Parametric excitation of spin waves by noise modulation of their frequencies

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The parametric excitation of spin waves in ferrite by noise modulation of a magnetic field in the radio frequency range is investigated. The dependence of the parametric instability threshold on the spectral width and intensity of the noise is determined. The nonlinear ferrite susceptibilities are measured. It is shown that the amplitude of the parametric spin waves beyond the threshold is limited by the phase mechanism.

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INTRODUCTION

In most experiments on the parametric excitation of spin waves in ferrites and antiferromagnets, the noise level is very low.[1,2] This is due to the high stability of the pump generator and the low thermal- and self-noise of the parametric spin waves (PSW). Another interesting case of the parametric instability in solids is the excitation of phonons of high energy by laser radiation.[3] Here the pump is broadband in comparison with the relaxation frequency of the phonons and the noise level is high.

In the present work, we have investigated the parametric excitation of spin waves under conditions of controlled noise, which allows us to track the transition from the coherent regime to the noise regime. The reason for the incoherence was the modulation of the spectrum of the waves ω_k by rf noise.

For an estimate of the effect of noise on the threshold of parametric instability, we consider the equation of motion for the amplitudes of the waves $a_k \exp(-i\omega_p t/2)$ under the action of the pump $h\exp(-i\omega_p t)$ and of the noise component at the frequency $\omega_k(t) = \omega_k + \xi(t)$:

$$\frac{\partial a_k}{\partial t}+\left\{\gamma_k+i\left[\omega_k+\xi(t)-\frac{\omega_p}{2}\right]\right\}a_k+ihV_k a_{-k}^*=0. \quad (1)$$

Under conditions of sufficiently powerful noise, the amplitudes of the waves on the resonant surface $\omega_k = \omega_p/2$ are equal to

$$a_k\approx a_k^0 e^{i\Phi(t)}+hV_k a_{-k}^{0*}\int^t e^{-i\Phi(t')}dt', \quad (2)$$

with accuracy to within small terms. Here

$$\Phi(t)=\int^t \xi(t')dt'.$$

The probability of dissipation of the PSW per unit time is $2\gamma_k\langle|a_k|^2\rangle$, and the probability of creation of a pair of waves by the pump is equal to

$$2\,\mathrm{Im}\langle hV_k a_k^* a_{-k}^*\rangle=2|hV_k|^2|a_k|^2\left\langle e^{i\Phi(t)}\int^t e^{i\Phi(t')}dt'\right\rangle$$

$$\approx 2|hV_k|^2|a_k|^2\tau_\Phi,$$

$\tau_\Phi = (\Delta\varepsilon)^{-1}$ is the characteristic correlation time of the random process $\exp[i\Phi(t)]$. The threshold is achieved in the case in which the dissipation of the PSW is compensated by their creation:

$$|h_c V_k|^2=\gamma_k\Delta\varepsilon. \quad (3)$$

The following estimate can be obtained for $\Delta\varepsilon^4$:

$$\Delta\varepsilon\sim\bar\xi^2(\bar\xi+\Delta\omega)^{-1}, \quad (4)$$

where $\bar\xi = \langle\xi^2\rangle^{1/2}$, and $\Delta\omega^{-1}$ is the correlation time of the process $\xi(t)$.

In Sec. 1 we obtain the parametric instability threshold in the case in which $\xi(t)$ is Gaussian noise (1.19), (1.22). The second part of Sec. 1 is devoted to the experimental study of the dependence of the threshold on the noise level and on its spectral width $\Delta\omega$ following parametric excitation of spin waves by a longitudinal microwave pump in samples of yttrium iron garnet. It is shown that at large $\Delta\omega$ the threshold is determined by the maximum spectral noise power $|h_c V|^2\sim\gamma\max\{\xi^2_\omega\}$, and an experimental test of the formula (1.19) is carried out.

The second section is devoted to the investigation of the steady state of the PSW beyond threshold. As is known, parametric excitation of waves by a monochromatic pump beyond the instability threshold produces a narrow packet of PSW whose number is limited by a self-consistent renormalization of the pump to the threshold level (S theory, phase mechanism). As a result, the width of the PSW packet in **k** space turns out to be much smaller than the dimensions of the region in which the threshold is exceeded, and in the absence of noise the packet width is simply zero. In this case, the amplitudes of the PSW can be described by means of dynamical equations.[1] The amplitudes of the PSW can be perturbed by the noise in two ways: first, the thermal- and self-noise of the PSW lead to the appearance of a random stimulating force acting on the PSW[2]; second, the parameters of the system, such as the pump field or the wave spectrum ω_k, can fluctuate. Just this latter case is studied in our work.

The degree of incoherence of the pump in most physically interesting cases is characterized by the correlation time τ. If this time is large in comparison with the relaxation time of the PSW, $\gamma^{-1}(\gamma\tau\gg 1)$, then the pump can be regarded as coherent. In the opposite case ($\gamma\tau\ll 1$), the pump is a noise. It is not difficult to establish the fact that the fluctuation of the wave spectrum $\omega_k(t) = \omega_k + \xi(t)$ is equivalent to the fluctuation of a phase of the parametric pump

$$\tilde h(t)=h\exp[i\Phi(t)], \quad (5)$$

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where $\Phi(t)$ is expressed by Eq. (2), and $\tau = \tau_\Phi = \Delta\varepsilon^{-1} - (4)$.

Recently, two theoretical works have been published that were devoted to the nonlinear theory of parametric excitation of waves by an incoherent pump.[5,6] In one of them, a generalization of the phase mechanism of limiting of the number of PSW is proposed to include the case of a noise pump. The authors of the second work have assumed that the principal mechanism for the limitation of the number of PSW beyond threshold is in this case the nonlinear damping of the waves, while phase correlation in pairs of PSW is completely absent. In the works cited, predictions have been made concerning the number of PSW, meaning also concerning the susceptibility of the system of PSW beyond the threshold.

