Hydrodynamical theory of multiple processes in light of contemporary experimental data

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Improved calculations are given for certain characteristics of multiple processes in the framework of the hydrodynamical theory. The principal attention is devoted to the dependence of the average transverse momentum $\langle p_\perp \rangle$ on the energy of the colliding particles. This dependence characterizes the interaction of the constituents (quarks and gluons) in the hydrodynamical stage of the process. The distributions of the secondary particles in pseudorapidity are also studied. The calculated characteristics are used for comparison with experimental data obtained in accelerators of the new generation (the SPS collider and the ISR). Satisfactory agreement between the experimental results and the theoretical predictions is observed.

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1. INTRODUCTION

Thirty years ago Landau[1] formulated the hydrodynamical theory of multiple production of particles. In this theory the collision process is broken down into three stages.

1. In the initial stage a large number of constituent particles[1] arise in a small volume. The mean free path of the constituents is small in comparison with the dimensions of the system, and statistical equilibrium is established in it.

2. In the second stage an expansion of the system occurs, which is described by the relativistic hydrodynamics of an ideal liquid (the hydrodynamical stage).

3. In the third stage after expansion and cooling the mean free path of the constituents is comparable with the dimensions of the system and transformation of the constituents into particles occurs.

All stages of the process are to a greater or lesser degree modeled in terms of statistical physics. The theory involves also a nonstatistical element—the choice of the initial conditions in the form of a Lorentz-contracted volume with longitudinal dimension $r_\parallel \sim m_p/m_\pi\sqrt{s}$ and transverse dimensions $r_0 \sim 1/m_\pi$ ($\sqrt{s}$ is the total energy of the colliding particles in the center-of-mass system, m_π is the pion mass, and $\hbar = c = 1$).[2)]

Although in the hydrodynamical theory a microscopic process is modeled in macroscopic terms (the pressure p and energy density $\mathscr{E}$), this theory has served as the basis for important predictions which, after many years and only after creation of new generations of accelerators, have finally been verified experimentally. The principal predictions which have been confirmed by experiment are as follows:

1. The dependence of the average multiplicity $\langle N \rangle$ on s (or on E_L—the initial energy in the laboratory system; see the data on the dependence of $\langle N \rangle$ on s up to $E_L \sim 10^3$ GeV in Ref. 2 and up to $E_L \sim 10^5$ GeV in Ref. 3).

2. The differential dependences dN/dy and $dN/d\eta$ (y and η are the rapidity and pseudorapidity[3)]); see the data at energy $E_L \sim 10^3$ GeV (Refs. 5 and 6).

3. A bound on the average transverse momentum $\langle p_\perp \rangle$ and its distribution. This fact, which is of very great importance for high energy physics, was predicted on the basis of the hydrodynamical theory.[7]

4. The dependence of $\langle p_\perp \rangle$ on the mass of the secondary particles.[8,9]

5. Universality of the distributions in the transverse mass $m_\perp = (p_\perp^2 + m^2)^{1/2}$ independent of the type of secondary particle[10] (m is the particle mass).

6. The weak dependence of $\langle N \rangle$ on the atomic number of the target nucleus (recent data can be found in Ref. 11).

7. An exceptionally weak dependence of $\langle p_\perp \rangle$ on s or E_L. This prediction has fundamental significance for the hydrodynamical treatment and will be discussed in Section 2.

Thus, the hydrodynamical theory has led to a number of important predictions which have received quantitative confirmation in accelerators of the lastest generations (the ISR with energy $\sqrt{s} \sim 50$–60 GeV and the SPS Collider with energy $\sqrt{s} \sim 540$ GeV).

However, in spite of the important heuristic achievements of the hydrodynamical theory, it has not received universal acceptance. Evidently this situation is the result of three arguments:

1. The microscopic diagram method is more adequate for the interaction of elementary particles.

2. The choice of initial conditions in the form of a Lorentz-contracted disk is only in crude agreement with the uncertainty principle.[12,13]

3. The assumption of complete statistical equilibrium is inconsistent with the presence of leading particles which preserve the quantum numbers of the primary hadrons. Let us consider these arguments in more detail.

