One-dimensional Peierls insulator in an alternating electric field

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A study is made of the problem of a one-dimensional Peierls insulator with a doubled lattice period [of the *trans*-$(CH)_x$ type] in an alternating electric field. It is shown that in the frequency region $\lambda \sim \bar{\omega}$ ($\bar{\omega}$ is the activation frequency for collective oscillations of the order parameter Δ of the Peierls insulator) there is parametric excitation of "optical phonons." At optical frequencies in the region $\lambda \sim \Delta$ the field propagates freely in the chain for $\lambda < 2\Delta$, while for $\lambda > 2\Delta$ the optical phonons become weakly damped on account of the decay of the ground state and the excitation of dissipative current in the system.

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INTRODUCTION

There has of late been a considerable growth of interest in quasi-one-dimensional conducting systems which undergo a Peierls transition to an insulator phase as the temperature is lowered. The clearest example of such a material is probably the conducting polymer polyacetylene $(CH)_x$. According to the current understanding, polyacetylene is a Peierls insulator with a doubled period of the lattice of carbon atoms. The width of the conduction band of $(CH)_x$ is ~ 10 eV, and the Peierls gap in the one-electron spectrum of $(CH)_x$ is $2\Delta \approx 1.5$ eV (Ref. 1). There are two modifications of polyacetylene: *trans*-$(CH)_x$ and *cis*-$(CH)_x$. In the *trans* phase the gap in the spectrum is of a purely Peierls origin, while in the *cis* phase the gap has a "hard" component[2] due to the overlap of the wave functions of the electrons of the filled orbitals. At the present time the *trans* modification is the more thoroughly studied from the experimental standpoint, and for this modification there are theoretical models which give a good description of the known facts.[3,4] For the *cis* phase the physical picture is not as clear, although certain theoretical conclusions [the luminescence of *cis*-$(CH)_x$ under optical excitation as opposed to the photoconductivity of *trans*-$(CH)_x$][2,5] can be regarded as experimentally confirmed.[6] For these reasons we shall concentrate on the trans-polyacetylene type of Peierls dielectric.

There have been many experimental studies of the behavior of *trans*-$(CH)_x$ in an alternating electric field. These studies can be divided into two groups—optical measurements and low-frequency measurements. The latter term is somewhat arbitrary, as this group also includes studies in the infrared region. Each of these groups can in turn be divided into two main areas: effects due to the presence of topological defects (solitons and polarons)[2–4] in $(CH)_x$, and phenomena present in homogeneous samples. We shall not touch upon the interaction of defects with the alternating field but shall not consider the behavior of a homogeneous Peierls insulator under nonsteady conditions.

The optical absorption of homogeneous *trans*–$(CH)_x$ has been analyzed in a number of theoretical papers,[7] where it was shown that the absorption coefficient γ has a square-root anomaly, $\gamma_{2\Delta} \sim (\lambda - 2\Delta)^{-1/2}$, at frequencies $\lambda \gtrsim 2\Delta$. To obtain this result, the authors of those papers[7] introduced into the calculation an explicit expression, extraneous to the model, for the matrix element of the interband transition. In the present paper the same result is obtained systematically in the framework of a continuum model for the Peierls insulator.

A more interesting and hitherto unstudied question is that of the role of collective oscillations of the Peierls insulator in the absorption of an alternating signal. It is known[8] that for a Peierls insulator with a double lattice period the order parameter is real and that there exists only one branch of collective oscillations (aside from the usual acoustic phonons, which are not coupled to oscillations in the alternated chain)—"optical phonons" with a frequency $\bar{\omega} \ll \Delta$. In a one-dimensional chain this mode is not optically and infrared active; it has been observed in $(CH)_x$ samples by Raman scattering.[9]

On the other hand, because the "optical phonons" are collective oscillations in the coupled electron–ion system, an external electromagnetic field should excite optical phonons by altering the spectrum of the conduction electrons of the Peierls insulator. In this paper we give a detailed analysis of the mechanism for this process. It is shown that for frequencies $\lambda \sim \omega$ the collective mode is excited by means of a parametric resonance and that the width of the excitation region depends on the amplitude of the field. The optical phonons of the Peierls insulator can therefore be detected directly by two-beam measurements in which a strong laser signal excites the phonons and a weak signal serves as the indicator.

