High-frequency effects produced by Andreev reflection of charge carriers in thin normal-metal layers

Yu. A. Kolesnichenko and V. G. Peschanskiĭ

Kharkov State University

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The high-frequency impedance of a thin normal-metal layer of thickness d much smaller than the mean free path l of the charge carriers but much larger than the skin-layer depth δ is investigated theoretically for the case when the metal is bounded on one side by a superconductor. It is shown that in magnetic fields of strength close to the value H_0 at which the characteristic radius of the charge trajectory is equal to the layer thickness d the appearance of the radio-frequency size-effect line is due to a change in the contribution of the current of the quasiparticles that interact with the interface $(n-s)$ between the normal metal and the superconductor. In the case of specular reflection of the changes from the sample surface, the change of the current is due to the return of the charges to the skin layer after each Andreev reflection, with the current decreasing at $H > H_0$ and decreasing at $H < H_0$. If the surface scattering is diffuse, the more substantial effect is the cutoff, in fields $H = H_0$, of the periodic trajectories of the charges moving along the $n-s$ interface. At high frequencies (HF), resonant absorption of the energy of the electromagnetic wave takes place at the cyclotron-resonance frequencies if $r < d < 2r$. In addition, a resonance is made possible by the electrons that glide along the mirror surface and enter periodically into the HF field spike produced in the layer at $r < d < 2r$ by Andreev reflections of the carriers. An experimental study of the HF properties of normal-metal layers in contact with superconductors permits not only observation of Andreev reflection but also determination of its probability and temperature dependence.

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As shown by Andreev,[1] reflection of charge carriers by the normal (n) and superconducting (s) phases is accompanied by a reversal of the charge, mass, and excitation rate if its energy $\xi(\mathbf{p}) = \varepsilon(\mathbf{p}) - \varepsilon_0$ is less than the gap in the superconductor [$\varepsilon(\mathbf{p})$ is the dispersion law of the quasiparticles, ε_0 is the Fermi energy, and $\mathbf{p}$ is the quasimomentum]. Many experimental results obtained to date confirm the existence of Andreev reflection. The most detailed data on the type of the interaction of particles with the $n-s$ interface can be obtained with effects in which the principal role is played by a select group of carriers incident at a definite angle on the boundary between the phases.

Andreev reflection was directly observed using the RF size effect,[2] tarnsverse focusing,[3] and geometric resonance and absorption of ultrasound.[4] An analysis of the transverse-focusing peaks observed by Tsoi and associates[3] agrees with theoretical results, while observed singularities in the absorption of ultrasound by superconductors in the intermediate state[4] were explained in Refs. 6 and 7.

In this paper we investigate theoretically the high-frequency (HF) impedance of a thin normal metal layer of thickness d much less than the carrier mean free path l but substantially larger than the skin-layer depth δ, which is bounded on one side by vacuum and on the other by the superconductor; this corresponds to the geometry used in the experiment by Krylov and Sharvin[2] (Fig. 1). They have noted that at $d = r$ (r is the characteristic Larmor radius) the carriers returning to the skin layer after one Andreev reflection (trajectory b in Fig. 1) contribute to a screening current of opposite sign. This is manifest bv an additional RF size-effect line in a magnetic field $H = H_0 = cp_0/ed$ (p_0 is the Fermi momentum).

We have analyzed in detail the effect of a change of the surface current on the layer impedance in the RF range. It was found that when the carriers are diffusely reflected by the sample boundary the substantial effect is the cutoff of the electron orbits with radius $r > d$, due to the change of the number of charges that absorb the RF-wave energy.

A number of effects connected with the specific character of the Andreev reflection are possible in the microwave region. If the normal-layer thickness satisfies the condition $r < d < 2r$, the period of motion of the excitations interacting with the $n-s$ interface (trajectory a in Fig. 1) coincides with the Larmor period T of the electron revolution, so that at external-field frequencies $\omega = n\Omega$ $(\Omega = 2\pi/T)$ a resonance similar to cyclotron resonance takes place.[8] It is easily seen that the very same carriers produce in the layer a narrow HF

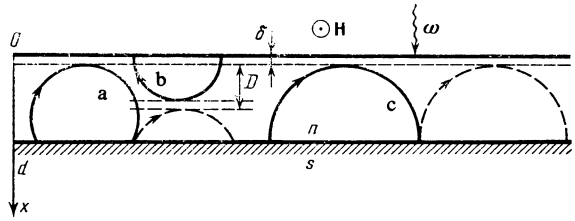

FIG. 1. Onset of an HF field spike in a normal-metal layer at a depth D in the case of Andreev reflection of carriers by the $n-s$ interface $x_s = d$. The electron and hole trajectories are designated by solid and dashed curves, respectively.

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field spike, similar to the spikes produced in the normal skin effect in normal-metal plates in a magnetic field parallel to the surfaces.[9–12] If the reflection by the external surface $x_s = 0$ is close to specular, the electrons gliding along this surface can return periodically, at a frequency $\Omega_0 = 2\pi/T_0$, to the spike (trajectory b in Fig. 1) and their interaction with the field in the spike is resonant. In weaker fields $H < H_0$, when the Larmor radius exceeds the layer thickness, $r > d$. The situation depends essentially on the type of interaction between the charge and the external boundary. In diffuse reflection the cyclotron-lines are cut off in fields $H \approx H_0$, and in specular reflection the positions of the resonance lines, relative to the magnetic field lines, change because of the dependence of the period of the motion of the effective charges on the layer thickness d. Since the resonance line is formed by carriers with external motion periods, an investigation of the impedance in the microwave region yields detailed information on the Andreev reflection of quasiparticles belonging to select sections of the Fermi surface.

