Creation of transversely polarized high-energy electrons and positrons in crystals

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It is shown that when high-energy γ quanta pass through a crystal at small angles to the crystallographic planes (axes) a new phenomenon arises: creation of transversely polarized electrons and positrons by unpolarized γ quanta. Estimates based on the theory developed in this paper for this phenomenon show that it can be used to obtain transversely polarized electrons and positrons with degree of polarization 50–90% and with energies of hundreds and thousands GeV in the case of incidence of the γ quanta on atomic planes, and starting with an energy of several tens of GeV in the case of incidence on atomic axes. Concrete calculations are made of the polarization, number, and angular distributions of positrons produced by 350-GeV γ quanta incident on the (110) family of planes of a tungsten plate of thickness 3×10 cm. The features of the manifestation of the described phenomenon in bent crystals are analyzed.

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I. INTRODUCTION

According to Refs. 1 and 2, a large group of polarization phenomena take place when electrons, positrons, and γ quanta pass through crystals. In particular, when electrons and positrons pass through a bent crystal, radiative self-polarization of the particle spin takes place. The interaction of the crystal with particles of sufficiently high energy moving in directions close to parallel to some family of planes (axes) is described by the potential of the planes (of the axes) which is averaged in the direction of these planes (axes). This interaction is characterized by a parameter $\chi = |V'|E/m^3$ ($\hbar = c = 1$), where e, m and E are the charge, mass and energy of the particles, V is the interplanar potential, and the prime denotes differentiation with respect to the direction $\mathbf{n}_x$ perpendicular to the plane. The estimate of the self-polarization length l_{pol} used in Ref. 1 is based on the theory of radiation in a slowly varying electromagnetic field.[3,4] The expression for l_{pol} can be written (at $\chi \ll 1$) in the form

$$l_{pol} \approx E/e^2m^2\chi^3. \quad (1)$$

The length l_{rad} over which the particle loses to radiation, in a slowly varying electromagnetic field, half its initial energy is equal to (see §90 of Ref. 3):

$$l_{rad} = E/e^2m^2\chi^2.$$

Obviously, the process of effective self-polarization will take place only if $l_{rad} \gtrsim l_{pol}$. We note that in radiative self-polarization in an accelerator the particle constantly receives additional energy. As can be seen from (1)

$$l_{pol}/l_{rad} \approx \chi^{-1}$$

and therefore at $\chi \ll 1$

$$l_{pol} \gg l_{rad}$$

and radiative self-polarization in a bent crystal will not be very effective. As χ approaches unity the effectiveness of radiative self-polarization of channeled particles increases, and the process is accompanied by emission of γ quanta having an energy comparable with the total energy of the particle.[1] A particle beam initially monochromatic in energy is then transformed into a beam having approximately half the initial energy and with an energy spread of the order of the beam energy. In view of the finite length of the crystal and the smaller difference of the probability of emission with spin flip by particles having oppositely directed polarizations, it is impossible for the beam to acquire the high degree of polarization (up to 92%) attainable in storage rings (see Ref. 4 as well as the estimates in Ref. 5).

It is shown in the present paper that passage of high-energy quanta through a crystal gives rise to a new phenomenon, production of electron-positron pairs with transverse polarization in the pair at a high degree of polarization ($\sim$50–90%). Consider the polarization of electrons and positrons produced by a γ quantum in a thin crystal plate. Let the γ quantum be incident on the crystal at a small angle relative to a certain family of crystallographic planes. If a definite electron (positron) emission direction is chosen, it is possible to make up out of their momenta $\mathbf{p}_1$ ($\mathbf{p}$) and of the γ-quantum momentum $\mathbf{k}$ a pseudovector $\mathbf{k}\times\mathbf{p}_1$ ($\mathbf{k}\times\mathbf{p}$) that characterizes the possible direction of the electron (positron) polarization. This polarization will on the average be parallel to the atomic planes and perpendicular to the quantum momentum. The mechanism that produces the polarization can be explained in the following manner.

The electrons (positrons) produced by an unpolarized quantum in a slowly varying electric field are polarized perpendicular to the γ-quantum momentum and to the field intensity (see below). The electric fields on opposite sides of atomic planes are oppositely directed. Therefore both the polarizations of the electrons (positrons) produced there and the directions of their deflection by the electric field will also be opposite. If we choose a crystal with thickness equal to half the length over which electrons (positrons) produced with a definite energy execute half an oscillation in the transverse direction, obviously electrons (positrons) with oppositely directed polarizations will leave the crystal in different directions. It is thus possible to obtain polarized beams of electrons and positrons. We note that for pair production in the Coulomb field of the nucleus one can use with sufficient accuracy the Born approximation, in which the cross section for the production, say, of a positron with polarization $\boldsymbol{\xi}$ can

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have no term proportional to $\xi \mathbf{k}\times\mathbf{p}$, since the scattering martix is Hermitian in this approximation (see Ref. 3, §71). For an exact quantitative description of this phenomenon we must find the cross section for the production of polarized electrons and positrons by an unpolarized γ quantum.

