

New type of boundary conditions for orientation deformations in homeotropic layers of nematic liquid crystals

M. I. Barnik, L. M. Blinov, T. V. Korkishko, V. A. Umanskiĭ, and V. G. Chigrinov

Research Institute for Organic Semiconductors and Dyes

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A new shape of the surface potential is proposed, in the form of the square of the elliptic sine of the angle of disorientation of the director of a nematic liquid crystal (NLC) on a substrate. Allowance for the new type of surface potential explains the apparent strong anchoring of the director to the substrate upon deformation of the homeotropically oriented NLC layer in an electric field (the B effect), which manifests itself in independence of the threshold voltage of the B effect on the thickness of the liquid-crystal layer. The experimentally observed dependences of the birefringence on the external voltage in an electric field parallel or perpendicular to the initial homeotropic orientation of the NLC director agree well with those calculated for the new type of potential.

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1. INTRODUCTION

An important parameter that influences the behavior of a liquid crystal in thin layers is the binding energy of its molecules with the bounding walls. The binding-energy concept was introduced by Rapini.¹ It is assumed that to change the director orientation of a nematic liquid crystal (NLC) on a solid surface it is necessary to expend a certain energy W to overcome the interaction between the NLC molecules and the substrate:

$$W = \frac{1}{2} W_0 \sin^2(\theta - \theta_0), \quad (1)$$

where θ_0 and θ are respectively the initial and final angles between the director orientation and the normal to the substrate plane and $W_0/2$ is the energy that must be expended for maximum deflection of the director on the surface.

The binding energy W_0 influences substantially the equilibrium distribution of the NLC director both in the absence (Refs. 2–4) and in the presence⁵ of external perturbations. In addition, under the action of an external electric or magnetic field it determines the dynamic⁶ and threshold^{7,8} characteristics of the electro-optical effects. In particular, it has been shown in Ref. 7 that if the NLC molecules are not strongly bound to the substrate the threshold voltage of the Fredericksz transition in an electric field depends on the binding energy W_0 and on the sample thickness L (Ref. 9):

$$U_n = U_n (1 + 2k_{ii}/LW_0)^{-1/2}, \quad U_n = 2\pi (\pi k_{ii}/|\epsilon_a|)^{1/2}, \quad (2)$$

where U_n is the threshold voltage of the Fredericksz transition in an electric field under tight-binding conditions ($W_0 \rightarrow \infty$), k_{ii} are the elastic constants ($k_{ii} = k_{11}$ or $k_{ii} = k_{33}$ for planar and homeotropic orientation of the NLC, respectively), $\epsilon_a = \epsilon_{\parallel} - \epsilon_{\perp}$ is the anisotropy of the NLC dielectric constant, and L is the thickness of the layer. The binding energy influences also the dependence of the relative birefringence the NLC director orientation angle θ_m at the center of the layer, and of the angle θ_b on the substrate, on the external voltage.⁷ Weakening of the director binding with the substrates causes the corresponding curves to become steeper with increasing voltage and to shift towards lower

voltages. The characteristic dependences of the relative birefringence δ and of the director orientation angle θ_b on the substrates in the B effect (Fig. 1) on ghd relative voltage are shown in Fig. 2. Similar threshold characteristics for the Fredericksz effect in a magnetic field were experimentally observed in Ref. 8. We note, however, that the situation in a magnetic field becomes more complicated, since the measured threshold of the Fredericksz transition in the case of tight binding depends linearly on the thickness of the NLC layer, and the accuracy of the experimental determination of the threshold compared with the case of an electric field is decreased as a result.

In accord with the foregoing, a relation of the type (2) might be the starting point for the determination of the binding energy from the threshold voltage of the Fredericksz effect. Certain measurements in electric¹⁰ and magnetic¹¹ fields, however, offer evidence that the threshold is independent of the NLC layer thickness. In accordance with (2), this suggests a tight binding ($W_0 \geq 0.1$ erg/cm²). For planar orientation this assumption is perfectly acceptable and was confirmed by a number of experiments.¹² For homeotropic orientation this assumption contradicts measurements made by various methods.^{4,5,8,13} In particular, the flexoelectric effect