§1. STATEMENT OF PROBLEM AND SOLUTION OF KINETIC EQUATION

Although the Andreev reflection is essentially a quantum effect, the motion of an excitation in an interval between two collisions is quasiclassical,[1] and if the normal phase has certain kinetic characteristics one can use the Boltzmann equation for the nonequilibrium increment

$$-\nu(\partial f_0^\nu/\partial\xi)\psi(\mathbf{r},\mathbf{p})e^{i\omega t}$$

to the Fermi distribution function

$$f_0^\nu(\xi) = {}^1/{}_2(1+\nu\,\mathrm{th}\,(\xi/2\theta))$$

of the electrons ($\nu = -1$) and of the holes ($\nu = 1$):

$$\left(-i\omega+\frac{1}{t_0}\right)\psi(\mathbf{r},\mathbf{p})+\mathbf{v}\frac{\partial\psi}{\partial\mathbf{r}}+\frac{e}{c}[\mathbf{v}\times\mathbf{H}]\frac{\partial\psi}{\partial\mathbf{p}}=e\mathbf{v}\mathbf{E}. \quad (1)$$

Here $\mathbf{p}$, $\mathbf{v} = \partial\varepsilon/\partial\mathbf{p}$, and $\mathbf{r}$ are the momentum, velocity, and coordinate of the quasiparticle; ω is the frequency of the external electromagnetic wave, t_0 is the average time between two collisions inside the volume, $\mathbf{E}$ is the electric field strength, t is the time, and θ is the temperature.

The boundary condition that connects the functions ψ of the incident and reflected carriers on the external surface of the sample can be written in the form

$$\psi^{\text{отр}}(0,\mathbf{p})=q(\mathbf{p})\psi^{\text{inc}}(0,\tilde{\mathbf{p}})+\chi(\mathbf{p}). \quad (2)$$

For a slightly rough metal surface, according to Fal'kovskiĭ's results,[13] the total probability of specular reflection is

$$q(\mathbf{p})=1-\int d\mathbf{p}'W(\mathbf{p},\mathbf{p}'),$$

$$\chi(\mathbf{p})=\int d\mathbf{p}'W(\mathbf{p},\mathbf{p}')\psi^{\text{inc}}(\mathbf{p}'),$$

where $W(\mathbf{p},\mathbf{p}')$ is the probability that a charge with momentum $\mathbf{p}'$ and incident on the boundary will have a momentum $\mathbf{p}$ after scattering. The momenta $\mathbf{p}$ and $\tilde{\mathbf{p}}$ are connected by the specular-reflection conditions:

$$\varepsilon(\mathbf{p})=\varepsilon(\tilde{\mathbf{p}}),\quad [\mathbf{n}\times\mathbf{p}]=[\mathbf{n}\times\tilde{\mathbf{p}}],$$

$\mathbf{n}$ is the inward normal to the sample surface $x_s = 0$. We note that relation (2) ensures automatically that the current will not flow through the sample surface.

The condition for Andreev scattering from the $n - s$ interface $x_s = d$

$$\psi^{\text{ref}}(d,\mathbf{p})=-\psi^{\text{inc}}(d,-\mathbf{p}) \quad (3)$$

corresponds to free flow of current through the interface.[14]

The solution of the kinetic equation

$$\psi(\mathbf{r},\mathbf{p}) =F(\mathbf{r}-\mathbf{r}(t))e^{i\omega^*(t-\lambda)}+\int_\lambda^t dt'e^{i\omega^*(t-t')}\mathbf{v}(t')\mathbf{E}(x+x(t')-x(t)) \quad (4)$$

contains an arbitrary function $F(\mathbf{r} - \mathbf{r}(t))$ of the characteristics; this function must be determined with the aid of the boundary conditions (2) and (3). Here $\omega^* = \omega + i/t_0$; λ is the instant of the last collision of the carrier with the plane $x_s = 0$ or $x_s = d$; t is the time of motion of the charge along the trajectory in the magnetic field;

$$\mathbf{v}(t)=\int^t \mathbf{v}(t')dt'.$$

In a magnetic field parallel to the external boundary and to the interface, the carrier motion in a plane perpendicular to the vector $\mathbf{H}$ is periodic, and the conditions (2) and (3) lead to a system of linear integral equations for the functions F_i corresponding to motion along one of the segments of the trajectories (Fig. 2) between two successive reflections.

For carriers interacting only with the interface (Fig. 2a) we have

$$F_i(\mathbf{r}-\mathbf{r}(t))=\frac{\alpha_k\varphi_i-\varphi_k}{1-\alpha_i\alpha_k},\quad i,k=1,2;\quad i\neq k, \quad (5)$$

where

$$\varphi_i=\int_{\lambda_i}^{\lambda_i'} dt'\exp[i\omega^*(\lambda_i'-t')]\mathbf{v}(t')\mathbf{E}(x_s+x(t')-x(\lambda_i')), \quad (6)$$

$$\alpha_i=\exp[i\omega^*(\lambda_i'-\lambda_i)], \quad (7)$$

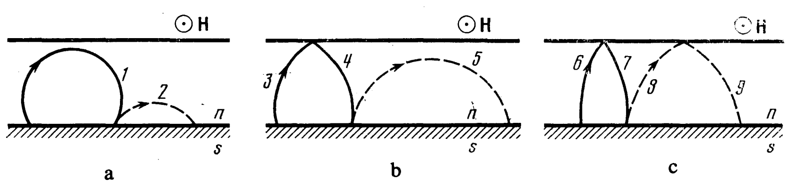

FIG. 2. Possible types of periodic trajectories of carriers interacting with an $n - s$ boundary. The numbers 1–9 indicate which of the functions F_i [Eqs. (5), (8), and (9)] corresponds to a given segment of the trajectory between two successive collisions of the charge with the layer boundaries.

and λ_i and λ_i' are the instants of two successive collisions of the quasiparticle with the boundaries ($\lambda_i' > \lambda_i$); $x_s = 0, d$ is the coordinate of the boundary with which the charge collides at the instant λ_i'. In the expressions for φ_i in (5) we must set x_s equal to d.

