

Dynamic transformations of domain-wall structure in an alternating magnetic field

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(Submitted 1 September 1983)

Zh. Eksp. Teor. Fiz. **86**, 1505–1515 (April 1984)

A magneto-optic method is used to investigate the nonlinear dynamic transformation of the structure of 180-degree domain walls in single crystals of yttrium iron garnet under the influence of a sinusoidal magnetic field imposed parallel to the magnetization vectors of the neighboring domains and to the surface of the slab. It is shown that a small-amplitude field causes out-of-phase oscillations of adjacent Bloch lines in the domain walls. When the field amplitude is increased to a certain critical value (several tenths of an oersted), a directed motion of the entire system of oscillating Bloch lines in the domain wall is observed, accompanied by the creation and annihilation of Bloch lines. It is established that the direction of motion of the oscillating Bloch lines in the presence of local Bloch-line pinning centers leads to an effective increase in the density of Bloch lines in the domain wall. It is found that there are certain resonant frequencies at which a weak field triggers the flipping of a large set of spins in a uniformly polarized domain wall, giving rise to isolated "dynamic" subdomains whose high-speed motion along the domain wall is accompanied by a periodic change in their dimensions. The resonant nucleation and directed motion of the subdomains are discussed in terms of nonlinear excitations of the soliton type in a system of spins localized in a domain wall.

Domain-wall displacement processes have a decisive influence on the many physical properties which make magnetically ordered crystals useful for solving a wide class of important practical problems. The foundations for analysis of the structure and dynamics of domain walls were laid almost a half-century ago in the famous paper of Landau and Lifshitz,¹ who considered a one-dimensional domain-wall model in which the distribution of the magnetization varies only in the direction perpendicular to the plane of the wall. The many subsequent studies have shown that such a model can be used successfully to describe the dynamical properties of domain walls in bulk uniaxial crystals with $\beta \gg 2\pi$ (β is the magnetic anisotropy constant) and even in certain multiaxial ferromagnets over a rather wide range of external magnetic fields.²⁻⁵

In the early 1970s, however, during an upsurge in interest in the study of domain-wall dynamics in magnetic-bubble materials in connection with their potential use as memory elements for computers, a number of experimental facts were discovered which contradict the predictions of the one-dimensional model. These facts pertain to the anomalous behavior of magnetic bubbles and domain walls in strong external magnetic fields exceeding a certain threshold value. To explain the anomalies, the concept of a dynamic transformation of the domain-wall structure was introduced, including the formation and motion of Bloch line segments separating subdomains of different polarity in the wall.²⁻⁵ Unfortunately, direct experimental study of such processes in magnetic-bubble materials is complicated and to this day has never been done.

It was shown in Refs. 6–8 that this interesting new type of nonlinear effect in a quasi-two-dimensional system of spins localized in a domain wall can be studied directly in multiaxial single crystals of yttrium iron garnet by using the Faraday effect to visualize the domain-wall structure. These crystals belong to a wide class of magnetically ordered mate-

rials with $\beta \ll 2\pi$ for which the Bloch lines are a necessary element of the structure of the ground state. In such materials the minimum of the free energy of a sample not subjected to external fields is attained when the domain walls break down under the influence of the surface magnetic charges into subdomains separated by vertical Bloch lines. The existing theories²⁻⁵ cannot be used to describe the dynamical properties of these two-dimensional domain walls. Upon the displacement of the domain walls, the gyroscopic force due to the external field causes the Bloch lines to begin moving as well. This means that the dynamic transformation of the domain-wall structure can govern its mobility even in the weakest of fields.⁹⁻¹¹

In an experimental study of the dynamical characteristics of Bloch lines in single crystals of yttrium iron garnet,¹² some new effects not predicted by the theory were observed: the directed displacement of the system of vertical Bloch lines in the wall and the resonant generation of these Bloch lines. In the present paper we report the data of a detailed study of these effects and discuss their possible causes.