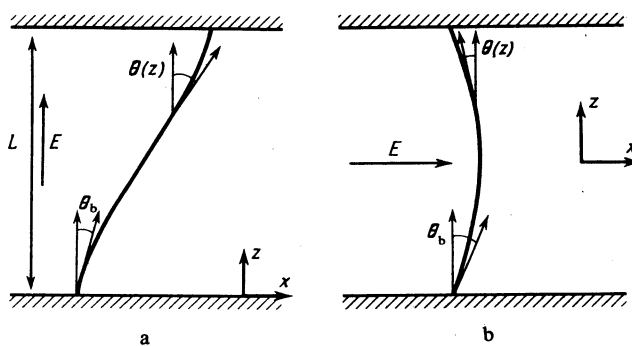


FIG. 1. Distribution of the director $\theta(z)$ in an NLC layer: a—Fredericksz transition (B effect)¹²; b—flexoeffect¹⁴; θ_b —angle at the boundary, E —external electric field.

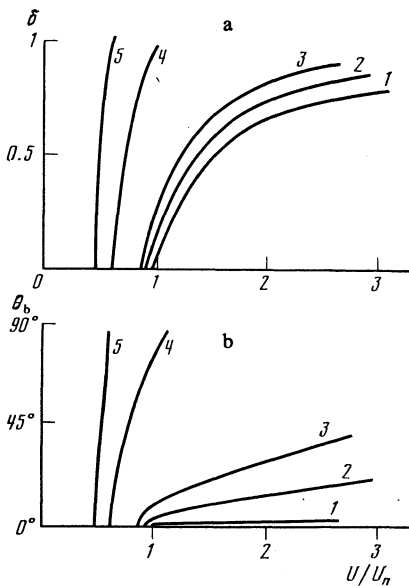


FIG. 2. Dependence of the relative birefringence (a) and of the angle of director orientation on the substrates (b) on the relative voltage U/U_n in the B effect. The numerical calculation was performed in accord with an algorithm analogous to that of Ref. 7, for the following parameters (MBBA): $k_{11} = 0.67 \cdot 10^{-6}$, $k_{33} = 0.83 \times 10^{-6}$, $\epsilon_1 = 5.25$, $\epsilon_{||} = 4.7$, $n_{||} = 1.76$, $n_{\perp} = 1.57$ (in cgs esu). The binding energy in the Rapini form (1) was used in the boundary condition. The NLC layer was $L = 22 \mu\text{m}$ thick. Binding energy $W_0 = 0.1$ (curve 1); 0.01 (2); 0.005 (3); 0.001 (4); 0.0005 (5) (in erg/cm^2).

in the Helfrich geometry¹⁴ (Fig. 2b) is possible only under conditions of non-tight binding of the NLC molecules with the substrate. The expression for the phase delay due to the flexoelectric effect, with allowance for the binding energy, takes in this case the form^{15,16}

$$\Delta\varphi = \Delta\varphi_0 \left(1 + \frac{LW_0}{2k_{33}}\right)^{-2}, \quad \Delta\varphi_0 = \frac{\pi L}{12\lambda} n_{\perp} \left(1 - \frac{n_{\perp}^2}{n_{||}^2}\right) \frac{U^2}{U_H^2}, \quad (3)$$

where $\Delta\varphi_0$ is the phase delay at zero binding energy, U is the voltage of the external electric field, $U_H = k_{33}/(e_{33} + m_p)$, with e_{33} the flexoelectric coefficient and m_p the surface polarization, while $n_{||}$ and n_{\perp} are the NLC refractive indices.

In all the studies known to us, the binding energy W is invariably used in the form of the Rapini potential (1), although in the general case W can be an arbitrary function of the square $(\theta - \theta_0)^2$ of the angle of disorientation of the director on the substrate, and expression (1) is only one of the possible approximations. In this paper, on the basis of investigations of orientational deformations of a homeotropically oriented NLC (B effect and flexoelectric effect) we show that the expression for the surface binding energy in the form (1) is not universal and propose a more general form of the surface potential that conforms to the experimental.

2. EXPERIMENTAL PROCEDURE

The orientational deformation of the NLC director in the B effect was investigated by measuring the phase delay between the ordinary and extraordinary light beams passing through the liquid-crystal layer. Transparent electrodes of

SnO_2 were deposited on the inner surfaces of the glass plates that bounded the NLC layers. These electrodes were coated with a thin layer of chromium stearylchloride to obtain geometric orientation. The NLC-layer thickness was set by Teflon liners. The phase delay and the threshold of the deformation were determined from the current-voltage characteristics obtained in crossed polarizers and recorded with a PDS-021 x - y plotter. The sweep rate of the voltage applied to the cell was $\approx 0.25 \text{ mV}/\text{sec}$.