In Sec. 2.1, we give the results of the experimental study of the nonlinear susceptibility of a ferrite as a function of the spectral noise power. It turns out that at supercriticalities $\gtrsim 3$ dB, the nonlinear susceptibility in the presence of noise is insignificantly reduced in comparison with the susceptibility in the coherent regime, and the real and imaginary parts of the susceptibility are close together in magnitude. This circumstance points to a common nonlinear mechanism that limits the number of PSW in the coherent and noise regimes, i.e., to reality of the phase mechanism.

The nonlinear susceptibility of the PSW is calculated in Sec. 2.2 on the basis of the results of the work of one of the authors.[5] While a comparison of the results of experiment with the predictions of the theory is given in Sec. 2.3.

As for Ref. 6, without going into a detailed analysis, we only remark that the stationary distribution of PSW found in it is unstable relative to a broadening in **k** space and therefore cannot be realized in experiment.

1. THRESHOLD OF PARAMETRIC INSTABILITY IN THE CASE OF NOISE MODULATION OF THE WAVE SPECTRUM

1.1. Theory

We consider the parametric instability of spin waves in a magnetic field $H(t)$:

$$H(t)=H_0+H_m(t)+h\cos\omega_p t \qquad (1.1)$$

directed along the z axis. H_0 is a constant magnetic field, $h\cos\omega_p t$ is the field of the monochromatic pump, $H_m(t)$ is the noise RF field. The equations of motion for the amplitudes of the PSW, $a_\mathbf{k}\exp(-i\omega_p t/2)$ in the field (1.1) have the form[1]

$$\frac{\partial a_\mathbf{k}}{\partial t}+\left\{\gamma_\mathbf{k}+i\left[\omega_\mathbf{k}-\frac{\omega_p}{2}+U_\mathbf{k}H_m(t)\right]\right\}a_\mathbf{k}+ihV_\mathbf{k}a_{-\mathbf{k}}^*=0, \qquad (1.2)$$

$\omega_\mathbf{k}$ is the dispersion law of spin waves, $\gamma_\mathbf{k}$ is their relaxation frequency, $U_\mathbf{k}$ and $V_\mathbf{k}$ are matrix elements of the interaction of the PSW with the noise field and the pump:

$$U_\mathbf{k}=g[1+(\omega_M/\omega_p)^2]^{1/2}, \quad |V_\mathbf{k}|=g\omega_M/2\omega_p, \qquad (1.3)$$

where g is the gyromagnetic ratio, $\omega_M = 4\pi gM$, and M is the equilibrium magnetization. We shall denote the quantity $U_\mathbf{k}H_m(t)$ by $\xi_\mathbf{k}(t)$.

For determination of the instability threshold, it is convenient to develop a diagram technique, similar to what was done in Ref. 7. In our case, in contrast to Ref. 7, the scattering of the PSW takes place from a randomly fluctuating magnetic field, and not from randomly distributed defects.

We consider the reaction of the amplitude of the PSW,

$$a_q\equiv a_{\mathbf{k}\omega}=\frac{1}{(2\pi)^{1/2}}\int a_\mathbf{k}(t)\,e^{i\omega t}dt \qquad (1.4)$$

to a small random force $f_{\mathbf{k}\omega}$. From (1.2), we obtain

$$(\omega-\overline{\omega}_\mathbf{k}+i\gamma_\mathbf{k})a_q-P_\mathbf{k}a_{\bar q}^*=f_q+\int\xi_{\mathbf{k}\omega'}a_{\mathbf{k}\omega''}\delta(\omega-\omega'-\omega'')\,d\omega'd\omega'',$$

$$-P_\mathbf{k}^*a_q-(\omega+\overline{\omega}_\mathbf{k}+i\gamma_\mathbf{k})a_{\bar q}^* \qquad (1.5)$$

$$=f_{\bar q}^*+\int\xi_{\mathbf{k}\omega'}^*a_{\mathbf{k}\omega''}^*\delta(\omega-\omega'-\omega'')\,d\omega'd\omega'',$$

where

$$q\equiv(\mathbf{k},\omega),\quad \bar q\equiv(-\mathbf{k},-\omega),\quad \overline{\omega}_\mathbf{k}=\omega_\mathbf{k}-\omega_p/2,\quad P_\mathbf{k}=hV_\mathbf{k},$$

$$\xi_{\mathbf{k}\omega}=(2\pi)^{-1}\int\xi_\mathbf{k}(t)\,e^{i\omega t}\,dt.$$

We introduce the normal and anomalous Green's functions G_q and L_q:

$$G_q\delta(q-q')=\langle\delta a_q/\delta f_{q'}\rangle,\quad L_q\delta(q+q')=\langle\delta a_q/\delta f_{q'}^*\rangle \qquad (1.6)$$

and the normal and anomalous correlations n_q and σ_q:

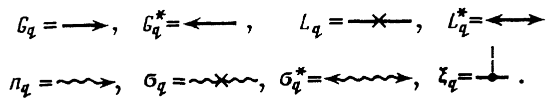
$$n_q\delta(q-q')=\langle a_q a_{q'}^*\rangle,\quad \sigma_q\delta(q+q')=\langle a_q a_{q'}\rangle. \qquad (1.7)$$

Allowing f_q to approach zero, we obtain the set of Dyson-Wylde equations for the Green's function and the correlators[7]:

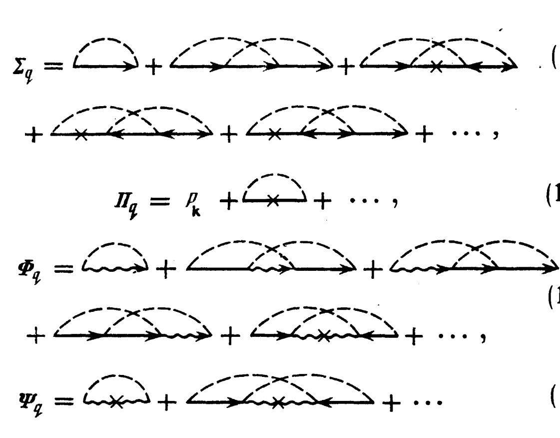
$$(\omega-\overline{\omega}_\mathbf{k}-\Sigma_q+i\gamma_\mathbf{k})G_q-\Pi_q L_q^*=1,$$
$$-\Pi_q G_q-(\omega+\overline{\omega}_\mathbf{k}+\Sigma_{\bar q}^*+i\gamma_\mathbf{k})L_q^*=0, \qquad (1.8)$$

$$n_q=|G_q|^2\Phi_q+|L_q|^2\Phi_{\bar q}^*+G_q\Psi_q L_{\bar q}^*+L_q\Psi_{\bar q}^*G_q^*,$$
$$\sigma_q=G_q\Phi_q L_{\bar q}+L_q\Phi_{\bar q}^*G_{\bar q}+G_q\Psi_q G_{\bar q}+L_q\Psi_{\bar q}^*L_{\bar q}. \qquad (1.9)$$

The quantities Σ_q, Π_q, Φ_q, Ψ_q can be represented in the form of a series in powers of the moments of the excitation $\xi_{\mathbf{k}\omega}$. Since the noise generator in our experiment produced a Gaussian noise, we limit ourselves to the consideration of just this case. We introduce the notation


(1.10)

In this notation, the series for the mass operators Σ_q, Π_q, Φ_q, Ψ_q have the form


(1.11)
(1.12)
(1.13)
(1.14)

The simplest case for study is Gaussian white noise, in which

$$\langle \xi_{k\omega}\xi_{k\omega'}\rangle = \xi_{k\omega}^2\delta(\omega+\omega'), \quad \xi_{k\omega}^2 = D/2\pi. \tag{1.15}$$

In this case, only the first terms in the diagram series (1.11)–(1.14) differ from zero; all the rest vanish by virtue of the analyticity of the Green's function in the upper half plane of the variable ω, i.e.,

$$\Sigma_q = -i\frac{D}{2}, \quad \Pi_q = P_k, \quad \Phi_q = \frac{D}{2\pi}n_k, \quad \Psi_q = \frac{D}{2\pi}\sigma_k,$$

or (1.16)

$$n_k = \int n_{k\omega}d\omega, \quad \sigma_k = \int \sigma_{k\omega}d\omega. \tag{1.17}$$

Substituting (1.16) in (1.8) and (1.9), we obtain a set of integral equations with degenerate kernel:

$$n_{k\omega} = \frac{D/2\pi}{|\Delta_{k\omega}|^2}\{[\Gamma_k^2 + (\omega+\bar{\omega}_k)^2 + |\Pi_k|^2]n_k + 2\Gamma_k \operatorname{Im}(\Pi_k^*\sigma_k) - 2(\omega+\bar{\omega}_k)\operatorname{Re}(\Pi_k^*\sigma_k)\},$$

$$\sigma_{k\omega} = \frac{D/2\pi}{|\Delta_{k\omega}|^2}\{-2i\Gamma_k\Pi_k n_k - (\Gamma_k^2 + \omega^2 - \bar{\omega}_k^2 - 2i\Gamma_k\bar{\omega}_k)\sigma_k + \Pi_k^2\sigma_k^*\}, \tag{1.18}$$

$$\Delta_{k\omega} = (\Gamma_k - i\omega)^2 - |\Pi_k|^2 + \bar{\omega}_k^2,$$

$$\Gamma_k = \gamma_k - \operatorname{Im}\Sigma_k = \gamma_k + D/2, \quad \Pi_k = P_k.$$

Integrating (1.18) with respect to ω, we obtain a set of linear algebraic equations for n_k, σ_k, σ_k^*. The threshold value of p_k is reached on the surface $\bar{\omega}_k = \omega_k = \omega_p/2 = 0$; here

$$|P_\Omega|^2 = \gamma_\Omega(\gamma_\Omega + D), \tag{1.19}$$

where Ω is the solid angle on the surface $\omega_k = \omega_p/2$. The threshold pump amplitude h_c is

$$h_c = \min_\Omega\left\{\frac{\gamma_\Omega(\gamma_\Omega+D)}{|V_\Omega|^2}\right\}. \tag{1.20}$$

In ferromagnets γ_Ω depends weakly on the angles while $|V_\Omega|$ is a maximum at $\theta = \pi/2$; therefore, the waves are excited primarily on the equator of the resonant surface $\omega_k = \omega_p/2$, as in the absence of noise. We note here that $h_c \to 0$ as $\gamma \to 0$. This circumstance is a general one and does not depend on the character of the noise. Actually, scattering from fluctuations of the magnetic field does not lead to dissipation of the PSW and therefore cannot compensate the creation of pairs of PSW by the pump.

