Radiative self-polarization of electron-positron beams in axial channeling

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The influence of the spontaneous electromagnetic radiation of electrons (or positrons) in axial channeling on the orientation of their spin is studied. It is shown that radiative self-polarization of the particles is possible and the conditions under which observation of this phenomenon is possible are pointed out.

1. INTRODUCTION

The spontaneous electromagnetic radiation by fermions moving in external fields can lead to the appearance of a referred orientation of the particle spin (radiative self-polarization). This effect was first predicted for electrons moving in a constant uniform magnetic field by Sokolov and Ternov.[1,2] At the present time the radiative self-polarization of electrons in a magnetic field has been studied in detail[3–7] and is explained in textbooks.[8,9]

Attempts have been made to observe a similar effect in the motion of electrons in electromagnetic fields of other configurations. For example, Ternov *et al.*[10] in a study of the spontaneous radiation of electrons in the field of a plane circularly polarized electromagnetic wave showed that radiative self-polarization does not occur in this case, although the nature of the classical motion is the same in Ref. 10 and in Refs. 1 and 2. This result indicates that the possibility of radiative self-polarization of the electron spin cannot be judged on the basis of the nature of the classical motion. This same conclusion has been drawn in Refs. 11 and 12 (see also Ref. 13), where the motion of an electron was considered in the approximation of a cylindrical rotator. The most important general result of Refs. 10–12 appears to us to be the conclusion that the total probability of transitions with spin flip is practically independent of the nature of the external field (and can be evaluated, for example, quasiclassically[4,6]), while the degree of polarization of a particle beam is extremely sensitive to the structure of the external field, a fact which determines the possibility of radiative self-polarization.

Up to the present time there has been no observation of physically interesting fields, other than a constant magnetic field, which lead to appreciable radiative self-polarization of electrons and positrons, and therefore the search for such fields presents considerable physical interest.

In the present paper we study the possibility of radiative self-polarization of electrons (or positrons) in axial channeling in crystals. Several authors[14–16] have considered the motion and radiation of charged particles in planar channeling in curved single crystals. The qualitative estimates of radiative self-polarization given in these studies are not in question, but the quantitative calculations of the degree of self-polarization carried out in these papers must be taken with a certain caution. In these studies the probability of transitions with spin flip was taken from results obtained for a magnetic field, whereas it is clear that in crystals the main role is played by electrostatic fields, which can give substantially different values for the degree of self-polarization of the particle beam. Here we show that the high-frequency radiation arising in channeling and first predicted by Kumakhov[17] opens up new possibilities for obtaining beams of polarized particles.

2. CHOICE OF EXTERNAL-FIELD MODEL AND CALCULATION OF TRANSITION MATRIX ELEMENTS

Let us consider an electron moving along the z axis of a cylindrical coordinate system r, φ, z and focused on the axis of the motion by an arbitrary axially symmetric electric field

$$E_r = -A_0'(r), \tag{1}$$

where $A_0(r)$ is an arbitrary function of r.

The Dirac equation for the field (1) permits four integrals of the motion: the total energy $E = c\hbar K$, the projections on the z axis of the momentum $p_z = \hbar k_3$ and the total angular momentum $J_z = \hbar(l - 1/2)$ (l is an integer), and the spin vector $\mathbf{T}$, where the spin operator $\mathbf{T}$ has the form (see Ref. 8)

$$\mathbf{T} = m_0 c \rho_3 \boldsymbol{\sigma} + \rho_1 \mathbf{P}, \tag{2}$$

and m_0 is the rest mass of the particle.