In the flexoelectric effect the electrodes were two parallel strips of aluminum or copper foil, which set simultaneously the thickness of the liquid-crystal layer. The NLC orientation was homeotropic; this was also accomplished by depositing a thin layer of chromium stearylchloride. The phase delay was measured with a Senarmont compensator built into a polarization microscope. The measurement procedure is described in detail in Ref. 17.

The substance investigated was methoxybenzylidene butyl aniline (MBBA) with elastic constants $k_{11} = 0.67 \cdot 10^{-6}$ and $k_{33} = 0.83 \cdot 10^{-6}$, dyn, refractive indices $n_{||} = 1.76$ and $n_{\perp} = 1.57$, and dielectric anisotropy $\epsilon_a = \epsilon_{||} - \epsilon_{\perp} = -0.55$ (the indices $||$ and \perp pertain to parallel and perpendicular directions relative to the NLC director). The thickness of the empty cell was determined with an interferometer¹⁸ accurate to $\approx 5\%$. The threshold voltage was measured accurate to $\approx 2\%$. The phase delay measurements with the Senarmont compensator were accurate to $\approx 5\%$.

3. EXPERIMENTAL RESULTS

Figure 3 shows plots of the phase delay for the B -effect vs the applied voltage at different NLC layer thicknesses. It can be seen from Fig. 3 that the threshold voltage of the B effect does not depend on the thickness of the NLC layer, whereas according to (2) it should decrease with decreasing thickness. This lowering of the threshold should be particularly strong if the binding is weak. The independence of the B -

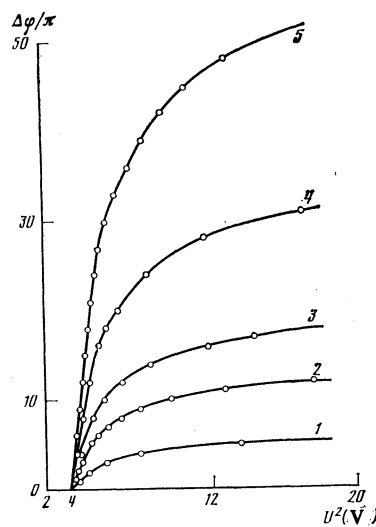


FIG. 3. Experimental plots of the phase delay (5) vs the external voltage for different layer thicknesses of the NLC (MBBA): $L = 10 \mu\text{m}$ (curve 1), 22 (2), 34 (3), 60 (4), 100 (5).

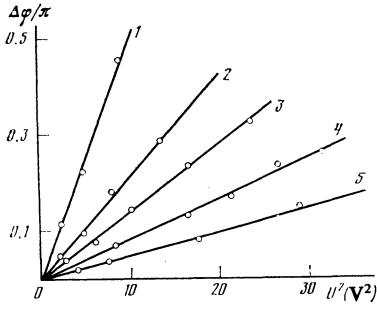


FIG. 4. Experimental dependences of the phases delay in the flexoeffect on the square of the external voltage for different layer thicknesses of the NLC (MBBA): $L = 100 \mu\text{m}$ (curve 1), 35 (2), 20 (3), 15 (4), 5 (5).

effect threshold voltage on the NLC layer thickness was observed experimentally in a range of L from 5 to $200 \mu\text{m}$. If the error in the measurement of the threshold voltage is $\approx 2\%$, the binding energy determined from (2) is $W_0 \sim 0.1 \text{ erg/cm}^2$.

Figure 4 shows plots of the phases delay in the flexoelectric effect against the square of the external voltage. The plots are straight lines in accord with (3). The quantity $\Delta\varphi / U^2 L$ is therefore a function of only the NLC parameters and of the binding energy. The value of $\Delta\varphi / U^2 L$ determined from Fig. 4 for different L are:

NLC layer thickness, μm	100	35	20	15	7
$(\Delta\varphi / U^2 L) \cdot 10^4, \text{ erg}^{-1}$	2.9	3.4	4.1	3.1	4.1

It can be seen that $\Delta\varphi / U^2 L$ is independent of L within the limits of the measurement error. Recognizing that the experimental error of $\Delta\varphi / U^2 L$ is $\approx 15\text{--}20\%$ and using Eq. (3) we find that the binding energy does not exceed in our case $W_0 \sim 10^{-3} \text{ erg/cm}^2$.¹⁷

Thus, actually the binding energy obtained from these two experiments at one and the same homeotropic orientation differs by two orders of magnitude. We shall attempt to resolve this discrepancy by introducing the binding energy in a different form, that of the elliptic sine of the misorientation angle $\theta - \theta_0$ of the director on the substrate:

$$W = \frac{1}{2} W_0 \text{sn}^2(\theta - \theta_0, k), \quad (4)$$

where $0 \leq k \leq 1$, and by considering the aforementioned effects with account taken of this new form of the potential.