2. ANALYSIS OF CROSS SECTION FOR PRODUCTION OF POLARIZED ELECTRONS AND POSITRONS BY A γ QUANTUM

Thus, let a γ quantum of energy ω be incident on a crystal of thickness l. A general expression for the cross section of pair production by a γ quantum having a polarization $\mathbf{e}$ in an arbitrary field is of the form (Ref. 3 §95).

$$d\sigma=2\pi\left|e(4\pi)^{1/2}\frac{1}{(2\omega)^{1/2}}M\right|^2\delta(\omega-E-E_1)\frac{d^3\mathbf{p}\,d^3\mathbf{p}_1}{(2\pi)^6},\quad(2)$$

where

$$M=\int\Psi_{E_1\mathbf{p}_1}^{(-)*}(\alpha\mathbf{e})e^{i\mathbf{kr}}\Psi_{-E-\mathbf{p}}^{(+)}d^3\mathbf{r},\quad(3)$$

$$\Psi_{E_1\mathbf{p}_1}^{(-)}=\frac{1}{\sqrt{2}}e^{i\mathbf{p}_1\mathbf{r}}\left(1-\frac{i\alpha\nabla}{2E_1}\right)u_{E_1\mathbf{p}_1}\varphi^{(-)}(\mathbf{r})\quad(4)$$

is the wave function of an electron having a momentum $\mathbf{p}_1$ and an energy E_1 and containing asymptotically far from the crystal (besides the plane wave) also a converging spherical wave,

$$\Psi_{-E-\mathbf{p}}^{(+)}=\frac{1}{\sqrt{2}}e^{-i\mathbf{pr}}\left(1+\frac{i\alpha\nabla}{2E}\right)u_{-E-\mathbf{p}}\varphi^{+}(\mathbf{r})\quad(5)$$

is the wave function of a positron with momentum $\mathbf{p}$ and energy E, containing asymptotically a diverging spherical wave.

In the standard representation,

$$\alpha=\begin{pmatrix}0&\sigma\\ \sigma&0\end{pmatrix},\quad u_{-E-\mathbf{p}}=\left(1-\frac{m}{E}\right)^{1/2}\begin{pmatrix}1\\ \dfrac{\mathbf{p}\sigma}{E-m}\end{pmatrix}w(-\xi),$$

$$u_{E_1\mathbf{p}_1}=\left(1+\frac{m}{E}\right)^{1/2}\begin{pmatrix}1\\ \dfrac{\mathbf{p}_1\sigma}{E_1+m}\end{pmatrix}w(\xi_1),\quad(6)$$

where $\sigma=(\sigma_i)$ are Pauli matrices, $\omega(\xi)$ is a two-dimensional spinor normalized to unity and describing the polarization state of a particle with a polarization vector ξ in its own reference frame, $\varphi^{(-)}(\mathbf{r})$ and $\varphi^{(+)}(\mathbf{r})$ are the solutions of the equations

$$\left[-\frac{\Delta}{2E}-\frac{E^2-m^2}{2E}+V\right]e^{-i\mathbf{pr}}\varphi^{(+)}(\mathbf{r})=0,$$

$$\left[-\frac{\Delta}{2E_1}-\frac{E_1^2-m^2}{2E_1}-V\right]e^{i\mathbf{p}_1\mathbf{r}}\varphi^{(-)}(\mathbf{r})=0,\quad(7)$$

the x axis is perpendicular to the family of atomic planes and the z axis is parallel to it. The solutions of (7) were obtained in Ref. 6. Substituting them in (2)–(6) we can obtain an exact expression for the number of electrons and positrons with polarizations ξ_1 and ξ, produced by a γ quantum having a polarization $\mathbf{e}$ and moving almost parallel to the family of the crystallographic planes (axes).[2] We shall consider hereafter the production of polarized particles with positrons in the planar case as the example. The required number of positrons of polarization ξ produced by the unpolarized γ quantum can be obtained by averaging the obtained number of electrons and positrons over the γ-quantum polarizations, summing over the electron polarization, and integrating over the electron momentum. As a result we obtain

$$\frac{d^2N}{dE\,d\Omega}=\frac{e^2E^2}{4\pi^2\omega}\sum_{fi\bar{n}}Q_{\bar{i}\bar{n}}(-p_x)\left[\frac{e^{iq_{z\bar{i}f}L}-1}{q_{z\bar{i}f}}\right]\left[\frac{e^{-iq_{z\bar{n}f}L}-1}{q_{z\bar{n}f}}\right]$$
$$\times\left\{\frac{\omega^2}{E_1^2}g_{\bar{i}f}g_{\bar{n}f}^*-\frac{2E}{E_1}[g_{\bar{i}f}g_{\bar{n}f}^*-(g_{\bar{i}f}\mathbf{n})(\mathbf{g}_{\xi}^{\bullet}\mathbf{n})]+\frac{\omega}{E_1}\xi i[g_{\bar{i}f}g_{\bar{n}f}^*]\right\},\quad(8)$$