The diagram method came into dominance after the well known work of Amati *et al.*,[14] which served as the beginning of the multiperipheral approach. In the initial version the exchange between vertices was accomplished by pions. As a result ρ mesons were formed, which eventually decayed into pions. However, this variant was found to contradict experiment (the predictions of a falloff of total cross sections and the plateau in the dN/dy distribution) and therefore it was modified into the multireggeon theory

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where exchange of pions was replaced by exchange of reggeons, i.e., by complex multiparticle formations essentially equivalent to a multiperipheral ladder. By this means (see for example Ref. 15) it was possible by including many Regge trajectories to obtain a constant total cross section.

However, for description of the set of experimental data it was then necessary to take into account also branching, i.e., multi-reggeon exchanges.[16,17] Since each reggeon itself is a multiparticle formation,[16] we can say that contemporary diagram description of multiple production requires simultaneous interaction of many particles. Furthermore this fact is an important prerequisite for establishment of statistical equilibrium. Therefore we are now seeing a certain convergence of the hydrodynamical theory and the diagram description of multiple processes (details can be found in Ref. 9). Apparently one can say that the diagram treatment and the hydrodynamical theory can be considered within certain limits to be microscopic and macroscopic theories of the same phenomenon.

Considering the question of initial conditions, it should be mentioned that the choice of the longitudinal dimensions ($r_\parallel \sim m_p/m_\pi\sqrt{s}$) contains a certain arbitrariness and either can be considered as an additional postulate of the theory or can be modified, depending on various model considerations (see for example Refs. 9 and 18). We emphasize that such modifications have relatively little effect on the theoretical distributions (except for the dependence of $\langle N \rangle$ on s).

The existence of leading particles can be justified (very crudely) on the basis of the Pauli principle[19] and also on the basis of the quark model. Experiments show that the average inelasticity coefficient $\langle K \rangle$ is equal to about 0.5 over a wide range of energies. We can therefore assume that a fraction of the energy $\sim 0.5\sqrt{s}$ goes into the statistical system. In the initial stage this system consists mainly of gluons.[18]

In conclusion it should be stated that the possible modifications of the hydrodynamical theory which we have mentioned are more moderate than the changes which the diagram method has undergone during its existence.

2. THE HYDRODYNAMICAL THEORY AND TRANSVERSE MOMENTUM

We mentioned above that the important prediction of a bound on $\langle p_\perp \rangle$ and an estimate of this quantity were made on the basis of the hydrodynamical theory. However, a detailed effect—the very weak increase of $\langle p_\perp \rangle$ with the energy E_L—is of more significance for verification of the hydrodynamical idea. The physical reason for this rise is the existence of the hydrodynamical stage of expansion of the system and the transition from quasi-one-dimensional motion to three-dimensional. The motion in the transverse direction is enhanced (but very slowly) with increase of the energy and produces a rise of $\langle p_\perp \rangle$. The very fact that $\langle p_\perp \rangle$ rises with increase of the energy reflects the existence of a relatively strong interaction of the constituents in the hydrodynamical stage. The magnitude of this effect will depend only weakly on the initial conditions and therefore also on the model of hadronization of the constituents, and is an important test of the principal element of the theory—the hydrodynamical stage.

However, it must also be mentioned that at the present time no theoretical values of the rise of $\langle p_\perp \rangle$ have been determined, as a consequence of exceptional difficulties in solution of three-dimensional problems in relativistic hydrodynamics. Various approximate solutions of the three-dimensional equations have been obtained in Refs. 1 and 20, but here the transverse momentum was not calculated. An investigation of the transverse momentum was carried out in Ref. 21, where the three-dimensional solutions were represented in parametric form and then the extremely complicated equations were solved numerically. However, in calculation of the distributions in $p_\perp$ a noninvariant expression was used for the single-particle distribution (see below), which violates energy conservation. This fact is evidently one of the reasons for the exaggeration of the values obtained for $\langle p_\perp \rangle$. We note also that in view of the complexity of the equations it is difficult here to take into account the influence of possible modifications of the boundary conditions on the numerical results. The nature of the rise of $\langle p_\perp \rangle$ was studied also in an article by Chaichian *et al.*[22] However, that work actually investigated the rise of the transverse mass of the elements of a liquid for angles not close to $\pi/2$, and not the transverse momenta of particles, and numerical values of $\langle p_\perp \rangle$ were not given.