4. THEORETICAL CALCULATION. COMPARISON WITH EXPERIMENT

We consider the deformation of a homeotropically oriented NLC layer of thickness L in an electric field \mathbf{E} . We denote by $\theta(z)$ the angle between the director $\mathbf{n} = [\sin\theta(z), 0, \cos\theta(z)]$ and the z axis perpendicular to the substrates (Fig. 1). The free energy of the NLC layer can then be written in the form¹²

$$\mathcal{F} = \frac{1}{2} \int_0^L \left[(k_{11} \sin^2 \theta + k_{33} \cos^2 \theta) \left(\frac{d\theta}{dz} \right)^2 + \frac{D_z^2}{4\pi} (\epsilon_{\perp} + \epsilon_a \cos^2 \theta)^{-1} \right] dz$$

$$+ \left\{ \sin \theta \cos \theta \left[(e_{11} + e_{33}) E - k_{13} \frac{d\theta}{dz} \right] - m_p \cos \theta E + W(\theta) \right\}_{z=L} + \left\{ -\sin \theta \cos \theta \left[(e_{11} + e_{33}) E - k_{13} \frac{d\theta}{dz} \right] + m_p \cos \theta E + W(\theta) \right\}_{z=0}, \quad (5)$$

where e_{11} and e_{33} are the flexoelectric moduli, k_{13} is the elastic modulus of second order,¹⁹ m_p is the surface polarization, $W(\theta)$ is the binding energy, and $D_z = (\epsilon_{\perp} + \epsilon_a \cos^2 \theta) E$ is the projection of the electromagnetic induction vector on the z axis ($\partial D_z / \partial z = 0$). We assume here that both substrates at $z = 0$ and $z = L$ are perfectly identical, i.e., they are characterized by the same binding energy with oppositely directed surface polarizations.

The equilibrium distribution of the director $\theta(z)$ corresponds to a minimum of the functional (5) and is consequently obtained from the solution of the following problem:

$$\frac{d^2\theta}{dz^2} (k_{11} \sin^2 \theta + k_{33} \cos^2 \theta) + \left(\frac{d\theta}{dz} \right)^2 (k_{11} - k_{33}) \sin \theta \cos \theta - \frac{D_z^2 \epsilon_a \sin 2\theta}{8\pi (\epsilon_{\perp} + \epsilon_a \cos^2 \theta)^2} = 0, \quad (6)$$

$$\left\{ \frac{d\theta}{dz} (k_{11} \sin^2 \theta + k_{33} \cos^2 \theta) - k_{13} \cos 2\theta \frac{d\theta}{dz} - k_{13} \sin \theta \cos \theta \frac{d^2\theta}{dz^2} \left(\frac{d\theta}{dz} \right)^{-1} + (e_{11} + e_{33}) E \sin \theta \cos \theta + m_p E \sin \theta \pm \frac{dW}{d\theta} \right\}_{z=L,0} = 0. \quad (7)$$

We consider only symmetrical solutions:

$$\theta|_{z=0} = \theta|_{z=L}, \quad \frac{d\theta}{dz} \Big|_{z=0} = -\frac{d\theta}{dz} \Big|_{z=L},$$

which correspond to a minimum of the free energy. The boundary conditions (7) then take the form

$$\left\{ \frac{d\theta}{dz} (k_{11} \sin^2 \theta + k_{33} \cos^2 \theta) - k_{13} \cos 2\theta \frac{d\theta}{dz} - k_{13} \sin \theta \cos \theta \frac{d^2\theta}{dz^2} \left(\frac{d\theta}{dz} \right)^{-1} - \frac{dW}{d\theta} \right\}_{z=0} = 0, \quad (8)$$

and $d\theta/dz|_{z=L/2} = 0$ (symmetry condition).