The Dirac wave functions of the electron ($e_0 = |e|$), which are eigenfunctions for these operators, are given in the textbook by Bagrov *et al.*[18] (Section 36, p. 119) and have the form

$$\Psi = \frac{N}{(rL)^{1/2}} e^{-icKt + ik_3 z + il\varphi} \begin{pmatrix} (\lambda + \zeta k_0)^{1/2} f e^{-i\varphi} \\ i\varepsilon (\lambda - \zeta k_0)^{1/2} g \\ \zeta\varepsilon (\lambda - \zeta k_0)^{1/2} f e^{-i\varphi} \\ i\zeta (\lambda + \zeta k_0)^{1/2} g \end{pmatrix}, \tag{3}$$

$$\lambda = (k_0^2 + k_3^2)^{1/2}, \quad \varepsilon = \operatorname{sign} k_3, \quad k_0 = m_0 c/\hbar.$$

Here L is the normalization length and $\zeta = \pm 1$ describes the orientation of the particle spin:

$$T_3 \Psi = \hbar \zeta \lambda \Psi. \tag{4}$$

The functions $f(r)$ and $g(r)$ are solutions of the following system of differential equations:

$$f' - \frac{l_0}{r} f + (\lambda + \zeta A) g = 0, \quad g' + \frac{l_0}{r} g + (\lambda - \zeta A) f = 0, \tag{5}$$
$$l_0 = l - 1/2, \quad A = K + e_0 A_0(r)/c\hbar.$$

The function (3) is normalized to unity if the normalization

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factor N is chosen from the condition

$$1=4\pi\lambda|N|^2\int_0^\infty (ff^+ + gg^+)\,dr. \tag{6}$$

The probability of spontaneous emission of a photon in transition from a state K, k_3, l, ζ (we shall assume for definiteness $l>0$) to a state $K', K_3', l'=l-\nu, \zeta'$ is calculated by the standard methods of quantum electrodynamics (see for example Refs. 5, 8, and 9) and is proportional to the square of the moduli of the matrix elements of the Dirac matrices $\bar{\alpha}_i$, where it is well known[5,8] that one must calculate the combinations

$$\bar{B}_1=-\bar{\alpha}_1\sin\varphi'+\bar{\alpha}_2\cos\varphi',$$
$$\bar{B}_2=\bar{\alpha}_1\cos\varphi'+\bar{\alpha}_2\sin\varphi',\quad \bar{B}_3=\bar{\alpha}_3, \tag{7}$$

and the direction of emission of a photon with frequency $\omega=c\varkappa$ will be given by the angles θ, φ' with inclusion of the conservation law $k_3'=k_3-\varkappa\cos\theta$. Representing $\bar{B}_i$ in the form

$$\bar{B}_i=2\pi NN'\int_0^\infty B_i\,dr, \tag{8}$$

it is easy to find from the functions (3) (with accuracy to an important phase factor) the following values:

$$B_1=iB(\zeta,\varepsilon)\,(f^{+\prime}gJ_{\nu+1}-\zeta\zeta'g^{+\prime}fJ_{\nu-1}),$$
$$B_2=B(\zeta,\varepsilon)\,(f^{+\prime}gJ_{\nu+1}+\zeta\zeta'g^{+\prime}fJ_{\nu-1});$$
$$B_3=\varepsilon B(-\zeta,-\varepsilon)\,(f^{+\prime}f-\zeta\zeta'g^{+\prime}g)J_\nu, \tag{9}$$
$$B(\zeta,\varepsilon)=[(\lambda'+\zeta'k_0)(\lambda+\zeta k_0)]^{1/2}$$
$$+\varepsilon\varepsilon'\zeta\zeta'[(\lambda'-\zeta'k_0)(\lambda-\zeta k_0)]^{1/2},$$

where $J_\alpha=J_\alpha(\varkappa r\sin\theta)$ is a Bessel function.

Integration over r in (8) is possible if one knows the form of the functions f and g, which requires that the actual form of the potential $A_0(r)$ be given.

3. ANALYSIS OF THE RADIATIVE SELF-POLARIZATION PROCESS IN AXIAL CHANNELING

The problem of finding a potential $A_0(r)$ which provides axial channeling is extremely complex.[19] No less complicated is the problem of integration of the system of equations (5), even if we assume that the potentials of the fields are known.