where ω, E, and E_1 are respectively the energies of the γ quanta, positrons, and electrons; in the planar case we have

$$g_{\bar{i}f}=\frac{1}{2E}(\mathbf{I}_{2\bar{i}f}+[\mathbf{p}-\mathbf{n}(\mathbf{pn})-\mathbf{n}_xp_x]J_{1\bar{i}f}+m\mathbf{n}J_{1\bar{i}f}),\quad(9)$$

$$J_{1\bar{i}f}=\int_0^d\psi_{f\varkappa_1}^*e^{ik_xx}\psi_{\bar{i}-\varkappa}dx,\quad I_{2\bar{i}f}=i\mathbf{n}_x\int_0^d\psi_{f\varkappa_1}^*e^{ik_xx}\nabla_x\psi_{\bar{i}-\varkappa}dx,\quad(10)$$

in the axial case:

$$g_{\bar{i}f}=\frac{1}{2E}(\mathbf{I}_{2\bar{i}f}+[\mathbf{n}_z-\mathbf{n}(\mathbf{nn}_z)]EJ_{1\bar{i}f}+m\mathbf{n}J_{1\bar{i}f}),\quad(9')$$

$$J_{1\bar{i}f}=\int_\Delta\psi_{f\varkappa_1}^*e^{i\mathbf{k}\rho}\psi_{\bar{i}-\varkappa}d^2\rho,\quad\mathbf{I}_{2\bar{i}f}=i\int_\Delta\psi_{f\varkappa_1}^*e^{i\mathbf{k}\rho}\nabla_\rho\psi_{\bar{i}-\varkappa}d^2\rho,\quad(10')$$

$\mathbf{n}$ is the direction of the γ-quantum momentum, $\mathbf{n}_x$ is a unit vector perpendicular to the family of atomic planes, $\psi_{\bar{i}-\varkappa}$ and $\psi_{j\varkappa_i}$ are respectively the wave functions of the transverse motion of the positrons and electrons, respectively, normalized to unity over the unit cell and satisfying, e.g., in the planar case, the equations

$$\left[-\frac{\Delta}{2E}+V(x)\right]\psi_{\bar{i}-\varkappa}=\varepsilon_{\bar{i}-\varkappa}\psi_{\bar{i}-\varkappa},$$
$$\left[-\frac{\Delta}{2E_1}-V(x)\right]\psi_{f\varkappa_1}=\varepsilon_{f\varkappa_1}\psi_{f\varkappa_1};\quad(11)$$

$\varepsilon_{\bar{i}-\varkappa}$ and $\varepsilon_{j\varkappa_i}$ are the energies of the transverse motion of the positron and electron in the lab, respectively,

$$Q_{\bar{i}\bar{n}}(-p_x)=c_{\bar{i}}(-p_x)c_{\bar{n}}^*(-p_x),\quad c_{\bar{i}}(-p_x)=\frac{1}{d^{1/2}}\int_0^d e^{-ip_xx}\psi_{\bar{i}-\varkappa}^*dx$$

d is the interplanar distance,

$$p_{z\bar{i}}=(E^2-m^2-2E\varepsilon_{\bar{i}-\varkappa}-p_y^2)^{1/2}\approx E-\frac{m^2}{2E}-\varepsilon_{\bar{i}-\varkappa}-\frac{p_y^2}{2E},$$

$$p_{1zf}=(E_1^2-m^2-2E_1\varepsilon_{f\varkappa_1}-p_{1y}^2)^{1/2}\approx E_1-\frac{m^2}{2E_1}-\varepsilon_{f\varkappa_1}-\frac{p_{1y}^2}{2E_1},\quad p_{1y}=k_y-p_y,\quad(12)$$

$$q_{z\bar{i}f}=p_{z\bar{i}}+p_{1zf}-k_z\approx\frac{\omega\theta^2}{2}-\frac{\omega m^2}{2EE_1}-\varepsilon_{f\varkappa_1}-\varepsilon_{\bar{i}-\varkappa}-\frac{p_y^2}{2E}-\frac{p_{1y}^2}{2E_1}.$$

We consider the last term in the curly brackets of (8) which takes into account the polarization of the produced positrons. We assume for simplicity that the quantum is incident on the crystal parallel to the planes. Equation (9) takes then the simpler form

$$g_{\bar{i}f}=\frac{1}{2E}(I_{2\bar{i}f}+[p_y\mathbf{n}_y+m\mathbf{n}]J_{1\bar{i}f}). \quad (13)$$