At optical frequencies the field propagates freely in the sample for $\lambda < 2\Delta$, without exciting the collective mode. When $\lambda > 2\Delta$, the phonons of the Peierls insulator become damped on account of the decay of the ground state and the production of electron–hole pairs. A weak field at low frequencies $\lambda \ll \bar{\omega}$ does not have any noticeable effects in a Peierls dielectric.

FORMULATION OF THE PROBLEM AND EXPOSITION OF THE FORMALISM

The microscopic treatment is based on the Lagrangian continuum model of a Peierls insulator with a doubled period[4,10] ($\hbar = v_F = 1$):

$$\mathscr{L} = i\bar{\Psi}_\nu \gamma_\mu D_\mu \Psi_\nu - \Delta \bar{\Psi}_\nu \Psi_\nu - \Delta^2/g^2 + \dot{\Delta}^2/g^2\omega_0^2. \tag{1}$$

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Here Ψ_ν are the spin wave functions of the electrons (holes), ν is the spin index, $\gamma_\mu = (\sigma_i, i\sigma_3)$, σ_i are the Pauli matrices, $D_\mu = \partial_\mu - ieA_\mu$, $A_\mu = (0, a\cos\lambda t)$, a is the amplitude of the alternating field, λ is its frequency, Δ is the order parameter of the Peierls insulator, g is the electron–phonon coupling constant, and ω_0 is the frequency of bare phonons with momentum $2k_F$. The first two terms describe the electron subsystem in the field of the lattice displacements Δ and in the external electric field A. The third term is the strain energy of the lattice in the harmonic approximation, and the last term in (1) is the kinetic energy of the Peierls lattice. The dependence of the vector potential A on the coordinate x can be neglected, since over the entire range of frequencies of interest here the wavelength k^{-1} of the field is large in comparison with the characteristic scale Δ^{-1} of the Peierls insulator ($k\Delta^{-1} \ll 1$). Since the existence temperatures of *trans*-$(CH)_x$ satisfy the conditions $\beta\Delta \gg 1$ and $\beta\omega \gg 1$, we shall study the behavior of the Peierls insulator by methods which correspond formally to $T = 0$.

The effective Lagrangian $\mathscr{L}_{\text{eff}}$ (see, e.g., Ref. 10) of model (1) is written in the standard form of a continuous integral over the fluctuating fields:

$$\exp\left\{i\int dx\,dt\,\mathscr{L}_{eff}\right\} = \int D\Psi_\nu D\bar{\Psi}_\nu \Delta \exp\left\{i\int dx\,dt\,\mathscr{L}\right\} = Z. \tag{2}$$

The imaginary and real parts of the effective Lagrangian determine the intensity of pair production by the field and the polarization of the ground state of the Peierls insulator, respectively.

The functional integral over the fermion fields $\bar{\Psi}_\nu$ and Ψ_ν can be evaluated exactly on account of the Gaussian structure of the functional $\mathscr{L}$ in (1). We shall evaluate the integral over the field Δ with the aid of perturbation theory, assuming

$$\Delta(x,t) = \Delta + \delta(x,t), \tag{3}$$

$\delta \ll \Delta$, where Δ is the equilibrium value of the order parameter and δ is its fluctuating part. In this case

$$Z = \exp\left[-i\int dx\,dt\,\frac{\Delta^2}{g^2} + \operatorname{Sp}\ln(i\gamma_\mu\partial_\mu - \Delta)\right] \times \int D\delta\exp\left\{-i\int dx\,dt\,l + \operatorname{Sp}\ln(i\gamma_\mu D_\mu - \Delta - \delta) - \operatorname{Sp}\ln(i\gamma_\mu\partial_\mu - \Delta)\right\}. \tag{4}$$

Here

$$l = \frac{\dot{\delta}^2}{(g\omega_0)^2} - \frac{\delta^2}{g^2} - 2\frac{\Delta\delta}{g^2}. \tag{5}$$

The functional determinants in (4) are conveniently evlauated in Euclidean space: $t \to -i\tau (0 \leqslant \tau \leqslant \beta)$.[10,11] Here the substitution $\lambda \to i\lambda$ transforms the vacuum–vacuum amplitude Z into the partition function of a two-dimensional model with a potential periodic along the τ axis. After evaluating the partition function, we shall perform an analytical continuation to the real-time axis $\lambda \to -i\lambda$ to obtain the final results.

