Possibility of observing optical radiation in spontaneous transitions of channeled particles

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We consider various types of optical radiation produced when electrons are channeled in a crystal: Cherenkov radiation of δ electrons, transition radiation, and emission in spontaneous transitions between transverse-energy levels. We show that an important role is played in a thick crystal by Cherenkov radiation of ϑ electrons. In a thin crystal, at angles close to 90°, it is possible to observe optical radiation connected with spontaneous transitions. This radiation permits observation, for the first time, of the anomalous Doppler effect previously considered by Frank and Ginzburg.

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1. INTRODUCTION

The passage of relativistic electrons through crystals, and particularly their electromagnetic radiation, is attracting great interest (see, e.g., Refs. 1 and 2).

So far, however, there were few investigations of optical radiation of relativistic electrons. In this paper we calculate for the first time the radiation of channeled δ electrons. We show that in a crystal several hundred micron thick this radiation predominates (the Cherenkov radiation of the primary electrons is blocked by total internal reflection).

At angles close to 90° it is possible to observe the optical radiation due to transitions between transverse-energy levels of channeled particles. A concrete experimental setup is proposed for the observation of the anomalous Doppler effect first considered by Frank and Ginzburg with emission from a moving quantum oscillator as an example.[3] Optimal for this purpose are a direction close to 90° and a crystal thickness on the order of the dechanneling length. We compare various optical-radiation mechanisms and the principal one from the viewpoint of the experimental conditions is determined.

2. CHERENKOV RADIATION

Far from the Cherenkov-radiation cone, the radiation-energy distribution at a crystal thickness L is given by[4]

$$\frac{dW}{d\Omega}=\frac{e^2}{c}\frac{R(\theta)}{2\pi^2 n(\psi-\psi_0)^2}\times\int_{\omega_1}^{\omega_2}\left\{1-[\cos a\omega+d\omega\sin a\omega]e^{-ad\omega^2/3}+\frac{2d}{a}\right\}d\omega,$$

$$a=\frac{nL}{c}(\psi-\psi_0)\sin\psi_0,\quad d=\frac{nL\langle\psi^2\rangle\sin\psi_0}{2c(\psi-\psi_0)}. \qquad (1)$$

Here $\sin\psi = n^{-1}\sin\theta$, where n is the refractive angle, ψ_0 is the Cherenkov-cone angle, θ is the observation angle ($d\Omega=\sin\theta d\theta d\varphi$, $\langle\psi^2\rangle^{1/2}$ is the mean squared multiple-scattering angle, $R(\theta)$ is the radiation transmission coefficient, and $\omega_2-\omega_1$ is the optical-frequency band under consideration.

The calculated number of emitted photons of the Cherenkov radiation as well as of the other considered types of radiation are listed in Table I for a frequency band with $\Delta\omega/\omega=0.1$ and for a crystal thickness 5 μm. We consider next transition radiation.

3. TRANSITION RADIATION

The angular distribution of the energy radiated by a charged particle passing through a plate of thickness L is described, for normal incidence of the electron, by the equa-

TABLE I. Angular density of various types of optical radiation at $\theta\approx 90°$

crystal, direction	Energy, MeV	λ_{anom}, nm	λ_{norm}, nm	Cherenkov radiation, kV/sr	kV/sr	δ-electron Cherenkov radiation, kV/sr	Radiation in spontaneous transitions, kV/sr·μm	
							norm.	anom.
C ⟨110⟩ 2s−2p	4	160	423	$1.2\cdot10^{-6}$	$1.4\cdot10^{-7}$	$7.0\cdot10^{-7}$	$5.1\cdot10^{-9}$	$3.7\cdot10^{-8}$
KCl ⟨100⟩ 4p−3s	4.9	29	429 202	$7.7\cdot10^{-5}$	$7.2\cdot10^{-7}$	$1.4\cdot10^{-5}$	$1.3\cdot10^{-8}$	–
KCl ⟨110⟩ 2−1	4.9	55	788 370	$7.7\cdot10^{-5}$	$7.2\cdot10^{-7}$	$1.0\cdot10^{-5}$	$9.3\cdot10^{-9}$	–
KCl ⟨100⟩ 3−2	4.9	115	1663 782	$7.7\cdot10^{-5}$	$7.2\cdot10^{-7}$	$1.0\cdot10^{-5}$	$5.4\cdot10^{-9}$	$2.4\cdot10^{-9}$