In a real situation, the rf noise is not white but has a finite range of frequencies. If the characteristic width λ of the correlator $\xi_{k\omega}^2$ is large in comparison with D, the threshold of the parametric instability differs little from (1.20) and can be determined by perturbation theory, taking into account the dependence of $\xi_{k\omega}^2$ on ω only in diagrams of fourth and second orders in ξ. For a Lorenzian form of ξ_ω^2,

$$\xi_{k\omega}^2 = \frac{D}{2\pi}\frac{\lambda^2}{\omega^2+\lambda^2} \tag{1.21}$$

we can obtain the following from (9), (10), (12)–(15), in place of (1.19):

$$|P_\Omega|^2 = (1 - 2D/\lambda)\gamma_\Omega(\gamma_\Omega + D). \tag{1.22}$$

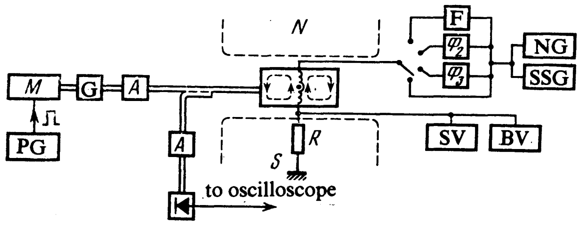


FIG. 1. Block diagram of the experimental setup. M = magnetron, PG = pulse generator, G = gate, A = attenuator, F = low-frequency filter, NG = noise generator, SSG = standard signal generator, SV = selective voltmeter, BV = broadband voltmeter.

For rectangular shape of ξ_ω^2 the threshold is determined by the equation (1.22) with accuracy to within terms $\sim(D/\lambda)^2$. It should be noted that if $\max \xi_\omega^2 = \xi_0^2 = D/2\pi$, the threshold cannot exceed (1.19) for any arbitrary shape of ξ_ω^2, since the decrease in the noise far from the center of the packet cannot lead to an increase of the threshold.

1.2. Experiment

The experiment was carried out on yttrium iron garnet crystals under conditions of parallel microwave pumping at a frequency of 9.4 GHz. Figure 1 shows a block diagram of the setup. The microwave system consists of a TE_{102} square-wave generator and the usual waveguide elements, assembled to operate "in reflection." The pump source is a pulsed magnetron, with a pulse frequency 25 GHz and duration 0.6 μs. The threshold of parametric excitation of the spin waves and their susceptibility were determined from the change of the signal reflected from the resonator. To modulate the field H in the sample, a coil was placed in the resonator with its axis parallel to the magnetic component of the microwave field. Current from a G2-37 noise generator flowed to the coil through a set of low frequency filters and a broadband amplifier. The generator assured a noise signal with a constant spectral density in the range $\Delta f = 0.7$ MHz. Since the characteristic frequency ω_k for the system of PSW in yttrium iron garnet at the various values of H lies in the limits of 0.2–0.5 MHz, the noise, with a band of 7 MHz, can be considered to be white in such a system with sufficient accuracy. Just such conditions also are of interest from the viewpoint of the arguments advanced in Sec. 1.1. The filters, which decrease the band of frequencies of the noise signal, are provided in the system to a check on the effect of the parameter $\gamma/\Delta f$. The noise spectrum, detected by means of a selective voltmeter V6-1 for four different values of the band Δf, is shown in Fig. 2.

The spectral density of the noise modulation of the field was determined in correspondence with Sec. 1.1:

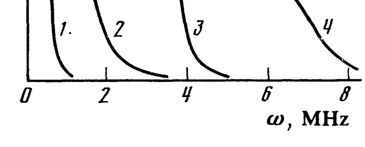


FIG. 2. Spectra of noise signals.

$$\langle H_m{}^2\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}J_\omega d\omega=2\int_0^{\infty}J_\omega df.$$

Assuming the shape of the spectrum in Fig. 2 to be rectangular, we obtain

$$J_\omega=\frac{\langle H_m{}^2\rangle}{2\Delta f}=\frac{K^2\langle i^2\rangle}{2\Delta f}. \tag{1.23}$$

Here $K = H/i$ is a calibrating coefficient for the field in the resonator, created by the coil in the passage of the current i through it. The coefficient K was determined from the shift in the resonant ferromagnetic-resonance field when direct current was turned on, and also by the method suggested in Ref. 8, from the change in the threshold of the parallel pumping in the case of sinusoidal modulation of the field H. In the latter case, a standard-signal generator was connected to the coil at the time of calibration. In a result, we assumed $K=(39\pm 4)\text{Oe}\cdot\text{A}^{-1}$. The current $\langle i^2\rangle^{1/2}$, determined with a broadband voltmeter by measuring the voltage drop across the resistance connected in series with the coil, was converted into $\langle H_m^2\rangle^{1/2}$ of J_ω by Eq. (1.35).

Figures 3 and 4 show the experimental dependences of the threshold of the parallel pumping on the effective value of the field for two values of the direct current: $H_0=H_c-100$ Oe and $H_0=H_c-10$ Oe, where $H_c=1680$ Oe for $\mathbf{M}\|\langle 100\rangle$ is the field corresponding to the minimum threshold. The curves were obtained with noise signals the shapes of the spectra of which are shown in Fig. 2. The maximum value of the spectral noise density of the most favorable shape ($\Delta f=7$ MHz), which was obtained without

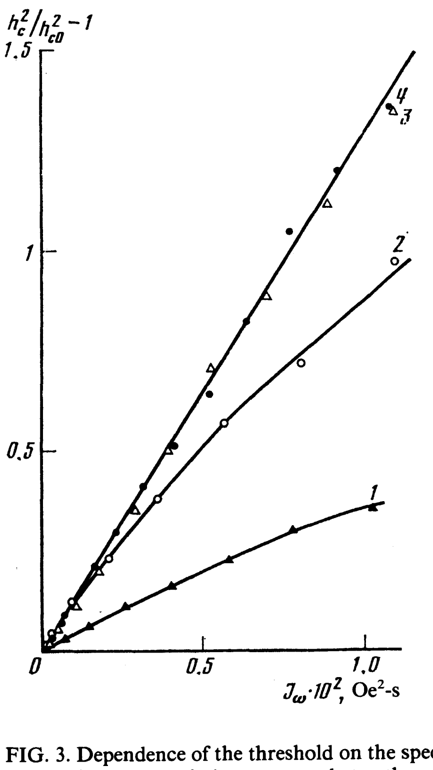


FIG. 3. Dependence of the threshold on the spectral density of the noise field. The curves 1–4 correspond to the curves 1–4 of Fig. 2. $H=H_c-100$ Oe.

