Flavor-changing interactions of Goldstone bosons with fermions

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In theories with gauge "horizontal" symmetry with respect to generations of fermions, the possibility is discussed of existence of a Goldstone boson whose interactions with the fermions are off-diagonal with respect to the flavors. The probabilities of $K^+ \to \pi^+ \alpha$ and $\mu \to e\alpha$ decays (α is a Goldstone boson) are found within the framework of the $SU(5)\times SU(3)$ theory recently proposed by Z. Berezhiani and Dzh. Chkarueli [JETP Lett. **35**, 612 (1982); Sov. J. Nucl. Phys **37**, 618 (1983)]. The experimental limit on the width of the $K^+ \to \pi^+ \alpha$ decay imposes in this case a restriction on the vacuum mean value responsible for the horizontal-symmetry breaking, $\langle \eta \rangle \gtrsim 10^{10}$ GeV. The possible nature of the coupling between a Goldstone boson and fermions is also discussed. It is shown that only pseudoscalar coupling is possible for interactions that are diagonal in flavor, whereas off-diagonal interactions can be both scalar and pseudoscalar.

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1. INTRODUCTION

Various types of Goldstone and pseudo-Goldstone bosons, connected with global-symmetry breaking, are considered at present in the literature: the axion,[1] majoron,[2] arion,[3] and pseudo-Goldstsone technicolor bosons.[4] The main purpose here is to study the possible existence of a Goldstsone boson (or a ultralight pseudo-Goldstone boson of the axion type) having an interaction that is nondiagonal in flavor with quarks and leptons. We shall see that such Goldstone bosons fit quite naturally in schemes in which the existence of generations of quarks and leptons is connected with a "horizontal" gauge symmetry group. In Sec. 3 of this paper we illustrate this with a very simple but nonrealistic model, while in Sec. 4 we consider the possible appearance of a Goldstone boson with an interaction that changes the flavors, within the framework of the $SU(5)\times SU(3)_h$ model,[5] in which the existence of three generations of fermions is connected with the $SU(3)_h$ horizontal-symmetry group. It must be noted that this model describes well the well known phenomenology of quarks and leptons, including the masses of the fermions and their mixings.[5]

Nonconservation of the flavors upon emission of the Goldstone boson α can lead to decays $K \to \pi + \alpha$ and $\mu \to e + \alpha$. Within the framework of the model of Ref. 5, we shall show that the existing experimental limit for the absence of the decay $K^+ \to \pi^+ + \alpha$ imposes a limitation on the vacuum mean value $\langle \eta \rangle$ with which the breaking of the horizontal symmetry is connected, $\langle \eta \rangle > 10^{10}$ GeV. This restriction is much more stringent than the standard restriction on neutral currents that change the flavors, where the characteristic scale turns out to be $\gtrsim 10^4$–10^5 GeV (Ref. 6); if the flavor-changing Goldstone bosons were really to exist, the decays $K \to \pi + \alpha$ (and $\mu \to e + \alpha$) would be incomparably easier to observe than other effects due to flavor-changing neutral currents ($\mu \to 3e$, $\mu \to e\gamma$, $K_L^0 \to e^\pm \mu^\mp$, $K^+ \to \pi^+ e^+ \mu^-$, $\mu^- N \to e - N$ etc., Ref. 6).

Besides the principal problem formulated above (the possible existence of a Goldstone with interaction that is nondiagonal in the flavors), we discuss in Sec. 2 the possible character of couplings that are diagonal in flavor. We shall verify that in contrast to the nondiagonal interaction, a Goldstone boson interaction diagonal in the flavors cannot be scalar, but only pseudoscalar. More accurately speaking, exchange of a Goldstone boson can never lead to a long-range action $\propto 1/r$ between fermions.

Yet, if a nonzero θ term exists in the theory, we transform in explicit fashion, by eliminating this term with the aid of chiral rotation of the quarks, the pseudoscalar couplings of the quarks with arbitrary massless bosons into scalar ones. Therefore, if a Goldstone massless boson having a pseudoscalar coupling with quarks existed initially in the theory (and such a boson indeed can exist[3]), a scalar coupling arises after eliminating the θ term. We shall verify, however, that the resultant scalar coupling of the quarks has a somewhat arbitrary character. Obviously, at any rate for quarks, owing to the confinement, the premise of long-range action $\propto 1/r$ is meaningless. For physical nucleons, however, (as well as other colorless physical particles), the scalar coupling and long-range action $\propto 1/r$ do not exist.

