decay that appear at times $2T$, $3T$,... have traversed well II, respectively 2, 3, etc. times. For sufficiently large $t=m_0T$, where $m_0\gg 1$, it is essential to allow for the fact that the levels in well II are discrete and have separation $\sim 1/T$. This results in the oscillatory behavior described by (2.8), which "feels" the discrete nature of the levels in the well and completely "ignores" the decay time τ of the quasistationary state.

We note that, for $m_c\ll m_0$, which is equivalent to $2\tau\gg T$ [see (2.6b)], the transition probability is given by the simple formula

$$|p(t)|^2=\frac{6^{5/3}}{\pi^2}\Gamma^2(^1/_3)(\tau/t)^{2/3}|1-\exp(-2iS(E_0))|^{-2}, \tag{2.9}$$

which describes the decay of the system to the discrete levels of well II. In the opposite limiting case for which $T\gg 2\tau$, we again have (2.9), but with a different coefficient: it differs by the factor $\frac{1}{9}(T/2\tau)^{2/3}$.

We note in conclusion that the exponential law defined by (2.8) and (2.9) is similar to (1.9) in that it is an oscillatory function of the external field. This is a consequence of the presence of $S(E_0)$.

3. POSSIBLE EXPERIMENTAL OBSERVATION OF INTERFERENCE EFFECTS DURING THE DECAY OF A QUASISTATIONARY STATE

In this Section, we shall discuss possible experimental detection of the effects discussed in the last two Sections, and the attendent difficulties, in terms of simple examples. The effects can be observed when the system under consideration has a metastable state, i.e., they can be looked for in all branches of physics, ranging from nuclear physics to solid-state physics and biophysics.

Let us first consider a model that is valid for both α-decay of the nucleus and for the motion of an atom, molecule, or electron in a metastable state, when these systems undergo a change in their atomic or ionic configuration, for example, in the course of a chemical reaction. To be specific, we shall consider α-decay. Suppose that an α-particle source is placed in a uniformly charged material with charged-particle density ρ. If the charge on the medium is negative, the α-decay process will occur in an external field that ensures that the motion of the α particle during the decay of the quasistationary state is finite. The effective potential in which the α particle moves is spherically symmetric and can be written in the form

$$U_{\mathrm{eff}}(r)=U_0(r)+Fr^2+\frac{1}{2m}\frac{l(l+1)}{r^2}, \tag{3.1}$$

where $U_0(r)$ is the potential due to the nucleus that is the source of the α particles. The second term on the right-hand side of (3.1) is the interaction energy between the α particle (charge $2e$) and the charged sphere of density ρ:

$$F=(8\pi/3)\rho e^2. \tag{3.1a}$$

The third term on the right-hand side is the usual centrifugal energy. The equation for the function $\chi(r)=rR(r)$, where $R(r)$ is the radial part of the wave function, is the one-dimensional Schrödinger equation[3] with the potential $U_{\mathrm{eff}}(r)$. The action S for the α particle outside the nucleus can then be written in the form

$$S=\int_{r_-}^{r_+}dr[2M(E-U_{\mathrm{eff}}(r))]^{1/2}, \tag{3.2}$$

where r_- and r_+ correspond to the zeros of the integrand. Outside the range of nuclear forces, $E-U_0(r)=E_0$ is a positive constant independent of r. The energy E_0 is of the order of a few MeV. If we neglect the nuclear radius r_0, the integral in (3.2) can be evaluated exactly:

$$S=\frac{\pi}{2}(2MF)^{1/2}\left\{\left[\frac{E_0}{(2F)^2}-\frac{L}{F}\right]^{1/2}-\left[\frac{L}{F}\right]^{1/2}\right\},$$
$$L=\frac{1}{2m}l(l+1). \tag{3.3}$$

Since the decay constant τ in (1.9a) increases exponentially with increasing L, we shall confine our attention to small values of L (actually, $L = 0$):

$$S=(\pi/4)E_0(2M/F)^{1/2}. \tag{3.3a}$$

The necessary condition for the motion of the α particle to be finite for a given radius R of the system is

$$E_0<FR^2. \tag{3.4}$$

For given R, this yields the following condition for the density:

$$\rho(\text{cm}^{-3})>\frac{3E_0}{8\pi e^2R^2}\approx 0.8\cdot 10^{12}\frac{E_0}{R^2}\left(\frac{\text{MeV}}{\text{cm}^2}\right). \tag{3.4a}$$

When $R = 10$ cm, this corresponds to $\rho\sim 10^{10}$ cm^{-3}, and the dependence on R is very strong. For example, when $R = 100$ cm, we already have $\rho\sim 10^8$ cm^{-3}. The voltage that is then produced in the system is determined by E_0 and amounts to 10^6 V. The electric field produces the following pressure in the system:

$$P=\frac{1}{8\pi}\left[\frac{8\pi}{3}\rho eR\right]^2, \tag{3.5}$$

which can be expressed with the aid of (3.4) in the following form: $P=\frac{1}{3}\rho E_0$ or $P(\text{Mbar}) = 0.5\cdot 10^{-18}\rho E_0$ (MeV/cm^3). This means that, for particles of about 1 MeV, the pressure produced in the system is about 1 Mbar for the charged-particle density $\rho\sim 10^{18}$ cm^{-3}. The minimum size of the system is then $R\sim 0.01$ cm.