If the charge interacts with two boundaries of the normal layer and collides twice with the $n-s$ boundary after specular reflection from the face $x_s = 0$ (Fig. 2b), the function F_i assumes three values:

$$\begin{aligned} &F_3=A_1[\alpha_5(\varphi_4+q\alpha_4\varphi_3)-\varphi_5],\\ &F_4=A_1[\alpha_3(\alpha_5\varphi_4-\varphi_5)+\varphi_3],\\ &F_5=A_1[q\alpha_4(\alpha_3\varphi_5-\varphi_3)-\varphi_4],\\ &A_1^{-1}=1-q\alpha_3\alpha_4\alpha_5. \end{aligned} \tag{8}$$

If the carriers do not collide twice in succession with the same face of the metallic layer (Fig. 2c), F_i takes on four different values:

$$\begin{aligned} &F_i=A_2\{q'\alpha_m[\alpha_l(\varphi_k+q\alpha_k\varphi_i)-\varphi_l]-\varphi_m\},\\ &F_k=A_2q\{\varphi_i-\alpha_i[\varphi_m-q'\alpha_m(\alpha_l\varphi_k-\varphi_l)]\}, \end{aligned} \tag{9}$$

$$A_2^{-1}=1-qq'\alpha_6\alpha_7\alpha_8\alpha_9;\quad q\equiv q(\varepsilon_0,p_z,\lambda_k);\quad q'\equiv q(\varepsilon_0-p_z,\lambda_m),$$
$$i,\ l=6,8;\ k,\ m=7,9;\ i\neq l;\ k\neq m;\ k-i=1;\ m-l=1,$$

where p_z is the projection of the momentum on the magnetic-field direction. We shall not present here for the function F the well known value corresponding to quasiparticles that move periodically along the metal surface (see, e.g., Ref. 15). The distribution function of the electrons in the volume can be obtained from (4), in which we must put $\lambda = -\infty$.

We have left out of (8) and (9) the terms due to allowance for the integral term in the boundary condition (2). Strictly speaking, such an approximation is valid only for carriers whose collision angles α with the external boundary is small compared with the angle width Δ of the scattering indicatrix, so that the arrival term can be neglected in the surface-collision integral. Inasmuch as under anomalous-skin-effect conditions the carriers interacting effectively with the electromagnetic field are those incident on the face $x_s = 0$ at small angles $\alpha \lesssim (\delta/r)^{1/2}$, the condition $\alpha < \Delta$ for such quasiparticle is satisfied as a rule in samples actually used in experiments. In those cases when the surface quality can be significantly improved and Δ therefore decreased ($\alpha > \Delta$), the boundary condition (2) takes the form of a second-order differential equation. The solution of the kinetic problems, as shown by Fal'kovskiĭ,[16] then become much more complicated, but this does not lead to qualitatively new effects.

We shall consider below a situation in which an HF field spike does not come close to the $n-s$ boundary and consequently the electromagnetic-wave amplitude at the superconductor boundary is small. In this case the influence of the superconductor on the total impedance of the sample is connected mainly with the change of the dynamics of the carrier in the screening normal-metal layer, which collide with the boundary $x_s = d$ and interact with the HF field in the skin layer. Therefore at $d - D > \delta$ (D is the distance from the HF field spike to the plane $x_s = 0$) there is no need to solve the microscopic problem of penetration of the electromagnetic field into the superconductor,[1)] and it suffices to take into account the presence of Andreev reflection from the plane $x_s = d$.

§2. ASYMPTOTIC DENSITY OF THE HF CURRENT

Maxwell's equation in the Fourier representation

$$k^2\mathscr{E}_\mu(k)+2E_\mu'(0)-2kE_\mu(d)\sin kd-2E_\mu'(d)\cos kd = \frac{4\pi i\omega}{c^2}j_\mu(k), \tag{10}$$

where

$$\begin{Bmatrix} \mathscr{E}_\mu(k) \\ j_\mu(k) \end{Bmatrix} = 2\int_0^d dx\cos kx \begin{Bmatrix} E_\mu(x) \\ j_\mu(x) \end{Bmatrix}$$

is an integral equation, since the relation between the Fourier components of the current $j_\mu(k)$ and of the electric field $\mathscr{E}_\mu(k)$ is nonlocal:

$$j_\mu(k)=\int_0^\infty K_{\mu\nu}(k,k')\mathscr{E}_\nu(k')\,dk',\quad \mu,\nu=y,z. \tag{11}$$

The relation between the field $E_\mu(d)$ and its derivative $E_\mu'(d)$ is determined from the solution of the boundary-value problem at $x_s = d$. However, in the approximation in the anomaly parameter $\delta/d \ll 1$, which will be considered below, the impedance terms containing $E_\mu(d)$ and $E_\mu'(d)$ are small and can be omitted.

Solution of the kinetic equation yields the high-frequency conductivity tensor $K_{\mu\nu}(k,k')$, which is the kernel of the integral operator that relates $j_\mu(k)$ and $\mathscr{E}_\mu(k)$. In the anomalous skin effect, when $\delta \ll (r,d) \ll l$, the significant values are $k \sim \delta^{-1}$ and to determine the surface-impedance tensor $Z_{\mu\nu}$ it suffices to know the asymptotic expression for $K_{\mu\nu}(k,k')$ at large k and k'. We shall assume that the time of flight of the carriers through the narrow skin layer is substantially shorter than its effective free path time $1/|\omega^*|$, i.e., that the following inequality holds

$$|\omega^*/\Omega|(\delta/r)^{1/2} \ll 1$$

(Ω is the frequency of the quasiparticle in the field $\mathbf{H}$). Without dwelling on the standard procedure of calculating the asymptotic values of the tensor $K_{\mu\nu}(k,k')$, we present only the final results.

