β decay in the field of an intense electromagnetic wave

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A simple method is proposed for calculation of the probabilities of quantum processes with formation of nonrelativistic charged particles in the field of an intense electromagnetic wave. On the basis of the method developed, a detailed discussion of β decay of nuclei in the field of a wave is given for the case of small energy release $\varepsilon_0 \ll m$, and also in the cases in which the nucleus is stable in the absence of a field. Effects of the possible difference of the neutrino mass from zero are studied.

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1. INTRODUCTION

Study of quantum processes in intense electromagnetic fields can provide valuable information on the properties of the particles taking part in these processes and on their interactions. The importance of study of such processes is due also to possible applications in astrophysics. In recent years as a result of development of laser technology there has been increased interest in processes occurring in strong fields.

A number of studies have been devoted to processes of weak interaction in intense electromagnetic fields. There have been discussions in particular of pion and muon decay in strong fields,[1–3] and of neutron decay in constant crossed electric and magnetic fields[4] and in the field of an electromagnetic wave.[5] Becker *et al.*[6] have studied the β decay of the ^{3}H nucleus in the field of a plane electromagnetic wave of circular polarization. These authors concluded that in the case of fields achievable at the focus of the radiation of modern high-power lasers an increase of the probability of β decay of tritium by 10^2–10^4 times is possible. Similarly it was concluded in Ref. 5 that an appreciable increase in the probability of β decay of the neutron is possible. However, an accurate calculation of the probability of β decay of nuclei in the field of an electromagnetic wave carried out in Refs. 7 and 8 showed that these results are incorrect. We note that the erroneous nature of the conclusions of Refs. 5 and 6 can be seen even from simple estimates (see below).

In Refs. 7 and 8 the discussion was carried out in the limit of a weak field $\chi \equiv eF/2\varepsilon_0(2m\varepsilon_0)^{1/2} \ll 1$, where F and ω are the field-strength amplitude and frequency of the electromagnetic wave, ε_0 is the kinetic energy release in β decay, and m is the mass of the electron) in two cases: $\omega \to 0$ (the limit of constant crossed electric and magnetic fields) and $\omega \sim \varepsilon_0$.

In the present work we study in detail the β decay of nuclei in the field of a circularly polarized wave in the case in which the emitted electron has nonrelativistic energies ($\varepsilon_0 \ll m$). For arbitrary values of the intensity and frequency of the external field we have obtained an expression for the β-decay probability in the form of a single integral. We have studied analytically the cases of weak fields ($\chi \ll 1$) and strong fields ($\chi \gg 1$) and also the β decay of nuclei which are stable in the absence of a field. On the assumption $\omega \ll \varepsilon_0$ we have studied the dependence of the β-decay probability on the frequency of the field. We have considered effects due to a possible difference of the neutrino rest mass m_ν from zero.

In this work we present a simple method of calculating the probabilities of quantum processes with formation of a nonrelativistic electron, which permits the cumbersome calculations which are usually encountered in the standard approach to be avoided.

2. CALCULATION OF β-DECAY PROBABILITY IN AN EXTERNAL FIELD

The influence of the field of an intense electromagnetic wave on the characteristics of β decay can be taken into account by replacing the wave function of free motion of the produced electron by the exact solution of the Dirac equation in the field of a plane electromagnetic wave.[1–8] Here the influence of the Coulomb field of the daughter nucleus on the electron wave function is not taken into account, an approximation which is permissible in the case of small nuclear charges and relatively large energy release ε_0. The conditions under which the Coulomb interaction can be neglected are discussed in more detail in Sec. 7.

As will be shown below, with the field intensities achievable at the present time under laboratory conditions, an appreciable change of the probability of β decay can arise only in the case $\varepsilon_0 \ll m$. Therefore in the present work we use the nonrelativistic wave function of an electron in the field of an electromagnetic wave, which in the dipole approximation can be represented in the form

$$\Psi_\mathbf{k}(\mathbf{r},t)=\exp\left\{i\mathbf{kr}-\frac{i}{2m}\int^t[\mathbf{k}-e\mathbf{A}(t')]^2\,dt'\right\}. \tag{1}$$

The wave function (1) is the exact solution of the Schrödinger equation for a particle in a time-varying uniform electric field described by a vector potential $\mathbf{A}(t)$. We consider a circularly polarized wave $\mathbf{A}(t)=A_0\{\cos\omega t, \sin\omega t, 0\}$, which simplifies the calculation and facilitates comparison with the results of other studies. The amplitude of the electric-field strength F is related to A_0 by the equation $F=\omega A_0$.

The standard method of calculating the probabilities of quantum processes in strong fields[1–8] is to expand the wave function of the particle in the field (or the amplitude of the process) in Fourier series, which permits the probability of the process to be expressed in the form of a sum of partial

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probabilities corresponding to absorption from the wave or emission into the wave of n quanta:

$$dW=\sum_{n=n_0}^{\infty} W_n \delta(E_i+n\omega-E_f)\,d\nu_f, \qquad (2)$$

where $d\nu_f$ is the density of final states. The coefficients of the expansion of the wave function (1) (or of the corresponding relativistic wave functions) are Bessel functions of the form $J_n((eF/m\omega^2)k_\perp)$, and therefore the right-hand side of Eq. (2) contains sums of the squares of Bessel functions with weights which depend on n, which are then integrated over the electron momentum. Calculation of the corresponding expressions involves considerable computational difficulties. Use of the uniform asymptotic behavior of Bessel functions at large values of the index and replacement of the summation in (2) by integration[1,2] will not lead to any substantial improvement of the situation.