Choosing the wave functions of the electrons and positrons to be real, we have

$$\xi i[g_{\bar{i}f}\times g^*_{\bar{n}f}]=m\xi_y\left(\int_0^d \psi_{\bar{i}-\varkappa}\nabla_x\psi_{f\varkappa_i}\,dx\right.$$

$$\left.\times\int_0^d \psi_{\bar{n}-\varkappa}\psi_{f\varkappa_i}\,dx+\int_0^d \psi_{\bar{n}-\varkappa}\nabla_x\psi_{f\varkappa_i}\,dx\int_0^d \psi_{\bar{i}-\varkappa}\psi_{f\varkappa_i}\,dx\right). \quad (14)$$

We consider the case of a potential that is symmetric about the center of the channel—in this case the wave functions of the positrons and electrons are either symmetric or antisymmetric. Differentiation changes the symmetry of the wave functions, therefore Eq. (14) is not zero only if the wave functions $\psi_{\bar{i}-x}$ and $\psi_{\bar{n}x_i}$ have different symmetries. In that case the only pair of integrals that is nonzero and positive is the one in which the integral with the derivative contains a wave function whose symmetry is opposite that of the electron wave function ψ_{fx_i}. It follows from this that the proposed effect will not be observed in sufficiently thick crystals for which

$$-(q_{z\bar{i}+1f}-q_{z\bar{i}f})L=(\varepsilon_{\bar{i}+1-\varkappa}-\varepsilon_{\bar{i}-\varkappa})L\gg 1,$$

i.e., whose thickness is much larger than the length over which the electron (positron) executes one oscillation in the transverse direction. In this case the product of the square brackets in (8) differs from zero and goes over into $2\pi L\delta(q_{z\bar{i}f})$, only if $q_{z\bar{i}f}=q_{z\bar{n}f}$, and this is possible only if $\bar{n}=\bar{i}$, but in this case (14) is equal to zero. It will be seen from the analysis that follow that the effect considered manifests itself best when $(\varepsilon_{\bar{i}+1-\varkappa}-\varepsilon_{\bar{i}-\varkappa})L\approx\pi$. We note that in the Born approximation the cross section cannot contain a term proportional to only one polarization vector of the electron or positron. The reason is that the scattering matrix is Hermitian in this approximation.[3]

We analyze now the angular dependence of the last term in (8)—it is contained in the coefficient $Q_{\bar{i}\bar{n}}(-P_x)$. Let the wave function $\psi_{\bar{i}-\varkappa}$ be symmetric and $\psi_{\bar{n}-\varkappa}$ antisymmetric about the channel center; then

$$Q_{\bar{i}\bar{n}}(-p_x)=\int_0^d \psi_{\bar{i}-\varkappa}(x)\cos(p_xx)\,dx\int_0^d \psi_{\bar{n}-\varkappa}(x)\sin(p_xx)\,dx.$$

It is now obvious that the number of positrons with definite polarization is not symmetric about the symmetry plane of the channel. Therefore, separating the positrons that move in a definite direction, we obtain a beam with nonzero polarization. An exact calculation of the effect considered can be carried out in the case of zero incidence angle of the γ-quantum beam on the atomic planes. To calculate the integrals (10) in this case it suffices to know the wave functions of the electrons and positrons only near the turning points, for outside this region one of the wave functions contained in the integrals (10) attenuates exponentially. To simplify the analysis we derive first expressions that describe the production of polarized positrons (electrons) in a slowly varying electric (magnetic) field.

3. PRODUCTION OF POLARIZED ELECTRONS AND POSITRONS IN A SLOWLY VARYING ELECTROMAGNETIC FIELD

To calculate the probability of production of polarized positrons by γ quanta in a slowly varying magnetic field we shall use a method described in Ref. 3, §§91 and 92. We regard a field as slowly varying if pair production in it can be described by an expression derived from the expression for the pair-production probability in a uniform field by replacing the constant field by a varying one. We note that the pair-production probability does not depend on the nature of the field (electron or magnetic), but is determined by the value of the acceleration that the field can impart to a unit charge. We shall therefore hereafter, without loss of generality, consider pair production in a uniform and constant electrric field of intensity $\vec{\mathscr{E}}$. Let this field be directed along the vector $\mathbf{n}_x$, let the γ quantum be incident in the direction of a vector $\mathbf{n}$ perpendicular to $\mathbf{n}_x$. We direct the coordinate axes along the vectors $\mathbf{n}_x$, $\mathbf{n}_y=[\mathbf{n}\mathbf{n}_x]$, $\mathbf{n}_x=\mathbf{n}$. The expression for the pair production probability, just as in §91 of Ref. 3, can be obtained from the expression [see Eqs. (90.10), (90.20), and (90.21) of Ref. 3, as well as 22 of Ref. 4]:

$$dw=\frac{e^2\omega^2\,d\omega\,d\Omega}{4\pi^2}\int_{-\infty}^{\infty}R_2^*\left(\frac{\tau}{2}\right)R_1\left(-\frac{\tau}{2}\right)L(\tau)\,d\tau, \quad (13')$$

where

$$R_2^*R_1=\mathrm{Sp}\frac{1+\xi\sigma}{2}((\mathbf{A}_2-i[\mathbf{B}_2\times\sigma])\mathbf{e})\frac{1+\xi_1\sigma}{2}((\mathbf{A}_1+i[\mathbf{B}_1\times\sigma])\mathbf{e}^*),$$

$$\mathbf{A}_1=\mathbf{A}\left(-\frac{\tau}{2}\right),\quad \mathbf{A}_2=\mathbf{A}\left(\frac{\tau}{2}\right), \quad (14')$$

$$\mathbf{B}_1=\mathbf{B}\left(-\frac{\tau}{2}\right),\quad \mathbf{B}_2=\mathbf{B}\left(\frac{\tau}{2}\right),$$

$$\mathbf{A}(\tau)=\frac{E+E_1}{2E_1}\mathbf{v}(\tau),\quad \mathbf{B}(\tau)=\frac{\omega}{2E_1}\left(\mathbf{n}-\mathbf{v}(\tau)+\mathbf{v}(\tau)\frac{m}{E}\right),$$

$\mathbf{v}(\tau)$ is the positron velocity at the instant of time τ, $\mathbf{v}(0)\|\mathbf{n}$, by making the substitution (see (91.1) of Ref. 3)

$$E,\ \mathbf{p}\to -E,\ -\mathbf{p};\qquad \omega,\ \mathbf{k}\to-\omega,\ -\mathbf{k};\qquad \xi\to-\xi;\qquad \mathbf{e}^*\to\mathbf{e}, \quad (15)$$

multiplying (13) by $(p^2/E^2)dE/d\omega$ and retaining the previous meaning of the notation. We shall investigate the creation of polarized particles with positrons as the example. To obtain from (13)–(15) the corresponding probability, the expression must be averaged overhe polarization of the γ quantum and summed over the polarization of the electron. Then $R_2^*R_1$ is then replaced by

$$\frac{1}{2}\sum_{\mathbf{e},\xi}R_2^*R_1=\frac{1}{2}\sum_{\mathbf{e}}\{(\mathbf{A}_2\mathbf{e})(\mathbf{A}_1\mathbf{e}^*)+[\mathbf{e}\times\mathbf{B}_2][\mathbf{e}^*\times\mathbf{B}_1]\}$$

$$+\frac{i}{2}\sum_{\mathbf{e}}\xi\{(\mathbf{A}_2\mathbf{e})[\mathbf{e}^*\mathbf{B}_1]-(\mathbf{A}_1\mathbf{e}^*)[\mathbf{e}\times\mathbf{B}_2]+[[\mathbf{e}\times\mathbf{B}_2][\mathbf{e}^*\times\mathbf{B}_1]]\}. \quad (16)$$

Using the results of §§90 and 91 of Ref. 3, we can employ the equality

$$\frac{e^2\omega E^2\,dE}{4\pi^2}\int\frac{1}{2}\sum_{e}\{(\mathbf{A}_2\mathbf{e})(\mathbf{A}_1\mathbf{e}^*)+[\mathbf{e}\times\mathbf{B}_2][\mathbf{e}^*\times\mathbf{B}_1]\}L(\tau)\,d\Omega d\tau$$
$$=\frac{\alpha m^2\,dE}{2\pi^{1/2}\omega}\left\{\int_x^\infty\Phi(\xi)\,d\xi+\left(\frac{2}{x}-\varkappa x^{1/2}\right)\Phi'(x)\right\}, \quad (17)$$

where $\Phi(x)$ is an Airy function defined as

$$\Phi(x)=\frac{1}{2\pi^{1/2}}\int_{-\infty}^{\infty}\exp\{i(xt+t^3/3)\}\,dt, \quad (18)$$
$$x=(m^3\omega/e\mathscr{E}EE_1)^{2/3}, \quad \varkappa=e\mathscr{E}\omega/m^3. \quad (19)$$

We transform the second sum in the right-hand side of (16) by using the equalities

$$\sum_{e}e_ie_k^*=[\delta_{ik}-n_in_k], \quad \mathbf{v}_{1,2}=\mathbf{n}\left(1-\frac{m^2}{2E^2}\right)\mp\frac{\mathbf{n}_x e\mathscr{E}\tau}{2E}. \quad (20)$$