We note that for the symmetric solution

$$\frac{d^2\theta}{dz^2} \Big|_{z=0} = \frac{d^2\theta}{dz^2} \Big|_{z=L}.$$

We write the potential energy (4) of the interaction of the NLC molecules with the substrate in the form

$$W = \frac{1}{2} W_0 \sin^2 \zeta(\theta), \quad (9)$$

$$\theta = \frac{\pi}{2} \frac{F(\zeta, k)}{F(\pi/2, k)}, \quad F(\zeta, k) = \int_0^{\zeta} \frac{d\alpha}{(1 - k^2 \sin^2 \alpha)^{1/2}}.$$

Here F is an elliptic integral of the first kind. Variation of the parameter k in the range $0 \leq k \leq 1$ corresponds here to a change of the half-width L_w of the potential "energy well" (Fig. 5a) in accordance with the expression

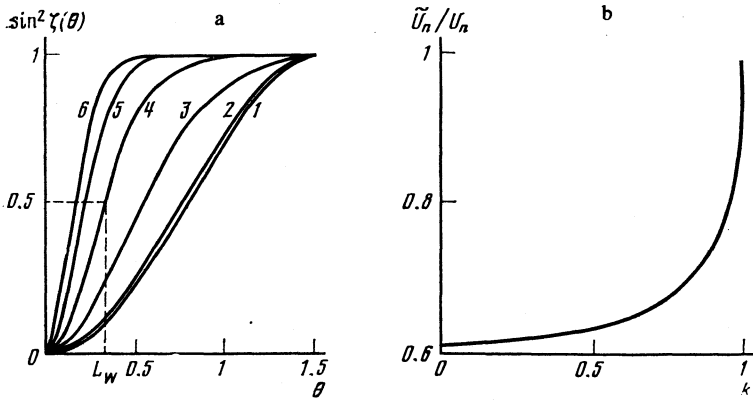


FIG. 5. a—Shape of plot of the binding energy (9) (of the surface potential) calculated numerically for different values of the parameter k (curve 1): 0.5 (2); 0.95 (3); 0.999 (4); 0.99999 (5); 0.99999999 (6). The quantity L_w is equal to the half-width of the “energy well.” b—Dependence of the relative threshold voltage in the B effect on the parameter k that characterizes the form of the binding energy (9). The parameters of MBBA were used in the numerical calculation (see caption of Fig. 2). NLC layer thickness $L = 22 \mu\text{m}$, binding energy $W_0 = 10^{-3} \text{ erg/cm}^2$.

$$L_w \approx 2.59 \left[\ln \frac{8}{1-k} \right]^{-1}, \quad 0 < 1-k \ll 1, \quad (10)$$

which is obtained from (9) by using the expansion of the binding energy $W(\theta)$ in powers of the angle θ as $k \rightarrow 1$:

$$W \approx \frac{W_0}{2} \left(\theta'^2 - \frac{1+k^2}{3} \theta'^4 \right),$$

$$\theta' = \frac{2}{\pi} F \left(\frac{\pi}{2}, k \right) \theta \approx \frac{1}{\pi} \theta \ln \frac{8}{1-k} \ll 1. \quad (11)$$

We note that $k=0$ corresponds to the Rapini potential (1) and that $k \rightarrow 1$ corresponds to an infinitely narrow potential well near $\theta=0$. The quantity $W_0/2$ is equal to the height of the “potential barrier.”

Deformation of the director $\theta(z)$ can occur only when the external voltage exceeds a certain threshold U_n . Putting $|\theta| \ll 1$ in (6) and (8) we obtain in accord with (11)

$$\frac{d^2\theta}{dz'^2} + q^2\theta = 0, \quad \left\{ \frac{d\theta}{dz'}(1-k_s) - \frac{LW_0'}{k_{33}}\theta \right\}_{z'=0} = 0, \quad \left. \frac{d\theta}{dz'} \right|_{z'=L} = 0, \quad (12)$$

where

$$z' = \frac{z}{L}, \quad k_s = \frac{k_{13}}{k_{33}}, \quad q^2 = -\frac{\epsilon_a U^2}{4\pi k_{33}}, \quad W_0' \approx W_0 \left(\frac{1}{\pi} \ln \frac{8}{1-k} \right)^2;$$

W_0' is the effective binding energy. We used here the fact that at $LW_0'/k_{33} \gg q^2 \approx 1$ we have

$$-k_s \theta \frac{d^2\theta}{dz'^2} \left(\frac{d\theta}{dz'} \right)^{-1} \sim k_s q^2 \frac{(k_{33}-k_{13})}{LW_0'} \theta \ll \frac{LW_0'}{k_{33}} \theta,$$

and this term can consequently be neglected.