In the Euclidean space (x,τ) the spur of the logarithm of the elliptic operator $\hat{M}_e$ is conveniently evaluated by the generalized-ζ-function method[11]:

$$\operatorname{Sp}\ln\hat{M}_e = -\zeta'(0) - \zeta(0)\ln C_R^2, \tag{6}$$

where

$$\zeta(s) = \sum_\alpha \lambda_\alpha^{-s}, \tag{7}$$

λ_α are the eigenvalues of the operator $\hat{M}_e$, and C_R is a normalization constant proportional to the cutoff energy k_F. We shall calculate $\zeta(s)$ for the eigenvalues of the squared operator

$$\hat{K}_0 = -(i\gamma_\mu\partial_\mu - \Delta)_e(i\gamma_\mu\partial_\mu + \Delta)_e. \tag{8}$$

Then, if fluctuations of the order parameter are not taken into account,

$$Z_0 = \exp\left\{-\int dx\,d\tau\,\frac{\Delta^2}{g^2} + \frac{1}{2}\operatorname{Sp}\ln\hat{K}_0\right\}, \tag{9}$$

$$\hat{K}_0 = -\partial_\tau^2 - \partial_x^2 + \Delta^2. \tag{10}$$

The eigenvalues of the operator $\hat{K}_0$ are

$$\lambda_n = k^2 + \omega_n^2 + \Delta^2, \tag{11}$$

where ω_n is the Matsubara Fermi frequency, and the value of the equilibrium order parameter Δ_0 is determined from the self-consistency equations for a Peierls dielectric,[10]

$$\left.\frac{\partial\ln Z_0}{\partial\Delta}\right|_{\Delta=\Delta_0} = 0. \tag{12}$$

It is easily verified that evaluation of (9) with the aid of the formulas given above yields the standard result of the theory of the Peierls insulator for $T = 0$:

$$\mathscr{L}_{eff} = -\frac{\Delta^2}{2\pi}\left\{\ln\frac{\Delta^2}{\Delta_0^2} - 1\right\}, \tag{13}$$

where $\Delta_0 = C_R\exp(-\pi/g^2)$. Comparison of (13) with the calculations of Ref. 4 shows that $C_R = 2k_F$.

Now Z is represented in the form

$$Z = Z_0\int D\delta\exp\left\{\int dx\,d\tau\,l_E + \frac{1}{2}\operatorname{Sp}\ln\hat{K} - \frac{1}{2}\operatorname{Sp}\ln\hat{K}_0\right\}, \tag{14}$$

where $\hat{K} = \hat{K}_0 + \hat{K}_1$, with

$$\hat{K}_1 = \sigma_1\dot{\delta} + \sigma_3\delta' + 2\Delta_0\delta + \delta^2 + 2ieA(\tau)\partial_x + e^2A^2(\tau) + e\sigma_2\dot{A}(\tau), \tag{15}$$

and l_E is obtained from (5) by the substitution $t \to -i\tau$.

Let us study the reaction of the Peierls insulator to an external field in the linear-response approximation. To do this we expand the "electronic" terms in the argument of the exponential (14) out to second order in A, assuming that $eA/\Delta_0 \ll 1$. For $(CH)_x$ the linear-response approximation is actually good for rather strong fields—all the way up to $E \sim 10^5$ V/cm. To detertmine the spectrum of small oscillations in the Peierls insulator we retain in expansion (14) the terms quadratic in $\delta(x,\tau)$. We find

$$\begin{aligned}\tfrac{1}{2}\operatorname{Sp}\ln(\hat{K}_0 + \hat{K}_1) - \tfrac{1}{2}\operatorname{Sp}\ln\hat{K}_0 &\approx \tfrac{1}{2}\operatorname{Sp}\hat{K}_0^{-1}\hat{K}_1 - \tfrac{1}{4}\operatorname{Sp}\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1 \\ &+ \tfrac{1}{6}\operatorname{Sp}\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1 \\ &- \tfrac{1}{8}\operatorname{Sp}\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1.\end{aligned} \tag{16}$$

The operation of taking the spur is written explicitly as

$$\operatorname{Sp}\hat{M} = \sum_\nu \operatorname{Tr}\int dx\,d\tau\langle x\tau|\hat{M}|x\tau\rangle, \tag{17}$$

where Tr denotes the taking of the trace over the matrix indices of the operator $\hat{M}$. We introduce the Green function of the operator $\hat{K}_0$:

$$\hat{G}_0(x\tau|x'\tau')=\langle x\tau|\hat{K}_0^{-1}|x'\tau'\rangle=J(x\tau|x'\tau')\hat{I}. \tag{18}$$

Here $\hat{I}$ is the unit 2×2 matrix, and

$$J(x\tau|x'\tau')=\frac{1}{2\pi\beta}\int dk\sum\frac{\exp[ik(x-x')-i\omega_n(\tau-\tau')]}{k^2+\omega_n^2+\Delta^2}. \tag{19}$$

Explicit expressions for the terms in (16) to quadratic order in δ and A are given in the Appendix.

