Possible enhancement of the weak interaction of neutrinos in a plasma

V. N. Oraevskiĭ and V. B. Semikoz

Institute of Terrestrial Magnetism, the Ionosphere, and the Propagation of Radio Waves, USSR Academy of Sciences

(Submitted 9 July 1983; resubmitted 13 October 1983)
Zh. Eksp. Teor. Fiz. **86**, 796–808 (March 1984)

In the framework of the standard model of electroweak interactions, a system with broken symmetry of the electromagnetic and weak interactions is considered. It is a Boltzmann plasma with neutrino background. It is shown that in such a medium and in the presence of polarization effects of $Z\gamma$ mixing of the excitations of the neutral fields a long-range weak force mechanism exists which differs from the well-known case of high-temperature restoration of symmetry. Dispersion relations are obtained for excitations of the Yang-Mills fields in the system, and for Weinberg angle $\theta_W \neq 30°$ the excitations of the massive neutral field δZ, which have a spectrum identical to electromagnetic oscillations, also have a massless (long-range) nature. The screening of test particles in the medium is calculated, allowance being made for the screening of a hypothetically massive neutrino coupled effectively by the long-range forces to the electrons within the Debye sphere. The polarization losses of neutrino energy expended on the excitation of the longitudinal oscillations of the neutral Z and A fields are calculated. At relatively high plasma densities ($\omega_p > E_\nu$) these losses exceed the collisional losses in direct νe collisions, which means that there is an enhancement of the weak interaction of the neutrinos with the medium through resonant excitation in it of collective degrees of freedom.

1. The standard model of electroweak interactions,[1] for which more and more new experimental confirmations are currently being found, is becoming as perfect a theory as quantum electrodynamics. It is sufficient to mention here the calculation of the radiative corrections in the model (see, for example, Ref. 2).

If the Lagrangian of Abelian electrodynamics can be taken as the basis of the theory of an ordinary plasma, the same can be done using the unified theory of electromagnetic and weak interactions[1] for an extended system of many particles including a neutrino component as subsystem.

The thermodynamics of a many-particle system in theories with spontaneously broken symmetry, in particular in the standard model,[1] has already been considered on a number of occasions[3,4] in connection with the study of various phase transitions (with the inclusion of hadrons; see Ref. 5). However, these studies were concerned only with the thermodynamically equilibrium state of a system in the range of temperatures and densities characteristic, for example, of the early stage of the Universe (big bang). Excitations of a medium due to fluctuations of the Yang-Mills fields were not considered. At the same time, fluctuations, which always exist in a medium, can grow in a system in the presence of large-scale inhomogeneities (particle streams, external fields), and it is therefore important to study the dispersion characteristics, in particular the spectra of boson excitations of the background equilibrium medium.

This problem has been solved in statistical QCD for a quark-gluon plasma with massless Yang-Mills fields.[6] So far as we know, there has as yet been no corresponding generalization to the case of many particles in theories with spontaneously broken symmetry.

The aim of the present paper is to find the dispersion characteristics, the screening, and the energy losses of test particles in an ordinary (Boltzmann) plasma with a background of neutrinos that are not in thermodynamic equilibrium with the matter. In the special case of a relic background, inequality of the temperatures ($T_e \neq T_\nu$) does not mean there are instabilities in the electron-neutrino system, as in, for example, an ordinary collisionless plasma with unequal electron and ion temperatures ($T_e \neq T_i$). Real instabilities arise when the medium ceases to be homogeneous or isotropic. Although the system as a whole is not in the state with minimum energy (as in the examples analyzed in Refs. 3 and 4), it does remain in equilibrium in the subsystems of the charged and neutral particles, which do not interact through direct collisions. If fluctuation currents of stable leptons arise in such a medium, they can be sources of fluctuations of the Yang-Mills fields, the spectrum of which is determined by the required dispersion relations.

By studying in this paper a Boltzmann plasma with neutrino background we simplify the problem by reducing the number of plasma components (the positron density is low, $n_0^+ \to 0$), and our aim is to exhibit in a simple example a new (collective) mechanism of neutrino-matter interaction. As neutrino component we can also consider nonequilibrium neutrino fluxes and even an isolated neutrino in the plasma.

The paper is arranged as follows. In Sec. 2, we consider the restrictions associated with the choice of the many-particle system, and we discuss the part played by the nonlinear self-interaction of the Yang-Mills fields, which influences the solutions of the tree approximation under equilibrium conditions (in the absence of fluctuations).

In Sec. 3, we formulate the basic equations that describe the dynamics of the Boltzmann plasma with a background of relic neutrinos. In Sec. 4, we obtain dispersion relations for

 0038-5646/84/030465-07$04.00

excitations of the vector fields for arbitrary values of the Weinberg angle θ_W both in the absence ($\theta_W = 30°$) and presence of $Z\gamma$ mixing ($\theta_W \neq 30°$).

In Sec. 5, we calculate the screening of test particles—with electric charge Q and neutrinos—and in Sec. 6 we calculate the polarization losses of the neutrinos in the Boltzmann plasma. The interpretation of these results in Sec. 7 shows that there is a collective mechanism of long-range weak interaction in the medium with manifest breaking of the symmetry of the electromagnetic and weak interactions. As a result, it is possible to use the self-consistent field approximation in deriving collisionless kinetic equations for the electron-neutrino plasma.

In the final Sec. 8 we discuss the possibility of using the obtained results to calculate the neutrino emission of a collapsing star.