The system (12) has a solution $\tilde{\theta} = \tilde{A} \cos qz' + \tilde{B} \sin qz'$ with nontrivial coefficients \tilde{A} and \tilde{B} under the condition

$$\frac{q}{2} \operatorname{tg} \frac{q}{2} = \frac{1}{2} \frac{LW_0'}{k_{33}-k_{13}}, \quad (13)$$

which coincides at $k=k_s=0$ with the corresponding Rapini condition.¹ In the approximation $LW_0'/(k_{33}-k_{13}) \gg 1$ expression (13) can be reduced to the form

$$q \approx \pi [1 - 2(k_{33}-k_{13})/LW_0'],$$

so that the threshold voltage is

$$U_n \approx U_n \left(1 - \frac{2(k_{33}-k_{13})}{LW_0'} \right) \approx U_n \left(1 - \frac{(k_{33}-k_{13})}{LW_0'} \cdot 2.94 L w^2 \right). \quad (14)$$

Thus, taking (14) into account, the effective binding energy, which is estimated experimentally from the threshold voltage, is larger than the real binding energy by $W_0'/W_0 \sim 1/L_w^2$ times. For $k=0.9999999$ and $k_s=0$ this amounts to $W_0'/W_0 \approx 60$ and $L_w \approx 0.13$. Estimates of the threshold voltage in accord with (14) agree well with the result of the numerically obtained exact solution of the problem (6), (8) (Fig. 5b). Whereas the apparent binding energy determined from experiment is $W_0' \sim 0.1 \text{ erg/cm}^2$, the real value of the binding energy is $W_0 \sim 1.7 \cdot 10^{-3} \text{ erg/cm}^2$.

The exact solution $\theta(z)$ of the problem (7), (8) is given by the following relations:

$$\frac{2z}{L} \int_{\theta_b}^{\theta_m} f_1(x) dx = \int_{\theta_b}^{\theta} f_1(x) dx, \quad \theta(L-z) = \theta(z),$$

$$\int_{\theta_b}^{\theta_m} \frac{f_1(x) dx}{f_1(\theta_b)} \left\{ (\kappa \sin^2 \theta_b + 1) - k_s \cos 2\theta_b + k_s \sin^2 \theta_b \cos^2 \theta_b \right. \\ \left. \times (\kappa \sin^2 \theta_b + 1)^{-1} \left[\kappa \right. \right. \\ \left. \left. + \frac{(1+\gamma \sin^2 \theta_m)(1+\kappa \sin^2 \theta_b)}{(1+\gamma \sin^2 \theta_b)(\sin^2 \theta_m - \sin^2 \theta_b)} \right] \right\} \\ - \frac{W_0 L}{k_{33}} \sin \zeta_b \cos \zeta_b (1-k^2 \sin^2 \zeta_b)^{1/2} \frac{F(\pi/2, k)}{\pi} = 0, \quad (15)$$

where

$$f_1(x) = \left[\frac{(1+\gamma \sin^2 x)(1+\kappa \sin^2 x)}{\sin^2 \theta_m - \sin^2 x} \right]^{1/2},$$

$$\gamma = \epsilon_{\perp}/\epsilon_{\parallel} - 1, \quad \kappa = k_{11}/k_{33} - 1, \quad \zeta_r = \zeta(\theta_r).$$

In the derivation of (16) we used the relations

$$\frac{d\theta}{dz} = \frac{2}{L} [f_1(\theta)]^{-1} \int_{\theta_b}^{\theta_m} f_1(x) dx, \quad (17)$$

$$\frac{d^2\theta}{dz^2} = -(\kappa \sin^2 \theta + 1)^{-1} \sin \theta \cos \theta$$

$$\times \left[\kappa + \frac{(1+\gamma \sin^2 \theta_m)(1+\kappa \sin^2 \theta)}{(1+\gamma \sin^2 \theta)(\sin^2 \theta_m - \sin^2 \theta)} \right] \left(\frac{d\theta}{dz} \right)^2.$$