1. EXPERIMENTAL TECHNIQUES

The samples were cut in the form of thin slabs parallel to the {110} or {112} planes from single crystals of yttrium iron garnet grown from a molten solution. The 180-degree domain walls between domains magnetized in the planes of the slabs contained Bloch lines. In linearly polarized light they showed up as boundaries between "white" and "black" subdomains (in the slightly crossed Nicols of a microscope) by virtue of the Faraday effect (see Fig. 1). To study the dynamical properties of the Bloch lines, the image of a portion of the crystal, a local region containing a single line, was delimited by a slit, as shown in Fig. 1a, and projected onto the cathode of a photomultiplier. The photomultiplier signal, proportional to the displacement of the Bloch lines, was amplified and recorded with the apparatus described in

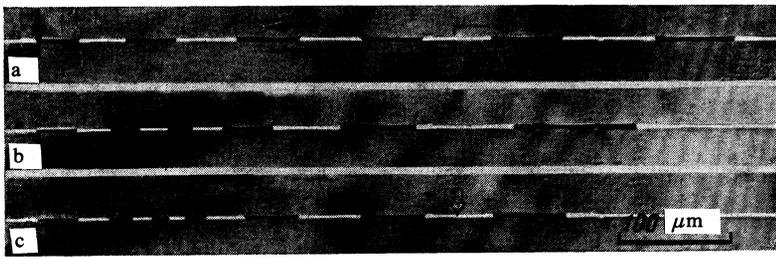


FIG. 1. A 180-degree domain wall in a (110) slab of yttrium iron garnet in polarized light (slightly crossed Nicols) under various conditions: a) initial state of the domain wall; the directions of the magnetizations in the domains are indicated by arrows, the photometered region is enclosed by dashed lines, and a Bloch line pinned at a defect is indicated by a vertical arrow; b) leftward displacement of the unpinned Bloch lines under the action of a single rf pulse; c) the appearance of a new subdomain (indicated by the double arrow) under the influence of a series of rf pulses (rf-pulse duration $\tau = 8 \mu\text{sec}$, $\nu = 1 \text{ MHz}$, $H_0 = 100 \text{ mOe}$).

Refs. 11 and 13. Periodic magneto-optic signals were recorded by a stroboscopic method, and single signals with a storage oscilloscope.

The magnetic field, produced by Helmholtz coils, acted along the magnetization in neighboring domains and caused a displacement of both the domain walls and also the Bloch lines within them. However, the motion of the domain walls within the confines of the image of the optical slit did not affect the value of the magneto-optic signal because the transillumination intensities of the field of view in slightly crossed Nicols in the neighboring domains were the same. The change in the signal was due solely to the displacement of the Bloch lines.

2. EXPERIMENTAL RESULTS

1. Directed motion of Bloch lines in an alternating field.

It has been shown^{11,14} that under the conditions usually realized in studies of the domain-wall dynamics in ferromagnetic materials, free or driven (including resonant) oscillations are excited in the system of Bloch lines. At relatively small field amplitudes H_0 these oscillations occur about the original equilibrium positions of the Bloch lines. When H_0 exceeds a certain critical value, however, one observes a directed (in each 180-degree domain wall) displacement of the entire system of oscillating Bloch lines. Figure 2 shows typical recording of the motion of two adjacent Bloch lines at a sinusoidal-field amplitude sufficient to initiate a continuous displacement of the equilibrium positions about which the Bloch lines oscillate. The magneto-optic signals represented