As a result we obtain

$$\sum_{e}\xi[[\mathbf{e}\times\mathbf{B}_2][\mathbf{e}^*\times\mathbf{B}_1]]=(\xi-\mathbf{n}(\xi\mathbf{n}))[\mathbf{B}_2\times\mathbf{B}_1]=\frac{m\omega^2e\mathscr{E}\tau}{4E^2E_1^2}(\xi\mathbf{n}_y), \quad (21)$$

$$\sum_{e}\xi\{(\mathbf{A}_2\mathbf{e})[\mathbf{e}\times\mathbf{B}^2]-(\mathbf{A}_1\mathbf{e}^*)[\mathbf{e}^*\times\mathbf{B}_2]\}=-\frac{m\omega(E+E_1)e\mathscr{E}\tau}{4E^2E_1^2}(\xi\mathbf{n}_y). \quad (22)$$

Substituting (19) and (20) in (16) and (13), making the substitution (15), integrating with respect to $d\tau$ and with respect to the angles $d\Omega$, and multiplying the result by $(p^2/\omega^2)dE/d\omega$, we obtain ultimately

$$dw=\frac{\alpha m^2\,dE}{2\pi^{1/2}\omega^2}\left\{\int_x^\infty\Phi(\xi)\,d\xi+\left(\frac{2}{x}-\varkappa x^{1/2}\right)\Phi'(x)+\frac{(\xi\mathbf{n}_y)\,\omega\Phi(x)}{Ex^{1/2}}\right\}. \quad (23)$$

Expressions (19) and (23) describe the probability, differential with respect to positron energy, of production of a positron with polarization ξ and of an electron with arbitrary polarization, by a γ quantum moving perpendicular to a uniform electric field. The only important quantity in the derivation of (23) was the positron acceleration imparted by the electromagnetic field and contained in the equation that describes the change of the positron velocity (20). In the case of an electric field it is equal to $e\mathscr{E}/E$. Obviously, the result (23) remains valid also in a uniform field that is a superposition of uniform electric and magnetic fields, if $e\mathscr{E}/E$ in (17)–(23) is replaced by the acceleration produced by this field in the direction perpendicular to the motion of the γ quantum. The differential probability for pair production of a γ quantum, accompanied by a positron polarization ξ, taking the form (23) in the case of an electric field, was first obtained by Ritus[7] by another method. The Airy function calculated from Eq. (18) at $x\sim1$ (at $x\gg1$ pair production is exponentially suppressed), is formed at $t\sim1$. This corresponds to a distance

$$l_{\text{coh}}=EE_1/\omega m^2. \quad (24)$$

If the field changes over the trajectory l_{coh} of the produced particles or over the time l_{coh} ($\hbar=c=1$) by an amount small compared with its value, pair production in such a field is described by Eq. (23) obtained for the case of a uniform field. Such a field was named slowly varying above.

4. PRODUCTION OF TRANSVERSELY POLARIZED ELECTRONS AND POSITRONS BY γ QUANTA IN CRYSTALS

The effect considered is of interest in connection with the possibility of obtaining transversely polarized electrons and positrons. It is therefore of interest to consider this effect at γ-quantum energies at which pair production is most intense, i.e., at $\varkappa\gtrsim1$ or at $\omega\gtrsim m^3/V'$ ($\omega>200$ GeV in tungsten). At such high energies the particles produced in the field of the planes are deflected by this field through angles $\sim(V_{\max}/\omega)^{1/2}$, much larger than the characteristic angle of the initial divergence m/ω of the particles produced by the γ quantum. To describe pair production in this case one can use the theory of pair production in a slowly varying electromagnetic field (see Ref. 3 and §77 of Ref. 8). Indeed, the change of the field over the length l_{coh} at a γ-quantum incidence angle $\theta\ll m^2d/\omega$ on the family of crystallographic planes is many times smaller than the field itself. We, however, are interested in even smaller (by a factor $2(V_{\max}/\omega)^{1/2}$) incidence angles of the γ quantum. It is precisely at these angles that particles with opposite directions of the polarizations diverge by the largest angles, of the order of $(V_{\max}/\omega)^{1/2}$. Using expression (23) for the probability of positron production with polarization ξ and calculating numerically the motion of the produced particles in the potential of the atomic planes, we can obtain the energy and angular distributions of the polarized particles leaving the crystal. We have carried out a similar calculation of the characteristics of the positrons produced by a γ quantum of energy $\omega=350$ GeV in a tungsten crystal 4×10^{-4} cm thick for the case of zero incidence angle of the γ quantum of the (110) family of atomic planes. The crystal thickness was chosen to be approximately half the length over which positrons produced with average energy $\omega/2$ execute one oscillation in the transverse direction. In this case of zero incidence angle of the γ quantum the only particle produced are those with a transverse motion energy less than $V_{\max}$. The motion of positrons having this transverse energy is very close to harmonic. Therefore the crystal length most suitable for obtaining polarized positrons is approximately the formula

$$l_{\text{cr}}=\frac{T}{2}=\frac{\pi d}{2}\left(\frac{\omega}{2V_{max}}\right)^{1/2}. \quad (25)$$