2. The exact Lagrange equations for the vector fields $A_\mu, Z_\mu, W_\mu^\pm$ in the model[1] have the form[1)]

$$\Box A_\mu = e\bar{\psi}_e\gamma_\mu\psi_e + J_\mu^{\text{nonlin}},$$

$$(\Box + M_Z^2)Z_\mu = \frac{e}{\sin 2\theta_W}\bar{\psi}_\nu\gamma_\mu\frac{(1-\gamma_5)}{2}\psi_\nu - \frac{e}{2\sin 2\theta_W}\bar{\psi}_e\gamma_\mu(2\cos 2\theta_W - 1 - \gamma_5)\psi_e - \frac{J_\mu^{\text{nonlin}}}{\text{tg}\,\theta_W}, \quad (2)$$

$$(\Box + M_W^2)W_\mu^+ = -\frac{e}{\sqrt{2}\sin\theta_W}\bar{\psi}_e\gamma_\mu\frac{(1-\gamma_5)}{2}\psi_\nu + I_\mu^{\text{nonlin}}, \quad (3)$$

$$(\Box + M_W^2)W_\mu^- = -\frac{e}{\sqrt{2}\sin\theta_W}\bar{\psi}_\nu\gamma_\mu\frac{(1-\gamma_5)}{2}\psi_e + (I_\mu^{\text{nonlin}})^+. \quad (4)$$

Here, e is the electric charge ($e^2 = 4\pi/137$), and M_Z and M_W are the masses of the Z and W bosons; the Hermitian neutral current J_μ^{nonlin} and charged current I_μ^{nonlin}, which are nonlinear in the field amplitudes and arise from the self-interaction of the Yang-Mills fields, are determined by

$$J_\mu^{\text{nonlin}} = ie\left[W^{-\nu}\left(\frac{\partial W_\nu^+}{\partial x^\mu} - 2\frac{\partial W_\mu^+}{\partial x^\nu}\right) - W^{+\nu}\left(\frac{\partial W_\nu^-}{\partial x^\mu} - 2\frac{\partial W_\mu^-}{\partial x^\nu}\right)\right] + e^2[2A_\mu(W^+W^-) - W_\mu^+(AW^-) - W_\mu^-(AW^+)] + \frac{e^2}{\tan\theta_W}[W_\mu^-(ZW^+) - W_\mu^+(ZW^-) - 2Z_\mu(W^+W^-)], \quad (5)$$

$$I_\mu^{\text{nonlin}} = ie\left[\left(A^\nu - \frac{Z^\nu}{\tan\theta_W}\right)\left(\frac{\partial W_\mu^+}{\partial x^\nu} - \frac{\partial W_\nu^+}{\partial x^\mu}\right) - \left(A^{\mu\nu} - \frac{Z^{\mu\nu}}{\tan\theta_W}\right)W_\nu^+\right] - \frac{e^2}{\sin^2\theta_W}[(W^+)^2W_\mu^- - (W^+W^-)W_\mu^+] + \frac{e^2}{\tan^2\theta_W}[Z^2W_\mu^+ - (ZW^+)Z_\mu] + e^2[A^2W_\mu^+ - (AW^+)A_\mu] + \frac{e^2}{\tan\theta_W} \times [Z_\mu(AW^+) + A_\mu(ZW^+) - 2(AZ)W_\mu^+]. \quad (6)$$

Quantizing the Fermi and Bose fields in Eqs. (1)–(6) and using subsequent averaging by means of the nonequilibrium statistical operator $\hat{\rho}(t)$, we can go over to a many-particle description of the nonequilibrium system of leptons and vector fields. In the general case, the nonequilibrium system of leptons will be described by kinetic equations for distribution functions determined in the standard manner. For example, in the simple case of a classical electron plasma (with a background of fixed ions) and a neutrino component the transition to the many-particle description has the usual form:

$$\text{Sp}\,[\hat{\rho}\hat{\bar{\psi}}_e\gamma_\mu\hat{\psi}_e] \to \int p_\mu f^-(\mathbf{p},\mathbf{r},t)\frac{d^3p}{p_0},$$

$$\text{Sp}\left[\hat{\rho}\hat{\bar{\psi}}_\nu\gamma_\mu\frac{(1-\gamma_5)}{2}\hat{\psi}_\nu\right] \to \int p_\mu f^\nu(\mathbf{p},\mathbf{r},t)\frac{d^3p}{p_0}.$$

Here, for massless neutrinos we use the Wigner distribution function with noncommuting variables $\mathbf{p}$ and $\mathbf{r}$ in the argument, and summation over the spin variables is understood if we are not interested in spin waves in the neutrino gas.

For bosons, the transition to the many-particle description appears much more complicated, in the first place because of the additional nonlinearity due to the contribution of the self-interaction of the Yang-Mills fields (5), (6), which is absent in the case of an ordinary Abelian plasma. However, in the linear response theory developed below, when nonequilibrium fluxes (electron or neutrino streams) are assumed to be absent, the problem simplifies appreciably.

Linearization with respect to the perturbations δA_μ, δZ_μ, $\delta W_\mu^\pm$ of the vector fields, which is possible in the absence of particle streams, does not free us from the need to take into account the contribution of the massive bosons to the polarization of the medium even in the finding of the linear response. We make one further simplifying step.

We consider as an example an equilibrium unbounded isotropic and homogeneous medium consisting of a Boltzmann plasma of electrons with fixed ions ensuring electrical neutrality,

$$n_0^- = n_i, \quad (7)$$

and relic neutrinos with density n_0^ν and, in general, a different temperature: $T_e \neq T_\nu$. The simplification in the calculations of the dispersion characteristics of such a medium is associated with the restriction to the range of densities and temperatures typical of a real plasma:

$$n_0^{4/3}/M_W \ll T \ll M_W. \quad (8)$$

It is well known that a Rayleigh-Jeans distribution of plasmons corresponds to low frequencies of boson excitations: $\omega \ll T$. On the other hand, in a weakly inhomogeneous medium the wave vector k is bounded above either by the electron mass m_e (Boltzmann plasma, $T_e \ll m_e$) or the temperature T (ultrarelativistic hot plasma[2)]: $m_e \ll T \ll M_W$). In accordance with the inequalities (8), these requirements are adequate for the consideration of small momentum transfers:

$$q^2 \ll M_W^2 \quad (q^2 = \omega^2 - k^2). \quad (8')$$

The use of the inequalities (8) and (8′) makes it possible:

a) restriction to a classical description of the Yang-Mills fields without recourse to the quantization procedure for them;

b) ignoring the restoration of symmetry[3] or Bose condensation of the charged bosons[4];

c) ignoring the contribution of the real W bosons to the dispersion characteristics of the medium. These particles decay too rapidly, even if they are produced by energetic electrons and neutrinos from the tails of the Fermi distributions with temperature $T \ll M_W$.