by curves 1 and 2 reflect the change in the intensity of the light transmitted through local regions of the crystal each containing one Bloch line. On the left is shown a schematic diagram of the domain wall, with the positions of the photometered regions of the crystal for each of the curves indicated by the dashed boxes. On both of the curves an increase in the intensity J of the magneto-optic signal corresponds to an increase in the volume of the "white" subdomain in the field of view of the microscope. Comparison of the curves shows that the adjacent lines bounding a single subdomain execute out-of-phase oscillations about an equilibrium position under the influence of an alternating magnetic field. This indicates that the spins in adjacent Bloch lines rotate in opposite directions. In Refs. 11 and 14, on the basis of visual observations during oscillations of Bloch lines under conditions of large displacements of the domain walls at low field frequencies ν , an erroneous (in light of the result shown in Fig. 2) conclusion was reached about the character of the rotation of the magnetization in adjacent Bloch lines. Figure 2 clearly shows that the Bloch lines, oscillating out of phase, gradually move in the same direction over the entire time that the magnetic field is acting.

This motion of the Bloch lines was recorded with an apparatus in which the periodic signal is detected by a stroboscopic oscilloscope,¹¹ under conditions such that the sinusoidal field was applied to the crystal periodically (in the form of rf pulses¹¹) for very short ($8\text{-}\mu\text{sec}$) time intervals, so that the Bloch line returned to its original position after each pulse was over.

When the length of the rf pulses or the field amplitude was increased, the displacement of the Bloch lines became irreversible and could no longer be recorded by the stroboscopic method. Moreover, under these conditions we could not follow the change in the substructure of a domain wall by direct observation. Therefore, for studying the irreversible displacement of the oscillating Bloch lines we used the technique of recording the signal with a storage oscilloscope in single-sweep operation (either with an internal trigger or synchronized with the exciting rf pulse).

The oscilloscope trace in Fig. 3a illustrates the one-time change which occurred in the intensity of the light transmitted by a local region (indicated by the rectangle in Fig. 1a) of the crystal under the action of the rf magnetic-field pulse shown in Fig. 3b. The observed decrease in the intensity of the light to a certain value which is maintained even after the field pulse has ended reflects the irreversible leftward (in Fig. 1a) shift of a single Bloch line separating a "black" and a

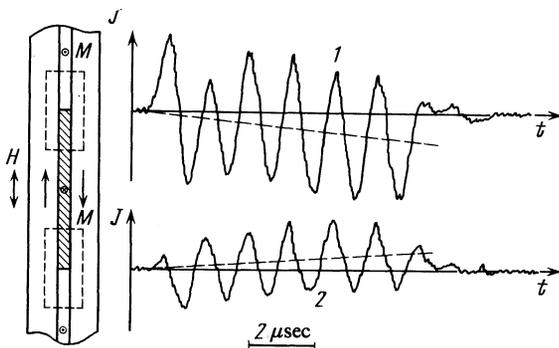


FIG. 2. Magneto-optic signals (curves 1 and 2) due to the motion of two adjacent Bloch lines (shown schematically at the left, with the photometered regions indicated) during the action of an rf pulse of magnetic field H along the magnetizations M in the domains ($\tau = 8 \mu\text{sec}$, $\nu = 780 \text{ kHz}$, $H_0 = 60 \text{ mOe}$).

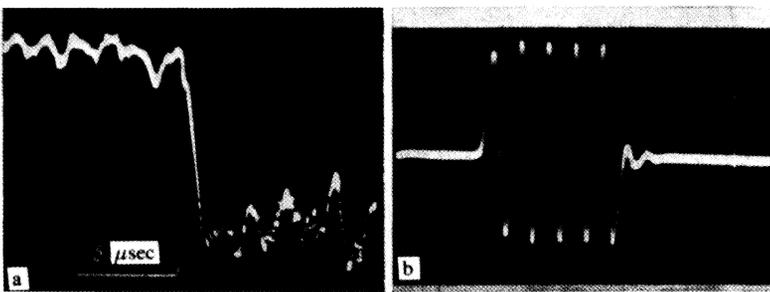


FIG. 3. Single-sweep oscilloscope trace reflecting the irreversible displacement of one Bloch line (a) under the influence of an rf magnetic-field pulse of amplitude $H_0 = 80$ mOe and $\nu = 850$ kHz (b).