Figure 1 shows the dependence of the degree of polarization of the positrons deflected by the interplanar potential to the right relative to the γ momentum on the positron energy. It can be seen from the figure that at $E<\omega/2$ the degree of positron polarization exceeds 60%, and at $E=0{,}2$ it reaches 90%. Figure 2 shows the dependence of the number of electron-positron pairs produced per γ quantum on the positron energy. This plot is obviously symmetric about the vertical straight line $E=\omega/2$. Only 1.4×10^{-3} pair is produced per γ quantum in this case. This value decreases with increasing γ-quantum energy. The reason is that with increasing γ-quantum energy, and hence with increasing $\varkappa(x)$, section with an

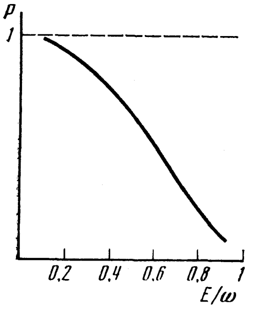

FIG. 1. Dependence of the degree of polarization of the produced electrons (positrons) on their energy.

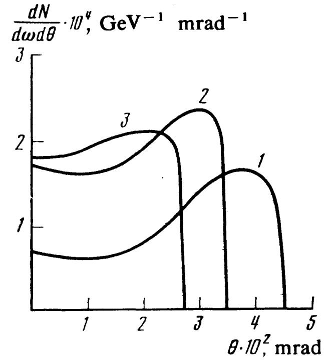

FIG. 3. Dependence of the number of positrons emitted from the crystal on the emission angle relative to the crystallographic plane.

ever decreasing interplanar potential participate in the intense pair production as $\varkappa(x)$, approaches unity. It is known (see Ref. 3, §91) that with increasing $\varkappa$ the intensity of pair production begins to decrease. This decrease, however, sets in at $\varkappa > 11$ and proceeds very slowly, in proportion to $\varkappa^{-1/2}$, and therefore cannot influence noticeably the growth of the number of pairs produced in the interplanar potential. The increase of the γ-quantum energy and of $\varkappa$ makes it possible to use crystals having a less steep potential than tungsten, and therefore a larger optimal thickness l_{cr} than tungsten (see (25)). In addition, as seen from (25), l_{cr} also increases with increasing ω if the crystal does not change. All the described factors make it possible to obtain more than 10^{-2} pairs per γ quantum at the described experimental setup. Figure 3 shows the angular distributions of positrons produced on interplanar-potential sections located to the left of the channel centers and deflected by the plane potential to the right, curves 1, 2, and 3 show the distributions of positrons with respective energies 0.3ω, 0.5ω, and 0.7ω. Obviously, the angular distribution of the positrons produced on interplanar-potential sections to the right of the channel centers are represented by the plots of Fig. 3 but reflected from the $\theta = 0$ axis.

An equation that describes the production of polarized electrons can be obtained from (23) by replacing ξ with $-\xi$ and E with E_1. The plots, obtained on this basis, of the electron polarization and of the number of produced pairs against the electron energy and emission angles will be of the same form as Figs. 1–3 for positrons. The difference between the planar potentials of electrons and positrons causes the angular distributions of the electrons to decrease with increasing electron emission angle, in contrast to the positron angular distributions shown in Fig. 3.

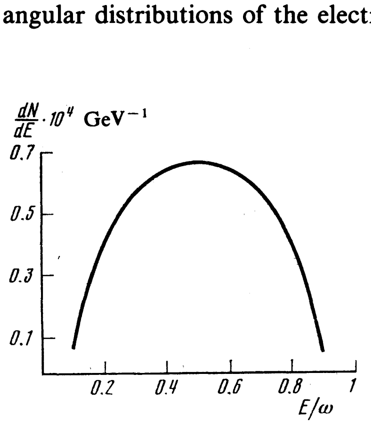

FIG. 2. Dependence of the number of produced pairs on the energy of the produced electron (positron).

We have described pair production above by using a model wherein the intracrystalline field is represented by the averaged potential of the plane. Account was thus taken only of cases of pair production with coherent momentum transfer to many nuclei of the crystal. However, cases of pair production with incoherent momentum transfer are not taken into account by this model. We note that the unpolarized electrons produced in such processes at zero γ-quantum incidence angle will, in contrast to positrons, be emitted from the crystal in an angle interval $\sim m/E$ and will not change the degree of polarization of the electrons produced in the averaged potential and emitted at larger angles, as shown above. The fraction of particles produced in incoherent transfer of momentum to an individual nucleus can be qualitatively estimated by starting from the pair-production cross section in the Born approximation. These estimates show that in the γ-quantum energy region of interest to us, in which $\varkappa \gtrsim 1$, the contribution of the unpolarized particles is large (it reaches several times ten percent) only for substances with $Z \gtrsim 70$, including tungsten. In substances with small Z, such as diamond or silicon, in which the γ quantum produces a pair at an arbitrary crystal orientation over a length on the order of 10 cm, the contribution of the incoherent pair production is negligibly small.