As a result, in the considered linear response theory the contribution of the nonlinear self-interaction of the Yang-Mills fields (5), (6) is important only in the finding of the classical solutions of the tree approximation in the absence of fluctuations (under equilibrium conditions). As in the original case,[4] the only nontrivial scalar components of the neutral fields $C_0^{A,Z}$ (tree solution) can be taken into account exactly. They do not occur in the condition (7) of electrical neutrality because of the absence of a contribution of real $W^\pm$ bosons ($n_0^W = 0$). In the equation for the scalar component of the Z field we take into account the compensation of the term $C_0^Z M_Z^2$ by the density of the weak charge j_0^Z, which under the considered conditions (cf. Ref. 4) has the form

$$j_0{}^Z = \frac{2e}{\sin 2\theta_W}\left[n_0{}^\nu - \frac{(2\cos 2\theta_W - 1)}{4} n_0{}^-\right]. \qquad (9)$$

The remarks made above about the part played by the self-interaction of the Yang-Mills field makes it clear why in what follows we write down only the linear parts of the vectors of the electric (δE_i^A) and quasielectric ($\delta E_i^{Z,W}$) fields in the wave equations. The latter [see (11) and (12) below] are a direct consequence of the basic microscopic equations of motion (1)–(4) and the procedure of macroscopic averaging described above. The right-hand sides of the averaged equations are determined by the induced lepton currents, which depend on the deviations $\delta f^{(-)}(\mathbf{p},\mathbf{r},t)$ and $\delta f^\nu(\mathbf{p},\mathbf{r},t)$ of the distribution functions from the equilibrium Maxwellian distribution for the electrons,

$$f_0{}^-(p) = \frac{n_0{}^-}{(2\pi m_e T_e)^{3/2}} \exp\left(-\frac{p^2}{2m_e T_e}\right), \qquad (10)$$

and the equilibrium Fermi distribution for the neutrinos,

$$f_0{}^\nu(p) = [(2\pi)^3(\exp(p/T_\nu)+1)]^{-1}. \qquad (10')$$

3. The wave equation for the excitations of the electromagnetic field has the usual form (in the Fourier representation)

$$[k_i k_j + q^2\delta_{ij}]\delta E_j{}^A(\omega,\mathbf{k}) = -ie\omega \int v_i \delta f^{(-)}(\mathbf{p},\mathbf{k},\omega)\, d^3p. \quad (11)$$

The limits of applicability of plasma electrodynamics are extended by the new equation

$$[k_i k_j + q^2\delta_{ij}]\delta E_j{}^Z(\omega,\mathbf{k}) - M_Z{}^2\left(\delta E_i{}^Z(\omega,\mathbf{k}) + \frac{k_i k_j}{q^2}\delta E_i{}^Z(\omega,\mathbf{k})\right)$$
$$= \frac{i\omega e(2\cos 2\theta_W - 1)}{2\sin 2\theta_W}\int v_i \delta f^-(\mathbf{p},\mathbf{k},\omega)\, d^3p$$
$$- \frac{i\omega e}{\sin 2\theta_W}\int c_i \delta f^\nu(\mathbf{p}, \quad \omega)\, d^3p, \qquad (12)$$

which describes the excitations of the neutral δZ field.[3)]

It remains to close the system of self-consistent equations (of the type of Vlasov equations) by writing down collisionless kinetic equations linearized with respect to the deviations from the distributions (10) and (10′).

The equation

$$\frac{\partial \delta f^-}{\partial t} + \mathbf{v}\frac{\partial \delta f^-}{\partial \mathbf{r}} + e\left\{\delta \mathbf{E}^A + \frac{[\mathbf{v}\delta\mathbf{B}^A]}{c}\right.$$
$$\left. - \frac{(2\cos 2\theta_W - 1)}{2\sin 2\theta_W}\left(\delta\mathbf{E}^Z + \frac{[\mathbf{v}\delta\mathbf{B}^Z]}{c}\right)\right\}\frac{\partial f_0{}^-}{\partial \mathbf{p}} = 0 \qquad (13)$$

for the electrons differs from the ordinary Boltzmann equation by the addition of the Lorentz force due to the presence of excitations of the δZ field.

On the basis of direct comparison of the Lagrangian of the standard model[1] and the Lagrangian of Abelian electrodynamics one can readily understand the origin of the last terms in Eq. (13) and in the kinetic equation for the Wigner distribution function $\delta f^\nu(\mathbf{p},\mathbf{r},t)$ of the neutrinos:

$$\frac{\partial \delta f^\nu}{\partial t} + \mathbf{c}\frac{\partial \delta f^\nu}{\partial \mathbf{r}} + \frac{e}{2\sin 2\theta_W}\left(\delta\mathbf{E}^Z + \frac{[\mathbf{c}\delta\mathbf{B}^Z]}{c}\right)\frac{\partial f_0{}^\nu}{\partial \mathbf{p}} = 0. \quad (14)$$

We have made a rigorous derivation of more general (quantum) collisionless equations, and this will be given in a separate paper. On the right-hand sides of the wave equations we can include external electromagnetic, $\mathbf{j}_0^A$, and weak, $\mathbf{j}_0^Z$, neutral currents, which makes it possible to analyze perturbations developing in the medium in the presence of streams of both charged particles and neutrinos.