Formulas (A1)–(A4) in the Appendix contain terms which do not depend on the order-parameter fluctuations δ. These terms introduce in the effective Lagrangian a field-dependent correction due to the polarization of the Peierls insulator by the external source:

$$\delta\mathscr{L}_{eff}\int dx\,d\tau=e^2a^2\int dx\,d\tau J(x\tau|x\tau)$$
$$-e^2\int dx\,dx'\int d\tau\,d\tau'\Big\{A(\tau)A(\tau')|J(x\tau|x',\tau')|^2 \tag{20}$$
$$+4A(\tau)A(\tau')\left|\frac{\partial J(x\tau|x'\tau')}{\partial x'}\right|^2\Big\}.$$

One is easily convinced that after the analytical continuation $\lambda\to-i\lambda$ the correction $\delta\mathscr{L}_{\rm eff}$ at frequencies $\lambda<2\Delta_0$ is a real quantity:

$$\delta\mathscr{L}_{eff}(\lambda<2\Delta_0)=\frac{\Delta_0^2e^2a^2}{\pi\lambda}\frac{\operatorname{arctg}(\lambda/2(\Delta_0^2-\lambda^2/4)^{1/2})}{(\Delta_0^2-\lambda^2/4)^{1/2}}. \tag{21}$$

If the frequency of the field exceeds the threshold, $\lambda>2\Delta_0$, then $\delta\mathscr{L}_{\rm eff}$ acquires an imaginary part which is proportional to the absorbed power:

$$\delta\mathscr{L}_{eff}(\lambda>2\Delta_0)=i\frac{\Delta_0^2e^2a^2}{2\lambda}\left(\frac{\lambda^2}{4}-\Delta_0^2\right)^{-1/2}$$
$$-\frac{\Delta_0^2e^2a^2}{\pi\lambda}\left(\frac{\lambda^2}{4}-\Delta_0^2\right)^{-1/2}\ln\frac{\lambda/2+(\lambda^2/4-\Delta_0^2)^{1/2}}{\Delta_0}. \tag{22}$$

It follows from (22) that the absorption coefficient has the behavior $\gamma_\lambda\sim(\lambda^2-4\Delta_0^2)^{-1/2}$. It is easily verified that the coefficients of the response in (22) satisfy the Kramers-Kronig relations. The function $\gamma(\lambda)$ agrees with that obtained in Ref. 7, but in contrast to those papers here we have not found it necessary to introduce an interband-absorption matrix element from outside the theory.

The remaining terms in (14)–(16) and (A1)–(A4) are due to fluctuations of the order parameter and give the quantum correction to the effective Lagrangian. Rather than proceeding to evaluate this correction, let us find the spectrum of small oscillations of the order parameter of the Peierls insulator in the presence of a field.

COLLECTIVE-MODE SPECTRUM OF A PEIERLS INSULATOR IN A FIELD

Let us first show how the spectrum[8] is obtained with our formalism for the case $A=0$. The quadratic [in $\delta(x,\tau)$] form in the argument of the exponential in (14) is

$$\mathscr{L}^{(\delta)}=l_E+(2\Delta_0\delta(x,\tau)+\delta^2(x,\tau))J(x\tau|x\tau)$$
$$-\int dx'd\tau'\{\delta'(x,\tau)\delta'(x',\tau')$$
$$+4\Delta_0^2\delta(x,\tau)\delta(x',\tau')\}|J(x\tau|x'\tau')|^2. \tag{23}$$