In this sense, the situation with exact Goldstone bosons differs in principle from the case of pseudo-Goldstone bosons of the axion type, whose mass is due to anomaly. In the latter case, a scalar interaction is possible in principle, leading to a potential $e^{-m_a r}/r$ (m_a is the axion mass), albeit only when *CP* symmetry is broken.

2. INTERACTIONS DIAGONAL IN THE FLAVORS

In this section we explain why exchange of Goldstone bosons cannot lead to a long-range action $\propto 1/r$ between nucleons (or any other physical particles), although scalar coupling can exist in a certain arbitrary sense in the interaction of Goldstone bosons with quarks. We consdier first the perfectly trivial case, when there are no strong fermion interactions. Let, e.g., there exist a fermion field ψ interacting with a complex scalar field φ:

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$$-\mathcal{L}_{int}=h\bar{\psi}_R\psi_L\varphi+h^*\bar{\psi}_L\psi_R\varphi^*, \quad h=|h|e^{i\nu}, \tag{1}$$

where the coupling constant h is regarded as complex and its phase is purposefully not included in the redefinition of the phase φ of the field.

If the self-action of the field φ has symmetry $\varphi\to e^{-i\theta}\varphi$, the theory is invariant to the chiral group

$$\varphi\to e^{i\theta}\varphi, \qquad \psi\to\exp(-i\theta\gamma_5/2)\psi, \tag{2}$$

which can be spontaneously broken only if the scalar field develops vacuum mean values which we shall assume for the sake of argument to be real. Let

$$\varphi=(H+i\chi)/\sqrt{2}, \qquad \langle H\rangle=v; \tag{3}$$

then χ is a Goldstone boson with the following interaction with a fermion:

$$-\mathcal{L}_{int}=m(\bar{\psi}e^{i\nu\gamma_5}\psi)+\frac{m}{v}\chi(\bar{\psi}i\gamma_5e^{i\nu\gamma_5}\psi), \quad m=\frac{|h|v}{\sqrt{2}}. \tag{4}$$

As seen from (4), formally χ interacts both with a pseudoscalar $\bar{\psi}i\gamma_5\psi$ ($\propto\cos\nu$), and with a scalar ($\propto\sin\nu$). If, however, a chiral transformation $\psi\to\exp[-(1/2)\,i\nu\gamma_5]\psi$ of the fermion field is carried out, which reduces the mass term of the field ψ to standard form, then only the pseudo-scalar interaction $(m/v)\chi(\bar{\psi}i\gamma_5\psi)$ remains. This trivial example shows that the statement that the interaction is pseudoscalar is somewhat arbitrary and has an objective meaning only if it is stipulated that the mass terms have been reduced to a standard form free of γ_5. More general is the statement that there exists a phase shift $\pi/2$ in the mass terms and in the interaction: $m(\bar{\psi}e^{i\nu\gamma_5}\psi)$ and $m/v\chi(\bar{\psi}\exp(i(\nu+\pi/2)\gamma_5)\psi)$. Physically the absence of scalar coupling at the standard form of the mass term manifests itself in the absence of long-range action $\propto 1/r$ between the fermions (the pseudoscalar exchange yields a spin-independent potential of the form

$$r^{-3}[\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2-3(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n})]).$$

It is easily seen that the phase shift $\pi/2$ between the mass term and the interaction has a rather general cause. When the field χ interacts with the pseudoscalar $\bar{\psi}\gamma_5\psi$ and the mass term is of standard form the transformation $\chi\to\chi+\varepsilon$, where ε is a small constant increment, the fermion mass does not change in first-order approximation in ϵ:

$$m\to m(1+\varepsilon^2/v^2)^{1/2}\approx m.$$

This property is a consequence of the initial global symmetry of the theory: observable quantities should be invariant to the shift of the Goldstone field. (The seeming absence of invariance in higher orders in ε is due to the fact that actually the Goldstone field is the phase of the field φ and coincides with $\mathrm{Im}\varphi$ only in first-order in χ/v). The scalar coupling of the field χ (again for the standard form of the mass term) is impossible, for in this case the shift $\chi\to\chi+\varepsilon$ is accompanied by $m\to m(1+\varepsilon/v)$. We see that the "mutual orthogonality" of the two terms in (4), which ensures absence of long-range action $\propto 1/r$ between the fermions, is a necessary consequence of the symmetry inherent in the theory.

We consider now the case of several fermions (several flavors). If the mass matrix is reduced to a form diagonal in the flavors and free of γ_5, the arguments presented above show that an interaction with the Goldstone bosons without change of flavor can have only a pseudoscalar character, whereas when the flavors change there can exist, generally speaking, both scalar and pseduoscalar coupling. Indeed, in the case of the shift $\chi\to\chi+\varepsilon$, when calculating the eigenvalues of the mass matrix (the physical masses of the fermions) we can neglect in first order in ε the appearance of off-diagonal terms $\propto\varepsilon$ in the mass matrix. In the models considered below we shall verify in fact that scalar flavor-changing couplings are possible, as well as that the scalar couplings that are diagonal in the flavors vanish once the mass matrix is reduced to a form diagonal in the flavors and free of γ_5, i.e., precisely on the physical states.