4. Eliminating by means of (13) and (14) the quantities $\delta f^{(-,\nu)}$ on the right-hand sides of Eqs. (11) and (12), we obtain a simple system of homogeneous linear algebraic equations for the intensities $\delta\,\mathbf{E}^{A,Z}$ of the neutral fields. Equating to zero the determinant of this system, we find dispersion relations for the longitudinal and transverse perturbations of the Yang-Mills fields. The permittivity tensors of the isotropic medium that occur in the dispersion relations,

$$\varepsilon_{ij}^{A,\nu} = \varepsilon_l^{A,\nu}\frac{k_i k_j}{k^2} + \varepsilon_{tr}^{A,\nu}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right),$$

are determined from (13) and (14) and are equal to the expressions (cf. Ref. 7)

$$\varepsilon_{ij}{}^\nu - \delta_{ij} = \frac{e^2}{\omega^2\sin^2 2\theta_W}\int \frac{[c_i(\omega - \mathbf{kc})\delta_{kj} + k_k c_i c_j]}{\omega - \mathbf{kc}}\frac{\partial f_0{}^\nu}{\partial p^k}d^3p, \quad (15)$$

$$\varepsilon_{ij}{}^A - \delta_{ij} = \frac{e^2}{\omega^2}\int \frac{[v_i(\omega - \mathbf{kv})\delta_{kj} + k_k v_i v_j]}{\omega - \mathbf{kv}}\frac{\partial f_0^{(-)}}{\partial p^k}d^3p, \qquad (16)$$

where the equilibrium distributions $f_0^{(-,\nu)}$ are determined in (10) and (10′).

We must distinguish two cases. If the Weinberg angle θ_W is equal to 30° and there is no $Z\gamma$ mixing [see Eqs. (11)–(14) above], then we have the usual equations for longitudinal and transverse oscillations in an Abelian plasma,

$$\varepsilon_l{}^A|_{\theta_W=30^\circ} = 0, \qquad \varepsilon_{tr}{}^A|_{\theta_W=30^\circ} - (k/\omega)^2 = 0,$$

which exist independently of the excitations of the Z field and are determined by the equations

$$\varepsilon_l{}^\nu - M_Z{}^2/q^2 = 0 \text{ and } \varepsilon_{tr}{}^\nu - (M_Z{}^2 + k^2)/\omega^2 = 0.$$

At the same time, as in the case of $\delta W^\pm$ oscillations (see footnote 3), oscillations of the Z field are not excited at the long wavelengths corresponding to the restriction (8′).

The situation is different for Weinberg angle $\theta_W \neq 30°$. We find directly from (11)–(14) that the longitudinal excitations of the δZ field are determined by the equation

$$\frac{1}{\varepsilon_l^A}\left\{\left(\varepsilon_l^\nu-\frac{M_Z^2}{q^2}\right)\varepsilon_l^A+\frac{(2\cos 2\theta_W-1)^2}{4\sin^2 2\theta_W}(\varepsilon_l^A-1)\right\}=0, \tag{17a}$$

and the transverse excitations are described by

$$\frac{1}{\varepsilon_{tr}^A-\left(\frac{k}{\omega}\right)^2}\left\{\left(\varepsilon_{tr}^\nu-\frac{M_Z^2+k^2}{\omega^2}\right)\left(\varepsilon_{tr}^A-\left(\frac{k}{\omega}\right)^2\right)+\frac{(2\cos 2\theta_W-1)^2}{4\sin^2 2\theta_W}(\varepsilon_{tr}^A-1)\right\}=0. \tag{17b}$$

For the δA field we obtain similar relations:

$$\frac{2\sin 2\theta_W}{(2\cos 2\theta_W-1)(1-\varepsilon_l^A)}\left\{\left(\varepsilon_l^\nu-\frac{M_Z^2}{q^2}\right)\varepsilon_l^A+\frac{(2\cos 2\theta_W-1)^2}{4\sin^2 2\theta_W}(\varepsilon_l^A-1)\right\}=0, \tag{18a}$$

$$\frac{2\sin 2\theta_W}{(2\cos 2\theta_W-1)(1-\varepsilon_{tr}^A)}\left\{\left(\varepsilon_{tr}^\nu-\frac{M_Z^2+k^2}{\omega^2}\right)\left(\varepsilon_{tr}^A-\left(\frac{k}{\omega}\right)^2\right)+\frac{(2\cos 2\theta_W-1)^2}{4\sin^2 2\theta_W}(\varepsilon_{tr}^A-1)\right\}=0, \tag{18b}$$

which differ from (17) only by the first factors, which do not have poles[4)] for $\theta_W \neq 30°$.

In the considered range of temperatures and densities (8) the modification of the well-known dispersion relations of an Abelian plasma by taking into account the weak interaction in (17) and (18) is slight. For example, the dispersion relation for the longitudinal δZ and δA excitations obtained from (17a) and (18a),

$$\omega_{Z,A}^2\approx\omega_{pe}^2\left[1+3k^2r_D^2+\frac{(2\cos 2\theta_W-1)^2}{4\sin^2 2\theta_W}\frac{(k^2-\omega_{pe}^2)}{M_Z^2}\right],\qquad \omega_{pe}^2=\frac{4\pi e^2 n_0^-}{m_e} \tag{19}$$

contains a correction (in the form of the third term) which is small compared with the ordinary contribution of spatial dispersion.

Besides the oscillation branch (19) there exists formally a further solution of Eqs. (17a) and (18a):

$$\omega_Z^2=k^2+M_Z^2+\frac{(2\cos 2\theta_W-1)^2}{4\sin^2 2\theta_W}\omega_{pe}^2, \tag{19′}$$

which it is natural to associate with longitudinal δZ oscillations not excited in the range (8′) of small momentum transfers.

A more important change in the results of Abelian plasma electrodynamics is the appearance in the presence of $Z\gamma$ mixing ($\theta_W \neq 30°$) of a new channel of energy dissipation of longitudinal oscillations of the neutral fields, including the electromagnetic field A. This is the collisionless damping of δZ and δA excitations on the relic neutrinos.