Expansion of the wave function (1) in Fourier series in the time variable essentially means going over to the energy representation, which can simplify the calculation only in the case in which energy is conserved. In the case in which the system is in a strong nonstationary external field, use of the energy representation in calculation of the total probabilities of processes is undesirable. In the present work all calculations are carried out without expansion of the wave function in Fourier series, which permits a substantial simplification of the calculation.

The probability of the process is determined by the square of the amplitude modulus and contains a double integral, with respect to time, of the square of the matrix element of the transition operator. Transforming in this interval from the variables t_1 and t_2 to the variables $t=(t_1+t_2)/2$ and $\tau=(t_1-t_2)/2$ and performing the integration over t, we obtain for the β-decay probability in the field of a wave per unit time the expression

$$W=\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3p}{(2\pi)^3}|M_{fi}|^2\,2\int_{-\infty}^{\infty}J_0\left(2\frac{eF}{m\omega^2}k_\perp\sin\omega\tau\right)$$
$$\times\exp\left\{+2i\left(\frac{k^2}{2m}+\varepsilon_k+E_\nu-\varepsilon_0\right)\tau\right\}d\tau. \qquad (3)$$

Here k and p are the electron and neutrino momenta, $k_\perp$ is the electron momentum component in the plane of rotation of the electric field strength vector, E_ν is the neutrino energy, $\varepsilon_k=e^2F^2/2m\omega^2$ is the average energy of oscillation of the electron in the field, $\varepsilon_0=M_i-M_f-m$ is the kinetic energy release, M_i and M_f are the masses of the parent and daughter nuclei (we neglect the recoil energy of the daughter nucleus), and M_{fi} is the time-independent part of the matrix element of the transition operator (in this case—of the weak interaction Hamiltonian). As follows from (1), the values of M_{fi} coincide with the corresponding expressions in the case of ordinary β decay calculated with electron wave functions in the form of plane waves. In the approximation of allowed β transitions the values of $|M_{fi}|^2$ do not depend on the momenta of the electron and neutrino and can be taken outside the integral sign in Eq. (3).

The transformation from (3) to the expression for the probability W in the standard approach using a Fourier expansion can be accomplished by means of the formula[9]

$$J_0(2x\sin y)=\sum_{n=-\infty}^{\infty}J_n^2(x)\,e^{2in y}.$$

In calculating the probability of β decay in a field it is convenient first to perform the integration over the electron and neutrino momenta. The first integral is carried out easily in cylindrical coordinates, and the second is trivial. As a result, at $m_\nu=0$ Eq. (3) becomes

$$W=2^{-5}\pi^{-4}(\pi i)^{1/2}m^{3/2}|M_{fi}|^2\omega^{7/2}\int_{-\infty}^{\infty}\frac{dx}{(x+i0)^{7/2}}$$
$$\times\exp\left\{-i\left[\delta x+\gamma\left(\frac{\sin^2x}{x}-x\right)\right]\right\}. \qquad (4)$$

Here we have used the notation $x=\omega\tau$, $\delta=2\varepsilon_0/\omega$, $\gamma=e^2F^2/m\omega^3$. As we shall see below, the principal parameter characterizing the probability of β decay in an external field is the quantity

$$\chi=(\gamma/\delta^3)^{1/2}=eF/2\varepsilon_0(2m\varepsilon_0)^{1/2}.$$

This formula gives the probability of β decay for arbitrary values of the field-strength amplitude F in the wave and of the frequency ω. The integral in (4) can be calculated numerically. However, in all cases of practical interest this integral is easily studied analytically. Setting $\gamma=0$ in (4), we obtain the probability of β decay in the absence of an external field:

$$W_0=|M_{fi}|^2(2m)^{3/2}(4/105\pi^3)\,\varepsilon_0^{7/2}. \qquad (5)$$

The characteristic frequencies of laser radiation are of the order $\omega\sim1$ eV, and therefore we can assume $\delta\gg1$. Here the integral in (4) gets its main contribution in the region $|x|\ll1$, which permits the expression $\sin^2x$ in the argument of the exponential under the integral sign to be expanded in series. Retaining the first two terms in this expansion, we obtain

$$W\approx2^{-5}\pi^{-4}(\pi i)^{1/2}m^{3/2}|M_{fi}|^2\omega^{7/2}$$
$$\times\int_{-\infty}^{\infty}\frac{dx}{(x+i0)^{7/2}}\exp\left\{-i\left[\delta x-\gamma\frac{x^3}{3}\right]\right\}. \qquad (6)$$

The integral in (6) has the form of an Airy integral, and its asymptotic behavior in various cases is investigated by the usual methods (see for example Ref. 10). The case in which $\sin^2x$ in the argument of the exponential in (4) cannot be expanded in series is discussed in Sec. 5.

3. β DECAY IN A WEAK FIELD

Before turning to calculation of the β-decay probability in the case of relatively weak fields, we shall give simple estimates of this quantity.