The interference effect described in the first two Sections of this paper can then be observed in the system under consideration. To ensure that they are more clearly defined, the period of motion in well I must be greater than the time constant τ characterizing the decay of the system:

$$T\gg\tau. \tag{3.6}$$

Since the velocity of the α particle emitted by the nucleus is $v\sim 10^9$ cm/s, we find that, for, say, $R\lesssim 10^2$ cm, we have $\tau\lesssim 10^{-7}$ s. For $R\lesssim 10^{-2}$ cm, which corresponds to a pressure of about 1 Mbar in the system (3.5), we have $\tau\lesssim 10^{-11}$ s. Condition (3.6) is not actually a necessary condition for the observation of the above effects: when (3.6) is satisfied the scale of the higher harmonics in (1.9) is greater than that for the zeroth harmonic; however, as we have seen, the ratio of T to τ can be arbitrary without affecting the possibility that the asymptotic laws of Sec. 2 will be observed.

A more serious aspect of the problem is as follows: the action S (in units of Planck's constant) is exceedingly high:

$$S\approx 4\cdot 10^{18}E_0/\sqrt{\rho}\,[\text{MeV}\cdot\text{cm}^{3/2}]. \tag{3.7}$$

This means that, for $E_0\sim 1$ MeV and high densities $\rho\sim 10^{18}$ cm^{-3} that correspond to an electric-field pressure of about 1 Mbar, we have $S\sim 10^9$, i.e., all the effects described in Secs. 1 and 2 are substantially weakened in this situation: they are all multiplied by the factor S^{-1} that is due to averaging over the two spatial coordinates, each of which introduces the factor $S^{-1/2}$ near the extremum of S. In reality, this is the usual situation in measurements of different oscillatory effects in the solid state as well: averaging over momentum space is unavoidable. This is the reason why, for example, in the de Haas–van Alphen effect, one measures not the oscillatory corrections to the thermodynamic potential, but the second derivative of the thermodynamic potential, i.e., the susceptibility which contains the factor S^2 that is large as compared with the thermodynamic potential. In our case, we can also measure the derivative of the effect: we have to apply an additional weak electric field to the system, which gives the factor S when the first derivative of the effect is measured. When $S\sim 10^9$ and the voltage is $V\sim 10^6$ V, the additional field is of the scale of mV. It is clear from (3.7) that, other things being equal, the effect is conveniently measured for high charged-particle densities ρ and low E_0 (the energy of the α particle leaving the nucleus).

Precisely the same effect occurs in the electric field in the planar geometry. In this case, one isolates purely geometrically the action S_{extr} along the field, which may have definite advantages from the external point of view.

An analogous effect occurs in the uniform magnetic field as well: the motion of the α particle is finite in the direction perpendicular to the field. The field strength is then determined by the condition

$$cMv/eH<R, \tag{3.8}$$

where the left-hand side is equal to the radius of the orbit in the magnetic field, and the right-hand side is the size of the installation. This condition gives a magnetic field $H = 4$ kOe when the linear dimensions of the apparatus are 1 m. As in the previous case, we must measure the derivatives of the effect. This can be done by applying both a constant magnetic field and, say, a low-frequency electromagnetic field of small amplitude.

As we have seen, the complexity involved in the observation of these effects is due to the high energy E_0 of the α particles and their large mass. If we take the electron as the emitted particle (low mass), with an energy of about 1 eV, the magnitude of S is reduced by a factor of 10^7 as compared with (3.7), so that S can be reduced down to 10^3–10^4, or even less, for reasonable external fields. Examples of this kind of system are provided by the autoionization states of negative ions,[6] say, the molecular hydrogen ion H_2^-, with an energy of about 3 eV. This also applies to solvated and hydrated electrons that appear in biological tissues and solutions under exposure to hard radiation.[7] The above effects can also be observed during the motion of atoms, but only at low energies of about 1 eV, which occurs, for example, during absorption on a surface. A possible system is that of hydrogen adsorbed on a metal surface. Hydrogen escaping from the surface forms molecules, and this is accompanied by the release of energy of the order of a few eV. The motion of the hydrogen molecules in the perpendicular direction can be restricted by a second metal surface.

Another possible object that is very different from that considered above is the decay of an excited quasistationary state of an atom that emits a photon. The photon may be returned to the system by a mirror. Vibration of the mirror produced, for example, by ultrasonic waves, can be used in measurements of the derivatives of the effect.

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