Recall that the validity of mean field theory for Peierls insulators guarantees the condition $\alpha^2=(g\omega_0)^2/\pi\Delta_0\ll 1$. The physical meaning of this inequality is simple: The electronic spectrum of the Peierls insulator forms over a time $\sim\Delta_0^{-1}$ at a fixed configuration of the lattice, and over times $\sim(g\omega_0)^{-1}$ the lattice fluctuates about the mean value of the displacement Δ_0. The condition $\alpha\ll 1$ (for polyacetylene $\alpha\sim 0.1$) permits one to make the last term in (23) local in τ. In fact, the characteristic value of $\tau\sim(g\omega_0)^{-1}$, while in the kernel $|J(x\tau|x'\tau')|^2$ the scale of the difference $\tau-\tau'\sim\Delta_0^{-1}$. Inside the curly brackets one can therefore make the replacement $\delta(x',\tau')\to\delta(x',\tau)$. Let us turn now to the spatial dependence of the kernel. It is easy to see that $|J(x\tau|x'\tau')|^2$ diverges when the arguments come into coincidence. This means that the characteristic values of the wave numbers k in the kernel are $k\sim k_F(|x-x'|\sim k_F^{-1}\ll\Delta_0^{-1})$. At the same time, the spatial scale of the fluctuations $\delta(x,\tau)$ is Δ_0^{-1}. Consequently, one can write

$$\delta(x',\tau)\approx\delta(x,\tau)+(x'-x)\delta'(x,\tau)+{}^1/_2(x-x')^2\delta''(x,\tau). \tag{24}$$

Going over to the variables $y=x'-x$, $\theta=\tau'-\tau$, we have for $\beta\Delta_0\gg 1$,

$$\int dx'd\tau'\{\delta'(x,\tau)\delta'(x',\tau')+4\Delta_0^2\delta(x,\tau)\delta(x',\tau')\}$$
$$\times|J(x\tau|x'\tau')|^2\approx\frac{1}{(2\pi\beta)^2}\int_{-\infty}^{\infty}dy\int_{-\tau}^{\beta-\tau}d\theta\{\delta'^2(x,\tau)$$
$$+4\Delta_0^2\delta^2(x,\tau)+2\Delta_0^2\delta(x,\tau)\delta''(x,\tau)y^2\}$$
$$\sum_{n,m}\iint_{-\infty}^{\infty}dk\,dq\frac{\exp[i(q-k)y+i(\omega_n-\omega_m)\theta]}{(k^2+\Delta_0^2+\omega_n^2)(q^2+\Delta_0^2+\omega_m^2)}$$
$$\approx\frac{1}{4\pi\Delta_0^2}(\delta'^2+4\Delta_0^2\delta^2)+\frac{\delta\delta''}{6\pi\Delta_0^2}. \tag{25}$$

We now invoke in (23) the easily verified self-consistency condition

$$2J(x\tau|x\tau)=1/g^2 \tag{26}$$

and obtain the final expression for the quadratic [in $\delta(x,\tau)$] form in (14):

$$\mathscr{L}^{(\delta)}=-\frac{\dot{\delta}^2}{(g\omega_0)^2}-\frac{\delta'^2}{12\pi\Delta_0^2}-\frac{\delta^2}{\pi}\rightarrow\delta(x,\tau)\Big\{-\frac{1}{\pi}$$
$$+\frac{1}{12\pi\Delta_0^2}\frac{\partial^2}{\partial x^2}+\frac{1}{(g\omega_0)^2}\frac{\partial^2}{\partial\tau^2}\Big\}\delta(x,\tau). \tag{27}$$

Analytical continuation of the spectrum of the differential operator in (27) to the real-frequency axis yields the spectrum of Ref. 8. Let us now consider how the spectrum (27) changes when the field $A(\tau)$ is taken into account.

1. Low frequencies $\lambda\ll\Delta_0$

In the low-frequency region the fields $A(\tau)$ in (A1)–(A4) vary smoothly on the scale of changes in the kernels $\tau-\tau'\sim\Delta_0^{-1}$. Therefore, all the expressions become local in τ, and locality in x is attained in the long-wavelength approximation (24) $k\ll\Delta_0$. Following the scheme given above, we obtain

$$\mathscr{L}^{(\delta)}(A)=-\frac{\dot{\delta}^2}{(g\omega_0)^2}-\frac{\delta'^2}{12\pi\Delta_0^2}-\frac{\delta^2}{\pi}+\frac{\delta^2}{3\pi}\frac{e^2A^2(\tau)}{\Delta_0^2}. \tag{28}$$