The total decay rate of the oscillations (19) can be obtained from Eqs. (17a) and (18a) by means of Eqs. (15) and (16) and is

$$\gamma_{A,Z}\approx-\frac{\omega_{pe}}{2}\left\{\operatorname{Im}\varepsilon_l^A|_{\omega=\omega_{pe}}+\left(\frac{2\cos 2\theta_W-1}{2\sin 2\theta_W}\right)^2\frac{(k^2-\omega_{pe}^2)^2}{M_Z^4}\operatorname{Im}\varepsilon_l^\nu|_{\omega=\omega_{pe}}\right\}, \tag{20}$$

where the imaginary parts of the Abelian and neutrino permittivities are, respectively,

$$\operatorname{Im}\varepsilon_l^A=\left(\frac{\pi}{2}\right)^{1/2}\Theta(k^2-\omega^2)\frac{\omega\omega_{pe}^2}{(kv_{Te})^3}\exp\left(-\frac{\omega^2}{2k^2v_{Te}^2}\right)$$

and

$$\operatorname{Im}\varepsilon_l^\nu=\frac{\pi^2e^2\omega T_\nu^2\Theta(k^2-\omega^2)}{6k^3},\qquad v_{Te}=\left(\frac{T}{m_e}\right)^{1/2},$$

$$\Theta(k^2-\omega^2)=1 \text{ for } k>\omega,\qquad \Theta(k^2-\omega^2)=0 \text{ for } k<\omega.$$

Because of the nonexponential dependence of the imaginary part $\operatorname{Im}\varepsilon_l^\nu$ on the neutrino temperature T_ν and wavelength k^{-1}, this dissipation mechanism is (for sufficiently large wavelengths) more important than the well-known mechanism of Landau damping on the electrons [see the first term in Eq. (20)].

5. We consider the screening of a test charged particle and an electrically neutral particle in the medium. To calculate the fields of test particles, we must write down the analogs of the Poisson equations for the scalar neutral components δA_0 and δZ_0 determined by the electric (δj_0^A) and weak (δj_0^Z) charges of a given test particle.

The system of the corresponding equations has the form

$$\varepsilon_l^A\delta A_0(\omega,\mathbf{k})+\frac{(2\cos 2\theta_W-1)}{2\sin 2\theta_W}(1-\varepsilon_l^A)\delta Z_0(\omega,\mathbf{k})=-\frac{\delta j_0^A(\omega,\mathbf{k})}{q^2}, \tag{21}$$

$$\frac{(2\cos 2\theta_W-1)}{2\sin 2\theta_W}(1-\varepsilon_l^A)\delta A_0(\omega,\mathbf{k})+\left[\varepsilon_l^\nu+\left(\frac{2\cos 2\theta_W-1}{2\sin 2\theta_W}\right)^2\times(\varepsilon_l^A-1)-\frac{M_Z^2}{q^2}\right]\delta Z_0(\omega,k)=-\frac{\delta j_0^Z(\omega,\mathbf{k})}{q^2}. \tag{22}$$

If the particle has electric charge Q, the sources of the fields on the right-hand sides of Eqs. (21) and (22) are, respectively, $\delta j_0^A=Q$ and

$$\delta j_0^Z=Q(2\cos 2\theta_W-1)/2\sin 2\theta_W,$$

where the last relation for the weak charge δj_0^Z follows from Eq. (9) and takes into account the exact compensation of the background external charge j_0^Z by the equilibrium terms of the tree approximation $C_0^Z M_Z^2$.

From Eqs. (21) and (22) we readily find in this case expressions for the static fields

$$\delta A_0(r)=(2\pi)^{-3}\int d^3k e^{i\mathbf{kr}}\delta A_0(0,\mathbf{k}),$$

$$\delta Z_0(r)=(2\pi)^{-3}\int d^3k e^{i\mathbf{kr}}\delta Z_0(0,\mathbf{k}),$$

which determine the screening of the electric charge Q in the considered medium. The potential of the electrostatic field is

$$\delta A_0(r)=\frac{Qe^{-r/r_D}}{r}\left[1-\left(\frac{2\cos 2\theta_W-1}{2\sin 2\theta_W}\right)^2 (M_Z r_D)^{-2}\right]$$
$$+\frac{2}{(M_Z r_D)^2}\left(\frac{2\cos 2\theta_W-1}{2\sin 2\theta_W}\right)^2\frac{Qe^{-M_Z r}}{r} \qquad (23)$$

and for an ordinary plasma, which has large Debye radius $r_D \gg M_Z^{-1}$, differs little from the well-known value in the Abelian electrodynamics of continuous media: $(Q/r)\exp(-r/r_D)$. For Weinberg angle $\theta_W = 30°$ there is no difference at all. In this sense, the electrodynamics of an ordinary plasma is an exact special case of the standard model[1] in the absence of $Z\gamma$ mixing [see Eqs. (11)–(14) above].

In addition, the electric charge Q in the presence of $Z\gamma$ mixing ($\theta_W \neq 30°$) produces a neutral field Z of small amplitude of the form[5)]

$$\delta Z_0(r)=\frac{Q(2\cos 2\theta_W-1)}{4\sin 2\theta_W}\left[\frac{e^{-r/r_D}}{(M_Z r_D)^2 r}-\frac{e^{-M_Z r}}{r}\right]. \qquad (24)$$

In both Eq. (23) and Eq. (24) the last terms are exponentially small at the large distances $r\sim r_D \gg M_Z^{-1}$ corresponding to the basic approximation (8′), and must be omitted. On transition to large momentum transfers, corresponding to short distances $r\sim M_W^{-1}$, it is necessary to take into account the contribution of the charged $W^\pm$ bosons to the polarization of the medium. This significantly changes the form of these terms, and their magnitude in a dense gas ($r_D \sim M_Z^{-1}$) will be of the same order as the main contributions in Eqs. (23) and (24).

If we ignore the relatively weak interaction of the right-handed neutrinos with matter, we can assert that the equations used here are also valid if the neutrinos have a small Dirac mass $\delta m_\nu \neq 0$. This makes it possible to calculate the screening of a test neutrino at rest in the considered medium on the basis of the same equations (21) and (22) as in the case of an electrically charged particle. At the same time, ignoring the mean square electromagnetic radius of the neutrinos, which is $\sim g^2 M_W^{-2}$ (see the explanations in Sec. 7), we can use in Eqs. (21) and (22) the values of the Fourier components of the external charges: $\delta j_0^A = 0$, $\delta j_0^Z = e/\sin 2\theta_W$.

