

Experimental possibilities of precision measurements of the hyperfine splitting energy in muonic atoms and crystals

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Laser-induced (CO_2 , $\lambda \approx 10 \mu\text{m}$) transitions between hyperfine structure (hfs) levels of muonic atoms and molecules are considered. It is shown that the Doppler width of the line decreases substantially in elastic collisions of muonic molecules, thereby significantly lowering the laser power needed to realize the indicated transitions. Estimates show that an effective change of the level populations of the hfs of $pp\mu$ and $pd\mu$ molecules is perfectly realistic in experiments with a hydrogen target in a muon beam. Realization of this possibility will permit: 1) a study of the spin dependence of the reactions $\mu^- + p \rightarrow n + \nu_\mu$ and $p + d \rightarrow {}^3\text{He} + \gamma$; 2) measure with accuracy 10^{-5} eV the hyperfine splitting in the molecules $pp\mu$ and $pd\mu$ and determine the contribution due to QED effects and three-particle relativistic forces.

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Progress in the development of high-power lasers¹ raises the question of experimentally determining the structure details of exotic atoms such as muonic, pionic, kaonic, and others. So far, the only example of research in this direction is the measurement² of the difference $2S_{1/2} - 2P_{3/2}$ in the system ${}^4\text{He}\mu$, as a result of which it was possible to verify the contribution made to this energy difference as a result of vacuum polarization, with relative accuracy $\sim 10^{-3}$.

Another important physical quantity that would be highly desirable to measure by a resonant method with an infrared laser is the hyperfine splitting in muonic hydrogen (the atoms $p\mu$, $d\mu$, $t\mu$). In these atoms the Bohr radius is smaller by a factor $m_\mu/m_e \approx 200$ than in ordinary hydrogen, and therefore the hyperfine splitting ΔE_{hfs} is much more sensitive to distortion of the Coulomb field of the nucleus as a result of the vacuum polarization, to the size of the nucleus, and to its polarizability.³ Since the interaction of the muon and proton spins is of the point type, the quantity ΔE_{hfs} is sensitive to the details of the proton structure, which can usually be observed only when the protons interact with high-energy electrons. Accordingly, the measurement of ΔE_{hfs} for the $d\mu$ and $t\mu$ atoms is of interest from the viewpoint of determining the characteristics of the deuteron and the triton. Unfortunately, as will be shown later, at the present state of the art the prognosis for investigations of the fine structure of muonic hydrogen with the aid of lasers is most unfavorable. However, in view of the great theoretical importance of the problem, we examine briefly the prospects.

The equation for hyperfine splitting of the muonic hydrogen atom in the ground state is of the form^{3,4}

$$\Delta E_{\text{hfs}} = \Delta E^0 (1 + \delta_1 + \delta_2 + \delta_3), \quad (1)$$

where ΔE^0 is determined by the Fermi formula

$$\Delta E^0 = \frac{8\beta_\mu\beta_N}{3a_\mu^3} g_N (2I+1) \left(1 + \frac{m_\mu}{m_N}\right)^{-3}. \quad (2)$$

The notation used in (1) and (2) is the following: β_μ and β_N are the muon and nuclear magnetons; g_N and I are the gyromagnetic ratios and the nuclear spin; m_μ and m_N are the

masses of the muon and of the nucleus; $a_\mu \approx \hbar^2/m_\mu e^2$ is the Bohr radius of the mu-atom; δ_1 , δ_2 , δ_3 are corrections that take into account relativistic effects and QED effects (δ_1), the nuclear charge distribution (δ_2) and the polarizability of the nucleus (δ_3).

It can be seen from (2) that $\Delta E_{\text{hfs}} \sim m_\mu^2$. This quantity is four orders larger in muonic atoms than in ordinary hydrogen and amounts to $\Delta E_{\text{hfs}} \sim 0.1$ eV. The contribution of the corrections is $\delta_1 - \delta_2 \sim m_\mu$, therefore the influence of the nuclear charge distribution in muonic atoms is of the order of one percent,⁴ and the correction for the polarizability should be $\delta_3 \sim 10^{-3}$ (Refs. 3 and 4). It is therefore obvious that measurements of ΔE_{hfs} with relative accuracy 10^{-4} are of great interest.