However, simple qualitative reasoning already shows that spontaneous radiation in axial channeling must lead to a preferred orientation of the spin. In fact, the actual experimental conditions are such that the energy of the transverse motion must be much less than the energy of the longitudial motion, i.e., the electron velocity v_3 along the z axis is close to the velocity of light ($v_3=c\beta_3, \beta_3=k_3/K\approx 1$), while the average transverse velocity $v_\perp=c\beta_\perp$ ($\beta_\perp^2=\beta^2-\beta_3^2$) is small. Here β^2 is determined from the condition $k_0(1-\beta^2)^{-1/2}=\bar{A}$, where $\bar{A}$ is the average value of A over the functions (3). In other words, the main contribution to the electron energy is from the momentum k_3 and in the equations (5) we have $A\sim k_3$, $\lambda\sim k_3$. Then we can obtain from Eq. (5)

$$f(r)\sim(1+\zeta)\psi(r),\quad g(r)\sim\zeta(1-\zeta)\psi(r), \tag{10}$$

where $\psi(r)$ is some function of r. Then, for example, for B_1 we have (for transitions with spin flip $\zeta'=-\zeta$)

$$B_1\sim\left(J_\nu'+\zeta\frac{\nu}{\varkappa r\sin\theta}J_\nu\right)\psi^2(r). \tag{11}$$

From (11) it follows that, whatever the nature of the function $\psi(r)$, the dependence of B_1 on the initial orientation of the spin is substantial and an appreciable preferred orientation of the spin should take place (at least for large k_3). If we consider a nonrelativistic electron ($\beta_3\ll 1$), then a similar conclusion is obtained from analysis of the coefficients $B(\zeta,\varepsilon)$ in the formulas (9).

Thus, at all energies of electrons (or positrons) we must expect the degree of radiative self-polarization of the particles as a result of spontaneous radiation in axial channeling to be appreciable.

A very important question is that of the absolute value of the probability of transitions with spin flip. We shall give an estimate of this quantity, proceeding from the model of a rigid cylindrical rotator.

In this model (see Refs. 11 and 12) it is assumed that the motion occurs over the surface of a cylinder of radius $r=R$. This corresponds to the case in which in the equations (5) the derivatives with respect to r of the functions f and g are assumed equal to zero and the integrals in Eqs. (6) and (8) are removed. Without loss of generality we can always choose the potential A_0 such that $A_0(R)=0$. In this case $A=K$.

From Eq. (5) we find

$$f=\left[\frac{1}{2}\left(1+\zeta\frac{\lambda}{K}\right)\right]^{1/2},\quad g=\zeta\left[\frac{1}{2}\left(1-\zeta\frac{\lambda}{K}\right)\right]^{1/2},$$
$$K^2=k_0^2+k_3^2+\left(\frac{l_0}{R}\right)^2, \tag{12}$$

and from (6) it follows that

$$4\pi\lambda|N|^2=1. \tag{13}$$

We note from (12) one obtains estimates (10) in complete agreement with the general reasoning given above. It is now straightforward to obtain exact (in our model) expressions for the total (integrated over angles and summed over ν) probability of transitions per unit time with spin flip. These exact expressions have a rather cumbersome form and are not given here. We note here only that the total probability W_ζ of transitions with spin flip per unit time will depend on the initial spin orientation ζ and is a function of the invariant parameter β_0 determined by the relation

$$\beta_0^2=\frac{\beta^2-\beta_3^2}{1-\beta_3^2},\quad \beta^2=1-\left(\frac{m_0c^2}{E}\right)^2, \tag{14}$$

where $E=c\hbar K$ is the energy of the electron (or positron). To evaluate β_0 it is convenient to introduce the entry angle γ of the particle into the channel, i.e., the angle between the initial electron velocity and the z axis. Obviously $\beta_3=\beta\cos\gamma$ and from Eq. (14) we find

$$\beta_0^2=q^2/(1+q^2),\quad q=\beta(1-\beta^2)^{-1/2}\sin\gamma. \tag{15}$$

If we expand the exact expressions for the probability in the Planck constant $\hbar$, it is easy to see that the quantum parameter is actually the quantity

$$\xi = q(1+q^2)^{1/2}/k_0 R. \tag{16}$$

This is a well known quantum parameter in the theory of synchrotron radiation,[5,6,8] which by the way is quite natural. Under actual experimental conditions the value of ξ is small, and therefore we shall give the expression for W_ζ for the case $\xi \ll 1$.