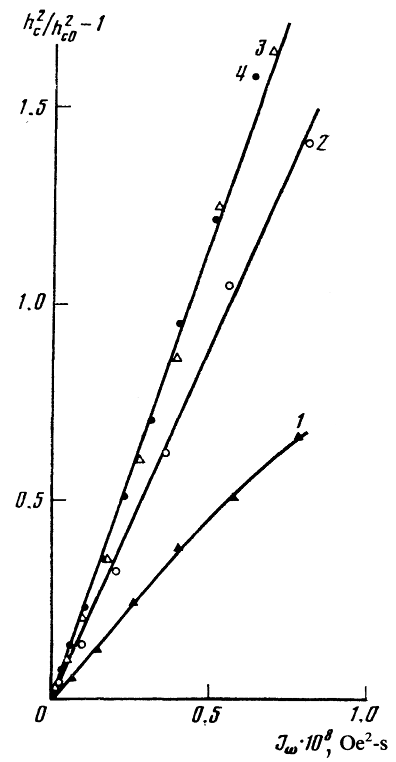


FIG. 4. Dependence of the threshold on the spectral density of the noise field. The curves 1–4 correspond to curves 1–4 of Fig. 2. $H=H_c-10$ Oe.

distortion of the spectrum, amounted to $J_\omega=1.2\times 10^{-8}$ $\text{Oe}^2\cdot\text{s}$, which corresponds to an effective noise pump with a frequency band $D\approx 10^7\ \text{s}^{-1}$. Significantly greater J_ω and D were achieved at lesser Δf($D_{\max}\approx 10^8\ \text{s}^{-1}$ at $\Delta f=0.7$ MHz), but at $\Delta f\lesssim 4$ MHz the quantitative interpretation of the results was difficult because the condition of applicability of Eq. (1.20) turns out to be violated.

The character of the threshold curves manifests itself most distinctly in terms of variables h_c^2/h_{c0}^2-1 and J_ω, a linear connection is predicted by Eq. (1.22) transformed with account of $h_{c0}V=\gamma$:

$$\frac{|h_cV|^2}{\gamma^2}-1=\left(\frac{h_c}{h_{c0}}\right)^2-1=\frac{U^2}{\gamma}J_\omega. \tag{1.24}$$

We note that the threshold curves in the new coordinates, in accord with theory, become straight lines only at sufficiently large Δf, when the noise can be regarded as white. The coincidence of plots 3 and 4 in Figs. 3 and 4 indicates that the principal factor determining the threshold is, in accord with (1.20), the spectral density of the noise field J_ω and not the mean square value $\langle H_m^2\rangle$ which determines the total noise power—the broadening of the spectrum from 4 to 7 MHz does not lead to a change of the threshold. With decrease in the width of the spectrum, when $\Delta\omega$ becomes commensurate with D, the threshold becomes increasingly dependent on $\Delta\omega$, and consequently also on $\langle H_m^2\rangle$. Here, in correspondence with (1.22), a deviation of the threshold curves from linear is observed, and it occurs earlier and is stronger for the set of curves at $H_0=H_c-100$ Oe, where the value of γ is larger.

The slopes of the lines at $\Delta f = 7$ MHz in Figs. 3 and 4 are different, and their ratio is equal to 1.75. This fact is explained by the dependence of the quantity γ in (1.24) on the constant field. The ratio γ_1/γ_2 for the two values of H in Figs. 3 and 4 can be defined as the ratio of the two values of the threshold field h_1/h_2 in the absence of noise. The ratio of the thresholds, determined from the two points for $H_m = 0$ in Fig. 3, is equal to 1.7, which agrees with the found ratio of the slopes.

We now compare the numerical values of the slopes of the theoretical and experimental threshold plots. In accord with (1.24), this quantity is equal to $\varkappa = U^2/\gamma$, where U is the coefficient of coupling of the modulation field with the system of PSW (1.3). For yttrium garnet, under the conditions of our experiment, $U = 2\times 10^7$ s^{-1} Oe^{-1}, and for the slope of the straight line at the value $2\gamma = 2\pi(0.75)\times 10^6$ s^{-1} (see, for example, Ref. 9), we obtain the value $\varkappa = 1.7\times 10^8$ s^{-1} Oe^{-2}. Experiment (Fig. 4) yields $\varkappa_{\rm exp} = (1.3 \pm 0.3) \times 10^8$ s^{-2} Oe^{-2}. In spite of the large error in the determination of $\varkappa_{\rm exp}$, which is connected with the large error in the calibration of the modulation coil, the satisfactory agreement of the estimate given above with the experimental results should be noted.

2. NONLINEAR SUSCEPTIBILITY OF A FERRITE BEYOND THE THRESHOLD OF PARAMETRIC EXCITATION

The state of a system of PSW beyond threshold is characterized by the complex susceptibility χ—the ratio of the magnetic moment of the PSW to the pump field:

$$\chi = \chi' + i\chi'' = m_z(\omega_p)/h. \quad (2.1)$$

The imaginary part of the susceptibility χ'' is the ratio of the power absorbed by the ferrite to the incident power, while the real part χ' characterizes the phase shift of the magnetization relative to the phase of the pump (for further details see Ref. 1).