Change in the probability of β decay in the presence of an external field can occur as follows. Let there be produced virtually in β decay an electron with energy differing by an amount $\Delta\varepsilon$ from the value ε determined by conservation of energy. The lifetime of this virtual electron will be $\tau_0\sim(\Delta\varepsilon)^{-1}$. If during this time the electron acquires from the field an energy equal to $\Delta\varepsilon$, it can become real. Taking it into account that the energy accumulated in a time τ_0 is

$\delta\varepsilon \sim eF(\varepsilon/m)^{1/2}\tau_0$, we find that the parameter $(\delta\varepsilon/\varepsilon)$ which characterizes the change of the probability of the process in the field essentially coincides with the quantity χ defined above. The meaning of this result is simple: the decay probability is influenced only by the energy which the electron acquires in the field at distances from the nucleus of the order of its de Broglie wavelength $(2m\varepsilon_0)^{-1/2}$, i.e., in the region of formation of the electron produced in β decay. The energy obtained at larger distances can change the angular and energy distributions of the electrons but not influence the probability of β decay, since this process has already been completed.

To estimate the value of W we can assume that it is determined by the probability of ordinary β decay W_0 taken at an energy $\varepsilon_0 + \delta\varepsilon_D$, where $\delta\varepsilon_D$ is the energy acquired by the electron at distances of the order of its de Broglie wavelength $\lambda\!\!\!^{-}_D$. In relatively weak fields $(\chi \ll 1)$ we have $\delta\varepsilon_D \ll \varepsilon_0$, and the quantity $W_0(\varepsilon_0 + \delta\varepsilon_D)$ can be expanded in powers of $\delta\varepsilon_D$:

$$W \approx W_0(\varepsilon_0+\delta\varepsilon_D) \approx W_0(\varepsilon_0) + W_0'(\varepsilon_0)\delta\varepsilon_D + {}^1/_2 W''(\varepsilon_0)(\delta\varepsilon_D)^2. \tag{7}$$

The quantity $\delta\varepsilon_D \sim eF(t_0)(2m\varepsilon_0)^{-1/2}$ will depend on the phase of the field ωt_0 at the moment of creation of the electron. Averaging over this phase and over the directions of emission of the electron, we obtain from (7) with inclusion of (5)

$$W \approx (2m)^{3/2}\left(\frac{4}{105\pi^3}\right)|M_{fi}|^2\left[\varepsilon_0^{7/2} + \frac{1}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\varepsilon_0^{3/2}\frac{e^2F^2}{3(2m\varepsilon_0)}\right]$$
$$= W_0(\varepsilon_0)\left[1+\frac{35}{6}\chi^2\right]. \tag{8}$$

The value of $\delta\varepsilon_D$ is proportional to the cosine of the angle between the vectors $\mathbf{F}(t_0)$ and $\mathbf{k}$. For this reason averaging over the directions of emission of the electron leads to vanishing of the term in (8) linear in χ, even in the case of a constant field when averaging over the phase of the field need not be carried out.

Let us consider now the integral (4) in the limit $\gamma \ll \delta^3 (\chi \ll 1)$. Since $\delta \gg 1$, the value of γ in this case can be quite large. The parameter γ can be written in the form $\gamma = (F/F_c)^2 \cdot (m/\omega)^3$, where $F_c = m^2/e \approx 1.3\cdot10^{16}$ V/cm is the critical field strength at which creation of e^-e^+ pairs from the vacuum begins. Setting $\omega \sim 1$ eV, we obtain $\gamma \approx 1$ at $F \approx 5\cdot10^7$ V/cm.

In the integral entering into (4) the important regions are $|x| \lesssim \delta^{-1} \ll 1$ and the regions of the stationary-phase points which lie outside this region. The contributions from the stationary points are rapidly oscillating functions of the field-strength amplitude F in the wave and on averaging over small fluctuations these quantities give zero contribution. In the special case $\delta \ll \gamma$ the stationary-phase points can be found analytically:

$$x_{1,2} = \pm\xi^{-1} \equiv \pm(\delta/\gamma)^{1/2}.$$

The corresponding contribution to the probability of the process is $W_0(105/16)\chi^4\cos(2/3\chi)$. We note that the existence of such rapidly oscillating contributions for $\chi \ll 1$ in the case of muon and pion decays in the field of a wave was observed in Refs. 1 and 2.

Neglecting these oscillating contributions, we obtain from Eq. (6) the following expression:

$$W(\chi) = W_0\sum_{m=0}^{\infty}\frac{\Gamma(^9/_2)}{m!3^m\cdot\Gamma(^9/_2-3m)}\chi^{2m}. \tag{9}$$

From comparison of (9) and (8) it follows that the simple estimate given above satisfactorily reproduces the coefficient of χ^2 in the expansion (9).

Since the parameter χ does not depend on the frequency ω of the field, Eq. (9) gives essentially the probability of β decay in a constant field. The dependence of W on ω can be obtained easily by taking into account the next terms of the expansion of $\sin^2 x$ in series in the argument of the exponential in (4), which gives

$$W = W_0\left[1+\frac{35}{8}\chi^2\left(1-\frac{1}{30\delta^2}+O(\delta^{-4})\right)+O(\chi^4)\right]. \tag{10}$$

Thus, the dependence of W on ω for $\chi \ll 1$ is determined by the parameter $\delta^{-2} \ll 1$, i.e., the decay probabilities in the field of the wave and in a constant field of corresponding strength have similar values.