1) The carrier reflection by the metal surface is close to specular, i.e., the effective carriers satisfy the relation

$$1-q \ll |\omega^*/\Omega|(\delta/r)^{1/2}.$$

At $\delta \ll r$ this condition is not too stringent, since for electrons that do not leave the skin layer the angle α of approach to the interface between the normal metal and the vacuum is smaller than or of the order of $(\delta/r)^{1/2}$. Inasmuch as at small α the diffuseness parameter is $1 - q(\alpha) \approx q'(0)\alpha$, the foregoing inequality is equivalent to

$$q'(0) \ll |\omega^*/\Omega|.$$

In this case the HF conductivity tensor can be represented by a sum of four terms:

$$K_{\mu\nu}(k,k')=K_n(k,k')+\overset{(1)}{K_{ns}}(k,k')+\overset{(2)}{K_{ns}}(k,k')+\tilde{K}_n(k,k'). \tag{12}$$

The kernel $K_n(k,k')$ is connected with the carriers that glide over the surface of the metal and make the main contribution to the formation of the screening HF current:

$$K_n(k,k') = \frac{k_0^{5/2}}{\beta(kk)^{1/2}}\left\{\left\langle \alpha\hat{S}\left(\frac{T}{2}-\tau,\frac{T}{2};x\left(\frac{T}{2}\right)\right)f_n - \frac{1}{(k+k')^{1/2}}\right\rangle\right\}. \quad (13)$$

Here

$$k_0^{5/2} = -\frac{4\pi^{3/2}\omega e^3 H}{i(ch)^3\omega}\left\langle \frac{v_\mu(T/2)v_\nu(T/2)}{|v_x'(T/2)|^{1/2}}\right\rangle, \quad (14)$$

$$\hat{S}(a,b;x)f = \frac{1}{\pi}\int_a^b d\lambda f'(\lambda)\frac{\sin(k-k')(x-x(\lambda))}{k-k'}, \quad (15)$$

$$\alpha = \beta\rho_{\mu\nu}^{\pm}\left(\frac{T}{2}\right)k_0^{-5/2}, \quad \beta = \frac{4\pi\omega}{c^2},$$

$$\rho_{\mu\nu}^{\pm}(t) = \frac{4\pi e^3 H}{ch^3}\frac{v_\mu(t,p_z)v_\nu(t,\pm p_z)}{|v_x'(t,p_z)v_x'(t,\pm p_z)|^{1/2}}, \quad (16)$$

$$f_n(\lambda) = \{\exp[-i\omega^*(T-2\lambda)]-q(\lambda)\}^{-1},$$

and τ satisfies the condition

$$d-x(\tau)+x(0)=0. \quad (17)$$

The angle brackets $\langle\ldots\rangle$ in (13) and (14) denote integration along that strip on the Fermi surface on which $v_x = 0$, while h is Planck's constant. We assume that on the trajectory of the carriers that do not interact with the layer boundary there are only two stationary phase points $t = 0$ and $T/2$, where $v_x'(0) = v_x'(T/2) = 0$, while $v_s(0) > 0$ and $v_x(T/2) < 0$.

The terms $K_{ns}^{(1)}(k,k')$ and $K_{ns}^{(2)}(k,k')$ in the HF electric conductivity are due to the carriers that interact with the $n-s$ boundary. For quasiparticles whose Larmor trajectory radius $r(p_z) < d < 2r(p_z)$ $(\tau > T/4)$ we have

$$K_{ns}(k,k') = \left\langle \theta(d-r(p_z))\theta(2r(p_z)-d)\rho_{\mu\nu}^{+}(0)\right.$$
$$\left.\times\left[\frac{1}{(kk')^{1/2}}\hat{S}\left(0,\frac{T}{2}-\tau;x(\tau)\right)f_{ns} - \frac{2}{\pi^2}f_{ns}(\tau)\frac{\ln(k/k')}{k^2-k'^2}\right]\right\rangle; \quad (18)$$

$$K_{ns}^{(2)}(k,k') = -\frac{2}{\pi}\left\langle \theta(d-r(p_z))\theta(2r(p_z)-d)\rho_{\mu\nu}^{-}(0)\right.$$
$$\left.\times e^{i\omega^* T/2}f_{ns}(\tau)\frac{\cos kD-\cos k'D}{k^2-k'^2}\right\rangle, \quad (19)$$

where

$$f_{ns}(\lambda) = [e^{-i\omega^*(T-2\lambda')}-1]^{-1},$$

and $\lambda'(\lambda)$ satisfies the equation

$$d-x(T/2-\lambda)+x(\lambda')=0 \quad (\lambda'(T/2-\tau)=0), \quad (20)$$

where $\theta(x)$ is the Heaviside function and

$$D(\tau,p_z) = x(\tau,p_z)-x(T/2-\tau,-p_z).$$

The electric conductivity of the carriers for which $r(p_z) > d$ $(\tau < T/4)$ can be written in the form

$$K_{ns}^{(1)}(k,k')$$
$$= \left\langle \theta(r(p_z)-d)\rho_{\mu\nu}^{+}(0)\left[\frac{1}{(kk')^{1/2}}\hat{S}(0,\tau;x(\tau))f_{ns}^{(1)}\right.\right.$$
$$\left.\left. -\frac{2}{\pi^2}f_{ns}^{(1)}(\tau)\frac{\ln k/k'}{k^2-k'^2}\right]\right\rangle; \quad (21)$$

$$K_{ns}^{(2)}(k,k')$$
$$= \begin{cases}\dfrac{4}{\pi^2}\left\langle \theta(r(p_z)-d)\rho_{\mu\nu}^{-}(0)e^{i\omega^* T/2}f_{ns}^{(1)}(\tau)\dfrac{C(kD)-C(k'D)}{k^2-k'^2}\right\rangle, \\ 0, \ |kD|, |k'D| \gg 1,\end{cases}$$

$$|kD|, |k'D| \leqslant 1, \quad (22)$$

where

$$f_{ns}^{(1)}(\lambda) = \{\exp[-i\omega^*(T-2\lambda'(\lambda)-2\lambda'(T/2-\lambda))]-1\}^{-1},$$

$$C(kD) = \int_1^\infty \frac{dt}{t^2-1}\cos kD(t^2-1). \quad (23)$$

The term $\tilde{K}_n(k,k')$ takes into account the contribution made to $K_{\mu\nu}(k,k')$ by the electrons which do not collide with the $n-s$ boundary and whose orbit diameter is $2r(p_z) < d$.