In arriving at (28) we have dropped terms containing $\dot{A}^2$, since they are small [having a factor $(\lambda/\Delta_0)^2 \ll 1$] in comparison to the terms proportional to $A^2(\tau)$. We have also neglected corrections $\sim A^2\partial^2/\partial x^2$, since $k \ll \Delta_0$. Therefore, the last term in (28), which, as we shall show, leads to parametric excitation of optical phonons in the Peierls insulator, has its origins in diamagnetic effects. In real time ($\tau \to it$, $\lambda \to -i\lambda$) the equation of motion for the collective mode is of the form

$$\ddot{\delta} + \frac{(g\omega_0)^2}{\pi}\delta\left\{1 + \frac{k^2}{k_0^2} + h\cos 2\lambda t\right\} = 0, \tag{29}$$

where $k^2 = 12\Delta_0^2$ and $h = e^2a^2/6\Delta_0^2 \ll 1$. Equation (29) describes a parametric resonance of in the system of "optical phonons." Since $k \ll k_0$, it is actually meaningful to consider a homogeneous parametric resonance. It is well known (see, e.g., Refs. 12 and 13) that in the vicinity of a parametric resonance

$$\lambda \approx \bar{\omega}/p \quad (\bar{\omega} = g\omega_0/\pi^{1/2},\ p = 1, 2\ldots)$$

all the solutions of (29) are unstable and require for stability that terms of higher order in δ be added to (29). In our case, as a calculation shows, the additional terms are

$$(g\omega_0)^2/\pi\,[-3\delta^2/2\Delta_0 + 23\delta^3/6\Delta_0^2].$$

With the nonlinear terms taken into account, the amplitude δ_0 in the vicinity of the first demultiplication resonance $\lambda = \bar{\omega}$ behaves as

$$\delta \approx \delta_0\cos(\bar{\omega}t) + \delta_1,$$

$$\delta_1 = -\frac{3}{4\Delta_0}\delta_0^2, \quad \delta_0^2 = \left(\frac{8}{23}\Delta_0^2\right)\left(\frac{\lambda^2 - \bar{\omega}^2}{\bar{\omega}^2} \pm \frac{h}{2}\right). \tag{30}$$

The constant term δ_1 is considerably smaller than δ_0 and will be unimportant in what follows. The maximum value of the amplitude of δ is much smaller than Δ_0, and solution (30) satisfies the small-fluctuation condition used in obtaining Eq. (29) and the nonlinear corrections to it.

Formulas (30) describe the parametric excitation of optical phonons in a Peierls insulator by a weak alternating electric field. As we know, an important feature of a parametric resonance is that the excitation region depends on the amplitude of the field: $\bar{\omega}(1 - h/4) < \lambda < \langle\bar{\omega}(1 + h/4)$. It follows from (30) that on the right-hand edge of the parametric-resonance frequency interval, $\lambda = \bar{\omega}(1 + h/4)$, the amplitude δ_0 increases discontinuously by an amount $\delta_0 \sim ea \ll \Delta_0$. In the low-temperature region $\beta\delta_0 > 1$, this effect could be manifested, for example, in a dependence of the Debye-Waller factor on the field amplitude.

2. High frequencies $\lambda \sim \Delta_0$

In this case the characteristic scale of changes in the field $A(\tau)$ is of the same order as in the electron kernels. Therefore, although the locality of $\delta(x,\tau)$ with respect to τ remains, the kernels are renormalized on account of the field. As a result

$$\mathscr{L}^{(\delta)}(A) = -\frac{\dot{\delta}^2}{(g\omega_0)^2} - \frac{\delta'^2}{12\pi\Delta_0^2} - \frac{\delta^2}{\pi} - \frac{2e^2A^2(\tau)}{\pi\Delta_0}\delta$$
$$+\frac{\pi}{2}e^2a^2(\delta^2 + 2\Delta_0\delta)\frac{1}{(2\pi)^2}\int dk\,(\lambda^2 + 4k^2)$$
$$\times(\lambda^2 + 12(k^2 + \Delta_0^2))(k^2 + \Delta_0^2)^{-3/2}(\lambda^2 + 4(k^2 + \Delta_0^2))^{-2}$$
$$-\frac{\pi}{2}e^2a^2\Delta_0^2\delta^2\frac{1}{(2\pi)^2}\int dk\,(\lambda^2 + 4k^2)$$
$$\times\frac{3\lambda^4 + 36\lambda^2(k^2 + \Delta_0^2) + 160(k^2 + \Delta_0^2)^2}{(k^2 + \Delta_0^2)^{5/2}(\lambda^2 + 4(k^2 + \Delta_0^2))^3}. \tag{31}$$