Regarding the crystal as a region of space occupied by a slowly varying electric field, one can propose other possibilities of realizing production of polarized particles in the slowly varying magnetic field considered in the preceding section. Thus, if the quantum is incident on a crystal at a small angle to a family of atomic axes, particles of opposite polarization will also be emitted from the crystal in different directions. The potential of the axes is 5–10 times deeper than the potential of the planes at equal temperatures. Therefore production of polarized electrons and positrons in the axial case is possible at γ-quantum energies smaller by several times than in the planar case. From Eq. (25), which is valid for the estimate of the optimal crystal length, it can be seen that in the axial case this length will be less than in the planar case. In addition, in the former case the particles emitted in definite directions will be produced on crystal cross section segments that are smaller than in the latter case. As a result of these factors, the number of polarized particles obtained per gamma quantum in the axial case is approximately one order smaller than in the planar case. The upper limit of the number of polarized particles produced by a γ quantum in the axial case after passing once through a plate is 10^{-3}, although the effect of production of polarized electrons and positrons can be observed starting with γ-quantum energies of several times ten GeV.

Let us analyze in conclusion the production of transversely polarized particles in a bent crystal. The fact that in a bent crystal unpolarized γ quanta should produce transversely polarized electrons and positrons was noted in Ref. 2. Particle motion in a bent channel can be described as motion in a potential

$$V_1(x)=V(x)+\frac{E(d/2-x)}{r}, \qquad \text{(a)}$$

where r is the crystal curvature radius. Such a potential is shown in Fig. 4 for the case of the (110) family of planes in tungsten, $E = 200$ GeV, and $r = 30$ cm. The points o, d, c, and o_1, d_1, c_1 represent the local maxima and minima of the potentials $V(x)$ and $V1(x)$, respectively. The pair production due to the electric field will be described as before by the potential $V(x)$. The positrons produced on the segment $bd_1(V_1(o) = V_1(b))$, will not land in the channel, and at the same time this is the steepest segment of the potential $V(x)$ with negative direction of the field intensity. On the other hand, the positrons produced on the segment o_1c, i.e., practically all the particles produced on the segment with positive direction of the field intensity, will move along the bent channel and will leave the crystal strongly deflected, if the crystal is not so long that they are forced to leave the channel as a result of multiple scattering. Such a picture is of course correct of the γ-quantum moves at a small angle to the line $V=0$ (we recall that in an unbent crystal the line $V=0$ coincides with the minimum of the potential energy of the positrons (with the maximum of the potential energy of the electrons)). In the opposite case all the particles will not land in the channel. It is easy to note that the largest deflection to the left will be experienced by just the positrons produced on those crystal segments where the γ quantum travels at a small angle (less than $2(eV_{\max}/\omega)^{1/2}$)) to the line $V=0$ whereas the positrons produced on the same segment and having opposite polarization directions land in the bent channel and can be deflected through angles larger than $2(eV_{\max}/\omega)^{1/2}$. At such high γ-quantum energies as considered by us, the necessary crystal bending angle is quite large ($r \gtrsim 10$ cm).

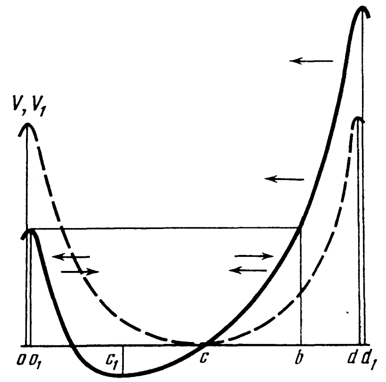

FIG. 4. Averaged potential $V(x)$ of the planes (dashed line) and the potential ($V_1(x) = V(x) + E(d/2 - x)/r$ used to describe the motion of charged particles in a bent interplanar channel.

5. CONCLUSION

The analysis in the preceding section shows that the effect considered by us in Secs. 2 and 3, production of transversely polarized electrons and positrons, as well as the effect of radiative self-polarization proposed in Ref. 1 makes it possible to obtain beams of transversely polarized particles of high energy (hundreds and thousands of GeV). Besides the shortcoming common to both methods, namely the broad energy spectrum of the obtained polarized particles and the substantial decrease (by an approximate factor of two) of the energy of the final particles compared with the energy of the initial ones, the phenomenon considered above has the advantage of high degree of polarization of the obtained particles. This makes the predicted effect promising for obtaining beams of polarized electrons and positrons needed for research into high-energy physics.

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