2) The reflection of the carriers by the metal surface is substantially different from specular, i.e., a considerable part of the charges that make the main contribution to the high-frequency current satisfy the inequality

$$1-q \gg |\omega^*/\Omega|(\delta/r)^{1/2}.$$

In this case the contribution made to the HF electric conductivity by the subsurface electrons is small, and the tensor $K_{\mu\nu}(k,k')$ is determined mainly by the charges that interact with the $n-s$ boundary:

$$K_{\mu\nu}(k,k') = \left\langle \theta(d-r(p_z))\theta(2r(p_z)-d)\right.$$
$$\left.\times\rho_{\mu\nu}^{+}(0)\tilde{f}_{ns}\frac{1}{\pi(kk')^{1/2}}\left[\frac{\sin(k-k')D}{k-k'}-\frac{1}{k+k'}\right]\right\rangle$$
$$-\frac{1}{\pi}\left\langle \theta(d-r(p_z))\theta(2r(p_z)-d)\rho_{\mu\nu}^{-}(0)\right.$$
$$\times\tilde{f}_{ns}\frac{1}{(kk')^{1/2}}$$
$$\left.\times\left[\frac{\cos kD-\cos k'D}{k-k'}+\frac{\sin kD+\sin k'D-\cos(k-k')D}{k+k'}\right]\right\rangle$$
$$-\frac{1}{2}\left\langle \theta(r(p_z)-d)\rho_{\mu\nu}^{+}(0)\left[\frac{1}{(kk')^{1/2}}\hat{S}(0,\tau;x(\tau))e^{2i\omega^*\lambda}\right.\right.$$
$$\left.\left.-\frac{2}{\pi^2}e^{2i\omega^*\lambda}\frac{\ln(k/k')}{k^2-k'^2}\right]\right\rangle + \tilde{K}(k,k'), \quad (24)$$

where $\tilde{f}_{ns} = (1/2)\cot(\omega^* T/2)$.

To avoid lengthy equations we shall assume hereafter that the electric vector of the linearly polarized wave is directed along one of the axes for which the tensor $K_{\mu\nu}(k,k')$ is diagonal, and for simplicity we shall omit the tensor indices.

§3. HIGH-FREQUENCY IMPEDANCE

To calculate the surface impedance of a plane parallel layer of the normal metal bordering on a superconductor

$$Z = \frac{4i\omega}{c^2}\frac{1}{E'(0)}\int_0^\infty dk\mathscr{E}(k), \quad (25)$$

it is necessary to find the solution of Maxwell's equation for the Fourier components of the electric field $\mathscr{E}(k)$. Starting from the structure (12), (24) of the kernel $K(k,k')$, it is convenient to seek the solution of Eq. (10) in the form of the sum

$$\mathscr{E}(k)=\mathscr{E}_0(k)+\mathscr{E}_1(k)+\mathscr{E}_2(k), \tag{26}$$

where $\mathscr{E}_0(k)$ is the Fourier component that describes the field of the main skin layer, the function $\mathscr{E}_1(k)$ is responsible for the formation of the HF field spike at the depth D, and $\mathscr{E}_2(k)$ is a small addition that takes into account the influence of the spike on the field in the skin layer.

1) When the carriers are reflected by the interface between a nearly specular normal metal and vacuum, a large surface current is produced mainly by electrons that do not leave the skin layer during the entire free-path time. This permits a perturbation-theory solution of the Maxwell equation by representing $\mathscr{E}_0(k)$ as the sum

$$\mathscr{E}_0(k)=\tilde{\mathscr{E}}_0(k)+\Delta\mathscr{E}_0(k), \tag{27}$$

where $\Delta\mathscr{E}_0(k)$ is a small increment to $\tilde{\mathscr{E}}_0(k)$ due to the carriers that undergo Andreev reflections and can resonantly absorb the energy of the HF wave. Retaining in (12) only the first term, we transform to the dimensionless wave vector $\xi = k/k_0$ and to the dimensionless Fourier component of the field $F_0(\xi) = -k_0^2(2E'(0))^{-1}\tilde{\mathscr{E}}_0(k)$. The integral equation (10) for the function $F_0(\xi)$ can be solved with the aid of the Mellin transformation

$$F_0(\xi)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} dz\,\xi^z M_0(z),\quad -2<a=\mathrm{Re}\,z<\frac{1}{2} \tag{28}$$

by a method proposed by Hartmann and Luttinger.[18] The explicit form of the Mellin transform $M_0(z)$ we obtained in Ref. 19:

$$M_0(z)=\left(\frac{4}{25(2\pi)^{1/2}}\right)^{2(z+2)/5}\Gamma^{-1}\left(\frac{7}{5}\right)\exp\left[\frac{\pi i(z+2)}{5}\right]$$
$$\times\cos\left(\frac{\pi z}{2}\right)\Gamma(z+1)\Gamma\left(\frac{1-2z}{5}\right)\Gamma\left(\frac{3-2z}{5}\right),$$

and the layer impedance, in first-order approximation in the anomaly parameter, is expressed in terms of the value of $M_0(z)$ at the point $z = -1$:

$$Z_0=-\frac{8i\omega}{c^2k_0}M_0(-1), \tag{29}$$

$$M_0(-1)=\frac{1}{2}\left(\frac{5}{4\pi}\right)^{1/5}\sin\left(\frac{2\pi}{5}\right)\Gamma^2\left(\frac{3}{5}\right)\exp\left(\frac{\pi i}{5}\right).$$

Using Eq. (28) and expression (19) for the "spike" part of the kernel of the integral operator, it is easy to find the function $\mathscr{E}_1(k)$ and hence the distribution of the HF electric field near the spike at a depth D.