“white” subdomain in the domain wall through a certain distance along the photometered part of the crystal. At the high noise level that is unavoidable in this method of measuring the Bloch-line displacement one cannot detect in the magneto-optic signal the weak oscillations due to oscillations of the Bloch lines.

Evidence of the fact that all the lines in the domain wall (and not just one) shift irreversibly under these conditions can be seen in the single-sweep oscilloscope trace shown in Fig. 4a. This trace is a record of the change in the magneto-optic signal over a short time interval (300 msec) during the continuous application of periodically repeating (every 12.5 msec) rf magnetic-field pulses. The almost vertical lines on the trace correspond to the motion of the Bloch lines during the rf pulses. The places on the trace where the intensity of the signal decreased (increased) with time correspond to the gradual entry of a black (white) subdomain into the photometered region. The extreme values of the signal intensity correspond to times at which only a black or only a white

subdomain was found in the photometered region. In view of this it is easy to see that over the measured time of periodic application of the sinusoidal field, 6 subdomains passed through the slit-delimited region of the crystal.

A sequence of photographs taken of the domain wall between single applications of the rf pulses showed that all the subdomains were displaced in the same direction after each brief application of the sinusoidal magnetic field; this confirms our interpretation of the oscilloscope trace of Fig. 4a.

Figure 4b shows a single-sweep trace corresponding to the continuous application of a sinusoidal field along the magnetization vectors in the adjacent domains. By reflecting the sequential alternation of black and white subdomains moving through the photometered region, this trace conclusively proves the existence of unidirectional motion of the oscillating Bloch lines under the given conditions. The high noise level apparent here, as in the previous single-sweep traces, does not allow one to follow on this curve the weak modulations created by the oscillations of the Bloch lines. The magneto-optic signal (whose shape depends on both the relative size of the slit and subdomains and on the velocity of the subdomains) not strictly periodic because the domain wall contains subdomains having different lengths and moving at different speeds (see Fig. 1a and also Fig. 4a). Both of these circumstances are apparently due mainly to defects in the crystal lattice.

By studying different slabs of yttrium iron garnet cut along the crystallographic planes $\{110\}$ or $\{112\}$, we found that directed motion of oscillating Bloch lines occurred in the majority of 180-degree domain walls, with the Bloch lines in adjacent domain walls moving in opposite directions. This effect was produced both by an alternating field throughout the investigated frequency range (from a few hundred Hz to several MHz) and also by square pulses, and it began after the amplitude of the field exceeded a certain critical value H_0^c which depended on ν . In the low-frequency range (from ~ 0.1 to ~ 300 kHz) we were able to reliably establish a decrease in H_0^c with increasing ν . This decrease is apparently due to the fact that the amplitudes of the domain-wall oscillations (the domain wall also oscillates in the field) have a spectrum of the relaxation type¹³: The amplitude of the domain-wall oscillations increases with decreasing frequency of the field. The displacement of the domain wall compensates the external magnetic field in the crystal (with some time delay) and thus weakens its effect on the Bloch lines.