No consistent calculations of ΔE_{hfs} for muonic atoms have been made so far. In Ref. 4 the author confined himself to an accuracy 10^{-3} eV, sufficient to examine the kinetics of muonic catalysis. The calculated values of ΔE_{hfs} , in electron volts, are:

$p\mu$	$d\mu$	$t\mu$
0.1817	0.0491	0.2402

When muonic atoms are formed, the hyperfine-structure (hfs) states with spin $F_1 = I + \frac{1}{2}$ and $F_2 = I - \frac{1}{2}$ are statistically populated, but subsequently, owing to spin-exchange collisions^{5,6}

$$p\mu(\uparrow\uparrow) + p(\downarrow) \rightarrow p\mu(\downarrow\uparrow) + p(\uparrow), \quad (3)$$

$$d\mu(\uparrow\uparrow) + d(\downarrow) \rightarrow d\mu(\downarrow\uparrow) + d(\uparrow), \quad (4)$$

$$t\mu(\uparrow\uparrow) + t(\downarrow) \rightarrow t\mu(\downarrow\uparrow) + t(\uparrow) \quad (5)$$

they go over to a lower hfs state.¹¹ For thermalized mu-atoms these transitions are irreversible. The rates⁶ of reactions (3)–(5) are given in Table I. It can be seen from the data in the table that the rate of the transition $F_1 \rightarrow F_2$ in spin-exchange collisions is large compared with the muon decay rate $\lambda_0 = 4.55 \cdot 10^5 \text{ sec}^{-1}$. This means that at moderate hydrogen densities (pressures $P \gtrsim 10$ atm) practically all the mu-atoms go over into the lower hfs state.

If the laser-induced transitions $F_1 F_2$ lead to a noticeable change of the population of the hfs states, this manifests itself in various processes, a fact that can be used to identify these transitions.

In analogy with the measurements of ΔE_{hfs} in muonium,⁷ it would be possible to use the dependence of the residual muon polarization on the populations of the hfs states of the mu-atom. However, owing to the high rate of depolarization on account of spin-exchange collisions, this procedure encounters great difficulties.

Another process whose probability depends strongly on the relative orientation of the spins of the muon and proton (deuteron, triton) is nuclear mu-capture.⁸ We shall consider only the reaction of mu-capture by a proton:



The measured value⁹ of the rate of reaction (6) from the state with $F=0$ is, according to the theory,⁸ $\lambda_{\text{capt}}^{\mu p}(F=0) = 660 \pm 48 \text{ sec}^{-1}$. For the rate of mu-capture from the triplet state of the $p\mu$ atom, only the theoretical estimate $\lambda_{\text{capt}}^{\mu p}(F=1) \approx 15 \text{ sec}^{-1}$ was obtained.

Owing to the high rate of the transition $F_1 = 1 \rightarrow F_2 = 0$ in liquid and gaseous hydrogen ($p \gtrsim 0.1 \text{ atm}$), mu capture is realized from the state $F=0$. The induced change of the population of the hfs states leads, obviously, to a decrease in the mu-capture probability. At saturation when the rate of the induced transitions $\Omega \gtrsim \lambda_p^0 \varphi$, the probability of mu-capture is decreased by approximately one-half.

An interesting possibility of recording transitions between the hfs states of the atoms $d\mu$ and $t\mu$ arises in connection with the resonant character of the production of the molecules $dd\mu$ and $dt\mu$.^{6,10-12} The induced transition between the hfs levels of the mu-atoms changes the position of the resonance in the cross section for the production of the muonic molecule, and this manifests itself in a change of the yield of the corresponding muonic-catalysis nuclear reaction.