This result is easy to understand from qualitative considerations. The dependence of probability W on the field frequency ω is determined by the parameter ωt_x, where t_x is the characteristic time of occurrence of the process. In this case t_x is the time of traversal by the electron of a distance equal to its de Broglie wavelength, i.e., $t_x \sim t_0 = \varepsilon_0^{-1}$. From time-reversal invariance it follows that the expansion should be carried out in even powers of the parameter $\omega/\varepsilon_0 = (2/\delta)$, which gives the desired estimate.

It should be noted that the expressions (9) and (10) for the probability of β decay in relatively weak fields cannot be obtained by means of perturbation theory in the electron interaction with an external field. To obtain these expressions we used the approximation $\gamma\delta^{-3} \ll 1$, whereas the perturbation-theory parameter, as can be seen from (3) and (4), is the quantity $\gamma\delta = [e^2F^2(2m\varepsilon_0)/m^2\omega^4]$, which can be very large.

4. β DECAY IN STRONG FIELD

Initially we shall give simple estimates of W in the case $\chi \gg 1$. In sufficiently strong fields the energy collected by the electron at distances of the order of its de Broglie wavelength $\lambda\!\!\!^{-}_D$ can become of the order of ε_0. Here it is necessary to consider that the quantity $\lambda\!\!\!^{-}_D$ itself changes with change of the electron energy. As the electron moves away from the nucleus its energy increases, and the value of $\lambda\!\!\!^{-}_D$ decreases and at some moment of time t_1 it becomes equal to the distance l_1 between the electron and the nucleus. The probability of β decay in the field is influenced only by the energy collected at a distance of the order of l_1; on further motion of the electron its distance from the nucleus becomes greater than $\lambda\!\!\!^{-}_D$ and the corresponding energy has no effect on the total probability of the process. The value of t_1 is easily estimated on the assumption that the energy collected at distances of the order of l_1 substantially exceeds ε_0; this gives

$t_1 \sim (m/e^2F^2)^{1/3}$. From this we obtain $l_1 \sim eFt_1^2/m = (eFm)^{-1/3}$. The energy obtained by the electron at distances of the order of l_1 is $\delta\tilde{\varepsilon}_D \sim t_1^{-1} = (e^2F^2/m)^{1/3}$, from which we obtain $(\delta\tilde{\varepsilon}_D)/\varepsilon_0 \sim \chi^{2/3}$. Thus, the principal parameter which determines the probability of the process for $\chi \gg 1$ is $\chi^{2/3}$.

It is easy to estimate the β-decay probability W for $\chi \gg 1$. We have

$$W \approx W_0(\varepsilon_0 + \delta\tilde{\varepsilon}_D) \approx W_0(\delta\tilde{\varepsilon}_D) \sim W_0(\varepsilon_0\chi^{2/3}) \sim W_0(\varepsilon_0)\chi^{7/3},$$

so that finally

$$W = C_1 W_0(\varepsilon_0)\chi^{7/3}, \qquad (11)$$

where C_1 is a constant of the order of unity. The dependence of W on the frequency of the field should be determined by the parameter $(\omega t_1)^2 \sim \delta^{-2}\chi^{-4/3}$.

Let us turn now to calculation of the probability of β decay in a field for $\chi \gg 1$. From Eq. (6) for $\gamma \gg \delta^3$ we obtain

$$W(\chi) = W_0\chi^{7/3}\left\{\frac{35}{144}3^{5/6}\pi^{-1/2}\sum_{m=0}^{\infty}\frac{(-1)^m 3^m}{m!\chi^{2m/3}} \times \sin\left(\frac{2}{3}\pi m + \frac{\pi}{6}\right)\Gamma\left(\frac{m}{3} - \frac{7}{6}\right)\right\}. \qquad (12)$$

In the case of sufficiently strong fields it is possible in principle to have β decay of nuclei which are stable in the absence of an external field and for which $\varepsilon_0 < 0$. A discussion of such processes for $\chi \ll 1$ is given in the next section. The result for the case $\chi \gg 1$ differs from (12) only in the absence of the factor $(-1)^m$ under the summation sign in the right-hand side of the formula. Here in W_0 and χ it is necessary to replace ε_0 by $|\varepsilon_0|$.

The dependence of W on the frequency ω can be obtained by taking into account the next terms of the expansion of $\sin^2 x$ in series in the argument of the exponential in (4), which gives

$$W = W_0(|\varepsilon_0|)\chi^{7/3}\left\{\frac{5\cdot 3^{5/6}\cdot\Gamma(5/6)}{8\sqrt{\pi}} \pm \frac{7\cdot 3^{1/6}\cdot\Gamma(1/6)}{16\sqrt{\pi}}\chi^{-2/3} + \frac{35\sqrt{3}}{48}\chi^{-4/3}\left(1 + \frac{2}{15\delta^2}\right) + O(\chi^{-2})\right\}. \qquad (13)$$

The upper sign refers to the case $\varepsilon_0 > 0$, and the lower sign to the case $\varepsilon_0 < 0$.

These results are in good agreement with the estimates given above. Note that the numerical value of the coefficient in front of $\chi^{7/3}$ in (13) is close to unity: $5\cdot 3^{5/6}\cdot\Gamma(5/6)/8\sqrt{\pi} \sim 0.994$.