In view of the importance of the dependence of $\langle p_\perp \rangle$ on E_L and the existence of new experimental data obtained in the SPS Collider ($E_L \sim 150$ TeV), we have carried out new calculations of $\langle p_\perp \rangle$ over a wide range of energies. These calculations are based on the following ideas:

1. A significant part of the hydrodynamical expansion is quasi-one-dimensional motion which is described by an exact solution.[4]

2. The motion in the transverse direction is determined mainly by the thermal motion.

3. The quasi-one-dimensional solution is valid as long as the path of an element of the liquid in the transverse direction is small in comparison with the transverse dimension of the system r_0. The surface of transition from the one-dimensional state to the three-dimensional stage is determined by the condition

$$t^2 - x^2 = r_0^2. \tag{1}$$

4. If we have $t^2 - x^2 > r_0^2$, the motion (three-dimensional) is quasi-inertial, i.e., close to conical emission. The velocity of an element along the X axis (the emission axis) is almost constant.

The condition (1) characterizes the joining of the one-dimensional and three-dimensional stages and is a relativistically invariant generalization of the inequality $t \lesssim r_0^2/(t-x)$ which in accordance with Ref. 1 determines the applicability of the one-dimensional approximation. Indeed, since the elements of the liquid are moving with velocities $V \sim 1$, we have $t + x \sim 2t$; then in order of magnitude the inequality given above goes over into our equation (1). We recall that the inequality $t \lesssim r_0^2/(t-x)$ was obtained[1] from approximate relations between the components of the energy-momentum tensor T_{ik} perpendicular to the axis of the motion.

The condition (1) determines the relation between the temperature $\tau = \ln(T/T_0)$ and the rapidity of the elements of the liquid $y_1 = \operatorname{arctanh} V_x$ in the region of the transition to the three-dimensional stage. This condition can be written in the form

$$\left(\frac{\partial\chi}{\partial\tau}\right)^2-\left(\frac{\partial\chi}{\partial y_1}\right)^2=r_0^2e^{2\tau}. \tag{2}$$

The quantity χ is described by the exact solution[4] of the one-dimensional equations:

$$\chi=\frac{\Delta}{2c_s}e^{\cdot}\int\limits_{c_sy_1}^{\tau}\exp\left(-\frac{1+c_s^2}{2c_s^2}\tau'\right)I_0\left(\frac{1-c_s^2}{2c_s^2}[(\tau')^2-c_s^2y_1^2]^{1/2}\right)d\tau'. \tag{3}$$

Here c_s is the velocity of sound and $\Delta = 4m_p r_0/\sqrt{s}$ is the longitudinal dimension of the initial volume with allowance for the effect of leading and shock compression. From the conditions (2) and (3) we can obtain the distribution of the entropy in the rapidity y:

$$\frac{dS}{dy_1}\infty e^{-\tau}\Phi(\tau, y_1),$$

$$\Phi(\tau,y_1)=\left[\left(\frac{\partial\psi}{\partial y_1}\right)^2-c_s^2\left(\frac{\partial\psi}{\partial\tau}\right)^2\right]\left[\frac{\partial\chi}{\partial y_1}\frac{\partial\psi}{\partial y_1}-\frac{\partial\chi}{\partial\tau}\frac{\partial\psi}{\partial\tau}\right]^{-1}, \tag{4}$$

where $\psi = \partial\chi/\partial t - \chi$, and τ and y_1 are related by the condition (2). We note that Landau[1] obtained the distribution of the entropy and energy in the angles in the asymptotic approximation (high energies and $|\tau| > |y_1|$). Distributions analogous to the solutions of Ref. 1 can be obtained in the asymptotic approximation from the relations (2)–(4). Indeed, setting

$$\partial\chi/\partial\tau\sim\Delta\exp\left[-\tau+(\tau^2-c_s^2y_1^2)^{1/2}\right],$$

we obtain

$$dS\infty dN\infty\exp\left[-L+(L^2-y_1^2)^{1/2}\right],\quad L=\ln\left(\sqrt{s}/4m_\pi\right). \tag{5}$$