$$W_\zeta = \frac{1}{2\tau}\left[1+\zeta\frac{\chi(\beta_0)}{\varphi(\beta_0)}\right], \quad \tau = \frac{T_0 E}{m_0 c^2 \xi^3 \varphi(\beta_0)}, \quad T_0 = \frac{\hbar^2}{e^2 m_0 c}. \tag{17}$$

Here the functions $\chi(x)$ and $\varphi(x)$ are given by the expressions

$$\varphi(x) = \frac{1}{2}\varepsilon_0^{5/2}\sum_{\nu=1}^{\infty}\nu^3\int_0^{\pi}\left[\cos^2\theta(1+\varepsilon_0\cos^2\theta)J_\nu'^2(z_0) + \frac{\varepsilon^2+\varepsilon_0\cos^2\theta}{x^2\sin^2\theta}J_\nu^2(z_0)\right]\sin\theta\, d\theta,$$

$$\chi(x) = \varepsilon_0^3\sum_{\nu=1}^{\infty}\nu^3\int_0^{\pi}\frac{1+\varepsilon}{x}\cos^2\theta J_\nu'(z_0)J_\nu(z_0)\,d\theta, \tag{18}$$

$$\varepsilon_0 = 1-x^2,\ \varepsilon = 1-x^2\sin^2\theta,\ z_0 = \nu x\sin\theta.$$

The function $\chi(x)$ can be calculated explicitly:

$$\chi(x) = (96x^3)^{-1}\left[9(1-x^2)\ln\frac{1+x}{1-x} - x(18-80x^2+46x^4)\right], \tag{19}$$

while for $\varphi(x)$ an explicit expression of this type is not known. It is easy to investigate the simplest properties of the functions $\chi(x)$ and $\varphi(x)$. In the interval from $x = 0$ to $x = 1$ both functions are bounded monotonic functions of x; here $\chi(x)$ falls off and $\varphi(x)$ increases with increase of x:

$$1/3 = \chi(0) > \chi(x) > \chi(1) = 1/6,$$
$$7/15 = \varphi(0) < \varphi(x) < \varphi(1) = 7\sqrt{3}/16. \tag{20}$$

Thus, if we create conditions under which the electrons (or positrons) in the beam all (or almost all) have $l > 0$, then regardless of the initial orientation of the spins of the particles in the beam, at large times $t > \tau$ a preferred orientation of the spin is established, and the fraction of electrons with spins β will be

$$n_\zeta = 1/2[1-\zeta\chi(\beta_0)/\varphi(\beta_0)]. \tag{21}$$

The nature of the change of the beam polarization with time is determined by a simple relaxation process and is well known (see for example Refs. 1–3), and the quantity τ is the relaxation time. From Eqs. (20) and (21) it follows that

$$n_{-1} = 6/7,\ n_1 = 1/7\ (\beta_0 \ll 1),$$

$$n_{-1} = \frac{63+8\sqrt{3}}{126} \approx 0{,}61, \quad n_1 = \frac{63-8\sqrt{3}}{126} \approx 0{,}39 \quad (\beta_0 \sim 1). \tag{22}$$

For positrons it is necessary in Eq. (21) to make the substitution $\zeta \to -\zeta$. If $l < 0$, then all conclusions remain in force also with the substitution $\zeta \to -\zeta$. Assuming that the motion of a particle along the z axis is relativistic ($v_3 \approx c$), we find that if the quantity $S_0 = c\beta_3\tau$ does not exceed the range of the channeled particles, radiative self-polarization actually is established in the course of channeling. The variation of the fraction $n_\zeta(s)$ of the particles in the beam with spin ζ as a function of the path traversed during channeling S can be obtained easily from well known formulas[1–3] and has the form

$$n_\zeta(S) = n_\zeta + (n_\zeta^0 - n_\zeta)l^{-S/S_0}, \tag{23}$$

where n_ζ is given by Eq. (21). For $S = 0$ we set $n_1(0) = n_1^0$, $n_{-1}(0) = n_{-1}^0$, and it is necessary that $n_1^0 + n_{-1}^0 = 1$