We consider first cylindrical dispersion of the carriers, with the Fermi-surface-cylinder axis coinciding with the magnetic field direction. In this case, which is apparently close to the conditions under which Andreev reflection was observed in tin with the aid of the RF size effect,[2] we have

$$E_1(x)=\frac{2i}{\pi}\frac{E'(0)}{k_0^2}\left(\frac{\eta}{k_1}\right)^3\int_0^\infty\frac{dk}{(k/k_1)^3-i}\{F_0(k)\sin k(D-x)$$
$$+G(k)\cos k(D-x)\}; \tag{30}$$

$$G(k)=\frac{2}{\pi}\int_0^\infty dx\frac{F_0(kx)}{x^2-1}, \tag{31}$$

$$\eta^3=\beta\rho(0)e^{i\omega^*T/2}f_{ns}(\tau)b_z, \tag{32}$$

$$k_1^3=\beta\rho(0)[f_n(\tau_0)+f_{ns}(\tau)]b_z, \tag{33}$$

b_z/h is the period of the reciprocal lattice in the direction of the z axis, and τ_0 satisfies the equation

$$x(T/2)-x(\tau_0)-D(\tau)=0 \tag{34}$$

and determines the period $T_0 = T - 2\tau_0$ of the motion of the electrons that return to the spike after specular reflection from the surface $x_0 = 0$.

The amplitude of the spike is a maximum near the center ($|D-x|\sim|k_0|^{-1}$), where it is equal to

$$E_1(D)=-\frac{E'(0)}{k_0}\frac{1}{3\sqrt{2}}\left(\frac{\eta}{(k_0k_1)^{1/2}}\right)^3e^{i\pi/4}. \tag{35}$$

If the spike (31) is far from the surface ($|k_0D|\gg1$), the influence of the carriers that interact with the $n-s$ boundary is described by the additions $\Delta\mathscr{E}_0$ and $\mathscr{E}_2(k)$ to the function $\tilde{\mathscr{E}}_0(k)$, which take into account the terms (18) and (19) in the total HF electric conductivity (12). It can be easily shown that the corresponding corrections to the impedance Z_0 (29) can be found by perturbation theory if the symmetry of the kernel $K_n(k,k')$ is used. Thus,

$$\Delta Z_0=\frac{4i\omega}{c^2}\frac{1}{E'(0)}\int_0^\infty dk\Delta\mathscr{E}_0(k)=C_1\hat{Z}\left(\frac{\eta_1}{k_0}\right)^3, \tag{36}$$

where

$$C_1=1.99\cdot10^{-2}e^{4\pi i/5},\quad \hat{Z}=\frac{8\omega}{c^2k_0},\quad \eta_1^3=\beta\rho(0)f_{ns}(\tau)b_z. \tag{37}$$

The impedance correction necessitated by the reaction of the spike on the main skin layer appears in second-order perturbation theory. Knowing the distribution of the field $\mathscr{E}_1(k)$ in the spike we obtain for $\mathscr{E}_2(k)$ the equation

$$k^2\mathscr{E}_2(k)-i\beta\int_0^\infty dk'K_n(k,k')\mathscr{E}_2(k')$$
$$=i\beta\int_0^\infty dk'K_{ns}^{(2)}(k,k')\mathscr{E}_1(k') \tag{38}$$

with the aid of which, in analogy with (36), we get

$$\Delta Z_2=\frac{4i\omega}{c^2}\frac{1}{E'(0)}\int_0^\infty dk\mathscr{E}_2(k)=C_2\hat{Z}\left(\frac{\eta^3}{k_1k_0^2}\right)^2,$$

$$C_2=\frac{\pi}{48\sqrt{3}}e^{5\pi i/6}. \tag{39}$$

It is easily noted that ΔZ_2 is $|k_0r|^{1/6}\gg1$ times smaller than ΔZ_0 in the RF region ($\omega\ll\Omega$).

In a narrow range of magnetic fields in which the spike emerges to the metal surface, its effect on the impedance is described by the terms $\varepsilon_1(k)$, which can no longer be regarded as rapidly oscillating. In this case we obtain

$$\Delta Z_1=A(k_0D)\hat{Z}(\chi/k_0)^3, \tag{40}$$

where

$$A(k_0D) \approx -\frac{2}{\pi}\int_0^\infty d\xi \int_0^\infty d\xi' \frac{\cos(k_0D\xi)-\cos(k_0D\xi')}{\xi^2-\xi'^2}F_0(\xi)F_0(\xi'),$$

$$|k_0D|\leqslant 1, \quad \tau \geqslant \frac{T}{4},$$

$$A(k_0D) \approx \frac{4}{\pi^2}\int_0^\infty d\xi \int_0^\infty d\xi' \frac{C(k_0D\xi)-C(k_0D\xi')}{\xi^2-\xi'^2}F_0(\xi)F_0(\xi'), \tag{41}$$