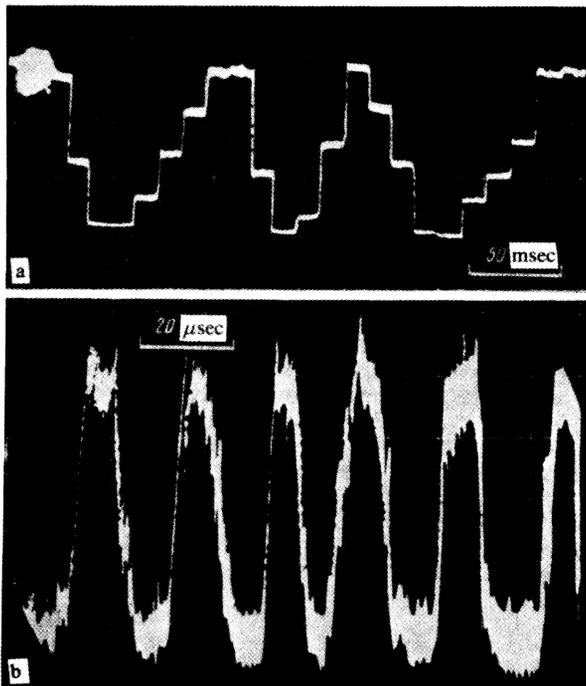


FIG. 4. Single-sweep oscilloscope traces recorded during the application to the crystal of rf pulses of a sinusoidal field (pulse duration $\tau = 2.35$ μ sec, $H_0 = 90$ mOe, $\nu = 850$ kHz) acting over an interval of about 12.5 msec (a) and during the application of a continuous sinusoidal field ($H_0 = 100$ mOe, $\nu = 850$ kHz) (b).

The velocity of the directed motion of the equilibrium positions of the oscillating Bloch lines turns out to be rather large. In particular, it can easily be determined from Fig. 4b that the average velocity of the subdomains passing through the slit (the extrema of the sinusoid correspond to the times at which the center of the slit coincides with the centroids of the black and white subdomains, which are about $50 \mu\text{m}$ long) is $\sim 5 \text{ m/sec}$. This value is comparable to the velocity of domain walls at the same field amplitudes.¹³ The instantaneous velocities of the oscillating Bloch lines under these conditions at certain times are of course much greater than their velocity of directed motion.

2. Generation of Bloch lines. Oscillograms of the type shown in Fig. 4b yield a ready estimate of the number of subdomains (or Bloch lines) passing through the slit-delimited section of the domain wall in a unit time. This number, equal to the number of half-periods of oscillation of the signal during the given time, comes to $(0.4-2) \cdot 10^5$ subdomains/sec for the case shown in Fig. 4b. The continuous directed motion of such a large number of subdomains could not have been due solely to the motion of subdomains present in the initial state of the domain wall, but implies that Bloch lines were rapidly created in the domain wall on one side of the slit and annihilated at the same high rate on the other side of the slit. It should be noted that the nucleation rate of the subdomains was lower than the frequency of the external magnetic field.

In a number of cases the generation and directed motion of Bloch lines led to their accumulation in individual regions of the domain wall. As the frequency ν of the external field was increased in the low-frequency region at a fixed amplitude above the critical value H_0^c , regions with an elevated density of quasistatic Bloch lines were observed in several domain walls.¹¹ Using short rf pulses, we were able to study the kinetics of the transformation of the domain-wall structure under these conditions.

In the segment of 180-degree domain wall shown in Fig. 1a, one of the Bloch lines (indicated by an arrow) is pinned by some sort of blocking center. The leftward displacement of the remaining Bloch lines under the action of a sinusoidal field applied briefly along the magnetization in the domains led at first to an increase in the lengths of the subdomains in the right-hand part of the photograph (Fig. 1b) to values significantly larger than in the original state. As a result, subsequent applications of rf field pulses initiated the creation of a new subdomain (indicated by the double arrow in Fig. 1c) within the nonequilibrium subdomain, which consequently split up into three parts. The concatenation of events such as these led to a substantial increase in the number of Bloch lines on the whole segment of domain wall.

The nucleation of a new subdomain occurred over a time which was much longer than the oscillation period of the external field. Unfortunately, no new-subdomain nuclei were detected under dynamical conditions. In rare cases we managed to observe the growth of nuclei localized about defects in the crystal. Figure 5a shows a segment of domain wall in the form of a rather wide, white band whose thickness varies on account of the inclination of the domain wall to the surface of the sample. When an alternating field at $\nu = 750$