As in the analysis of the classical contribution to the absorption (22), one must consider two regions: $\lambda < 2\Delta_0$ and $\lambda > 2\Delta_0$. If $2\Delta_0 - \lambda \sim \Delta_0$, after analytical continuation the last two terms in (31) are real and can be dropped, and the equation of motion assumes the simple form

$$\ddot{\delta} + \frac{(g\omega_0)^2}{\pi}\delta\left\{1 + \frac{k^2}{k_0^2}\right\} = \frac{e^2a^2}{\pi\Delta_0}(g\omega_0)^2\cos 2\lambda t. \tag{32}$$

This equation describes the free propagation of light through a Peierls insulator because the driving term in (32) is not at resonance with the natural frequency $\bar{\omega}$. If $\lambda - 2\Delta_0 \sim \Delta_0$, then the last two terms in (32) acquire an imaginary part which dictates a damping of the collective mode Γ_λ:

$$1 + \frac{k^2}{k_0^2} \to 1 + i\Gamma_\lambda + \frac{k^2}{k_0^2},$$

$$\Gamma_\lambda = e^2a^2\left\{\frac{2}{\lambda(\lambda^2/4 - \Delta_0^2)^{1/2}} + \frac{3\Delta_0^2}{\lambda(\lambda^2/4 - \Delta_0^2)^{3/2}}\right. \tag{33}$$

$$+\frac{26\Delta_0^4}{\lambda^3(\lambda^2/4 - \Delta_0^2)^{3/2}} + \frac{3\Delta_0^4}{\lambda(\lambda^2/4 - \Delta_0^2)^{5/2}} - \frac{19\Delta_0^2}{\lambda^3(\lambda^2/4 - \Delta_0^2)^{1/2}}$$
$$\left.-\frac{210\Delta_0^4}{\lambda^5(\lambda^2/4 - \Delta_0^2)^{1/2}}\right\} \sim e^2a^2/\Delta_0^2 \ll 1. \tag{34}$$

This damping is due to the appearance of dissipative current in the system.

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APPENDIX

Here we give the explicit form of the terms in expansion (16):

$$\tfrac{1}{2}\mathrm{Sp}\,\hat{K}_0^{-1}\hat{K}_1$$
$$= \mathrm{Tr}\int dx\,d\tau\,\hat{G}_0(x\tau|x\tau)\{2\Delta_0\delta(x,\tau) + \delta^2(x,\tau) + \tfrac{1}{2}e^2a^2\};$$

$$\tfrac{1}{4}\mathrm{Sp}\,\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1$$
$$= \frac{e^2}{2}\mathrm{Tr}\int dx\,dx'\,d\tau\,d\tau'\{A(\tau)A(\tau')\sigma_2\hat{G}_0(x\tau|x'\tau')$$
$$\times\sigma_2\hat{G}_0(x'\tau'|x\tau) - 4A(\tau)A(\tau')\frac{\partial\hat{G}_0(x\tau|x\tau')}{\partial x'}\frac{\partial\hat{G}_0(x'\tau'|x\tau)}{\partial x}\right\}$$
$$+\frac{1}{2}\mathrm{Tr}\int dx\,dx'\,d\tau\,d\tau'\{\delta'(x,\tau)\delta'(x',\tau')$$
$$+4\Delta_0^2\delta(x,\tau)\delta(x',\tau')\} \tag{A1}$$

$$\times|\hat{G}_0(x\tau|x'\tau')|^2 + \frac{e^2}{2}\mathrm{Tr}\int dx\,dx'\,d\tau\,d\tau'\{A^2(\tau)(\delta^2(x',\tau')$$
$$+2\Delta_0\delta(x',\tau'))|\hat{G}_0(x\tau|x'\tau')|^2 + A^2(\tau')(\delta^2(x,\tau)$$
$$+2\Delta_0\delta(x,\tau))|\hat{G}_0(x\tau|x'\tau')|^2\}; \tag{A2}$$

$$
{}^{1}/{}_{6}\mathrm{Sp}\,\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1 = {}^{4}/{}_{3}e^2\Delta_0^2\,\mathrm{Tr}\int dx\ldots dx''\,d\tau\ldots d\tau''
$$

$$
\times\{\delta(x',\tau')\,\delta(x'',\tau'')\,A^2(\tau)\,\hat{G}_0(x\tau\mid x'\tau')\,\hat{G}_0(x'\tau'\mid x''\tau'')\,\hat{G}_0(x''\tau''\mid x\tau) + \text{cyclic permutations}\,\}
$$