$$|k_0D|\leqslant 1, \quad \tau \leqslant \frac{T}{4},$$

$$A(k_0D) \approx 0, \quad |k_0D| \gg 1.$$

Analysis of Eq. (41) shows that $\arg A = \frac{4}{3}\pi + \frac{1}{2}\pi[1+\mathrm{sign}(\tau - T/4)]$ changes by π at $H = H_0$. The absolute value $|A|$ is of the same order as the absolute value of the constant C_1 if $|k_0D| \sim 1$, and $|A| \ll C_1$ at $|k_0D| \ll 1$ or $|k_0D| \gg 1$. It is easily seen that it follows from (41) that $A(0) = 0$. Consequently, for a cylindrical dispersion law in magnetic fields $H \approx H_0$ satisfying the condition $r \approx d$, the addition ΔZ_1 due to the spike emerging from the surface changes by an amount of the same order as ΔZ_0. At $0 < r - d \approx \delta_0 = |k_0|^{-1}$ this circumstance is a manifestation of the contribution made to the current of the carriers that return to the skin layer after each collision with the $n-s$ boundary. In fields H weaker than H_0, when $0 < d - r \approx \delta_0$, the situation is reversed, and the subsurface current is increased by the contribution made to it by charges that land in the skin layer after the Andreev reflections. The term ΔZ_1 in the HF impedance behaves in analogy with the current. We note that in the case of specular reflection of the excitations from the metal surface the change of the impedance at $H \approx H_0$ is small compared with the principal term Z_0.

2) If the surface scattering is diffuse, the current in the skin layer is made up of charges that collide with the $n-s$ boundary and produce at $r < d < 2r(d - r > \delta, 2r - d \gg \delta)$ an HF field spike at a depth D. As the spike approaches the surface ($D \approx \delta$) the surface current decreases just as it does in specular reflection from the external boundary $x_s = 0$. At $q = 0$, however, what is more important is that in fields $H \leqslant H_0$ there are no carriers that return to the skin layer after Andreev reflections (the cutoff effect).

Since at $kD, k'D \gg 1$ the second term in expression (14) for the kernel $K(k,k')$ of the integral operator makes a substantially smaller contribution to the current (11) than the first, and $[\sin(k-k')D]/\pi(k-k')$ can be replaced by the δ function $\delta(k-k')$, Maxwell's equation (10) reduces to the integral equation solved in Ref. 16. By a standard calculation procedure we find that the equation for the surface impedance in the case near-diffuse scattering by the surface is of the form

$$Z = \frac{8\pi\omega}{c^2}\begin{cases} \dfrac{1}{\sqrt{3}}e^{-i\pi/3}\dfrac{1}{\tilde{k}}, & H > H_0 (d-r > |\tilde{k}|^{-1}), \\ \dfrac{\sqrt{3}}{4}\dfrac{1}{\tilde{k}_1}, & H \leqslant H_0, \end{cases} \tag{42}$$

where

$$\tilde{k}^3 = \beta\rho(0)\bar{f}_{ns}b_z, \tag{43}$$

$$\tilde{k}_1{}^3 = {}^1/_2\beta\rho(0)e^{2i\omega^*\tau}b_z. \tag{44}$$

In (42) the cubic-root branch $\tilde{k}$ and $\tilde{k}_1$ must be chosen to satisfy the condition $\mathrm{Re}\,Z > 0$. When the Larmor radius becomes equal to the layer thickness $r(H_0) = d$, cutoff takes place of the periodic trajectories of the carriers, and in the RF region ($\omega \ll \Omega$) this decreases by a factor l/r the impedance of a conductor with a cylindrical dispersion law.

If the Fermi surface has no cylindrical sections, the HF field spike in the normal-metal layer is formed only by a small group of carriers located near the sections $p_z = p_{ze}$ of the equal-energy surface $\varepsilon(\mathbf{p}) = \varepsilon_0$, sections that correspond to the extrema of $D(\tau, p_z)$, as functions of p_z. In this case, in magnetic fields in which $r(p_{ze}) - d \leqslant \delta$, the contribution to the surface current will be compensated for a small fraction $[\sim(\delta/r)^{1/2}]$ of all the charges entering the skin layer at small angle. Consequently, the change of the impedance will be proportional to the small parameter $(\delta/r)^{1/2} \ll 1$ for the same reason, the cutoff of the extremal arcs $r(p_{ze}) = d$ in diffuse scattering by the metal surface does not lead to an abrupt decrease of the impedance, which receives contributions from carriers with all possible values of p_z.

More information can be obtained in investigations of the Andreev reflection of electrons in metals with a complicated dispersion law from the high-frequency characteristics of the layer under resonance conditions. The reason is that the resonant singularities in the impedance are produced by select groups of carriers with extremal period $T(p_{z1})$ of motion in a magnetic field. As follows from (36), (39), and (40), in magnetic fields such that the diameter $2r(p_{z1})$ of the effective orbit is less than double the thickness of the metal layer in the normal state but such that $d < 2r(p_z)$, a number of resonance lines is produced, which coincide with the cyclotron-resonance line in a bulky conductor[8]:

$$\omega = n\Omega(p_{z1}), \quad \left.\frac{\partial T}{\partial p_z}\right|_{p_z = p_{z1}} = 0, \quad n = 1, 2, 3\ldots. \tag{45}$$

At almost-specular reflection of the charges by the metal surface, the resonant increment ΔZ_{res} to the surface impedances takes at small detunings from the resonance $\omega T(p_{z1}) = 2\pi n(1-\Delta)(|\Delta| \ll 1)$ the form

$$\Delta Z^{\mathrm{res}} = \Delta Z_0^{\mathrm{res}} + \Delta Z_1^{\mathrm{res}}; \tag{46}$$

$$\Delta Z_0^{\mathrm{res}} = C_1 Z\rho^+(0)\beta\Psi(\Delta, T, \gamma)|_{p_z = p_{z1}}, \tag{47}$$

$$\Delta Z_1^{\mathrm{res}} = Z\rho^-(0)(-1)^n A(k_0D)\beta\Psi(\Delta, T, \gamma)|_{p_z = p_{z1}},$$

$$0 < |k_0|D \leqslant 1,$$

$$\Delta Z_1^{\mathrm{res}} = Z\rho^-(0)C_2\frac{\rho^-(0)}{k_0^2[\rho^+(0)]^{4/3}}[\beta\Psi(\Delta, T, \gamma)]^{4/3}_{p_z = p_{z1}},$$

$$|k_0D| \gg 1, \tag{48}$$

where

$$\Psi(\Delta, T, \gamma) = \frac{1}{2n(2\varkappa)^{1/2}} \frac{(\zeta+s\Delta)^{1/2}+is(\zeta-s\Delta)^{1/2}}{\zeta},$$

$$\varkappa = \frac{1}{2T}\frac{\partial^2 T}{\partial p_z^2}, \quad s=\operatorname{sign}\varkappa, \quad \zeta=(\Delta^2+\gamma^2)^{1/2}, \quad \gamma=\frac{1}{\omega\tau}.$$