The foregoing statements can be explained differently in the following manner. The interaction with the pseudoscalar density $(\bar{\psi}i\gamma_5\psi)\chi$ can be written as an interaction with the divergence of the axial current: $(2m)^{-1}\partial_\mu(\bar{\psi}\gamma_\mu\gamma_5\psi)\cdot\chi$. It can be seen that after the transformation of the derivative the interaction contains not the field χ itself, but $\partial_\mu\chi$, as should be the case for a Goldstone particle. Similarly, a scalar coupling of the type $(\bar{\psi}_1\psi_2)\chi$ is rewritten in the form

$$i(m_1-m_2)^{-1}\partial_\mu(\bar{\psi}_1\gamma_\mu\psi_2)\chi\to -i(m_1-m_2)^{-1}(\bar{\psi}_1\gamma_\mu\psi_2)\partial_\mu\chi,$$

which also contains only the derivative $\partial_\mu\chi$. In the case of a scalar coupling that is diagonal in the flavors, it cannot be rewritten in terms of $\partial_\mu\chi$, since $\partial_\mu(\bar{\psi}\gamma_\mu\psi)=0$. It is interesting that for a nondiagonal coupling this can likewise not be done at $m_1=m_2$. As will be seen from the example presented below, in accordance with this remark the coupling constant h of the interaction $h(\bar{\psi}_1\psi_2)\chi$ vanishes at $m_1=m_2$.

In the arguments presented above we have ignored strong gauge interactions of the fermions. Allowance for them calls for caution, since the chiral invariance used above to reduce the mass matrix to standard form usuallyl does not take place either as a consequence of the anomaly or simply because of dynamic spontaneous violation, formation of a quark condensate ($\langle\bar{u}u\rangle\neq0$, $\langle\bar{d}d\rangle\neq0$). In chiral transformation of quarks, on the one hand, the "chiral phases" of the condensates change, i.e.,

$$\langle\bar{u}u\rangle\to\langle\bar{u}\exp(i\theta_u\gamma_5)u\rangle,$$
$$\langle\bar{d}d\rangle\to\langle\bar{d}\exp(i\theta_d\gamma_5)d\rangle,$$

and on the other hand, as result of the anomaly, a change takes place in the value of the θ term: $\theta\to\theta-\theta_u-\theta_d$.

To understand the resultant situation, we consider by way of example the Goldstone particle — arion — introduced in Refs. 3 and verify the following.

1) In the absence of anomaly the spontaneous symmetry breaking itself, i.e., the formation of $\langle\bar{u}u\rangle\neq0$ and $\langle\bar{d}d\rangle\neq0$, takes place in such a way that the mass matrix of the quarks turns out to be automatically γ_5-free, and the interaction with the arion is purely pseudoscalar.

2) In the presence of anomaly, the quark mass matrix can be reduced to a γ_5-free form by redefining the θ term; conversely, the θ term can be eliminated by introducing complex components in the mass matrix of the quarks. In this case, at $\alpha_{\rm eff}=0$, the interaction of the quarks with the arion can indeed have a scalar character. However, the inter-

action of any physical (colorless) states with an arion, such as nucleons, does not contain a scalar coupling, and there is consequently no long-range action $\propto 1/r$.

In the simplest variant, the arion can be described in the following manner (see the second reference 3). Let a field φ_1 that is a doublet in the $SU(2)_L$ group give mass to all the fermions (quarks and lepton), and let a doublet field φ_2 not interact with either the quarks or the leptons. Let also the interaction of the fields φ_1 and φ_2 with each other be such that independent phase rotations of the fiels φ_1 and φ_2 are possible [there are no terms $(\varphi_1{}^+\varphi_2)^2$ + h.c.) in the interaction]. The Lagrangian of the interaction of φ_1 with the quarks u and d (for simplicity, we leave out the other fermions) is of the form

$$-\mathscr{L}=h_d\bar{d}_R(d_L\varphi_1^{0*}+u_L\varphi_1^{+*})+h_u\bar{u}_R(u_L\varphi_1^0-d_L\varphi_1^+)+\text{H.c.}, \quad (5)$$

where the Yukawa constants

$$h_d=|h_d|e^{i\nu_d}, \qquad h_u=|h_u|e^{i\nu_u},$$

are assumed, as in (1), to be complex.