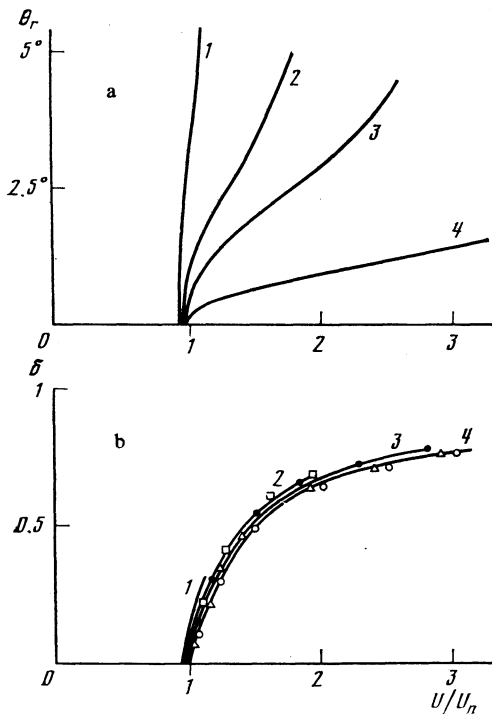


FIG. 6. Dependence of the director orientation angle on the substrates (a) and of the birefringence (b) on the external voltage in the B effect. The numerical calculation (solid curves) was carried out for the new form (9) of the binding energy at $k = 0.99999999$. NLC-layer thickness; $L = 10 \mu\text{m}$ (curve 1—theory, \square —experiment), 22(2, \bullet); 34 (3, \triangle); 100 (4, \circ).

The director orientation angles on the substrates $\theta_b = \theta|_{z=0} = \theta|_{z=L}$ and on the NLC layer $\theta_m = \theta|_{z=L/2}$ depend on the applied voltage U in the following manner:

$$U/U_n = (2/\pi) (1 + \gamma \sin^2 \theta_m)^{1/2} \int_{\theta_b}^{\theta_m} f_2(x) dx, \quad (18)$$

$$f_2(x) = (1 + \kappa \sin^2 x)^{1/2} [(1 + \gamma \sin^2 x) (\sin^2 \theta_m - \sin^2 x)]^{-1/2}.$$

Using expression (16) and (17) we can calculate the dependence of the inclination angle of the director on the substrate on the applied voltage U . The results of the numerical calculation of $\theta_b(U)$ for $k = 0.99999999$ at different NLC layer thicknesses are shown in Fig. 6a.

Another important field-dependent NLC-layer characteristic is the relative birefringence

$$\delta \left(\frac{U}{U_n} \right) = \Delta n \left(\frac{U}{U_n} \right) (\Delta n_{\text{max}})^{-1}, \quad (19)$$

where $\Delta n_{\text{max}} = n_{\parallel} = n_{\perp}$ is the maximum birefringence

$$\Delta n(U/U_n) = \int_0^L (n(z) - n_{\perp}) dz$$

is the birefringence corresponding to the given value of the external voltage $U > U_n$, and

$$n(z) = n_{\parallel} n_{\perp} [n_{\perp}^2 \sin^2 \theta(z) + n_{\parallel}^2 \cos^2 \theta(z)]^{-1/2}.$$

With allowance for (15), the equation for δ can be transformed into

$$\delta = \left(\int_{\theta_r}^{\theta_m} f_3(x) dx / \int_{\theta_r}^{\theta_m} f_1(x) dx - 1 \right) (n_{\parallel}/n_{\perp} - 1)^{-1}, \quad (20)$$

where

$$f_3(x) = \left[\frac{(1 + \gamma \sin^2 x) (1 + \kappa \sin^2 x)}{(1 + \nu \sin^2 x) (\sin^2 \theta_m - \sin^2 x)} \right]^{1/2}, \quad \nu = \frac{n_{\perp}^2}{n_{\parallel}^2} - 1.$$

The $\delta(U/U_n)$ curves obtained by numerical calculation are shown in Fig. 6b.

It can be seen from Figs. 6a and 6b that the B -effect threshold voltage depends little in practice on the NLC layer thickness or, equivalently, on the binding energy (a decrease of the layer thickness is equivalent to a decrease of the binding energy, see Eqs. (12) and (16)). This agrees with the results of a numerical investigation of the dependence of the phase delay on the voltage (see Fig. 3), but differs radically from the results of calculations under the Rapini assumption (see Fig. 2, $k = 0$). The theoretically obtained dependences of the birefringence on the voltage (Fig. 6b) also agree well with the experimental data obtained for different NLC layer thicknesses. We note that allowance for the elastic modulus of second order k_{13} , as shown by numerical calculations with account taken of the form of the surface potential, do not influence substantially the character of the dependences of δ and θ_b on U/U_n ; the dependences considered were therefore obtained for $k_{13} = 0$.