We proceed now to estimate the laser power needed to effect a noticeable change in the population of the hfs state of hydrogen mu-atoms. This problem was considered for a solitary muonic atom in Ref. 13. No account was taken there of the degenerate triplet state, and this can change the corresponding matrix elements of the transitions by 100–150%. This, however, is of little importance for the real situation, equally as allowance for relaxation processes in the hfs, since, as will be presently shown, the main factor that hinders the change of the population of the hfs levels with a laser is the Doppler line broadening. Bearing this circum-

stance in mind, we shall not solve the relaxation equations for the spin density matrix, but calculate the matrix element of the transition by elementary perturbation theory.¹⁴

We consider the interaction between an atom at rest and circularly polarized radiation: $B_x = B \cos \omega t$, $B_y = B \sin \omega t$ (ω is the frequency of the laser radiation). It is known that the Hamiltonian of the problem is of the form

$$\hat{V} = \frac{1}{2} B (\beta_\mu g_\mu \hat{s}_\mu^+ - \beta_p g_p \hat{s}_p^+) e^{-i\omega t} + \frac{1}{2} B (g_\mu \beta_\mu \hat{s}_\mu^- - \beta_p g_p \hat{s}_p^-) e^{i\omega t} \quad (7)$$

We have accordingly

$$\begin{aligned} \hat{V} |00\rangle = & 2^{-1/2} \hat{V} (|+-\rangle - |-+\rangle) = \frac{B e^{-i\omega t}}{2^{1/2}} (\beta_\mu g_\mu + \beta_p g_p) |++\rangle \\ & - \frac{B e^{i\omega t}}{2^{1/2}} (-\beta_\mu g_\mu - \beta_p g_p) |--\rangle. \end{aligned}$$

The operators s^\pm and the hfs states are defined in standard fashion. In states of the type $|+-\rangle$ the second index pertains to the muon.

We introduce the frequency

$$f [\text{rad/sec}] = \frac{1}{\hbar} \frac{B}{2^{1/2}} (\beta_\mu g_\mu + \beta_p g_p) = 3.96 \cdot 10^4 B \text{ (G)}. \quad (8)$$

Then

$$\langle 11 | \hat{V} | 00 \rangle = e^{-i\omega t} \hbar f, \quad \langle 1, -1 | \hat{V} | 00 \rangle = -e^{i\omega t} \hbar f,$$

only the element $\langle 11 | \hat{V} | 00 \rangle$ is significant near resonance.

It is known¹⁴ that the transitions occur at a frequency

$$\Omega = [(\omega - \omega_0)^2 + 4f^2]^{1/2}, \quad (9)$$

where ω_0 is the hyperfine splitting frequency.

If the mu-atom was at $t=0$ in a lower spin state, the population of the upper (triplet) state at the instant of time t is

$$|a(1, 1)|^2 = \frac{1}{2} [1 + (\omega - \omega_0)^2 / 4f^2]^{-1} (1 - \cos \Omega t) \quad (10)$$

which is a periodically varying function with period $T = \pi / \Omega$.

The minimum frequency of the transition is obviously determined from the condition

$$\Omega \geq 2\pi (\lambda_0 + \lambda_p^0 \varphi). \quad (11)$$

It follows from (8), (9), and (11) that at $\lambda_p^0 \varphi \lesssim \lambda_0$ (low pressures) the amplitude of the magnetic field intensity should be $B_0 \gtrsim 50 \text{ G}$, corresponding to a pulse power

$$W_0 \gtrsim 10^6 \text{ W/cm}^2. \quad (12)$$

TABLE I. Spin states of hydrogen mu-atoms.