From relations of the type (2)–(4) we can also extract more accurate solutions (not only asymptotic ones), expanding the Bessel function in (3) in series:

$$I_0(z)=1+\left(\frac{z}{2}\right)^2+\frac{1}{4}\left(\frac{z}{2}\right)^4+\dots,\quad z=\frac{1-c_s^2}{2c_s^2}(\tau^2-c_s^2y_1^2)^{1/2}$$

and calculating $\partial\chi/\partial\tau$, $\partial\chi/\partial y_1$. Estimates showed that up to SPS Collider energies ($z_{\max}\approx 3$) one can obtain an accuracy $\sim$5–7% by keeping only three terms of the expansion. As a result we obtain

$$\frac{\partial\chi}{\partial\tau}=\frac{\Delta}{2c_s}e^{-\tau}\left\{\frac{1}{2}e^{2(\tau-B)}+\frac{1}{2}I_0\left(\frac{1-c_s^2}{2c_s^2}(\tau^2-B^2)^{1/2}\right)\right.$$
$$+\frac{(B^2-\tau^2)(2\tau+1)}{128}$$
$$+\frac{1}{8}\left[e^{2(\tau-B)}\left(B+\frac{1}{2}\right)-\left(\tau+\frac{1}{2}\right)\right]$$
$$\left.+\frac{e^{2(\tau-B)}(2B^2+3B+3/2)}{128}-\frac{2\tau^2+3\tau+3/2}{128}\right\}$$

$$\frac{\partial\chi}{\partial y_1}=\frac{\Delta}{2c_s}e^{-\tau}\left\{-\frac{Bc_s}{4}e^{2(\tau-B)}\right.$$
$$+\frac{Bc_s}{2(\tau^2-B^2)^{1/2}}I_1\left[\frac{1-c_s^2}{2c_s^2}(\tau^2-B^2)^{1/2}\right]$$
$$-\frac{Bc_s(B^2-\tau^2)}{32}\frac{2\tau+1}{24}+\frac{Bc_s}{32}\left[\left(\tau+\frac{1}{2}\right)-e^{2(\tau-B)}\left(B+\frac{1}{2}\right)\right]$$
$$\left.+\frac{Bc_s}{32\cdot 24}\left[2\tau^2+3\tau+\frac{3}{2}-e^{2(\tau-B)}\left(2B^2+3B+\frac{3}{2}\right)-c_se^{2(\tau-B)}\right]\right\},$$
$$B=c_sy_1. \tag{6}$$

Using Eqs. (2) and (6) we can show that with accuracy about 10% the motion of the elements of the liquid, after the region of transition to the three-dimensional stage is reached, is inertial:

$$(t+x)/(t-x)\sim e^{2y_1}.$$

For calculation of the transverse momenta of the particles it is necessary to take into account in explicit form the thermal motion, which is this case plays the principal role.[23] Assuming that in the proper system of the element of the liquid the thermal motion is isotropic, we can write the invariant single-particle distribution for a relativistic gas in the form[24]

$$E\frac{dN}{d^3p}=\int f(x,p)p^\mu\, d\sigma_\mu, \tag{7}$$

where p_μ are the momenta of the particles, $d\sigma_\mu$ is the element of the three-dimensional hypersurface in the 4-volume, and $f(x,p)$ is the invariant distribution function:

$$f(x,p)=f(\bar{x},\bar{p})=\frac{g}{(2\pi)^3}\left[\exp\left(\frac{p^\mu u_\mu}{T}\right)-1\right]^{-1} \tag{8}$$

($\bar{x}$ and $\bar{p}$ are values in the rest system of the element).

The function (8) describes real particles (mainly pions) in the escape stage (at a temperature $T_f = m_\pi$). At higher values of the temperature T_0 the statistical system consists of constituents (mainly gluons) with a large number of degrees of freedom, which also determine the values of the average multiplicity $\langle N\rangle$ of secondary particles.[18]

At the present time it is assumed that at a temperature somewhat above $T_f = m_\pi$ there is a phase transition from the quark-gluon phase to the hadron phase, which possibly is a transition of second order. However, in view of the absence of a sufficiently well founded model of hadronization, this question is not discussed in the present article and we shall merely assume that the rapidity distributions of the hadrons preserve the distribution of the constituents.