It is interesting to note that the principal term in the expansions (12) and (13) does not depend on ε_0. Actually $W_0 \propto \varepsilon_0^{7/2}, \chi^{7/3} \propto (\varepsilon_0^{-3/2})^{7/3} \propto \varepsilon_0^{-7/2}$. This is due to the fact that in strong fields $\delta\tilde{\varepsilon}_D \gg \varepsilon_0$. For this reason $W(\chi)$ for $\chi \gg 1$ is almost independent of the sign of ε_0, i.e., the decay probability in this limit is the same for β-active and stable nuclei under the condition that the matrix elements M_{fi} are the same in the two cases. The strong field, in effect, pulls an electron out of the nucleus regardless of whether or not it is stable in the absence of the field.

5. β DECAY OF NUCLEI WHICH ARE STABLE IN THE ABSENCE OF AN EXTERNAL FIELD. THE CASE $\chi \ll 1$

In order that a nucleus which is stable in the absence of a field can undergo decay, the virtual electron must collect from the field an energy equal to $|\varepsilon_0|$. For this a time $t_2 = (2m|\varepsilon_0|)^{1/2}/eF$ is necessary. If the energy deficit $|\varepsilon_0|$ is such that the condition $t_2 \lesssim t_0 = |\varepsilon_0|^{-1}$ is satisfied, then β decay can occur with a relative probability of the order of unity, i.e., $W \sim W_0(|\varepsilon_0|)$. The condition $t_2 \lesssim |\varepsilon_0|^{-1}$ is equivalent to the requirement $\chi \gtrsim 1$; however, if $\chi \ll 1$ the probability of the process will be substantially smaller. In the case $\omega t_2 \ll 1$ the probability will be exponentially small since the probability that the electron will live longer than the time of its virtual existence $|\varepsilon_0|^{-1}$ is exponentially small. The results obtained in this case are similar to the case of tunneling autoionization of atoms in a constant electric field or ionization in the field of a strong wave at low frequencies.[11] Calculating the integral (6) for $\delta < 0$, $|\delta| \ll \gamma \ll |\delta|^3$ by the method of steepest descent, we obtain the following asymptotic expansion:

$$W(\chi) = W_0(|\varepsilon_0|)\frac{105}{32}\pi^{-1/2}\chi^4 e^{-\frac{2}{3\chi}} \times \sum_{k=0}^{\infty}(-\chi)^k \sum_{n=0}^{2k}\frac{\Gamma(9/2 + 2k - n)\Gamma(k + n + 1/2)}{n!(2k-n)!3^n\Gamma(9/2)}. \qquad (14)$$

Allowance for the dependence of W on the field frequency ω is made in the usual way and gives

$$W = W_0(|\varepsilon_0|)\frac{105}{32}\left(1 + \frac{5}{9\xi^2}\right)\chi^4\exp\left\{-\frac{2}{3\chi}\left(1 - \frac{1}{15\xi^2}\right)\right\}. \qquad (15)$$

Here

$$\xi = (\omega t_2)^{-1} = (\gamma/|\delta|)^{1/2} = eF/(2m|\varepsilon_0|)^{1/2}\omega \gg 1.$$

In the case $\xi \ll 1 (\omega t_2 \gg 1)$ the absorption of energy from the field by the electron has an essentially quantum nature. The total number K of absorbed photons is determined by the necessity of collecting an energy $|\varepsilon_0|$, i.e., $K \geq K_0 = |\varepsilon_0|/\omega$. Accordingly the probability of the process should contain a characteristic dependence on the field-strength amplitude of the wave:

$$W \propto F^{2|\varepsilon_0|/\omega} = F^{|\delta|}.$$

The value of W can be calculated by applying the saddle-point method to the integral in (4). For $\xi \ll 1$ we cannot use the simpler expression (6) since the region of imaginary x with large modulus is important in the integral. The saddle point is found from the transcendental equation.

$$\operatorname{sh}^2 z - \left(\operatorname{ch} z - \frac{\operatorname{sh} z}{z}\right)^2 = \frac{1}{\xi^2}, \qquad (16)$$

where $z = ix$. Unfortunately it is not possible to find this point analytically for arbitrary values of the parameters ξ as was done in Ref. 11 in treatment of the ionization of atoms by a strong wave. The reason for this is that we are considering a circularly polarized wave, whereas Ref. 11 considered a linearly polarized wave for which Eq. (16) reduces to the trivial equation $\sinh^2 z = \xi^{-2}$.

Calculating the saddle-point approximation for $\xi \ll 1$, we obtain from (4)

$$W\approx W_0(|\varepsilon_0|)\frac{105}{32}(\xi^{-1/2}\delta^{-4})\left(\ln\frac{1}{\xi}\right)^{-9/2}\xi^{|\delta|}$$

$$=W_0(|\varepsilon_0|)\frac{105}{32}\left(\frac{\omega}{2\varepsilon_0}\right)^4\left(\ln\frac{(2m|\varepsilon_0|)^{1/2}\omega}{eF}\right)^{-9/2}$$

$$\times\left(\frac{eF}{(2m|\varepsilon_0|)^{1/2}\omega}\right)^{2|\varepsilon_0|/\omega-1/2} \quad (17)$$