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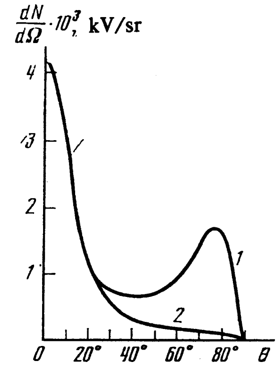

FIG. 1. Total angular density of Cherenkov radiation of primary and δ electrons and of transition radiation: 1—KCl, 2—diamond.

tion[5]

$$\frac{dW}{d\Omega}=\frac{e^2\beta^2}{\pi^2 c}\frac{\sin^2\theta\cos^2\theta}{(1-\beta^2\cos^2\theta)^2}\frac{(n^2-1)^2}{(1-\beta^2x^2)^2}\int_{\omega_1}^{\omega_2}|A(\omega,\theta)|^2\,d\omega, \quad (2)$$

where

$$A(\omega,\theta)=\Big[(x+y)(1\mp\beta x)(1\pm\beta x-\beta^2)\exp\left(-i\frac{\omega}{c}Lx\right)+(x-y)(1\pm\beta x)(1\mp\beta x-\beta^2)\exp\left(i\frac{\omega}{c}Lx\right)-2x(1\mp\beta\cos\theta)(1\pm\beta\cos\theta-\beta^2n^2)\exp\left(i\frac{\omega}{c}L\right)\Big]\times\Big[(x+y)^2\exp\left(-i\frac{\omega}{c}Lx\right)-(x-y)^2\exp\left(i\frac{\omega}{c}Lx\right)\Big]^{-1} \quad (3)$$

$$x=(n^2-\sin^2\theta)^{1/2},\quad y=n^2\cos\theta$$

(the upper and lower signs pertain here to backward and forward radiation). The angular distribution of the transition radiation is negligibly changed when the electron incidence angle deviates little from normal (within 1–3°).

Figure 1 shows the angular dependences of the number of emitted optical photons of the sum of the Cherenkov radiation of the primary and δ electrons and of the transition radiation. It can be seen that the minimum-intensity angle is $\approx 90°$. The spontaneous emission produced by channeling and considered below (Fig. 2) has no sharply pronounced

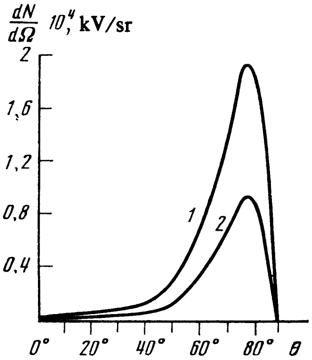

FIG. 2. Angular dependence of Cherenkov radiation of δ electrons when channeled (1) and in a non-oriented target (2) for KCl; $E = 5$ MeV.

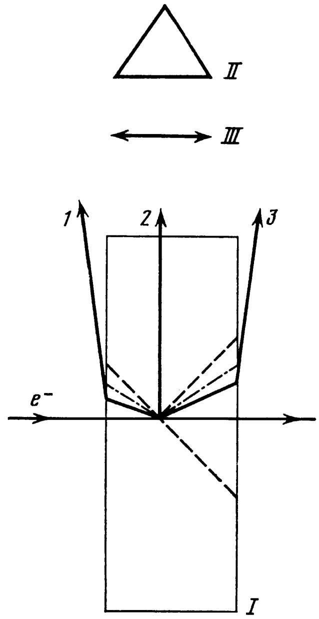

FIG. 3. Possible experimental setup: dashed line—Cherenkov cone; dash-dot—total internal reflection angle, 1—normal wave λ_1, 2—normal wave λ_2, 3—anomalous wave, I—crystal, II—detector, III—lens.

angular dependence. In this connection, the proposed experimental setup for the observation of this radiation is shown in Fig. 3. We examine now the characteristics of the spontaneous emission.