Atom of hfs states	$p\mu$		$d\mu$		$t\mu$	
	$F_1=1$	$F_2=0$	$F_1=3/2$	$F_2=1/2$	$F_1=1$	$F_2=0$
	$1/4$	$1/4$	$3/8$	$1/8$	$3/4$	$1/4$
Rate of $F_1 \rightarrow F_2$ transition in spin-exchange collisions. ⁶	$\lambda_p^0 = 1.3 \cdot 10^{10} \text{ c}^{-1}$		$\lambda_d^0 = 5 \cdot 10^7 \text{ c}^{-1}$		$\lambda_t^0 = 10^9 \text{ c}^{-1}$	

Note. The rates of the transition $F_1 \rightarrow F_2$ are given for liquid-hydrogen density $\rho_0 = 4.25 \cdot 10^{22} \text{ nuclei/cm}^3$ and can be recalculated for hydrogen gas with density ρ using $\lambda_p = \lambda_p^0 \varphi$, $\lambda_d = \lambda_d^0 \varphi$, $\lambda_t = \lambda_t^0 \varphi$, where $\varphi = \rho / \rho_0$.

At $\lambda_p^0 \varphi > \lambda_0$ (pressure $P > 0.1$ atm) the required power increases like the square of the pressure.

The thermal motion of the mu-atoms makes the resonant absorption frequencies dependent on the atom velocity. The distribution of the resonant frequencies in the ensemble is determined by the known formula^{15,16}

$$I(\omega) = (2\pi\Delta\omega_D^2)^{-1/2} \exp[-(\omega - \omega_0)^2 / 2\Delta\omega_D^2],$$

where $\omega_0 = 2.8 \cdot 10^{14}$ rad/sec is the resonant frequency for a mu-atom at rest and

$$\Delta\omega_D = \langle v_x^2 \rangle^{1/2} \omega_0 c^{-1} = (kT/m)^{1/2} \omega_0 c^{-1},$$

where v is the mu-atom velocity.

It can be seen from (10) that a noticeable population of the triplet state can be produced in an ensemble of muonic atoms provided that the obvious condition $\Delta\omega_D \lesssim 2f$ is satisfied. At room temperature ($T = 300$ K) we have $\Delta\omega_D \approx 5 \cdot 10^{-6} \omega_0$, while at $T = 300$ K (liquid hydrogen) $\Delta\omega_D \approx 1.5 \cdot 10^{-6} \omega_0$, i.e., $\Delta\omega_D \sim 10^9$ rad/sec, which is larger by two or three orders than the natural width $\Delta\omega_0 = 2\pi\lambda_0 = 2.5 \cdot 10^6$ rad/sec. It follows therefore that the necessary intensity values should be of the order of $(10^2 - 10^3) B_0 \sim 10^4$ G. The pulse power is accordingly $W \sim 10^{11}$ W/cm², or larger by five orders than the minimum estimate (12).

The conditions of registration of mu capture are such that the laser must be triggered after each stoppage of the muon in hydrogen, and to ensure a sufficiently high rate of statistics accumulation the triggering frequency must be not less than $n_\mu = 10$ sec⁻¹. The pulse duration is determined by the muon lifetime, i.e., should be $\Delta t \sim 1$ μ sec. Therefore the average continuous power of the laser is found to be of the order of

$$\bar{W} = nW\Delta t \sim 10 \text{ sec}^{-1} \cdot 10^{11} \text{ W/cm}^2 \cdot 10^{-6} \text{ sec} \sim \text{MW/cm}^2$$

which is beyond the present capability (one can realistically count on a power of the order of a kW/cm²).

The situation could turn out to be more attractive if it is recognized that elastic collisions of the mu-atoms in the course of their thermal motion decrease the Doppler line width. According to Ref. 16 the Doppler width decreases in the ratio

$$\alpha = 2\pi l / \lambda, \quad (13)$$

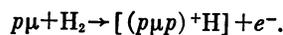
where λ is the radiation wavelength $l = 1/\rho\sigma$ is the mean free path between collisions, ρ is the hydrogen density, and σ is the cross section for elastic collisions. The wave-length of the transition between the hfs states of hydrogen mu atoms is

$\lambda \sim 10$ μ m, and the collision cross section $\sigma \sim 10^{19}$ cm², whence $\alpha \sim 1/\varphi$ (φ is the relative density), i.e., the decrease of the Doppler broadening is insignificant even at liquid-hydrogen density.