2.1. Experimental results

The measurements of χ' and χ'' were performed on the apparatus described in Sec. 1.2. Figures 5 and 6 show the experimental plots of the susceptibilities χ'' and χ' vs the supercriticality $\zeta = h/h_c$ at various values of the spectral density of the noise field J_ω and at $\Delta f = 7$ MHz. A feature of the set of plots is that at small ζ the noise significantly lowers the susceptibility, as a result of which the entire $\chi''(\zeta)$ curve and, in particular, its maximum, is shifted with increase in J_ω in the direction of larger ζ. At large supercriticalities, the character of all the curves for χ'' is practically unchanged from the character of the corresponding curve for the coherent pump. The behavior of the real part of the susceptibility χ'' falls off appreciably in the presence of noise, and at $\zeta \geqslant 2$ the value of χ' amounts to more than 50% of the real part of the susceptibility in the absence of the noise, while this ratio falls to 30% with increase in the supercriticality. The maximum value of χ' is close to the maximum value of χ''.

The results attest to the presence of phase correlations in the PSW system. In particular, they indicate that the PSW amplitude is not limited by nonlinear damping, since this mechanism yields $\chi' = 0$. Moreover, the similarity of the behavior of $\chi'(\zeta)$ and $\chi''(\zeta)$ to the susceptibilities obtained in the absence of noise shows that the mechanism of limitation of the PSW is the same in the two cases.

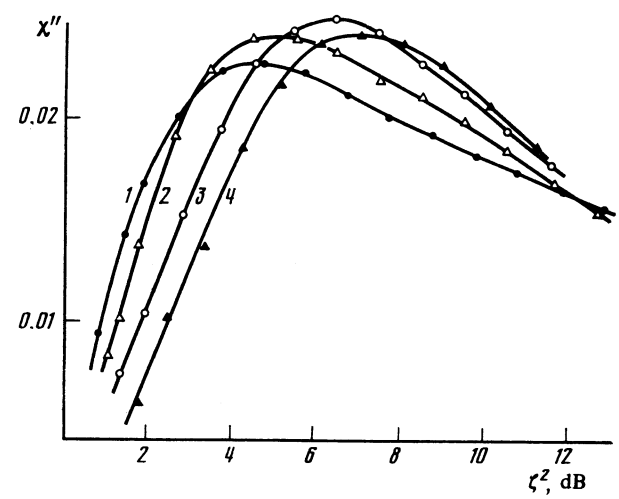


FIG. 5. Dependence of the imaginary part of the longitudinal microwave susceptibility on the supercriticality ($H = H_c - 100$ Oe): 1—$J_\omega = 0$; 2—$J_\omega = 1.7\times 10^{-9}$ Oe$^2\cdot$s; 3—$J_\omega = 3.4\times 10^{-9}$ Oe$^2\cdot$s; 4—$J_\omega = 5.1\times 10^{-9}$ Oe$^2\cdot$s.

For a more illustrative comparison of the obtained data with the results of calculation, we have plotted in Fig. 7 the experimental values of the quantity $\chi''(\zeta)\zeta^2$, which is proportional to the absorbed power, as a function of $\zeta^2 - 1$, in log-log scale, in which the Eqs. (2.11) and (2.12) predict a linear dependence. The graph is constructed for the greatest spectral density of the noise.

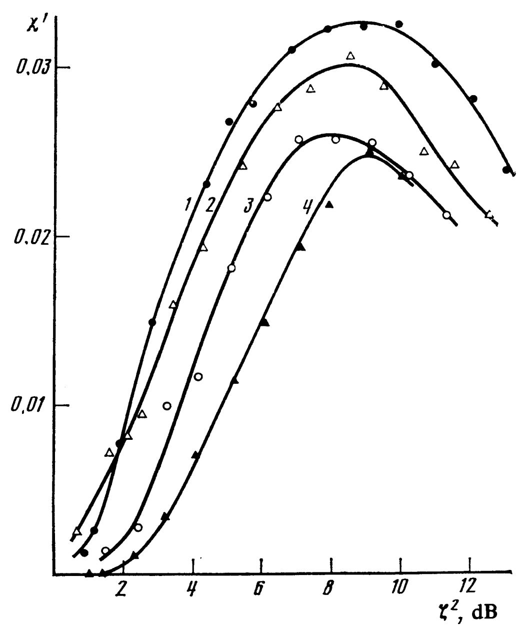


FIG. 6. Dependence of the real part of the longitudinal microwave susceptibility on the supercriticality ($H = H_c - 100$ Oe): 1—$J_\omega = 0$; 2—$J_\omega = 1.7\times 10^{-9}$ Oe$^2\cdot$s; 3—$J_\omega = 3.4\times 10^{-9}$ Oe$^2\cdot$s; 4—$J_\omega = 5.1\times 10^{-9}$ Oe$^2\cdot$s.

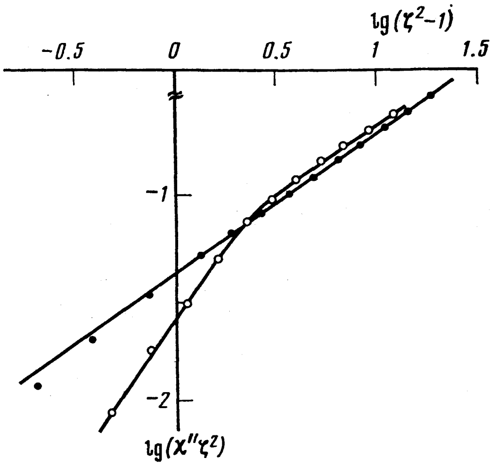


FIG. 7. Dependence of the absorbed power on the supercriticality in logarithmic coordinates; $H = H_c - 100$ Oe, $J_\omega = 5.1\times10^{-9}$ Oe$^2\cdot$s.