4. SPONTANEOUS OPTICAL EMISSION IN TRANSITIONS BETWEEN TRANSVERSE-ENERGY LEVELS

Transitions between transverse-energy levels of a channeled particle produce mainly x rays.[6] However, in view of the complicated character of the Doppler effect several radiation modes can exist in a medium with light dispersion. The high-frequency (Kumakhov) mode is emitted forward at angles $\theta\leqslant\gamma^{-1}$, where $\gamma=(E+mc^2)/mc^2$ is the Lorentz factor. There are also one or several additional optical modes[7] (of course, for transparent crystals), which are emitted at all angles.

For optical frequencies in the transparency region, the refractive index can be regarded as approximately constant: $n = \text{const} > 1$. The Doppler formula for the optical mode is then simplified:

$$\omega=\Omega_{ab}/(1-\beta n\cos\theta), \quad (4)$$

where $\hbar\Omega_{ab}=E_{\perp b}-E_{\perp a}$ is the difference of the transverse energies.

The angular distribution of the probability of the transparent emission of a photon of frequency ω per unit length is given, in the planar case, by the formula

$$dW_{ab}/dt=\Pi_b e^2|\Omega_{ab}|^2x_{ab}^2n\{(1-\beta n\cos\theta)^2-(1-\beta^2n^2)\sin^2\theta\cos^2\varphi\}\{2\pi\hbar c^4|1-\beta n\cos\theta|^4\}^{-1}, \quad (5)$$

where

$$x_{ab}=\int\psi_a^*x\psi_b\,dx, \quad (6)$$

where ω and Ω_{ab} connected by relation (4), the angle φ is measured from the oscillation plane, Π_b is the probability of populating the level b:

$$\Pi_b=\frac{1}{d_p}\left|\int e^{ip_\perp x}\psi_b(x)\,dx\right|^2,$$

and x_{ab} is the dipole matrix element. Here d_p is the distance between the planes, $p_\perp$ is the transverse momentum of the incident particle, ψ_a and ψ_b are the wave functions of the final and initial states of the transverse motion and are determined, just as the eigenvalues $E_{\perp a}$ and $E_{\perp b}$, from a Schrödinger equation with relativistic mass m:

$$\frac{\hbar^2}{2m}\nabla^2\psi+(E_\perp-U)\psi=0. \tag{7}$$

For axial channeling, $|x_{ab}|^2$ must be replaced by $|\mathbf{r}_{ab}|^2=r_{ab}{}^2/2$ and $\cos^2\varphi$ by 1/2, in the case when states connected with a single chain are considered and the potential can be regarded as axisymmetric. It is then necessary to take into account in U the centrifugal potential, and ∇^2 must be expressed in curvilinear cylindrical coordinates.

The most intense are transitions between bound states. For these states Eq. (7) can be solved in the tight-binding approximation with boundary conditions $|\psi|^2\to 0$ as x (or r)$\to\infty$. The equation was solved numerically.[1)]

It follows from (5) that, other conditions being equal, the probability is proportional to the square of the frequency ω. Equations (4) and (5) are valid also for the x-ray branch at $n=1$. Therefore the numbers of x-ray and optical photons emitted forward into a unit solid angle differ by approximately $(\omega/\Omega_{ab})^2\approx 4\gamma^4$ times.

As $\beta n\to 1$, the Cherenkov cone divides the optical branch in fact into two separate Doppler branches, normal ($\beta n\cos\theta<1$) and anomalous ($\beta n\cos\theta>1$).