Thus, the high rate of the spontaneous $F_1 \rightarrow F_2$ transitions in spin-exchange collisions and, most importantly, the Doppler line broadening, are the factors that prevent the measurement of the hyperfine splitting in the muonic-hydrogen atom.

It turns out that both these factors become insignificant on going from mu-atoms to muonic molecules. We shall consider only molecules (it is more correct to call them mu-molecular ions) $pp\mu$ and $pd\mu$, since in contrast to other similar systems $dd\mu$, $dt\mu$ and $tt\mu$ their lifetimes are large enough (of the order of microseconds).^{5,6}

Muonic molecules are produced when mu-atom collide with hydrogen molecules in reactions of the type^{5,6}



The important difference between muonic molecules and mu-atoms of hydrogen is that these molecules are charged systems. They therefore become rapidly ($\sim 10^{-13}$ sec) "overgrown" by electrons and hydrogen molecules to form molecular complexes ($pp\mu, e$), ($pp\mu, H_2$).¹⁷ These complexes are of the usual atomic size and the cross section for their elastic collisions with the hydrogen molecules is of the order of 10^{-15} cm². It follows from (13) that so large a cross section the spread of the emission frequencies decreases to the level of the natural width if hydrogen of relative density $\varphi \gtrsim 0.1$ is used.

Another important feature of the muonic molecules $pp\mu$ and $pd\mu$ is that their spin states can be regarded in good approximation as quenched, since there are no spontaneous transitions between these states.^{5,6} This means that the minimum rate of the induced transitions, needed to change the populations of the spin states of these molecules, should be $\Omega \sim 10^6$ sec⁻¹, or smaller by two or three orders than for mu-atoms.

The hfs levels of muonic molecules were calculated in Refs. 11 and 18, with accuracy $\sim 10^{-3}$ eV. It follows from these calculations that precision measurement of the hfs level differences of the hfs of mu-molecules is of great interest from the viewpoint of determining the contribution of three-particle relativistic forces and QED effects.

The $pp\mu$ molecule is produced in a state with orbital momentum $L = 1$ and a total spin of the two protons $S_p = 1$ at a rate $\lambda_{pp\mu}^0 = 2.5 \cdot 10^6$ sec⁻¹ (Refs. 5 and 6). The rate of the

TABLE II. Hyperfine structure of the levels of the $pp\mu$ muonic molecule in a state with $L = 1, v = 0$ (Refs. 11, 18).

Number, n	Total spin, $S_{pp\mu}$	Total angular momentum, J	Level energy*, meV	Population for $F_p = 0$	Rate of nuclear mu-capture, sec ⁻¹
1	3/2	3/2	46.8	0.0001	15
2		5/2	46.3	0	
3		1/2	42.7	0.0002	
4	1/2	1/2	-90.5	0.3330	500
5		3/2	-92.4	0.6667	

*Reckoned from the unsplit-state energy $\epsilon_{pp\mu}^0 (L = 1, v = 0) = -107$ eV.

orthopara transitions in $pp\mu$ is small compared with the muon decay rate.^{18,19} The level $L = 1, S_p = 1$ is split by the fine and hyperfine interactions into five sublevels whose energies and populations are given in Table II.

Just as in the $p\mu$ atom, the rate of nuclear capture of a muon in the $pp\mu$ system depends on the population of its spin states. Owing to the large rate of the $F_1 = 1 \rightarrow F_2 = 0$ transition, the $pp\mu$ molecule is produced in $p\mu + p$ collisions in practice only in hfs states with total spin $S_{pp\mu} = \frac{1}{2}$. In these two states the rate of nuclear mu-capture is $\lambda_{\text{capt}} \times (S_{pp\mu} = \frac{1}{2}) = 500 \text{ sec}^{-1}$ (Refs. 18, 19) whereas in the three hfs states with spin $S_{pp\mu} = 3/2$ the mu-capture rate is close to zero: $\lambda_{\text{capt}}(S_{pp\mu} = 3/2) = 15 \text{ sec}^{-1}$ (Ref. 18). The induced transitions $S_{pp\mu} = \frac{1}{2} \leftrightarrow S_{pp\mu} = 3/2$ decrease the mu-capture probability, and this can be used to determine the value of ΔE_{hfs} in the $pp\mu$ system.