Besides the "volume" cyclotron resonance, in a thin layer it is possible to have a unique resonance due to the motion of the electrons that land periodically in the field spike (30) after specular reflections from the interface between the normal metal and the vacuum. The resonant frequencies are given by the relation

$$\omega T_0(p_{z2}) = 2\pi n, \quad T_0 = T - 2\tau_0, \quad \left.\frac{\partial T_0}{\partial p_z}\right|_{p_z=p_{z2}} = 0, \tag{49}$$

and the increment to the resonance (39), which describes the resonance (49) at detunings $|\Delta| \ll 1$ ($\omega T_0 = 2\pi n(1-\Delta)$), can be represented in the form

$$\Delta Z_2^{\text{res}} = C_2 Z \left(\frac{\chi^3}{k_0^2}\right)^2 [\beta\rho^+(0)\Psi(\Delta, T_0, \gamma)]^{-2/3}_{p_z=p_{z2}}. \tag{50}$$

Resonant absorption of the energy of the HF field in magnetic fields that satisfy (49) recalls cyclotron resonance in a thin plate,[20] first theoretically investigated by one of us.

In magnetic fields such that $r(p_{z1}) \gg d$ the period of motion of the carriers colliding with two boundaries depends on the layer thickness d, and the positions of the resonance lines on the magnetic-field scale differ from the values given by the condition (45):

$$\omega T_1(p_{z3}) = 2\pi n, \quad T_1 = T - 2\tau', \quad \left.\frac{\partial T_1}{\partial p_z}\right|_{p_z=p_{z3}} = 0, \tag{51}$$

where $\tau' = \lambda'(\tau)$, and the $\lambda' = \lambda'(\lambda)$ dependence should be obtained with the aid of Eq. (20). The impedance increment that describes the resonance (51) is of the form

$$\Delta Z^{\text{res}} = C_1 \tilde{Z} \beta\rho^+(0)\Psi(\Delta, T_1, \gamma)|_{p_z=p_{z3}}. \tag{52}$$

If the reflection by the surface of the sample is close to diffuse, the resonant dependence of the surface impedance on the magnetic field is preserved at $r(p_{z1}) < d < 2r(p_{z1})$ and is connected as before with the carriers undergoing Andreev reflection. Near the resonant frequencies (45), expression (42) for the impedance Z takes the form

$$Z = \frac{2}{\sqrt{3}} \beta e^{-i\pi/3} [\beta\rho^+(0)\Psi(\Delta, T, \gamma)]^{-1/3}_{p_z=p_{z1}}. \tag{53}$$

At $d = r(p_z)$, cutoff of the lines of the cyclotron resonance (45), similar to that occurring in a normal-metal plate with diffuse faces, takes place in fields satisfying the condition $d = 2r(p_{z1})$.

In the derivation of Eqs. (47), (48), and (53), which describe the behavior of the impedance near the resonance (45), we have assumed that the probability Q of the Andreev reflection is equal to unity. If, however $Q < 1$, as is possible for example when the normal layer of one metal borders on superconducting substrate of another metal, the weak diffuseness $(1 - Q^2 \ll 1)$ of the carrier interaction with the $n - s$ boundary can be taken into account by introducing an additional broadening of the resonance lines $(1 - Q^2)/2\pi n$, i.e., γ in (47), (48), and (53) must be replaced by $\gamma' = \gamma + (1 - Q^2)/2\pi n$.

CONCLUSION

Anderson reflection of carriers from an $n - s$ boundary leads thus to an entirely different dependence of the surface impedance of a thin normal-metal layer on the magnetic field compared with the impedance of a thin metallic plate. At $r < d < 2r$, a narrow HF-field spike is produced inside the layer at a distance $D(H)$ from its surface. If the electrons are specularly reflected from the interface between the normal metal and vacuum, the carriers gliding over the boundary, landing periodically in the spike, produce the resonance that is not observed in either bulky or thin conductors in the normal state. In the same magnetic-field range, at any character of the scattering from the layer surface, resonance should be observed at frequencies corresponding to the cyclotron resonance.[8] In weak field **H**, at which $r \gg d$, the behavior of the impedance as a function of the magnetic field depends essentially on the state of the sample boundary. Thus, in the case of specular reflection the resonant dependence of Z on **H** is preserved, whereas for diffuse scattering the cyclotron resonance vanishes if $r \gg d$.

At radio frequencies in the magnetic field interval $|r - d| \leqslant \delta$ an abrupt change takes place in the contribution made to the subsurface layer by the carriers that interact with the $n - s$ interface; this manifests itself in the onset of an RF size-effect line at $r = d$. Such a line is most intense when it is due to motion of excitations belonging to cylindrical parts of the Fermi surface.

An experimental investigation of the high-frequency properties of thin normal-metal layers bordering on superconductors makes it thus possible not only to observe directly Andreev reflection of carriers, but also to gauge its probability and temperature from the amplitude and width of the resonance lines.

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[1)]At $d = D$ the electromagnetic field in a superconductor differs noticeably from zero at the Meissner depth, and the effects analyzed by Azbel'[17] manifest themselves in the impedance.

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