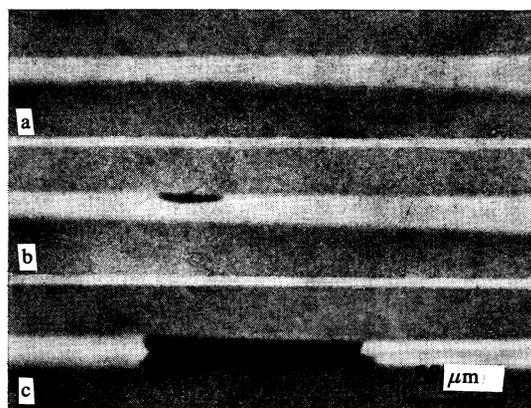


FIG. 5. a) Part of a subdomain of a 180-degree domain wall in a (110) slab of yttrium iron garnet; b) new-subdomain nucleus arising upon the application of a sinusoidal field ($H_0 = 50 \text{ mOe}$, $\nu = 750 \text{ kHz}$) and persisting after the field is turned off; c) transformation of the nucleus into a subdomain at $H_0 = 55 \text{ mOe}$.

kHz was applied to the sample and slowly increased in amplitude, a small region of dark contrast appeared in a subdomain near one of the surfaces of the crystal, increased in size, and remained after the field was turned off (Fig. 5b). As the amplitude of the field was increased, the dark region grew, and at a certain value of H_0 it suddenly became a new subdomain (Fig. 5c).

3. Generation and motion of solitary nonlinear excitations in the domain wall. The creation, annihilation, and directed motion of Bloch lines in the preceding discussion are processes which occur in a group of interacting Bloch lines. It is therefore of interest, on the one hand, to segregate these processes and, on the other hand, to study the possibility of exciting them in a spin system localized in a uniformly magnetized, unipolar domain wall.

For magnetizing the domain wall an additional, static external magnetic field H_z was applied to the sample perpendicular to the plane of the slab. To exclude random nucleation of subdomains due to thermal fluctuations, noise, or other causes, this field was maintained, at a low intensity (several oersteds), throughout the experiment. Figure 6a shows a typical single-sweep oscilloscope trace recorded during the simultaneous action of the static ($H_z = 8.5 \text{ Oe}$) and sinusoidal ($H_0 = 40 \text{ mOe}$, $\nu = 1.1 \text{ MHz}$) fields. Three peaks of various widths and amplitudes reflect the successive passage of three perturbations, characterized by practically reversed spins, through the photometered part of the domain wall. The amplitudes of the peaks on the oscilloscope trace are governed by the size of the spin-flip region in the domain wall, and the widths of the peaks by the propagation velocity of the perturbation. Several of the peaks observed under these conditions had characteristics similar to those in the demagnetized wall (Fig. 4b).

The excitations were generated most efficiently at $\nu_p = 1.1 \text{ MHz}$. As ν deviated from this value (to either larger or smaller values), the number of peaks per unit time was observed to decrease. Outside the frequency interval $\Delta\nu_p = 0.8-1.3 \text{ MHz}$ these peaks were not observed. Increasing the field H_z resulted in a shrinkage of the region of alternat-