We shall estimate the laser intensity needed for a noticeable change of the populations of the hfs states in $pp\mu$ by a procedure similar to that used by us for $p\mu$ atoms.

The Hamiltonian of the interaction with the radiation field is again of the form (7) where now, however, \hat{S}_p must be taken to mean the sum of the spins of the two protons: $\hat{S}_p = \hat{S}_{p1} + \hat{S}_{p2}$. The matrix elements of the operators \hat{S}_p^\pm and \hat{S}_μ^\pm can be represented with the aid of the Wigner-Eckart theorem in the form

$$\langle \alpha j m | \hat{S}_{p,\mu}^\pm | \beta j' m' \rangle = C_{j, \pm 1, j', m'}^{jm} (\alpha || \hat{S}_{p,\mu}^\pm || \beta). \quad (14)$$

Here $C_{j_1, m_1, j_2, m_2}^{jm}$ are Clebsch-Gordon coefficients and the quantities $(\alpha || \hat{S}_{p,\mu}^\pm || \beta)$ do not depend on the quantum numbers m and m' .

We present below the calculated values of $(\alpha || \hat{S}_{p,\mu}^\pm || \beta)$ (as usual, the matrix elements of the operator \hat{S}^- are determined from the condition $\langle \alpha | \hat{S}^- | b \rangle = (b | \hat{S}^+ | \alpha)^*$).

$$\begin{aligned} (5 || \hat{S}_\mu^+ || 3) &= ({}^2/{}_3)^{1/2}, & (5 || \hat{S}_p^+ || 3) &= -({}^2/{}_3)^{1/2}, \\ (4 || \hat{S}_\mu^+ || 3) &= ({}^8/{}_{27})^{1/2}, & (4 || \hat{S}_p^+ || 3) &= -({}^8/{}_{27})^{1/2}, \\ (4 || \hat{S}_\mu^+ || 1) &= -({}^{10}/{}_{27})^{1/2}, & (4 || \hat{S}_p^+ || 1) &= ({}^{10}/{}_{27})^{1/2}, \\ (2 || \hat{S}_\mu^+ || 3) &= ({}^2/{}_{27})^{1/2}, & (2 || \hat{S}_p^+ || 3) &= -({}^2/{}_{27})^{1/2}, \\ (2 || \hat{S}_\mu^+ || 1) &= -({}^{16}/{}_{27})^{1/2}, & (2 || \hat{S}_p^+ || 1) &= ({}^{16}/{}_{27})^{1/2}. \end{aligned}$$

It can be seen that the matrix elements of the operator \hat{V} for the $pp\mu$ molecule are of the same order of magnitude as for the $p\mu$ atom. This means, with account taken of the absence of spontaneous transitions between the hfs states of the $pp\mu$ molecule and of the narrowing down of the Doppler line to

the natural width, that the continuous average laser power should be of the order of a kW/cm², which is already attainable now.

The absence of spontaneous transitions between the hfs levels in $pp\mu$ makes it possible to realize a measurement program wherein the state with $S_{pp\mu} = \frac{1}{2}$ is first transformed into a state with $S_{pp\mu} = 3/2$, after which the neutron from the mu capture is registered. It can be seen from Fig. 1 that it is most advantageous to use the transitions $5 \rightarrow 1 \rightarrow 3$, for in this case the neutron counting rate is changed by three times (the initial population of state 5 is 67%). From the point of view of present-day capabilities of high-power tunable laser, the suitable transition is $5 \rightarrow 3$ (wavelength 9.2 μm).