The model considered has two U(1) symmetries:

1) $\varphi_1\to e^{i\theta_1}\varphi_1$, $d\to\exp(i\theta_1\gamma_5/2)d$, $u\to\exp(-i\theta_1\gamma_5/2)u$;

2) $\varphi_2\to e^{i\theta_2}\varphi_2$, the fermions are not transformed.

We note that the two symmetries are not spoiled by the anomaly, since the first of them constitutes chiral isotopic transformation: $q\to\exp(-i\theta_1\gamma_5\tau_3/2)q$.

Breaking of the symmetries described, i.e., the precipitation into the condensate $\langle\varphi_1{}^0\rangle\neq0$, $\langle\varphi_2{}^0\rangle\neq0$, leads to the appearnce of two Goldstone particles, of which the combination

$$g=(v_1H_1+v_2H_2)/v,$$
$$H_i=2^{1/2}\,\text{Im}\,\varphi_i^0, \quad v_i=2^{1/2}\langle\varphi_i^0\rangle, \quad v^2=v_1^2+v_2^2 \quad (6)$$

is absorbed by the Z boson. (For simplicity we assume v_i to be real and positive.) The orthogonal state

$$a=(v_1H_2-v_2H_1)/v \quad (7)$$

is a massless Goldstone particle (arion).

it is easy to find the interaction of the arion with quarks by starting from the interaction (5) of the field φ_1 (Ref. 3):

$$-\mathscr{L}_a=m_d(\bar{d}\exp(i\gamma_5\nu_d)d)+m_u(\bar{u}\exp(i\gamma_5\nu_u)u)$$
$$+\frac{v_2}{vv_1}a[m_d(\bar{d}i\gamma_5\exp(i\nu_d\gamma_5)d)$$
$$-m_u(\bar{u}i\gamma_5\exp(i\nu_u\gamma_5)u)],$$
$$m_d=|h_d|v_1/2^{1/2}, \qquad m_u=|h_u|v_1/2^{1/2}. \quad (8)$$

If the theory were to contain exact symmetry with respect to the independent chiral rotations u and d, we could reduce the mass terms to standard form with the aid of the transformations

$$d\to\exp(-i\nu_d\gamma_5/2)d, \quad u\to\exp(-i\nu_u\gamma_5/2)u.$$

In this case the interaction of the arion with the fermions would be purely pseudoscalar. As mentioned above, the chiral symmetry is broken, however, for two reasons.

1) Strong interaction of the quarks leads to dynamic breaking of the chiral symmetry because of formatin of condensates: $\langle u\bar{u}\rangle\neq0$, $\langle d\bar{d}\rangle\neq0$.

2) The theory contains an anomaly, as a result of which only chiral rotations of quarks with opposite phases, i.e., $\exp(i\theta\tau^3\gamma_5)$ transformatins, are possible.

Let us first forget the anomaly, but let $[\bar{u}u\rangle\neq0$ and $\langle\bar{d}d\rangle\neq0$. Dynamic breaking of the chiral symmetries

$$u\to\exp(i\theta_u\gamma_5/2)u, \quad d\to\exp(i\theta_d\gamma_5/2)d$$

leads to the appearance of the Goldstone particles π and η, which in the absence of anomaly and at $m_u=m_d=0$ are strictly massless. The zero mass of π nd η manifests itself in the fact that the energy of ther vacuum in the limit $m_u=m_d=0$ does not depend on the chiral phases of the condensates $\langle\bar{u}u\rangle$ and $\langle\bar{d}d\rangle$, i.e., on the quantities α_u and α_d defined by the equalities

$$\langle\bar{u}u\rangle=\rho_u\cos\alpha_u, \quad \langle\bar{d}d\rangle=\rho_d\cos\alpha_d,$$
$$\langle\bar{u}i\gamma_5u\rangle=\rho_u\sin\alpha_u, \quad \langle\bar{d}i\gamma_5d\rangle=\rho_d\sin\alpha_d. \quad (9)$$

Chiral rotations of the quarks:

$$u\to\exp(i\theta_u\gamma_5/2)u, \qquad d\to\exp(i\theta_d\gamma_5/2)d$$

leads to the transformations $\alpha_u\to\alpha_u-\theta_u$, $\alpha_d\to\alpha_d-\theta_d$. The usual choice of the phases of the condensate is the following: $\alpha_u=\alpha_d=0$. In this case, as is well known, $\langle\bar{u}u\rangle\approx\langle\bar{d}d\rangle\approx-(250\text{ MeV})^3$.