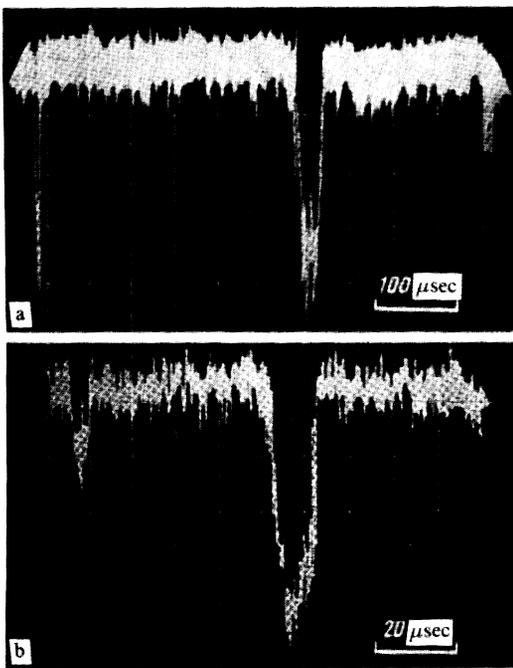


FIG. 6. Single-sweep oscilloscope traces recorded during the application of different magnetic fields to a magnetized domain wall: a) a sinusoidal field ($H_0 = 40$ mOe, $\nu = 1.1$ MHz); b) a sinusoidal ($H_0 = 25$ mOe, $\nu = 450$ kHz) and a single square field pulse 30 mOe high and $5 \mu\text{sec}$ long. In addition to these fields, which were applied along the magnetizations in the domains, a static field of 8.5 Oe was applied normal to the surface of the slab.

ing-field frequencies in which isolated peaks appeared on the traces. Decreasing the amplitude of the sinusoidal field had a similar effect.

In the same experimental arrangement we were also able to prove that these reversed-spin solitary excitations experience a directed motion even at sinusoidal-field frequencies which do not promote their creation. In these experiments, individual subdomains were created by additional square magnetic-field pulses H_p acting, like the sinusoidal field, along the magnetization in the domains. In Fig. 6b the time at which one such pulse was applied shows up on the oscilloscope trace as a small peak at the left on account of the displacement of the domain wall at the time of the pulse. The image of the domain wall partially overlapped the edge of the slit, causing a change in the magneto-optic signal. The large peak at the center of the trace is due to the passage through a slit of a nonlinear excitation which arose during the time that the pulsed field was applied to the crystal. No such oscilloscope traces reflecting a directed motion of solitary nonlinear excitations were obtained at sinusoidal-field frequencies outside the aforementioned optimum range $\Delta\nu_p$ unless single pulses H_p were first applied to the crystal.

Finally, we note that on the expanded single-sweep traces of the peaks corresponding to excitations moving in the sinusoidal field one can in spite of the high noise level make out the modulation of the signal at the frequency of the sinusoidal field. It is quite probable that these oscillations are due to periodic changes in the size of the reversed-spin

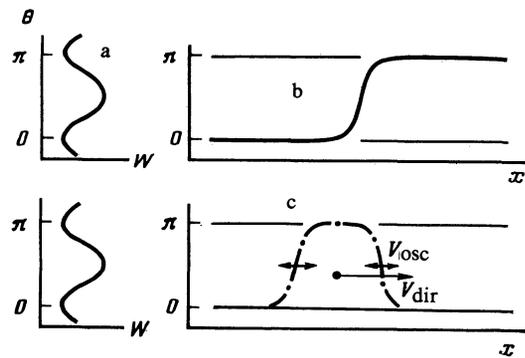


FIG. 7. a) Internal energy W of spins localized in a domain wall as a function of their direction (θ is the angle between the magnetization vector and the preferred direction); b) distribution along the length of the domain wall of the directions of the magnetization vectors at the center of the wall in a region containing a single Bloch line; c) initial state of the magnetized domain wall (solid curve) and the formation of a new subdomain (dot-and-dash curve). V_{dir} is the velocity of directed motion, V_{osc} the velocity of oscillatory motion.

regions (i.e., to oscillations of the Bloch lines bounding them).

3. DISCUSSION OF RESULTS

As we know, the ground state of a system of spins localized in a 180-degree domain wall in an infinite crystal (the state corresponding to a minimum of the internal energy W of the spin system) turns out to be degenerate with respect to the direction of the magnetization in the wall. Identical minimum values of W are attained when the spins in the central part of the domain wall point in either of two opposite directions (Fig. 7a). In a real crystal the equilibrium state in the absence of an external magnetic field is characterized by the presence of a domain wall which is split up by the surface magnetostatic field into subdomains separated by Bloch lines (Fig. 7b).