After integrating over the angles, the number of x-ray photons emitted on going from state a to b is $4\gamma^2$ times larger than that of the optical photons. The exact number of photons in the interval $\Delta\omega$ is

$$W_{ab}=\Pi_b\frac{e^2\cdot}{2\hbar c}\frac{x_{ab}{}^2\Omega_{ab}{}^2}{c^3\beta}\Delta\omega\left\{1+\frac{1}{\beta^2n^2}\right\}$$
$$\times\left\{1-\frac{1-\beta^2n^2}{1+\beta^2n^2}\frac{\Delta\omega}{\Omega_{ab}}+\frac{1}{3}\frac{(1-\beta^2n^2)^2}{1+\beta^2n^2}\frac{\Delta\omega^2}{\Omega_{ab}{}^2}\right\}\cdots \tag{8}$$

Table I lists the values of $dW_{ab}/d\Omega$ and W_{ab} for the strongest optical transitions.[2)] It can be seen from the table that observation of optical photons from spontaneous transitions of channeled particles against the background of the transition radiation is difficult even in the range where the latter is close to a minimum. It must be borne in mind that the crystal should be thin enough (several microns at an electron energy of several MeV), since the initial level population was used in the calculations. The detector entry angle must not exceed the critical channeling angle (Fig. 3).

5. CHERENKOV RADIATION OF δ ELECTRONS

One of the reasons why optical radiation is produced when electrons pass through crystals is the Cherenkov radiation of δ electrons. We consider first the characteristic of the δ-electron optical radiation when a beam of electrons passes through a non-oriented target. The total number W_0 of the optical photons emerging from a crystal having a length L less than the mean free path of the δ electrons is then given by

$$W_0(E)=\rho_0N_0L^2\frac{\Delta\omega}{\omega}\int_{\xi_{min}}^{\xi_{max}}\sin^2\delta f(\xi)\,[1-R(\theta)]\,\sigma(E,\xi)\sin\xi d\xi, \tag{9}$$

where the optical photons in the δ-electron Cherenkov cone which are emitted forward from the crystal constitutes the fraction

$$f(\xi)=\begin{cases}\dfrac{1}{\pi}\arcsin\alpha, & \operatorname{tg}\delta\operatorname{tg}\xi\cos\varphi_{cr}\geqslant b,\\[2mm] \dfrac{1}{2}+\dfrac{1}{\pi}\arcsin\alpha, & \operatorname{tg}\delta\operatorname{tg}\xi\cos\varphi_{cr}<b.\end{cases}$$

In these equations

$$b=[2\cos\xi\cos\varphi_{cr}\cos\delta-\cos^2\delta-\cos(\xi-\varphi_{cr})\cos(\xi+\varphi_{cr})]^{1/2},$$
$$\alpha=b/\operatorname{tg}\xi\operatorname{tg}\delta\sin\delta,$$

ξ is the δ-electron emission angle relative to the primary-electron trajectory; $\delta(\xi)=\arccos(1/n\beta)$ is the Cherenkov cone angle of a δ electron having a velocity $v=\beta c$. Next,

$$\sigma(E,\xi)=-\frac{2\pi e^4}{E^2}\left[\frac{\gamma^2}{\cos^4\xi}+\frac{\gamma-1}{\gamma+1}\frac{4}{\sin^4\xi}+\frac{4(\gamma-1)^3(\gamma+1)}{\gamma^2(1+\cos^2\xi+\gamma\sin^2\xi)}-\frac{2(\gamma-1)(2\gamma-1)}{\gamma^2\sin^2\xi\cos^2\xi}\right]$$

is the Møller cross section for δ-electron emission[8]; ρ_0 is the average density of the crystal electrons; N_0 is the number of optical photons emitted per unit length by an electron having a velocity $v\approx c$; E is the kinetic energy of the beam electron; $R(\theta)$ is the coefficient of internal reflection for a photon leaving the crystal at an angle θ; φ_{cr} is the total-internal-reflection angle; ξ_{min} and ξ_{max} are the minimum and maximum emission angles of the δ electrons, at which their Cherenkov radiation emerges from the crystal; $\Delta\omega/\omega$ is the width of the considered optical line.