The quantity $\langle p_\perp\rangle$ is determined from the relation

$$\langle p_\perp\rangle=\int p_\perp\, dN\Big/\int dN, \tag{9}$$

where dN is the number of particles in an element of phase space:

$$dN=Ap_\perp\, dp_\perp\, dy_1\, dy\left\{\exp\left[\frac{m_\perp \operatorname{ch}(y-y_1)}{T_t(y_1)}\right]-1\right\}^{-1}p^\mu\, d\sigma_\mu.$$

Here we have used the relation (8), where

$$p^\mu u_\mu = m_\perp \operatorname{ch}(y-y_1),$$

$m_\perp$ is the transverse mass, and y is the rapidity of the particles. The quantity $T_t(y_1)$ is the temperature in the region of the transition to the three-dimensional stage, which for high energies can be somewhat above the assumed escape temperature $T_j \sim m_\pi$. The further three-dimensional motion and cooling can, generally speaking, lead to some increase of $\langle p_\perp \rangle$ (see below).

A numerical calculation on the basis of Eqs. (2) and (6) permits one to obtain the relation $T_t(y_1)$. For example, for the SPS Collider energies we have $T_t(0) = 1.47 m_\pi$, $T_t(2) = 1.32 m_\pi$, $T_t(4.56) = m_\pi$, and for ISR energies:

$$T_t(0) = 1{,}05\, m_\pi, \quad T_t(1{,}9) = 1.02\, m_\pi.$$

The quantity $p^\mu d\sigma_\mu$ [with inclusion of (2)–(4)] has the form

$$p^\mu d\sigma_\mu = \pi r_0^2 m_\perp e^{-\tau_t}\left[\frac{\partial\chi}{\partial\tau}\operatorname{ch}(y-y_1) + \frac{\partial\chi}{\partial y_1}\operatorname{sh}(y-y_1)\right]\Phi(\tau, y_1), \quad \tau_t = \ln\frac{T_t}{T_0}. \qquad (11)$$

Substituting (11) into (10) and integrating over all variables, we obtain the value of $\langle p_\perp \rangle$. For simplification of the calculations we shall replace $m_\perp$ by $p_\perp$. In this case the integrals over $dp_\perp$ and dy_1 are calculated analytically. As a result we can write

$$\langle p_\perp \rangle = \left\{3\pi\zeta(4)\int_0^{y_{1\,max}} T_\tau(y_1)e^{2\tau_t(y_1)}\frac{\partial\chi}{\partial\tau}\Phi(\tau, y_1)\,dy_1\right\} \times \left\{4\zeta(3)\int_0^{y_{1\,max}} e^{2\tau_t(y_1)}\frac{\partial\chi}{\partial\tau}\Phi(\tau, y_1)\,dy_1\right\}^{-1}. \qquad (12)$$

The integrals over dy_1 in the numerator and denominator were calculated graphically (see Fig. 1). The ratio of these integrals (i.e., of the areas) for the SPS Collider energies is 1.3 and consequently

$$\langle p_\perp \rangle = 1.3 m_\pi (3\pi\zeta(4)/\zeta(3)) \sim 0{,}39 \text{ GeV}.$$

A similar calculation for ISR energies ($\sqrt{s} = 53$ GeV) gives $\langle p_\perp \rangle \sim 0.33$ GeV.

We have also carried out a more accurate calculation without the substitution $m_\perp \to p_\perp$. In this case the result is

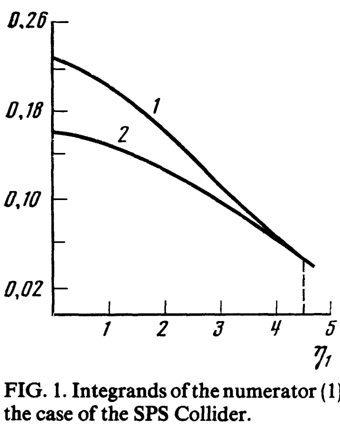

FIG. 1. Integrands of the numerator (1) and denominator (2) in Eq. (12) for the case of the SPS Collider.