The most interesting of the new results presented in this paper, we believe, was obtained in the experiments with a unipolar domain wall, in which only one direction, characterized by $\theta = 0$ (the solid line in Fig. 7c), was realized at its center in a static field applied perpendicular to the surface of the sample. In these experiments it was found that at certain definite frequencies a very weak alternating field parallel to the magnetization in the adjacent domains causes a large set of spins to flip, leading to the formation of a subdomain which moves along the domain wall at a rather high velocity. The motion of this "dynamic" subdomain is accompanied by oscillations of the Bloch lines bounding it (the dot-and-dash curve in Fig. 7c). The same processes of dynamic transformation of the domain-wall structure occur in walls which contain Bloch lines in the initial state.

It has thus been shown in the present paper that under certain conditions, nonlinear excitations having the form of bound states of two Bloch lines propagating in the same direction are a necessary element of the domain-wall structure.

At small amplitudes of the alternating magnetic field acting along the magnetization in the domains, only driven

oscillations of the Bloch lines and experimentally unobservable small oscillations of the magnetization (spin waves) occur in the domain wall throughout the investigated frequency range. When the amplitude exceeds a certain threshold value, nonlinear magnetization waves which can lead to a significant increase in the density of Bloch lines are excited in the domain wall.

Unfortunately, no theoretical analysis of the nonlinear Landau-Lifshitz equation describing the dynamics of the magnetization in our experimental situation has ever been carried out. As we have already mentioned, the transformation of the domain-wall structure has been considered only for twisted domain walls moving at high velocities under the influence of a static magnetic field in highly anisotropic magnetic films.

It seems to us that the microscopic picture of the events observed in single crystals of yttrium iron garnet can be described in terms of magnetic solitons by analogy with the treatment of nonlinear excitations in a uniformly magnetized ferromagnet in the volume as a whole (see the review by Kosevich¹⁵). In terms of these concepts it has been shown theoretically that in the general case the highly excited state of a ferromagnet is due to the presence of a magnon gas, of topological solitons (moving domain walls), and of dynamic magnetic solitons (the so-called bions or oscillations). Bions are a new type of collective excitation of a ferromagnet and are described in the language of a bound state of a large number of magnons. They are characterized, in particular, by a precession frequency ω of the spins about a definite direction and a translational velocity which does not depend on ω . The limiting states of this inhomogeneous excitation of the magnetization field of a magnet are, on the one hand, spin waves, and on the other hand, a bound state of two domain walls of opposite sign.

In a system of spins localized in a domain wall, the role of the topological solitons is played by the Bloch lines (Fig. 7b). The alternating magnetic field excites magnons which, under resonance conditions (in the optimum frequency interval) at a high density of the magnon gas, form bound states—peculiar nonlinear magnetization waves. At large amplitudes of the deviation of the magnetization vectors from the initial state they convert into a bound state of two Bloch lines. The bound state, decaying under the influence of the magnetostatic fields when the alternating field is turned off, is transformed into a remanent stationary subdomain. The frequency and amplitude of the external magnetic field are such that dynamic solitons are excited. The Bloch line bounding the subdomain contain not only oscillatory but also directed motion along the domain wall (Fig. 7c).

If this picture is in fact realized then experiments of this type may yield complete information on the characteristics of soliton-like nonlinear excitations of the magnetization in domain walls. From a single-sweep oscilloscope trace (Fig. 4b) one can estimate the velocity of the centroid of the coupled Bloch lines, their oscillation frequency, and the spatial characteristics and amplitude of the soliton. For proof of this qualitatively consistent picture of the evolution of the dynamic transformation of the domain-wall structure it will be necessary to develop a rigorous theory made specific to the experimental situation.

In conclusion we note that the nucleation of dynamic solitons most likely occurs in the parts of the crystal where the potential relief describing the rotation in the domain wall is distorted by defects of the crystal lattice.

We are deeply grateful to V. G. Bar'yakhtar, A. K. Zvezdin, A. V. Mikhaïlov, and V. L. Pokrovskii for discussion of the results.

¹For brevity we shall call these short-duration sinusoidal-field signals "rf pulses," a term used in radio technology.

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