Independence of the vacuum energy of the phases α_u and α_d means that the ground state energy is independent of the constant shift of the Goldstone fields π and η. Obviously, accurate to a factor, α_u and α_d simply coincide with the mean values of the fields π and η:

$$\alpha_u=\frac{\sqrt{2}}{f_\pi}\langle\pi_u(x)\rangle, \quad \alpha_d=\frac{\sqrt{2}}{f_\pi}\langle\pi_d(x)\rangle,$$
$$\eta(x)=2^{-1/2}(\pi_u(x)+\pi_d(x)), \qquad \pi(x)=2^{-1/2}(\pi_u(x)-\pi_d(x)), \quad (10)$$

where $f_\pi\approx90$ MeV is the axial constant. The proportionality coefficient in Eqs. (10) can be established by using the method of current algeba (see also Ref. 7). To this end it is possible, e.g., to use the PCAC relation $\partial_\mu J_\mu^5(x)=2^{1/2}f_\pi m_\pi^2\pi(x)$ and the standard formula for the π-meson mass. Then

$$\pi(x)=(2^{1/2}f_\pi m_\pi^2)^{-1}\partial_\mu J_\mu^5=2^{-1/2}f_\pi 2m_q(\bar{\psi}i\gamma_5\psi)/[-2m_q\langle\bar{\psi}\psi\rangle]$$
$$=2^{-1/2}f_\pi(\bar{\psi}i\gamma_5\psi)/|\langle\bar{\psi}\psi\rangle|.$$

Since in fact $m_u\neq0$ and $m_d\neq0$, the vacuum energy depends actually on α_u and α_d, in other words, at $m_u\neq0$ and $m_d\neq0$ the particles π and η have a nonzero mass. The dependence of the vacuum energy on α_u and α_d can be easily found from (5). Averaging (5) over the vacuum and using (9), we obtain in the tree approximation.

$$\mathscr{E}_{vac}=m_d\rho\cos(\nu_d-\alpha_d)+m_u\rho\cos(\nu_u-\alpha_u), \quad \rho_u=\rho_d=\rho. \quad (11)$$

Since α_u and α_d are mean values of the dynamic variables (10), they are determined from the minimum of the energy (11) which yields

$$\alpha_d=\nu_d,\quad \alpha_u=\nu_u. \tag{12}$$

We change now to new fields

$$d'=\exp(i\nu_d\gamma_5/2)\,d,\quad u'=\exp(i\nu_u\gamma_5/2)\,u.$$

In terms of these variables the Lagrangian (8) takes the standard form, i.e., the γ_5 dependence in the mass terms vanishes and the coupling of the arion with the fermions remains purely pseudoscalar. The vacuum means (9) in terms of u' and d' also become purely scalar:

$$\langle\bar{u}'u'\rangle=\rho,\quad \langle\bar{d}'d'\rangle=\rho,$$
$$\langle\bar{u}'i\gamma_5u'\rangle=0,\quad \langle\bar{d}'i\gamma_5d'\rangle=0. \tag{9'}$$

We see that despite the spontaneous violation of the chiral invariance, the phases of the vacuum mean values are ordered in such a way (Eq. 12) that it is possible by using one and the same chiral transformation to reduce the mass terms to standard form and simultaneously ensure the usual choice of chiral phases of the quark vacuum mean values [formual (9')]. The scalar coupling of the arion with the fermion vanishes in this case.

It is obvious from the example presented that the situation in which the anomaly is taken into account takes an entirely different form. In this cas even at $m_u=m_d=0$ the existence of an anomaly leads to a dependence of the vacuum energy on the phases of the condensate α_u and α_d, defined by Eqs. (9), and the minimum of the energy of the vacuum does not correspond to the condition (12), which ensures the absence of a scalar coupling between the arion and the quarks. We obtain below this minimum and verify that the scalar coupling of the arion with the quarks does indeed take place if the chiral phases of the quark fields are chosen such that the condenstate takes the form, i.e., $\langle\bar{q}q\rangle\neq0$ and $\langle\bar{q}i\gamma_5q\rangle=0$, and there is no θ term. We shall verify, however, that no scalar coupling of the arion with the nucleon exists also in this case. We present first the proof, and then track the manner in which the scalar coupling of the arion with the nucleon is cancelled, starting with the quark level.

The simplest proof of the absence of scalar coupling between the arion and nucleons has in fact already been presented: it consists in the remark that the nucleon mass, just as any other physical quantity, cannot change following a constant shift of the Goldstone field α. If we include in the effective Lagrangian of the interaction of the arion with the nucleon the scalar coupling

$$-\mathcal{L}_{eff}=m_N\bar{\psi}_N\psi_N+h\bar{\psi}_N\psi_N\cdot\alpha,$$

then the shift $\alpha\to\alpha+\varepsilon$ brings about $m_N\to m_N+h$; therefore the interaciton $\psi_N\psi_N\cdot\alpha$ is forbidden.