6. THE CASE OF NONZERO NEUTRINO MASS

In the last few years as a result of the experiment on measurement of the tritium β spectrum carried out at the Institute of Theoretical and Experimental Physics, which gave an indication of a nonzero neutrino mass,[12] the question of the value of m_ν has been extensively discussed in the literature. The only characteristic energy of β decay is the energy release ε_0; since in all known cases ε_0 is substantially greater than the suggested neutrino mass, study of the effects of a difference of m_ν from zero is difficult. In the case of β decay in the field of an electromagnetic wave we have the appearance of new quantities with dimensions of energy which can be made up of the field-strength amplitude F and the field frequency ω. If any characteristics of the process should turn out to be dependent on the ratios of m_ν to parameters of this type, study of β decay in a field could provide important information on the value of m_ν. Unfortunately, as will be shown below, the answer to this question is mainly in the negative, at least in regard to the total probability of the process. A substantial dependence on the neutrino mass arises only in cases when the probability of the process is itself very low.

Qualitatively the effects of a neutrino mass can be understood in the following way. In the estimates made previously we consider the lifetime of a virtual electron whose energy differs by an amount $\Delta\varepsilon$ from the value prescribed by conservation of energy. However, together with the electron a neutrino is also produced virtually. In the case $m_\nu=0$ the neutrino can in principle be created with zero energy and can live infinitely long, and therefore its existence will not introduce additional restrictions in the estimate. For $m_\nu\neq 0$ the neutrino energy will be $E_\nu>m_\nu$ and the estimates carried out above will be valid only in the case in which the characteristic time of the process is $t_x\ll\tau_\nu=m_\nu^{-1}$. Consequently the neutrino mass should enter into the probability of the process in the form of a parameter $(m_\nu t_x)^2$. We note that the dependence on the field frequency ω enters in the same way into the probability of the process.

The probability of β decay in the field of an electromagnetic wave for $m_\nu\neq 0$ can be found by substituting $E_\nu=(p^2+m_\nu^2)^{1/2}$ in (3); as the result we obtain instead of (4)

$$W(m_\nu)=(-2^{-4})\pi^{-4}(\pi i)^{1/2}m^{3/2}|M_{fi}|^2m_\nu^2\omega^{3/2}$$

$$\times\int_{-\infty}^{\infty}\frac{dx}{(x+i0)^{5/2}}K_2\left(-2i\frac{m_\nu}{\omega}x\right)\exp\left\{-i\left[\delta x+\gamma\left(\frac{\sin^2 x}{x}-x\right)\right]\right\}. \quad (18)$$

Here $K_2(z)$ is a modified Bessel function. Its argument is $(-2im_\nu x/\omega)=-2i(m_\nu\tau)$; thus, the dependence of W on m_ν is determined, as expected, by the characteristic times of occurrence of the process. Transition to the case $m_\nu\to 0$ can be accomplished using the asymptotic form $K_2(z)\approx 2z^{-2}-1/2$ of $K_2(z)$ for small z.

The value of $W(m_\nu)$ can be found in the general case by numerical calculation of the integral in Eq. (18); however, in all special cases of interest this integral is easily investigated analytically. Using the condition $m_\nu\ll\varepsilon_0$, for $\varepsilon_0>0$ and $\chi\ll 1$ we obtain

$$W(m_\nu)=W_0(m_\nu)\left\{1+\frac{35}{8}\chi^2\left[1+\frac{9}{4}\left(\frac{m_\nu}{\varepsilon_0}\right)^2-\frac{1}{30\delta^2}\right]+O\left(\left(\frac{m_\nu}{\varepsilon_0}\right)^4,\chi^4,\delta^{-4}\right)\right\}. \quad (19)$$

Here we have used the expression for the probability of β decay in the absence of a field for $m_\nu\neq 0$:

$$W_0(m_\nu)=W_0(0)\left[1-\frac{35}{16}\left(\frac{m_\nu}{\varepsilon_0}\right)^2+O\left(\left(\frac{m_\nu}{\varepsilon_0}\right)^4\right)\right].$$

In the case of strong fields ($\chi\gg 1$) the calculation gives

$$W(m_\nu)=W_0(|\varepsilon_0|,m_\nu=0)\left\{\frac{5\cdot 3^{5/6}\Gamma(5/6)}{8\sqrt{\pi}}\pm\frac{7\cdot 3^{1/6}\Gamma(1/6)}{16\sqrt{\pi}}\chi^{-2/3}+\frac{35\sqrt{3}}{48}\chi^{-4/3}\left[1+\frac{2}{15\delta^2}-\frac{1}{2}\left(\frac{m_\nu}{\varepsilon_0}\right)^2\right]+O\left(\chi^{-2},\left(\frac{m_\nu}{\varepsilon_0}\right)^4\right)\right\}. \quad (20)$$

Here the upper sign corresponds to $\varepsilon_0>0$ and the lower to $\varepsilon_0<0$.