Type of particle	$\langle p_\perp \rangle$, MeV	
	E_L=100 GeV	E_L=1000 GeV
π^-	320	340
π^+	333	350
K^-	400	420
K^0	420	430
P	450	470
All particles (without distinction as to type)	340	360

expressed in the form of integrals of series of the form $\Sigma_n K_n(m_\perp n/T_t)$, which converge rather rapidly. This gives some increase of $\langle p_\perp \rangle$: $\langle p_\perp \rangle \sim 0.43$ GeV for the SPS Collider and ~ 0.35 GeV for the ISR. The change of $\langle p_\perp \rangle$ in the three-dimensional stage was taken into account by replacing the argument of the exponential in (10) by the quantity

$$[m_\perp \operatorname{ch}(y-y_1)\operatorname{ch}\xi - p_\perp \operatorname{sh}\xi\cos\varphi]/m_\pi,$$

where $\operatorname{sh}\xi$ is the transverse hydrodynamical velocity, which was determined from comparison of the ratios $\int dE/\int dN$ at $T = T_t$ and $T = m_\pi$. This leads to a small increase of $\langle p_\perp \rangle$ for the SPS Collider: $\langle p_\perp \rangle = 0.45$ GeV. For ISR energies the increase is hardly noticeable. The calculation accuracy of our estimates[4)] is about 5%.

It was mentioned above that in Ref. 21 in calculation of $\langle p_\perp \rangle$ a noninvariant expression was used for the single-particle distribution:

$$\frac{EdN}{d^3p} = \int f(x, p)\langle E\rangle u^\mu\, d\sigma_\mu,$$

which leads to some violation of the conservation of energy. We emphasize, however, that Milekhin's article[21] has an important significance for the problem under discussion, since in it the rise of $\langle p_\perp \rangle$ with the energy E_L was noted for the first time.

According to the data reported at the Paris Conference (see Ref. 9), for an energy $\sim 10^5$ GeV (the SPS Collider) $\langle p_\perp \rangle \sim 0.42$ GeV,[5)] whereas for energies $E_L \sim 10^3$ GeV (ISR) $\langle p_\perp \rangle \sim 0.35$ GeV, which agrees reasonably well with the theoretical estimates. However, the very fact that $\langle p_\perp \rangle$ increases is more important. This is a serious argument in favor of existence of the hydrodynamical stage.

It is necessary to mention that the published data obtained in the SPS Collider are insufficiently accurate to completely exclude a rise of $\langle p_\perp \rangle$ as the result of an increase of the fraction of heavy particles. In order to reduce the probability of such an interpretation, we have given a table of data on $\langle p_\perp \rangle$ for particles of various types but at lower energies E_L. The table was taken from Ref. 25. These data indicate a definite rise [although it is small ($\sim$5–7%) for a change of the energy by an order of magnitude] in $\langle p_\perp \rangle$ for particles of various types (including pions). Apparently this tendency is preserved also with increase of the energies E_L.

3. DISTRIBUTIONS IN RAPIDITY AND PSEUDORAPIDITY

The number of particles in an element of 4-volume is given by Eq. (10). Integrating this expression over dy_1 and

$dp_\perp$, we can obtain the distribution of the particles in rapidity.

However, in an experiment one usually measures the distribution in the pseudorapidity η of the particles. Conversion from rapidities y to values of $p_\perp$ and η is made by means of the formula

$$y = \mathrm{Arth}\left[\frac{p_\perp}{(p_\perp^2+\mu_1^2)^{1/2}}\,\mathrm{th}\,\eta\right], \qquad (13)$$

where $\mu_1 = m_\pi/\mathrm{ch}\,\eta$. The distribution in particle pseudorapidity is given by the expression