Under these conditions, the solutions of the system (21), (22) have the following form. The quasielectrostatic Z field has the form

$$\delta Z_0(r)=\frac{e(2\cos 2\theta_W-1)}{(2\sin 2\theta_W)^3 (M_Z r_D)^4}\frac{e^{-r/r_D}}{r}+\frac{e}{2\sin 2\theta_W}\frac{e^{-M_Z r}}{r}, \qquad (25)$$

and the electrostatic A field, which exists in the absence of radiative corrections only by virtue of the $Z\gamma$ mixing, is

$$\delta A_0(r)=\frac{e(2\cos 2\theta_W-1)}{(2\sin 2\theta_W)^2(M_Z r_D)^2}\left[\frac{e^{-r/r_D}}{r}-\frac{e^{-M_Z r}}{r}\right]. \qquad (26)$$

As in the case of the electric charge, the contribution of the last terms in Eqs. (23) and (24) can be ignored under the conditions (8).

Note that the potential (26) produced by the test neutrino is an attractive potential for electrons [$-e\delta A_0(r) < 0$], and accordingly the field (24) creates an equal potential for attraction of neutrinos by an electron ($Q = -e$),

$$(e/\sin 2\theta_W)\,\delta Z_0(r) < 0$$

under the only condition that there is $Z\gamma$ mixing ($\theta_W \neq 30°$), the experimentally confirmed[8] inequality $\sin^2\theta_W < 0.25$ holding for the Weinberg angle (see footnote 4).

6. We determine the polarization energy losses of the neutrinos used in exciting the longitudinal A oscillations described by the dispersion relation (18a). For this, we calculate the energy flux of the electromagnetic field emitted by a neutrino ($\theta_W \neq 30°$) through the cylindrical surface of radius $\rho = b_0$ around its "trajectory."[6)] We take the neutrino velocity along the z axis: $c = (0,0,c)$. Then the energy losses per unit length are

$$\frac{dW_\nu^A}{dl}=\frac{b_0}{2}\int_{-\infty}^{\infty}\delta E_z^A \delta B_\varphi^A\,dz. \qquad (27)$$

Here, the components of the longitudinal and transverse fields $\delta E_i^A(\mathbf{r},t)$ can be determined from Eqs. (11) and (12), into which we substitute as sources the currents produced by the test neutrino:

$$\mathbf{j}_0^A=0,\qquad \mathbf{j}_0^Z(\omega,\mathbf{k})=\frac{2\pi e c\delta(\omega-\mathbf{kc})}{\sin 2\theta_W}.$$

Then, solving the system of equations, we find the electromagnetic field induced by the moving neutrino in the presence of $Z\gamma$ mixing:

$$\delta E_i^A(\mathbf{r},t)$$
$$=\frac{2ie(2\cos 2\theta_W-1)}{(2\sin 2\theta_W)^3(2\pi)^3}\left\{\int d^3k\exp[i(\mathbf{kr}-\omega t)](1-\varepsilon_l^A)k_i\right.$$
$$\times\left[k^2\left[\left(\varepsilon_l^\nu-\frac{M_Z^2}{q^2}\right)\varepsilon_l^A+\left(\frac{2\cos 2\theta_W-1}{2\sin 2\theta_W}\right)^2(\varepsilon_l^A-1)\right]\right]^{-1}$$
$$+\int d^3k\exp[i(\mathbf{kr}-\omega t)](1-\varepsilon_{tr}^A)\left[c_i-\frac{k_i(\mathbf{kc})}{k^2}\right]$$
$$\times\left[\omega\left[\left(\varepsilon_{tr}^\nu-\frac{M_Z^2+k^2}{\omega^2}\right)\right.\right.$$
$$\left.\left.\left.\times\left(\varepsilon_{tr}^A-\left(\frac{k}{\omega}\right)^2\right)+\left(\frac{2\cos 2\theta_W-1}{2\sin 2\theta_W}\right)^2(\varepsilon_{tr}^A-1)\right]\right]^{-1}\right\},$$
$$\omega=\mathbf{kc}. \qquad (28)$$

Here, the permittivities $\varepsilon_{l,tr}^{A,\nu}(\omega,\mathbf{k})$ are determined in accordance with Eqs. (15) and (16). Direct calculations by means of (28) of the losses (21) lead to the result

$$\frac{dW_\nu^A}{dl}=-\frac{(2\cos 2\theta_W-1)^2 e^2\omega_{pe}^2}{16\sin^4 2\theta_W}\left(\frac{\omega_{pe}}{M_Z}\right)^4\ln\frac{2c}{b_0\omega_{pe}\gamma}, \qquad (29)$$

where $\gamma = 0.577$ is Euler's constant, $\omega_{pe} = [(4\pi e_0^2 n_0^-)/m_e]^{1/2}$ is the Langmuir frequency, and $e_0^2 = (137)^{-1}$.

Note that the neutrino losses on the excitation of the longitudinal δZ oscillations described by the dispersion relation (17a) are of the same order as (29). For comparison, we give expressions for the polarization losses of a test electron in the same medium[7]:

$$\frac{dW_e^A}{dl}=-\frac{e^2\omega_{pe}^2}{v^2}\ln\frac{2v}{b_0\omega_{pe}\gamma} \qquad (30)$$

and for the collisional losses of neutrinos in direct νe collisions,

$$\frac{dW_{\nu e}^{\text{coll}}}{dl}=-\frac{e^2\omega_{pe}^2}{8\pi\sin^4\theta_W}\left(\frac{E_\nu}{M_W}\right)^4,\qquad E_\nu \ll \omega_{pe}, \qquad (31)$$

obtained by averaging over the electron distribution in the plasma.