Numerical estimates for electrons rotating with characteristic radii of axial channeling $R \sim 10^{-8}$ cm show that if we have conditions for which the value of q reaches unity, then from Eq. (16) it follows that $\xi \sim 10^{-2}$ (which corresponds to the approximation $\xi \ll 1$ used by us). For $E = 500$ MeV we have $\tau \sim 10^{-10}$ sec, $S_0 \sim 3$ cm. For $E = 5$ GeV it is possible to obtain $q \sim 2$, which increases ξ by more than a factor of four. As a result τ decreases by almost an order of magnitude and $S_0 \sim 0.3$–0.5 cm. These values of S_0 are achievable experimentally at the present time.

The results obtained here for a rotator model can be used for analysis of the self-polarization effect in motion of charged particles in curved crystals.

4. INFLUENCE OF A MAGNETIC FIELD ON THE RADIATIVE SELF-POLARIZATION IN AXIAL CHANNELING

Radiative self-polarization during axial channeling in the field (1) is possible if the charged-particle beam has a certain angular momentum of rotation. To achieve this level it is possible to use an axially symmetric magnetic field directed along the z axis, and here the magnetic field will enhance the radiative self-polarization. The solution (3) of the Dirac equation retains its form (see Ref. 18) if in addition to the electric field (1) there is a magnetic field which can be specified in the form

$$H_z = r^{-1}A_1'(r), \tag{24}$$

where A_1 is an arbitrary function of r. In this case there is a change of the system (5), in which it is necessary to make the substitution $l_0 \to l - 1/2 + e_0 A_1(r)/c\hbar$. Here all conclusions drawn in Section 3 regarding the possibility of radiative self-polarization under conditions of near-axial motion, and also the estimates (10) and (11), remain in force.

Special attention, in our opinion, must be given to the use of single-crystal ferromagnetic materials, in which the internal magnetic field (on ordering of the orientation of most of the domains) can reach extremely large values; in particular the condition $l > 0 (l < 0)$ will be realized automatically.

We shall consider one further aspect of the phenomenon studied here. Specifically, let there be only a constant and uniform magnetic field $H_z = H$, and let the electric field be equal to zero. In this case, expressing the orbit radius in the well known way[5] in terms of the magnetic field strength, we find

$$R = \frac{H_0\beta_0(1-\beta_0^2)^{-1/2}}{k_0 H}, \quad H_0 = \frac{m_0^2c^3}{e_0\hbar}, \quad \xi = \frac{H}{H_0}(1-\beta_0)^{-1/2}. \tag{25}$$

For example, if $H \sim 10^8$ G (a field of the explosive type) and $R \sim 10^{-2}$ cm, i.e., $(1-\beta_0^2)^{-1/2} \sim 10^{-3}$, then from (25) we have $\xi \sim 10^{-2}$ and $\tau \sim 10^{-10}$ sec. Thus, if an explosive field is maintained for a time of 10^{-10}–10^{-9} sec in a cylinder with $R \sim 10^{-2}$ cm and length 3–10 cm, then the electrons which have traversed the region of strong magnetic field are polarized.

This effect may turn out to be important also in astrophysics. Near the surface of neutron stars the magnetic fields can reach values close to H_0 in volumes whose linear dimensions are of the order of kilometers. Energetic electrons in traversing such a field will surely be polarized, which may lead to observable cosmological effects.

In conclusion we note that the use of specially pure and defect-free crystals, low temperatures, and other special precautions will apparently permit experimental realization of the conditions for observation of the self-polarization of electrons (or positrons) in axial channeling.

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