As follows from Eqs. (19) and (20), in strong fields and also in weak fields for $\varepsilon_0>0$ the probability of β decay will depend very weakly on the value of m_ν. In the case $\varepsilon_0<0$ and $\chi\ll 1$ for relatively low frequencies of the field ($\xi\gg 1$) we obtain

$$W(m_\nu)=W_0(|\varepsilon_0|,m_\nu=0)\frac{105}{32}\chi^4e^{-\frac{2}{3\chi}}\times\left[\frac{m_\nu^2}{2\varepsilon_0^2\chi^2}K_2\left(\frac{m_\nu}{|\varepsilon_0|}\frac{1}{\chi}\right)\right]. \quad (21)$$

In the case $(m_\nu/|\varepsilon_0|)\ll\chi\ll 1$ it follows from this that

$$W(m_\nu)\approx W_0(|\varepsilon_0|,m_\nu=0)\frac{105}{32}\chi^4e^{-\frac{2}{3\chi}}\times\left[1-\left(\frac{m_\nu}{\varepsilon_0}\right)^2\frac{1}{4\chi^2}\right], \quad (22)$$

i.e., the corrections due to the neutrino mass are small. However, in the case $(m_\nu/|\varepsilon_0|)\sim\chi\ll 1$ the result can change considerably. For example, for $(m_\nu/|\varepsilon_0|)=2\chi$ we have $W(m_\nu)\approx 0.5W(m_\nu=0)$. In the case $\chi\ll(m_\nu/|\varepsilon_0|)\ll 1$ the formula 21 gives

$$W\approx W_0(|\varepsilon_0|,m_\nu=0)\frac{105}{64}\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{m_\nu}{|\varepsilon_0|}\right)^{3/2}\chi^{5/2}\times\exp\left\{-\frac{2}{3\chi}\left(1+\frac{3}{2}\frac{m_\nu}{|\varepsilon_0|}\right)\right\}. \quad (23)$$

The obtained relations (21)–(23) refer to the tunneling limit $\xi\gg 1$. In the opposite "multiphoton" case $\xi\ll 1$ the calculation gives

$$W(m_\nu)\approx W_0(|\varepsilon_0|, m_\nu=0)\,\frac{105}{64}\,\delta^{-4}\left(\frac{m_\nu}{\varepsilon_0}\right)^2 \times K_2\left(2\frac{m_\nu}{\omega}\ln\frac{1}{\xi}\right)\left(\ln\frac{1}{\xi}\right)^{-5/2}\xi^{|\delta|-1/2}. \quad (24)$$

From this for $(m_\nu/\omega)\ll\left(\ln\frac{1}{\xi}\right)^{-1}\ll 1$ we obtain

$$W(m_\nu)\approx W_0(|\varepsilon_0|, m_\nu=0)\,\frac{105}{32}\,\delta^{-4}\left(\ln\frac{1}{\xi}\right)^{-9/2} \times\xi^{|\delta|-1/2}\left[1-\left(\frac{m_\nu}{\omega}\ln\frac{1}{\xi}\right)^2\right]. \quad (25)$$

In the case $(m_\nu/\omega)\ln 1/\xi\gg 1$ we have

$$W(m_\nu)\approx W_0(|\varepsilon_0|, m_\nu=0)\,\frac{105}{32}\,\delta^{-4}\left(\ln\frac{1}{\xi}\right)^{-3}\left(\frac{m_\nu}{\omega}\right)^{3/2} \times\xi^{|\delta|+2m_\nu/\omega-1/2}=W(m_\nu=0)\left(\frac{m_\nu}{\omega}\ln\frac{1}{\xi}\right)^{3/2}\xi^{2m_\nu/\omega}. \quad (26)$$

From these results it follows that in a number of cases a difference of the neutrino mass from zero can substantially change the probability of the process [for example, compare (23) with (15), and (26) with (17)]. However, this situation arises only for $\varepsilon_0<0$ when β decay does not occur in the absence of a field; here the probability of the process is itself small and it is essentially impossible to measure it.

7. DISCUSSION OF RESULTS

Let us discuss first the region of applicability of the results. We assumed that $\varepsilon_0\ll m$ and used the nonrelativistic wave function of an electron in the field of a wave. For validity of the nonrelativistic approximation it is necessary also that the energy obtained by the electron from the field during β decay be substantially smaller than its rest mass. In calculation of such β-decay characteristics as the electron energy spectrum, it is necessary to take into account the total energy acquired by the electron in the wave field; in the case $\xi\gg 1$ this energy can take on relativistic values. However, we are interested only in the total probability of the process, and therefore it is necessary to take into account only the energy which is acquired by the electron at distances from the nucleus of the order of its de Broglie wavelength $\lambda\!\!\!^{-}_D$. Let us estimate the value of this energy. In the case $\varepsilon_0>0$, $\chi\ll 1$ we have $\delta\varepsilon_D\sim\varepsilon_0\chi\ll\varepsilon_0\ll m$. In the case $\varepsilon_0<0$, $\chi\ll 1$ the characteristic value of the energy acquired by the electron is $|\varepsilon_0|\ll m$. Thus, for $\chi\ll 1$ the conditions of applicability of the nonrelativistic approximation are clearly satisfied. In the case $\chi\gg 1$ we have $\delta\tilde{\varepsilon}_D\sim(e^2F^2/m)^{1/3}$, and the condition $\delta\tilde{\varepsilon}_D\ll m$ places an upper limit on the field-strength amplitude in the wave: $F\ll F_c=m^2/e$.