$$
+ {}^{1}/{}_{3}\,e^2\mathrm{Tr}\int dx\ldots dx''\,d\tau\ldots d\tau''\,\{[\delta^2(x',\tau') + 2\Delta_0\,\delta(x',\tau')]
$$

$$
\times\dot{A}(\tau'')\,\dot{A}(\tau)\,\hat{G}_0(x\tau\mid x'\tau')\,\hat{G}_0(x'\tau'\mid x''\tau'')\,\sigma_2\hat{G}_0(x''\tau''\mid x\tau)\,\sigma_2 + \text{cyclic permutations}\,\}
$$

$$
-\frac{4e^2}{3}\,\mathrm{Tr}\int dx\ldots dx''\,d\tau\ldots d\tau''\,\Big\{[2\Delta_0\delta(x,\tau) + \delta^2(x,\tau)]\,A(\tau')\,A(\tau'')
$$

$$
\times\frac{\partial\hat{G}_0(x\tau\mid x\tau')}{\partial x'}\,\frac{\partial\hat{G}_0(x'\tau'\mid x''\tau'')}{\partial x''}\,\hat{G}_0(x''\tau''\mid x\tau) + \text{cyclic permutations}\Big\}
$$

$$
+\frac{e^2}{3}\,\mathrm{Tr}\int dx\ldots dx''\,d\tau\ldots d\tau''\,\{\delta'(x',\tau')\,\delta'(x'',\tau'')\,A^2(\tau)
$$

$$
\times\hat{G}_0(x\tau\mid x'\tau')\,\hat{G}_0(x'\tau'\mid x''\tau'')\,\hat{G}_0(x''\tau''\mid x\tau) + \text{cyclic permutations}\,\}; \qquad \text{(A3)}
$$

$$
\frac{1}{8}\,\mathrm{Sp}\,\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1\hat{K}_0^{-1}\hat{K}_1 = -\,4e^2\Delta_0^2\,\mathrm{Tr}\int dx\ldots dx'''\,d\tau\ldots d\tau'''
$$

$$
\times\Big\{\delta(x',\tau')\,\delta(x'',\tau'')\,A(\tau''')\,A(\tau)\,\hat{G}_0(x\tau\mid x'\tau')\,\hat{G}_0(x'\tau'\mid x''\tau'')\,\frac{\partial\hat{G}_0(x''\tau''\mid x'''\tau''')}{\partial x'''}
$$

$$
\times\frac{\partial\hat{G}_0(x'''\tau'''\mid x\tau)}{\partial x} + \text{cyclic permutations}\Big\} + e^2\Delta_0^2\,\mathrm{Tr}\int dx\ldots
$$

$$
\ldots dx'''\,d\tau\ldots d\tau'''\,\{\delta(x,\tau)\,\delta(x',\tau')\,\dot{A}(\tau'')\,\dot{A}(\tau''')\,\hat{G}_0(x\tau\mid x'\tau')\,\hat{G}_0(x'\tau')\mid x''\tau'')
$$

$$
\times\,\sigma_2\hat{G}_0(x''\tau''\mid x'''\tau''')\,\sigma_2\hat{G}_0(x'''\tau'''\mid x\tau) + \text{cyclic permutations}\,\}
$$

$$
-\,e^2\,\mathrm{Tr}\int dx\ldots dx'''\,d\tau\ldots d\tau'''\,\{\delta'(x,\tau)\,\delta'(x',\tau')\,A(\tau'')\,A(\tau''')
$$

$$
\times\hat{G}_0(x\tau\mid x'\tau')\,\hat{G}_0(x'\tau'\mid x''\tau'')\,\frac{\partial\hat{G}_0(x''\tau''\mid x'''\tau''')}{\partial x'''}\,\frac{\partial\hat{G}_0(x'''\tau'''\mid x\tau)}{\partial x} + \text{cyclic permutations}\Big\}
$$

$$
+\frac{e^2}{4}\,\mathrm{Tr}\int dx\ldots dx'''\,d\tau\ldots d\tau'''\,\{\delta'(x,\tau)\,\delta'(x',\tau')\,\dot{A}(\tau'')\,\dot{A}(\tau''')\,\hat{G}_0(x\tau\mid x'\tau')
$$

$$
\times\hat{G}_0(x'\tau'\mid x''\tau'')\,\sigma_2\hat{G}_0(x''\tau''\mid x'''\tau''')\,\sigma_2\hat{G}_0(x'''\tau'''\mid x\tau) + \text{cyclic permutations}\,\}. \qquad \text{(A4)}
$$

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