As one would expect for the conditions of an ordinary plasma used in the present problem ($\omega_{pe} \ll M_W$), the polarization losses of the charged particles exceed by many orders of magnitude the polarization losses of the neutrinos [cf. (29) and (30)]. On the other hand, the collisionless polarization losses of the neutrinos in a dense plasma, $\omega_{pe} \gg E_\nu$, may be appreciably greater than the collisional losses [cf. (29) and (31)].

7. We now examine the mechanism of long-range weak forces and the possibility of the self-consistent field approximation in the considered medium with manifest symmetry breaking of the electromagnetic and weak interactions. In theories in which the Yang-Mills fields are massless, for example, in statistical QCD[6] or in the standard model[1] but with symmetry restoration at high temperatures,[3] the self-consistent field approximation is ensured by two factors: the presence of the long-range interaction manifested in the same dependence of the forces on the distance in the form r^{-1}, and the smallness of the coupling constant. These two factors together make it possible to introduce an ideality parameter of the medium, which is equal to the ratio of the small interaction energy to the mean kinetic energy, or the plasma parameter

$$(n_0 r_D^3)^{-1} \ll 1. \qquad (32)$$

For example, in the case of statistical QCD, substituting in the inequality (32) the quark-gluon plasma density $n_0 \sim T^3$ and the Debye radius $r_D \sim (gT)^{-1}$, we see that the plasma description is valid only far from the confinement radius, where asymptotic freedom guarantees the coupling constant is small: $g \ll 1$.

In the standard model,[1] a small coupling constant is also guaranteed, but the long range of the weak forces with manifest symmetry breaking, in particular the long-range interaction of the neutrinos with the electrons of the medium, appears a paradox.

Nevertheless, the examples of the calculations in this paper show that in the presence of $Z\gamma$ mixing ($\theta_W \neq 30°$) such a long-range interaction exists. It is explained by the fact that the rare collision of a neutrino with a plasma electron realized through exchange of a massive Z^0 boson sets in motion not only the recoil electron but also, through the self-consistent long-range electromagnetic field, a large number of neighboring electrons at distance $r \gtrsim r_D \gg M_Z^{-1}$. However, not every perturbation in the density of the weak charge and current (electron component) produced in a rare νe collision of this kind can be the source of a fluctuation of the electromagnetic field. The requirement of parity conservation in electromagnetic interactions tells us that only the vector part of such a perturbation δj_μ^Z in the density of the weak current (which does not contain γ_5 matrices) can be the source of a long-range δA field. This part of the current is proportional to the factor ($2\cos 2\theta_W - 1$), which occurs in all the examples with neutrinos considered above and does not vanish when $\theta_W \neq 30°$.

The long-range interaction is weakened in the range of small momentum transfers (8′) by the presence in the matrix element of νe scattering of the Fermi weak interaction constant $G_F = e^2/4\sqrt{2}M_W^2 \sin^2\theta_W$, which occurs in the final results in the form of the small dimensionless parameter $(M_Z r_D)^{-2} \ll 1$.

In conclusion, we discuss also the possibility of long-range interaction for a neutrino due to its having a mean square electromagnetic radius $\langle r_\nu^2 \rangle \simeq g^2 M_W^{-2}$. It is easy to show[2] that in the limit of small momentum transfers $q^2 \ll M_W^2$ the photon Green's function $1/q^2$ (with which hopes of such a long-range interaction are associated) cancels exactly against the limiting value of the renormalized vertex operator

$$\lim_{q^2\to 0} \Gamma_R{}^A \sim g^2 q^2/M_W{}^2, \qquad g = e/\sin\theta_W.$$

As a result, as was to be expected, this contribution will be a correction of order $g^2 \ll 1$ to the heavy Z^0 boson exchange that we have taken into account and leads to the perturbation in the density of the vector weak current.

8. The expression (29) obtained for the polarization losses is general for different media with different plasma frequencies ω_{pe}. In particular, it holds for the degenerate ultrarelativistic gas formed in black-hole collapse.

In the regime of free fall of matter to the center of a collapsing star, neutronization of the matter releases neutrinos with energy $E_\nu \sim 30$–40 MeV, which appreciably exceeds the neutronization thresholds in the corresponding nuclear reactions.[9] To estimate the collisional losses of hard neutrinos ($E_\nu \gg p_F$) in the degenerate gas, we cannot use the expression (31) but must use the different result of averaging over the distributions of the electron before scattering ($f(p) \sim \Theta(p_F - p)$) and the scattered electron ($f(p') = 1 - \Theta(p_F - p')$), which is

$$\frac{dW_{e\nu}^{\text{coll}}}{dl} = -\frac{9e^2\omega_{pe}^2}{1024\pi^3 \sin^2\theta_W}\left(\frac{E_\nu}{M_W}\right)^2\left(\frac{p_F}{M_W}\right)^2, \qquad (33)$$

where $\omega_{pe} = (4\pi e_0^2\, p_F^2/3)^{1/2}$.

From comparison of Eqs. (29) and (33) it can be seen that the collisional losses will be decisive for the hard neutrinos, and these must be taken into account in determining the neutrino opacity of a collapsing star with matter falling freely to the center.

However, in the early stage of collapse the density increases more slowly than in the free-fall regime in the peripheral part of the star or in the central core, where the pressure gradient decelerates the compression. Then the rate of neutronization may be greater than the rate of establishment of the equilibrium state of the degenerate gas with momentum $p_F \gg 1$. As a result, all the neutronization reactions begin from the threshold, and the energy $E_\nu = E_F - Q$ carried away by the neutrinos is small compared with the threshold Q. In this collapse regime, the polarization losses (29) appreciably exceed the collisional losses, which are estimated for a degenerate gas in accordance with an expression of the type (31):

$$\frac{dW_{\nu e}^{\text{coll}}}{dl} = -\frac{e^2\omega_{pe}^2}{160\pi^3 \sin^4\theta_W}\left(\frac{E_\nu}{M_W}\right)^4, \quad E_\nu \ll p_F. \qquad (31')$$

Note that in (31′) we take into account only the incoherent scattering by electrons of soft neutrinos with wavelength λ_ν greatly exceeding the mean distance p_F^{-1} between the charged particles. The incoherent losses of the electron neutrinos on the fixed ions (charge Z, mass number A) are even less than the losses (31′) due to the additional factor $(3Z - A)^2 m_e / M_A$,[10] where $M_A = AM_p$ is the ion mass.