The angular distribution of the number of δ-electron Cherenkov-radiation photons is given by

$$\frac{dN}{d\Omega}=\frac{e^2}{\hbar c}\frac{L^2}{4\pi c}\Delta\omega\rho_0[1-R(\theta)]\int_{\xi_1}^{\xi_2}\sin^2\delta\frac{\sigma(E,\xi)}{\eta}\sin\xi d\xi, \tag{10}$$

where $d\Omega=\sin\theta d\theta d\varphi$ is the solid-angle element, while ξ_1 and ξ_2 are determined from the equation

$$\xi_{1,2}-\delta(\xi_{1,2})\mp\eta/2=\pm\arcsin(\sin\theta/n), \tag{11}$$

that defines those δ electrons which radiate into the given solid angle; η is the Cherenkov-radiation cone width and is goverened by the dispersion $n(\omega)$, by the multiple scattering of the electrons in the crystal, and by the diffraction divergence (in the approximation in which the radiation is uniformly distributed over the cone width.

We consider now the effect of channeling on the emergence of the δ electron. According to Fermi the inelastic-scattering probability is

$$\frac{dW}{dt}=\frac{2\pi}{\hbar}\left|\langle fF|\sum_l V_l|iI\rangle\right|^2\delta(E_{fF}-E_{iI}),\qquad(12)$$

where

$$\sum_l V_l=\sum_l\frac{e^2}{|\mathbf{r}-\mathbf{r}_l|^2}$$

is the potential energy of the interaction with the electrons (the perturbation), i and f are the initial and final states of the channeled particle, and I and F are the same for the crystal. From the energy conservation law, which is expressed in (12) by the δ-function, it follows that the energy transferred to the crystal electron is

$$\Delta E=\Delta E_\perp+\Delta E_\parallel=E_F-E_I.$$

Expanding the perturbation in a Fourier integral, we obtain

$$\frac{dW}{dt}=\frac{2\pi}{\hbar}\delta(E_F-E_I-E_{\perp f}+E_{\perp i}-E_{\parallel f}+E_{\parallel i})$$

$$\times\left|\int d\mathbf{q}V(q)\langle f|\exp i\mathbf{qr}|i\rangle\langle F|\sum_l\exp(-i\mathbf{q}\mathbf{r}_l)|I\rangle\right|^2,\qquad(13)$$

where $V(q)=4\pi e^2/q^2$. The integration over the particle longitudinal coordinates along which the motion is described by plane waves yields the conservation of the longitudinal momentum $\mathbf{q}_\parallel=\mathbf{p}_{\parallel i}-\mathbf{p}_{\parallel f}$.

Production of δ electrons corresponds to large energy transfers, such that the final states can be regarded as plane waves. Since these energies exceed greatly the binding energies of the atomic electrons as well as of the channeled ones, we can integrate with respect to the final momenta p_f of the scattered electron and with respect to q, assuming that $p_I\ll q$. In the upshot we get

$$\frac{dW_{ch}}{dt}=\frac{2\pi}{\hbar}\int d\mathbf{q}_\perp d\mathbf{q}_\perp'V(q)V^*(q')\psi_i^2(\mathbf{r}_\perp)d\mathbf{r}_\perp e^{i(\mathbf{q}-\mathbf{q}')\mathbf{r}_\perp}$$

$$\times e^{i(\mathbf{q}\mathbf{r}_l-\mathbf{q}'\mathbf{r}_l')}\psi_I(\mathbf{r}_l)\psi_I^*(\mathbf{r}_l')d\mathbf{r}_l\,d\mathbf{r}_l'$$

$$=\frac{2\pi}{\hbar}V(q)\int\psi_i^2(\mathbf{r}_\perp)\int\psi_I^2(\mathbf{r}_\perp,z)\,dz\,d\mathbf{r}_\perp.\qquad(14)$$

Comparing this result with the plane-wave case (i.e., replacing ψ_i by $\exp i\mathbf{p}_\perp\cdot\mathbf{r}_\perp/\hbar$), we see that the cross section for the production of the δ electrons can be obtained by multiplying the usual cross section dW_a by the overlap coefficients of the square of the wave function ψ_i and the crystal electron density averaged along the z axis:

$$dW_{ch}=QdW_a,$$

$$Q=\int\psi_i^2(\mathbf{r}_\perp)\int\psi_I^2(\mathbf{r}_\perp,z)\frac{dz}{d}\,d\mathbf{r}_\perp.$$