That there is no scalar coupling between an arion and a nucleon can be formally verified in the following manner. We consider the Lagrangian (8) of the interaction of quarks with an arion. From the equations of motion that follow for this Lagrangian we can see that

$$\partial_\mu(\bar{q}\tau^3\gamma_\mu\gamma_5q)=2i[m_u(\bar{u}i\gamma_5e^{i\nu_u\gamma_5}u)-m_d(\bar{d}i\gamma_5e^{i\nu_d\gamma_5}d)], \tag{13}$$

i.e., the arion $\alpha(x)$ interacts with the divergence of the isovector axial current. (This, of course, is obvious also without calculations, since a Goldstone boson must interact with the divergence of a conserved current.)

The matrix element over the nucleon states, which determines the interaction of the arion with the nucleon, is equal according to (8) and (13)

$$q_\mu\langle N|\bar{q}\tau^3\gamma_\mu\gamma_5q|N\rangle, \tag{14}$$

where q_μ is the momentum transfer. The scalar coupling of the nucleon $(\bar{\psi}_N\psi_N\cdot\alpha)$ with the arion would be obtained from the γ_5-free terms in this matrix element. Such terms, generally speaking, could be present, since parity is not conserved. The Lorentz structure of the matrix element (14) contains six terms, of which three violate parity: γ_μ, $\sigma_{\mu\nu}q_\nu$, and q_μ. The first two vanish when multiplied by q_μ, and the last makes a contribution $\propto q^2$, which vanishes as $q^2\to0$ and consequently does not lead to long-range action. Thus, the use of the observtion law (13) turns out to be sufficient for a formal proof of the absence of long-range action between the nucleons.

We now track, in explicit fashion, how the scalar coupling is producued between the quarks and the arion (choosing the chiral phases of the quarks to correspond to $\theta_{\rm eff}=0$) and how the scalar coupling of the nucleon with the arion vanishes notwithstanding.

In the presence of anomaly, the energy of the vacuum depends on the combination $\alpha_u+\alpha_d-\theta$, inasmuch as under the transformation

$$u\to\exp(i\gamma_5\theta_u/2)\,u,\quad d\to\exp(i\theta_d\gamma_5/2)\,d.$$

the phases of the condenstate and the θ term vary in the following manner:

$$\alpha_u\to\alpha_u-\theta_u,\quad \alpha_d\to\alpha_d-\theta_d,\quad \theta\to\theta-\theta_u-\theta_d. \tag{15}$$

The expression for the energy of the vacuum (8) must therefore be supplemented by the "anomalous terms"

$$\mathscr{E}_{vac}=m_d\rho\cos(\nu_d-\alpha_d)+m_u\rho\cos(\nu_u-\alpha_u)+V_{anom}(\alpha_u+\alpha_d-\theta). \tag{16}$$

At small m_d and m_u the last term, which reflects the presence of the anomaly, is larger than the first two, and it is therfore necessary in fact to minimize expression (16) subject to the additional condition that $M_{\rm anom}$ be a minimum, i.e., at

$$\alpha_u+\alpha_d-\theta=0. \tag{17}$$

This leads to the following values of α_u and α_d:

$$\operatorname{tg}\alpha_u=\frac{m_u\sin\nu_u-m_d\sin(\nu_d-\theta)}{m_u\cos\nu_u+m_d\cos(\nu_d-\theta)},$$
$$\operatorname{tg}\alpha_d=\frac{m_d\sin\nu_d-m_u\sin(\nu_u-\theta)}{m_d\cos\nu_d+m_u\cos(\nu_u-\theta)}. \tag{18}$$

The standard choice of the chiral phases corresponds to the usual form of the condensate, i.e., to the conditions

$$\langle\bar{u}u\rangle\neq0,\quad \langle\bar{d}d\rangle\neq0,\quad \langle\bar{u}i\gamma_5u\rangle=\langle\bar{d}i\gamma_5d\rangle=0.$$

For a transition to such phases from the initial phases (9), it is necessary to carry out the rotation

$$d\to\exp(i\alpha_d\gamma_5/2)\,d,\quad u\to\exp(i\alpha_u\gamma_5/2)\,u.$$

Of course, the mass terms of the quarks remain complex in this case, but since they are small this choice of the chiral phases is still the most reasonable. We note that in this case, according to (15) and (17), the θ term also vanishes, and since the condensate has the normal form, direct application of the formulas of the chiral perturbation theory, which uses the smallness of m_u and m_d, becomes possible. We shall use this circumstance below.