The features of the effect of the noise on the susceptibility beyond the threshold stand out clearly. This plot has two linear portions, and the slope at small $\zeta^2 - 1$ is equal to 1.4, which agrees with the exponent $n = 1.5$ in (2.11), while at large ω it approaches 0.7—the slope of the corresponding straight line for coherent pumping, shown for comparison in the same coordinates in Fig. 7. Possible reasons for the deviation of the exponent from $\frac{1}{2}$ in the latter case were discussed earlier in Ref. 9.

2.2. Nonlinear susceptibility of PSW in the case of a phase mechanism of amplitude limitation

For the study of parametric excitation of waves beyond threshold, we use the Hamiltonian of the system of PSW in the S-theory approximation[1]:

$$\mathscr{H}=\int\bar{\omega}_{\mathbf{k}}a_{\mathbf{k}}a_{\mathbf{k}}^{*}d\mathbf{k}+\frac{1}{2}\int(hV_{\mathbf{k}}a_{\mathbf{k}}^{*}a_{-\mathbf{k}}^{*}+\text{к. с.})\,d\mathbf{k}+\int T_{\mathbf{kk}'}|a_{\mathbf{k}}|^2|a_{\mathbf{k}'}|^2d\mathbf{k}d\mathbf{k}'+\frac{1}{2}\int S_{\mathbf{kk}'}a_{\mathbf{k}}^{*}a_{-\mathbf{k}}^{*}a_{\mathbf{k}'}a_{-\mathbf{k}'}d\mathbf{k}d\mathbf{k}'+\int\xi_{\mathbf{k}}(t)\,a_{\mathbf{k}}a_{\mathbf{k}}^{*}d\mathbf{k}. \quad (2.2)$$

The expression for the matrix elements $T_{\mathbf{kk}'}$ and $S_{\mathbf{kk}'}$ in a ferromagnet can be found in Ref. 1.

The growth rate of the parametric instability of spin waves in parallel pumping is maximal on the equator of the resonance surface $\omega_{\mathbf{k}} = \omega_p/2$, $\theta_{\mathbf{k}} = \pi/2$. Therefore, at not too large supercriticalities, a group of PSW is excited on the equator. In this case, the quantity $U_{\mathbf{k}}$ for all PSW is the same and the fluctuation of the spectrum

$$\xi_{\mathbf{k}}(t)=U_{\mathbf{k}}H_m(t)=\xi(t)$$

does not depend on **k**. By the substitution (6), the equations of motion for the new amplitudes $b_{\mathbf{k}}$ are reduced to the form

$$\frac{\partial b_{\mathbf{k}}}{\partial t}+\left[\gamma_{\mathbf{k}}+i\left(\bar{\omega}_{\mathbf{k}}+2\int T_{\mathbf{kk}'}|b_{\mathbf{k}'}|^2d\mathbf{k}'\right)\right]b_{\mathbf{k}} +i\left[\tilde{h}(t)\,V_{\mathbf{k}}+\int S_{\mathbf{kk}'}b_{\mathbf{k}'}b_{-\mathbf{k}'}d\mathbf{k}'\right]b_{-\mathbf{k}}^{*}=0, \quad (2.3)$$

$$b_{\mathbf{k}}=a_{\mathbf{k}}\exp\left[-i\int^{t}\xi(t')\,dt'\right],\quad \tilde{h}(t)=h\exp\left[-2i\int^{t}\xi(t')\,dt'\right].$$

Thus, the problem of the parametric excitation of waves by noise modulation of the frequency at not too large a supercriticality is equivalent to the problem of the parametric excitation of waves by the noise poimp $\tilde{h}(t)$. In such a case, we can use for the calculation of the susceptibility the results of Ref. 5, where the distribution function $n_{\mathbf{k}} = \langle|a_{\mathbf{k}}|^2\rangle$ of the PSW parametrically excited by the noise pump was determined in the S-theory approximation.

As is shown in Ref. 5, the distribution of the PSW in the case of incoherent pumping is described by the equation

$$\tilde{h}_{2\omega}^{2}|V^2|=P_{2\omega}^{2}\Big\{1-\mathrm{Re}\int Sn_{\omega'}(\omega'-\omega-i\gamma)^{-1}d\omega +\Big|\int Sn_{\omega'}(\omega'-\omega-i\gamma)^{-1}d\omega'\Big|^2\Big\}$$

jointly with the condition of external stability

$$P_{2\omega}^{2}=\tilde{h}_{c2\omega}^{2}|V|^2 \quad \text{at} \quad n_\omega>0, \quad (2.4)$$

$$P_{2\omega}^{2}<\tilde{h}_{c2\omega}^{2}|V|^2 \quad \text{at} \quad n_\omega=0. \quad (2.5)$$

Here n_ω is the distribution of the PSW over the eigenfrequencies:

$$n_{\omega_k}=\int\frac{n_{\mathbf{k}}k^2}{v_\Omega}d\Omega,$$

Ω is the solid angle at the resonant surface, v_Ω is the group velocity, the angular distribution of the PSW is singular in the polar angle θ and isotropic in the azimuthal angle φ, i.e., $n_{\mathbf{k}} = n_{\mathbf{k}}\delta(\theta-\pi/2)$, P_ω^2 is the autocorrelation function of the unrenormalized pump:

$$P_{\mathbf{k}}(t)=\tilde{h}(t)\,V_{\mathbf{k}}+\int S_{\mathbf{kk}'}b_{\mathbf{k}'}(t)\,b_{-\mathbf{k}'}(t)\,d\mathbf{k}', \quad (2.6)$$

S is the value of $S_{\mathbf{kk}}$ averaged over the distribution $n_{\mathbf{k}}$. As is seen from (2.5), the amplitude of the PSW is limited because of the self-consistent renormalization of the pump to the threshold level.