$$\frac{dN}{d\eta} = A_1 \int_{-y_{1\,max}}^{y_{1\,max}} \int_0^\infty \frac{dy_1\, dp_\perp^2\, m_\perp \,\mathrm{ch}\,\eta}{(p_\perp^2\,\mathrm{ch}^2\,\eta + m_\pi^2)^{1/2}}$$
$$\times\left\{\exp\left[\frac{m_\perp \mathrm{ch}(y-y_1)}{T_1(y_1)}\right]-1\right\}^{-1}\Phi(\tau, y_1)$$
$$\times e^{-\tau_1}\left[\frac{\partial\chi}{\partial\tau}\mathrm{ch}(y-y_1)+\frac{\partial\chi}{\partial y_1}\mathrm{sh}(y-y_1)\right], \qquad (14)$$

where the value of y is determined by Eq. (13).

Integration over $dp_\perp$ in (14) can be carried out analytically in a number of cases, and integration over dy_1 is carried out numerically.

The distribution for the energies of the SPS Collider was normalized to the experimental value ~ 29 particles,[26] which is close to the theoretical predictions for these energies.

The results of the calculations for energies $\sqrt{s} = 540$ and 53 GeV are given in Fig. 2. In the same figure we have shown experimental data taken from Refs. 26–28.

4. CONCLUSION

Thus, the hydrodynamical theory with small modifications describes reasonably well the latest experimental data.

Apparently the behavior most important for the hydrodynamical theory is the observed rise of $\langle p_\perp \rangle$. This can serve as a justification for use of statistical-hydrodynamical methods for description of multiple processes, at least in the stage of interaction of the constitutents.

The highly developed diagram model[16,17] which competes with these methods also in essence simulates the multiparticle interaction. This is apparently the reason for the successful description of the experimental data in the framework of two externally so different approaches. We note, however, that the question of calculation of the transverse momentum in the diagram model remains open. Modeling of the transition of hadrons into constituents (quarks an gluons) and of constitutents into hadrons remains a question which has not been finally solved for any model, in the following sense: as a result of the lack of theory of confinement, the process of hadronization of quarks cannot be calculated in terms of quantum chromodynamics. In the hydrodynamical theory, the hadronization quarks is described simply and elegantly as a phase transition. Unfortunately this description is in no way related to the basic ideas of quantum chromodynamics.

We note also that in the near future studies of interaction of relativistic nuclei are planned. In view of the complexity of these processes, the statistical-hydrodynamica approach will undoubtedly play an important role in their interpretation.

Apparently it is not accidental that some well known physicists[29] already analyze the interaction of relativistic nuclei on the basis of the hydrodynamical theory.[a)]

In conclusion the authors thank E. L. Feinberg for discussion of questions related to this work and P. Karlson for communicating the lastest results obtained at the SPS Collider.

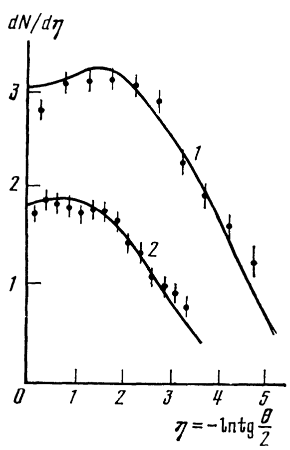

FIG. 2. Distribution of $dN/d\eta$ for the SPS Collider energies: $\sqrt{s} = 540$ GeV (1) and for ISR energies: $\sqrt{s} = 53$ GeV (2).

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1) At the present time the constituents should be identified with quarks and gluons.

2) Obviously extraction of the size of nucleons from statistical considerations is impossible.

3) In high-energy physics the pseudorapidity was first introduced by Landau,[1] and the rapidity by Khalatnikov.[4]

4) Note that in our calculations we have used a value of the velocity of sound $c_s = (1/3)^{1/2}$, which is an additional postulate of theory.

5) According to the data of P. Karlson (private communication) the latest results obtained in the SPS Collider lead to a value $\langle p_\perp \rangle = 0.44$ GeV. This value corresponds to the identified particles—pions.

6) In Refs. 9 and 29 use was made of initial conditions different from those adopted in Landau's article.[1] This circumstance emphasizes the arbitrariness in selection of initial conditions in the form of a Lorentz-contracted disk, although the theory with such conditions satisfactorily describes the experimental data (see below).

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