In the new "phase gauge" the Lagrangian (8) takes the form

$$-\mathscr{L}=m_d\bar{d}(\cos\beta_d+i\gamma_5\sin\beta_d)d+m_u\bar{u}(\cos\beta_u+i\gamma_5\sin\beta_u)u$$
$$+\frac{m_dv_2}{vv_1}\alpha\bar{d}(i\gamma_5\cos\beta_d-\sin\beta_d)d-\frac{m_uv_2}{vv_1}\alpha\bar{u}(i\gamma_5\cos\beta_u-\sin\beta_u)u, \quad (19)$$

where $\beta_d=\nu_d-\alpha_d$, $\beta_u=\nu_u-\alpha_u$, so that according to (18)

$$\operatorname{tg}\beta_d=\frac{m_u\sin(\nu-\theta)}{m_d+m_u\cos(\nu-\theta)},\quad \operatorname{tg}\beta_u=\frac{m_d\sin(\nu-\theta)}{m_u+m_d\cos(\nu-\theta)}$$

$$\nu=\nu_u+\nu_d. \quad (20)$$

The Lagrangian (19) contains both scalar and pseudoscalar couplings of the arion with the quarks. At first glance it may seem that the scalar coupling of the arion with the physical states, e.g., with a nucleon, is also inevitable here. Actually, when calculating the matrix element of the scalar density over the nuclear states

$$\langle N|m_u\sin\beta_u\bar{u}u-m_d\sin\beta_d\bar{d}d|N\rangle \quad (21)$$

it is natural to expect (21) to be proportional to the $\bar{\psi}_N\psi_N$-scalar nucleon density, resulting in a scalar interaction between the arion and the nucleon. We shall verify that actually the matrix element (21) is cancelled out by the matrix element over the nucleon states of the pseudoscalar part of the interaction (19). The point is that the matrix element of the pseudoscalar density has an enhancement $\sim 1/m_q$ ($q=u,d$) compared with the matrix element of the scalar density. Therefore the pseudoscalar matrix element must be calculated in the next order in m_q, taking into account also the parity nonconserving mass terms of the quarks. The nucleon matrix element of the pseudoscalar density can then contain the scalar $\bar{\psi}_N\psi_N$ and cancel out (21). The matrix element of the pseudoscalar density, in which account is taken of the parity violation on account of the quark mass terms proportional to γ_5, is of the form

$$-i\left\langle N\left|\lim_{q\to0}\int d^4z e^{iqz}T\{[m_d\cos\beta_d(\bar{d}(z)i\gamma_5d(z))\right.\right.$$
$$-m_u\cos\beta_u(\bar{u}(z)i\gamma_5u(z))]$$
$$\left.\left.\times[m_d\sin\beta_d(\bar{d}(0)i\gamma_5d(0))+m_u\sin\beta_u(\bar{u}(0)i\gamma_5u(0))]\}\right|N\right\rangle. \quad (22)$$

The first factor in the T product can be replaced in accordance with the equations of motion by the divergence of the axial current. Transferring the derivatives, we obtain in the limit $q=0$, as usual, standard equal-time commutators. Their calculation gives an expression that differs from (21) in sign. Thus, the scalar interaction $(\bar{\psi}_N\psi_N\cdot\alpha)$ is indeed cancelled out when account is taken of both the scalar and the pseudoscalar density in the interaction of the arion with the quarks. We note that although we did not need, to prove this cancellation, the explicit form of the angles of the chiral rotation α_a and α_u (18), this rotation was necessary to prove the absence of long-range nucleon action, since it is precisely our choice of the chiral phases which ensured the vanishing of the θ term and the definite parity of the vacuum corresponding to the standard choice of the condensate $\langle\bar{q}i\gamma_5q\rangle=0$. This concludes the discussion of the question of how the scalar coupling of the arion with the nucleon vanishes notwithstanding the presence of scalar coupling of the arion and the quarks.

So far we have considered strictly massless Goldstone particles. Yet, in the theory one frequency encounters light scalar particles corresponding to spontaneous breaking of a certain approximate symmetry. A classical example of such a pseudo-Goldstone particle, which acquires mass as a result of an anomaly in the axial quark field, is the axion.[1] Whereas the mass of the standard axion has a natural order of magnitude of hundreds of keV, the widely discussed at present "ghost" axion[8] can have a mass up to 10^{-8} eV, corresponding to a Compton wavelength ~ 20 m. Although exchange of such particles does not lead to a true (decreasing in power-law fashion) long-range action, interest attaches to the question whether an axion (or a ghost axion) can have a scalar interaction with nucleons and induce in them a "pseudo-long-range action" $\propto e^{-m_ar}/r$.