When considering the flexoelectric effect within the framework of the model (6), (7) the new form (4), (9) of the surface potential does not change the qualitative character of the deformation of the NLC director in an electric field. The director orientation angle at $\varepsilon_a = 0$ is equal in this case to

$$\theta(z) = \frac{U}{U_H} \left[\frac{LW'_0}{2k_{33}} + 1 - k_s \right]^{-1} \frac{z}{L}, \quad (21)$$

and the phase delay is

$$\Delta \varphi = \Delta \varphi_0 \left(\frac{LW'_0}{2k_{33}} + 1 - k_s \right)^{-2}. \quad (22)$$

Here

$$W'_0 = W_0 \left[\frac{2}{\pi} F \left(\frac{\pi}{2}, k \right) \right]^2$$

is the effective binding energy. In accord with this expression, taking into account the estimate $W'_0 \sim 10^{-3}$ erg/cm² for the effective binding energy, we obtain the real binding energy $W_0 \sim 1.7 \cdot 10^{-5}$ erg/cm². The difference between the estimated binding energies for the flexoelectric and B effects may be due to the influence of the current-conducting SnO₂ layer, which can increase the NLC binding energy in the B effect compared with the case of the flexoeffect, where the experimental conditions do not permit deposition of such a layer.

5. CONCLUSION

By introducing a new type of surface potential in the form of an elliptic sine (4), we can obtain a noncontradictory

explanation of the experimentally obtained estimates of the binding energy of a homeotropically oriented NLC, namely, the apparent strong binding in the case of the *B* effect and the weak binding that follows from investigations of the flexoelectric deformation. The new form of binding energy (4) proposed by us is only one of the possible approximations of the real form of the surface potential, the exact determination of which calls for further experimentation.

- ¹A. Rapini and M. Papoular, *J. de Phys. Colloc.* **30**, 54 (1969).
²G. Ruchenkov and M. Kleman, *J. Chem. Phys.* **64**, 404 (1976).
³G. Porte, *J. de Phys.* **37**, 1245 (1976).
⁴D. Riviere, Y. Levy, and E. Guyon, *J. de Phys. Lett.* **40**, 215 (1979).
⁵J. Sicart, *ibid.* **37**, 25 (1976).
⁶H. Yamada and C. Ishii, *Mol. Cryst. Liq. Cryst.* **76**, 113 (1981).
⁷J. Nehring, A. Kmetz, and T. Scheffer, *J. Appl. Phys.* **47**, 850 (1975).

- ⁸S. Naemara, *Appl. Phys. Lett.* **33**, 1 (1978).
⁹E. Guyon and W. Urban, in: *Nonemissive Electrooptic Displays*, Plenum, 1976, p. 121.
¹⁰M. Ohtsu, T. Akahane, and T. Taco, *Jpn. J. Appl. Phys.* **13**, 621 (1974).
¹¹S. Majoros, D. Brisbin, D. Johnson, *et al.*, *Phys. Rev. A* **20**, 1619 (1979).
¹²V. G. Chigrinov, *Kristallografiya* **27**, 404 (1982) [*Sov. Phys. Crystallogr.* **27**, 245 (1982)].
¹³G. Barbero, R. Malvano, and A. Stigazzi, *Nuovo Cimento* **B59**, 367 (1980).
¹⁴W. Helfrich, *Phys. Lett.* **A35**, 393 (1971).
¹⁵D. Schmidt, M. Schadt, and W. Helfrich, *Z. Naturforsch.* **A72**, 277 (1972).
¹⁶A. I. Derzanski, A. G. Petrov, and M. D. Mitov, *J. de Phys.* **39**, 273 (1977).
¹⁷B. A. Umanskiĭ, L. M. Blinov, and M. I. Barnik, *Kristallografiya* **27**, 729 (1982) [*Sov. Phys. Crystallogr.* **27**, 437 (1982)].
¹⁸L. M. Blinov, V. A. Kizel, V. G. Romyantsev, and V. V. Titov, *J. de Phys. Colloc.* **36**, 1 (1975).
¹⁹A. I. Derzhanski and A. L. Petrov, *Acta Phys. Polon.* **A19**, 3084 (1976).

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