In coherent elastic scattering, the neutrino energy is conserved[7)] (the energy transfer is zero, $\Delta E_\nu = 0$). Therefore, coherent scattering of neutrinos on the degenerate electrons is entirely absent because of the occupation of all electron states with momentum $p \leqslant p_F$. A change in the neutrino energy would lead to orthogonality of the final states in each successive multiple scattering, i.e., to the absence of an interference of the amplitudes and to simple summation of the cross sections (proportional to n_0, the particle density). This has already been taken into account in the calculation of the incoherent energy losses (31′).

Coherent scattering of the neutrinos on the nondegenerate ions is possible. However, this does not change the neutrino energy ($\Delta E_\nu = 0$), i.e., this contribution does not compete with the losses (29) and (31′). Thus, soft neutrinos with energy $E_\nu \ll \omega_{pe} < p_F$ lose virtually all the energy in the collapse process on the excitation of longitudinal plasma oscillations.

We thank D. Yu. Bardin for helpful discussions of questions associated with allowance for the radiative corrections in the standard model,[1] and Ya. A. Smorodinskiĭ for fruitful discussion of the results of the paper.

[1)]We use a system of units in which $\hbar = c = 1$, and the Feynman metric $A_\mu B^\mu = A_0 B_0 - \mathbf{A} \cdot \mathbf{B}$, $\mu = 0,1,2,3$; the standard representation of the Dirac γ matrices is used, and $\gamma_5 = \gamma_5^+ = i\gamma_0\gamma_1\gamma_2\gamma_3$.

[2)]The generalization of the results of the present paper to the case of a hot plasma presents no difficulty, since in the range of temperatures $m_e \ll T \ll M_W$ all the simplifications of the present problem are valid. Only the conditions of equilibrium are changed. In particular, in an equilibrium medium of electrons, positrons, and neutrinos ($T_e = T_\nu = T$, $n_0^- = n_0^+ = 2n_0^\nu$) not only the scalar components of the neutral Yang-Mills fields vanish (tree approximation) but also the weak charge of the system [$C \quad = j_0^Z = 0$; cf. Eq. (9)].

[3)]We do not give the macroscopic wave equations for the δE_i^W fields obtained from (3) and (4). In a homogeneous and isotropic medium the $\delta W^\pm$ excitations, described by the dispersion relation $\varepsilon_l^W - M_W^2/q^2 = 0$ for longitudinal oscillations and $\varepsilon_{tr}^W - (M_W^2 + k^2)/\omega^2 = 0$ for transverse oscillations, are absent in the range (8′) of small momentum transfers.

[4)]The external parameter of the standard model,[1] the Weinberg angle θ_W, can be calculated in a grand unification theory of higher symmetry, and the value obtained by the renormalization-group generalization of SU(5) theory, namely, $(\sin^2\theta_W)_{\rm th} \sim 0.2$ (instead of $\sin^2\theta_W = 3/8$ without renormalization), agrees well with the experimental result[8]: $(\sin^2\theta_W)_{\rm exp} = 0.224 \pm 0.020$.

[5)]In a hot lepton gas ($m_e \ll T \ll M_W$) Eq. (24) is augmented by the contribution of the pseudovector weak current, which does not vanish for any value of the Weinberg angle θ_W.

[6)]To use classical concepts it is, generally speaking, necessary to assume a nonvanishing neutrino mass $\delta m_\nu \neq 0$. The replacement of the velocity v by c in the present calculations is not fundamental in the derivation of the estimates of the energy losses.

[7)]In coherent scattering, only the direction of the neutrino momentum changes, and this is determined by the refractive index

$$n_\nu = 1 + \frac{2\pi}{p^2} \sum_a n_{0a} f_{\nu a}(0),$$

where $E_\nu = p$, ($m_\nu = 0$), and $f_{\nu a}(0)$ is the amplitude of (forward) scattering by an ions of species a.[10]

[1]S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in: Elementary Particle Physics, Almquist and Wiksells, Stockholm (1968), p. 367.
[2]D. Yu. Bardin, P. Ch. Christova, and O. M. Fedorenko, Nucl. Phys. B **197**, 1 (1982).
[3]D. A. Kirzhnits and A. D. Linde, Phys. Lett. B **42**, 471 (1972); A. D. Linde, Phys. Rev. D **14**, 3345 (1976).
[4]A. D. Linde, Phys. Lett. B **86**, 39 (1979).
[5]I. V. Krive, Yad. Fiz. **31**, 1259 (1980) [Sov. J. Nucl. Phys. **31**, 650 (1980)].
[6]É. V. Shuryak, Zh. Eksp. Teor. Fiz. **74**, 408 (1978) [Sov. Phys. JETP **47**, 212 (1978)]; O. K. Kalashnikov and V. V. Klimov, Yad. Fiz. **31**, 1357 (1980) [Sov. J. Nucl. Phys. **31**, 699 (1980)].
[7]A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, Élektrodinamika plazmy (Plasma Electrodynamics), Nauka, Moscow (1974), pp. 179, 656.
[8]C. Prescott, W. Atwood, R. Cottrell, *et al.*, Phys. Lett. B **84**, 524 (1979).
[9]Ya. B. Zel'dovich and I. D. Novikov, Teoriya tyagoteniya i évolyutsiya zvezd (The Theory of Gravitation and the Evolution of Stars), Nauka, Moscow (1971), p. 387.
[10]P. Langacker, J. P. Leveille, and J. Sheiman, Phys. Rev. D **27**, 1228 (1983).

Translated by Julian B. Barbour