All calculations in the present work were carried out in the dipole approximation in the interaction of the electron with the external field. The criterion of applicability of the dipole approximation is the condition $\omega l_x\ll 1$, where l_x is the characteristic dimension of the region of formation of the electron produced in β decay. Using the estimates of the value of l_x obtained above for special cases, it is easy to see that for $\omega\lesssim\varepsilon_0\ll m$ this condition is always satisfied.

A very important question is in regard to the influence of the Coulomb field of the daughter nucleus on the electron formed in β decay. Unfortunately there is no exact solution of the Schrödinger (or Dirac) equation for an electron in the combined field of an electromagnetic wave and a Coulomb potential. The existing attempts to take into account approximately the Coulomb interaction are as a rule based on perturbation theory and have not led to much success. In the electron wave function (1) used in the present work the Coulomb interaction is not taken into account. This approximation is permissible if the energy of the Coulomb interaction is small in comparison with the characteristic energy of the electron: $\varkappa\equiv\alpha Z/l_x\varepsilon_x\ll 1$.

It is easy to evaluate the parameter $\varkappa$ in each of the cases considered. For $\varepsilon_0>0,\chi\ll 1$ we obtain the ordinary Coulomb parameter

$$\varkappa=\alpha Z/v_0=\alpha Z(m/2\varepsilon_0)^{1/2};$$

for $\varepsilon_0<0,\chi\ll 1$ we have

$$\varkappa\sim\alpha Z\chi(m/2|\varepsilon_0|)^{1/2}.$$

In strong fields ($\chi\gg 1$) we obtain

$$\varkappa\sim\alpha Z(F_c/F)^{1/3}=\alpha Z\chi^{-1/3}(m/2|\varepsilon_0|)^{1/2}.$$

As follows from the results of the present work, the principal parameter which characterizes the influence of an external electromagnetic field on the probability of β decay is the quantity $\chi=(F/F_c)(m/2|\varepsilon_0|)^{3/2}$. The maximum values of field strength obtained in focusing of the radiation of the high-power lasers which exist at the present time reach $F\sim(10^{-5}\text{–}10^{-6})F_c$ according to Becker *et al.*[6]; it follows from this that an appreciable effect could be observed for $\varepsilon_0\lesssim 100$ eV. However, for all known β transitions the values of ε_0 are considerably greater than this. In particular, for the case discussed in Ref. 6 of tritium β decay $\varepsilon_0=18.6$ keV, $\chi\sim 10^{-4}$, and the relative change of the probability of the process is of the order 10^{-8}.

As we have mentioned above, the existence in the neutrino of a nonzero rest mass will make practically no change in the probability of β decay in a field for $m_\nu\ll\varepsilon_0,\varepsilon_0>0$. An appreciable change of the β-decay probability can occur only in cases in which the nucleus is stable in the absence of a field, i.e., $\varepsilon_0<0$. However, the β-decay probabilities themselves in this case are so small that they are essentially inaccessible to measurement.

As follows from the discussion carried out above, strong electromagnetic fields can substantially change the total probability of any quantum process only in the case when a low-energy electron is formed as the result of the process. Processes of this type are reactions near threshold. As an example we can mention the photoelectric effect in two fields, one of which is a weak field with frequency Ω not much greater than the ionization potential I while the second is a strong field with frequency $\omega\ll I$. The simple method of calculation proposed in the present work can be used to discuss a broad class of problems of this type.

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[1)]Here and below $\hbar=c=1$.

[1]A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **46**, 1768 (1964) [Sov. Phys. JETP **19**, 1191 (1964)].
[2]V.I. Ritus, Zh. Eksp. Teor. Fiz. **56**, 986 (1969) [Sov. Phys. JETP **29**, 532 (1969)].
[3]V. A. Lyul'ka, Zh. Eksp. Teor. Fiz. **69**, 800 (1975) [Sov. Phys. JETP **42**, 408 (1975)].
[4]I. M. Ternov, V. N. Rodionov, V. G. Zhulego, and A. I. Studenikin, Yad. Fiz. **28**, 1454 (1978) [Sov. J. Nucl. Phys. **28**, 747 (1978)].
[5]I. G. Baranov, Izv. vyzov Fizika **4**, 115 (1974) [Sov. Physics Journal].
[6]W. Becker, W. H. Louisell, *et al.*, Phys. Rev. Lett. **47**, 1262 (1981).
[7]I. M. Ternov, O. F. Dorofeev, and V. N. Rodionov, Preprint No. 08/1982, Moscow State University.
[8]I. M. Ternov, V. N. Rodionov, and O. F. Dorofeev, Preprint No.14/1982, Moscow State University.
[9]I. S. Gradshteĭn and I. M. Ryzhik, Tablitsy integralov, summ, ryadov i proizvedeniĭ, Moscow, Nauka, 1971. Russ. transl., earlier edition, I. S. Gradshteyn and I. M. Ryzhik (eds.), Tables of Integrals, Series, and Products, Academic Press, N.Y. (1965).
[10]G. N. Watson, A. Treatise on the Theory of Bessel Functions, Second Edition, Cambridge University Press (1966). Russ. transl., earlier edition, Moscow, IIL, 1949, Part 1.
[11]L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1945 (1964) [Sov. Phys. JETP **20**, 1307 (1965)].
[12]V. S. Kozik, V. A. Lyubimov, E. G. Novikov, *et al.*, Yad. Fiz. **32**, 301 (1980) [Sov. J. Nucl. Phys. **32**, 154 (1980)].

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