It is then necessary to replace the average crystal-electron density ρ_0 in the expression for the angular density of the radiation and for the total photon yield by

$$f_e=\sum_{i=1}\Pi_i\int_0^{2\pi}d\chi\int_0^{r_m/2}\psi_i^2(\mathbf{r}_\perp)\rho(r_\perp)r_\perp\,dr_\perp,\qquad(15)$$

where Π_i is the population of the i-th level and r_m is the distance to the nearest chain of atoms.

When calculating the increase of the δ-electron yield for channeled electrons we used the Moliere model for the potential and density of the atom electrons, and averaged over the z axis and over the thermal vibrations. The transverse-motion wave functions were calculated in the tight-binding approximation. In a KCl crystal at an electron energy 5 MeV, ten transverse-energy levels are produced. The greatest increase of the δ-electron yield is obtained for the 1s state, by approximately 20 times, and for example for the 2p state, by six times. The total increase of the δ-electron yield for channeled particles with allowance for the initial capture and for the level population is approximately 3.5.

Table II lists the angular densities and the total yield of the optical radiation in a frequency band with $\Delta\omega/\omega=0.1$ for oriented and non-oriented KCl crystals of different thickness. Figure 2 shows the angular distribution of the intensity of the Cherenkov radiation of electrons for channeled and non-channeled electrons at a crystal thickness equal to the characteristic dechanneling length (estimated at 2 μm in the calculations).

CONCLUSION

Analysis of the results shows that the optimum conditions for the observation of optical radiation in transitions between transverse energy levels of a channeled electron are a crystal thickness close to the dechanneling length and an observation angle close to 90° relative to the incident beam. The detector frequency resolution should be ~10% and the angular resolution of the order of the critical angle, i.e., ~1°. A possible experimental setup is shown in Fig. 3.

Attempts to measure the optical radiation[9,10] in spontaneous transitions of channeled electrons should be regarded as unsuccessful, since the target thickness and the observation angles were not suitably chosen (see the detailed criticism[11] of the premises of Ref. 9).

The angular distribution of the sum of the Cherenkov radiation of the primary and δ electrons and of the transition radiation do not contradict those observed in Ref. 9. It can be seen from Table II that for a thick crystal the orientation dependence turns out to be weaker than in the experi-

TABLE II. Angular density, kV/sr, of optical radiation of δ electrons in non-oriented target and in the case of channeling

Target thick-ness, μm	Non-oriented, KCl			Oriented, KCl ⟨100⟩		
	$\theta=0°$	$\theta=40°$	$\theta=85°$	$\theta=0°$	$\theta=40°$	$\theta=85°$
5	$6.0\cdot10^{-7}$	$2.0\cdot10^{-6}$	$7.5\cdot10^{-5}$	$1.2\cdot10^{-6}$	$4.0\cdot10^{-6}$	$1.5\cdot10^{-4}$
20	$4.8\cdot10^{-6}$	$1.6\cdot10^{-5}$	$6.0\cdot10^{-4}$	$5.8\cdot10^{-6}$	$1.9\cdot10^{-5}$	$7.2\cdot10^{-4}$
100	$5.3\cdot10^{-5}$	$1.8\cdot10^{-4}$	$6.7\cdot10^{-3}$	$5.5\cdot10^{-5}$	$1.9\cdot10^{-4}$	$7.0\cdot10^{-3}$

ments.[9,10] It appears that the effect observed in these experiments was partially due to hard radiation, more likely to the action of scattered electrons or δ electrons. Favoring this assumption is also the fact that no increase in the backward photon yield was observed,[10] since optical radiation should be reflected back from the crystal boundary.

1)The authors thank V. I. Telegin for kindly supplying the program.

2)A line (level) width $\Delta\omega/\omega = 0.1$, which is close to the actually observed values, was used.

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