In the case of coherent pumping $\tilde{h}_\omega^2\ h^2\delta(\omega-\omega_p)$ we obtain the standard solution of S theory[1]:

$$n_{\omega_k}=N_S\delta(\omega_k-\omega_p/2),\quad |S|N_S=\gamma(\zeta^2-1)^{1/2}. \quad (2.7)$$

Here ζ^2 is the ratio of the pump power to the threshold value.

In Ref. 5, the distribution of the PSW was studied in another extreme case, when the spectral width of the effective pumping $\Delta\varepsilon$ (4) is large in comparison with the PSW damping γ. In the limiting cases of large and small supercriticalities, for an effective pump of the form

$$\tilde{h}_{2\omega}^{2}|V|^2=\frac{\gamma}{2\pi}\zeta^2\left(1-\frac{\omega^2}{\nu^2}\right) \quad (2.8)$$

analytic solutions n_ω were obtained in Ref. 5.

Using the results of Ref. 5, we can obtain the following expression for the susceptibility in the case of incoherent pumping:

$$\chi=\frac{2|V|^2}{|S|}\frac{2}{|hV|^2}\int\frac{\int|S|n_{\omega'}(\omega'-\omega-i\gamma)^{-1}d\omega'}{\int|S|n_{\omega'}(\omega'-\omega-i\gamma)^{-1}d\omega'-1}\bar{h}_{2\omega}^2|V|^2d\omega. \quad (2.9)$$

A known expression for the susceptibility in the case of coherent pumping can easily be obtained from (2.7) and (2.9):

$$\chi_s''=\frac{2|V|^2}{|S|}\frac{(\zeta^2-1)^{1/2}}{\zeta^2},\quad \chi_s'=\frac{2|V|^2}{|S|}\frac{\zeta^2-1}{\zeta^2}. \quad (2.10)$$

In the case of noise pumping, using the results of Ref. 5, we obtain

$$\chi''=\frac{2|V|^2}{|S|}\frac{4}{3\sqrt{3}}\frac{(\zeta^2-1)^{3/2}}{\zeta^2},\quad \chi'=\frac{2|V|^2}{|S|}2(\zeta^2-1) \quad (2.11)$$

in the limit of small supercriticalities ($\zeta^2-1\lesssim 1$), and

$$\chi''=\frac{2|V|^2}{|S|}\frac{3\sqrt{3}}{16}\frac{(\zeta^2-1)^{1/2}}{\zeta^2},\quad \chi'=\frac{2|V|^2}{|S|} \quad (2.12)$$

in the limit of large supercriticalities ($\zeta^2-1\gtrsim 1$).

Strictly speaking, the expressions for the susceptibility (2.11)–(2.12) were obtained for a special form of the effective pumping (2.8). Actually, these formulas are valid in practice for any pumping spectrum, since the PSW distribution depends essentially on the behavior of $\tilde{h}^2_{2\omega}$ near the maximum. For example, for the spectral density $\xi(t)$ of the pumping in the case of white noise, numerical calculation shows that the difference of the susceptibility from (2.11) does not exceed 10% in the region of 0–5 dB.

2.3. Discussion of the results

It is seen from the (2.11) and (2.12) that the quantity $\chi''\zeta^2$ behaves like $(\zeta^2-1)^{3/2}$ at small supercriticalities and like $(\zeta^2-1)^{1/2}$ at large. Such a dependence is well substantiated experimentally (Fig. 7). The behavior of $\chi'(\zeta)$ (Fig. 6) also agrees well with the formulas (2.11) and (2.12). The decrease of $\chi''(\zeta)$ in the region $\zeta^2\gtrsim 8$ dB and the divergences that arise are connected with the creation of a second group of waves.[1]

We now consider the quantitative agreement of Eqs. (2.11) and (2.12) with the experimental results. Equation (2.11) for the imaginary part of the susceptibility is in excellent agreement with experiment. For example, for $\zeta^2=2$ dB it follows from it that $(\chi''/\chi_s'')_{\rm theor}=4(\zeta^2-1)/3\sqrt{3}=0.446$, while the ratio measured experimentally gave $(\chi''/\chi_s'')_{\rm exp}=0.44\ (\pm 7\%)$. In the region of large supercriticalities, a significant divergence from Eq. (2.12) takes place, from which it follows that $(\chi''/\chi_s'')_{\rm theor}=3\sqrt{3}/16=0.324$, while $(\chi''/\chi_s'')_{\rm exp}\approx 0.8$. As for the real part of the susceptibility, at large supercriticalities $(\chi'/\chi_s')_{\rm theor}=1$, while $(\chi'/\chi_s')_{\rm exp}=0.8$. Such an agreement can be regarded as satisfactory. In the region of small supercriticalities, there is a large divergence: at $\zeta^2=2$ dB we have $(\chi'/\chi_s')_{\rm theor}=2(\zeta^2-1)/3=0.48$, as against $(\chi'/\chi_s')_{\rm theor}\approx 0.1$.

In our view, the indicated divergences are due to the fact that under the experimental conditions, the effective pumping was insufficiently broadband, i.e.,

$$\frac{\Delta\varepsilon}{\gamma}\approx\frac{|h_cV|^2}{\gamma^2}\leqslant 3,$$

while the expressions (2.11) and (2.12) were obtained under the assumption that $\Delta\varepsilon/\gamma\gg 1$.

Nevertheless, the obtained qualitative agreement of the Eqs. (2.11) and (2.12) allows us to assume that the amplitude of the PSW under conditions of noise excitation is limited just as in coherent excitation because of the phase mechanism, and the existing theory correctly describes the features of the behavior of the PSW beyond threshold.

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