It is possible in principle (by using two lasers) to effect two transitions: $5 \rightarrow 3$ (9.2 μm) and $4 \rightarrow 3$ (9.3 μm) and obtain thus an almost pure state of the μp system with total spin $F_{\mu p} = 1$. Realization of this possibility will create exceptionally favorable conditions for the measurement of the rate of mu-capture by a proton in a triplet spin state ($\lambda_{\text{capt}}^{\mu p} (F = 1)$), and the density of the hydrogen used can be high ($\varphi \sim 1$). We note that in the "standard" (without a laser) method of measuring $\lambda_{\text{capt}}^{\mu p} (F = 1)$ it becomes necessary to use hydrogen of very low density ($\varphi \sim 10^{-4}$) and a contribution ($\frac{1}{3}$) of states with $F = 0$ is always present in this case.

Investigation by a resonant method of the hfs state of the other muonic molecule— $pd\mu$ —has the same advantages as for $pp\mu$, namely, there is no Doppler broadening and the spin states are quenched. The $pd\mu$ system is produced when muons are slowed down in an $\text{H}_2 + \text{D}_2$ mixture at a rate $\lambda_{pd\mu} = 6 \cdot 10^6 \text{ sec}^{-1}$ (Ref. 6) in a state with $L = 1$, and then go over rapidly (10^{-11} sec) into a state with $L = 0$ (Ref. 6). The energies of the hfs levels for this state are given in Table III, which lists also the level populations and the relative rates of the rate of the synthesis reaction



catalyzed by the muon. For the state with total angular momentum $J = 0$ the rate of the reaction (15) is $\lambda_{pd}^{\mu} = 0.3 \cdot 10^6 \text{ sec}^{-1}$ (Ref. 6), which is close to the muon-decay rate. Since the spin of the ${}^3\text{He}$ nucleus produced in the reaction (15) is $\frac{1}{2}$, the rate of the reaction from states corresponding to parallel orientation of the proton and deuteron spins should be suppressed by several orders compared with the rate of this reaction from states with total spin $S_{pd} = \frac{1}{2}$.⁶ It can be seen from Fig. 2 that different hfs levels of the pd molecule correspond to different contributions of states with $S_{pd} = \frac{1}{2}$ and

TABLE III. Hyperfine structure of the levels of the muon molecule $pd\mu$ in a state with $L = 0, v = 0$ (Ref. 11).

Number, n	Total spin, S_{pd}	Rate of reaction $p + d \rightarrow {}^3\text{He} + \gamma$	Population		Level energy, meV*
			$F_{d\mu} = 1/2$	stat. mixt. $F_{d\mu} = 3/2 \text{ and } 1/2$	
1	2	$\lambda_{1pd} \ll \lambda_0$	0	0.42	35.9
2	1	$\lambda_{2pd} = 0.14 \lambda_{3pd}$	0.38	0.25	17.3
3	0	$\lambda_{3pd} = 3 \cdot 10^3 \text{ c}^{-1}$	0.25	0.08	4.6
4	1	$\lambda_{4pd} = 0.86 \lambda_{3pd}$	0.37	0.25	-78.7

*Reckoned from the unsplit-state energy $\epsilon_{pd\mu} (L = 0, v = 0) = -222 \text{ eV}$.

$S_{pd} = 3/2$, therefore the effective rate of reaction (15) in $p\mu$ should depend on the population of its spin states.⁶ If a laser is used to change the population of the hfs levels, this will change the yield of the γ quanta from reaction (15). A suitable transition is $3 \rightarrow 1$ ($\lambda = 10.8 \mu\text{m}$) induction of which changes the γ -quantum yield from (15) by approximately two times. We note that a comparison of the characteristics of the reaction (15) at zero energies and, in particular its spin dependence, with the results of the investigation of the inverse reaction $\gamma + {}^3\text{He} \rightarrow p + d$ is of great interest from the viewpoint of the problem of checking invariance to time reversal.²⁰

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¹¹In (3) and (4), one arrow corresponds to the spin of the nucleus and the other to the muon spin.

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