The foregoing proof of the absence of scalar couplings of Goldstone particles is obviously not valid in the present case. Nonetheless it is easy to see that in the case of exact CP invariance of the theory the axion also has a pure pseudoscalar coupling: the axion is a particle with negative CP parity, therefore its scalar interaction with the nucleons would be evidence of CP violation. On the other hand if explicit CP violation is present in theory with the axion (as, e.g., in the model of Kobayashi aned Maskawa), no reason can be seen, generally speaking, for a scalar axion coupling not to occur.

Thus, Goldstone particles do not lead to a long-range action $\propto 1/r$. What will happen if we simply introduce into the theory a non-Goldstone spinless particle having a scalar coupling with fermions, and artificially set its mass equal to zero? Obviously, it will acquire a mass in the higher orders in particular, even when the simplest fermion loop is taken into acocunt. The only possibility under which the particle remains without mass is a successive cancellation, in all orders, of the contribution of the bosons and fermions. This possibility is naturally realized only in supersymmetry, where the masslessness of some supersymmetric partner of the considered particle can be ensured in the usual fashion—e.g., by chiral symmetry for the fermion or by "Goldstone character" for the pseudoscalar partner. Inasmuch as in a real world symmetry should, at any rate, be broken, the mass of the scalar particle will differ from zero. For the lower limit of the mass of such a scalar particle it is natural to expect, in order of magnitude,

$$m\approx h\Lambda, \quad (23)$$

where Λ is the scale at which the supersymmetry is broken and h is the Yukawa coupling constant. Let $h \sim m_q/m_{GUT} \sim 10^{-16}$, just as in the theory of the ghost axion. If $\Lambda \sim 1$ TeV, then $m \sim 10^{-4}$ eV, which corresponds to the long-range action radius $m^{-1} \sim 0.2$ cm. Up to these distances, the considered long–range action exceeds by six orders of magnitude the gravitational one.

3. SIMPLEST MODEL OF AN INTERACTION NOT DIAGONAL IN THE FLAVORS

We consider first the simplest albeit unrealistic model in which Goldstone boson has an interaction which is nondiagonal in the flavors, and discuss certain characteristic properties of this interaction.

In our model there exists a gauge "horizontal" group (group of "generations") $SU(3)_h$, but there are no usual gauge interactions connected with the group $SU(3)_c \times SU(2)_L \times U(1)$. Three left-helical fermions $l_L^\alpha = (e,\mu,\tau)_L$ are transformed in accordance with the triplet representative $SU(3)_h$, just as the charge-conjugate left-helical states $(l^c)_L^\alpha = (e^c,\mu^c,\tau^c)_L$. Obviously, the mass terms are transformed in this case like $3 \times 3 = \bar{3} + 6$ and can arise only as a result of spontaneous symmetry breaking. The permissible Yukawa couplings take the form

$$-\mathscr{L} = h((l^c)_L{}^\alpha c l_L{}^\beta)\zeta^\gamma \varepsilon_{\alpha\beta\gamma} + f((l^c)_L{}^\alpha c l_L{}^\beta)\omega_{\alpha\beta} + \text{H.c.}, \tag{24}$$

where the Higgs scalar fields are transformed in accordance with the triplet and antisextet representations $\zeta^\alpha \sim 3$ and $\omega_{\alpha\beta} \sim 6$. It is easily seen that the theory can have (subject to additional conditions on the sector of the interaction of the Higgs bosons) a global $U(1)$-symmetry of phase transformations $\exp(iY\theta)$, if we put

$$Y(l_L{}^\alpha) = Y(l_L{}^{c\alpha}) = 1, \qquad Y(\zeta^\alpha) = Y(\omega_{\alpha\beta}) = -2. \tag{25}$$

The Y-symmetry forbids, in the interaction Lagrangian of the Higgs bosons, terms of the form

$$\omega_{\alpha\beta}\zeta^\alpha\zeta^\beta, \quad \operatorname{Det}\omega = {}^1/_6[(\operatorname{Sp}\omega)^3 + 2\operatorname{Sp}\omega^3 - 3\operatorname{Sp}\omega\operatorname{Sp}\omega^2].$$

We propose next that the Higgs potential is constructed in such a way that the following components of the scalar fields differ from zero:

$$\langle\zeta\rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \qquad \langle\omega\rangle = \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{pmatrix} \tag{26}$$

This rather arbitrary assumption brings us closer to the realistic model considered in the next section. The coupling constants h and f in (24) will be assumed to be complex, and the vacuum mean values (26) real. A Goldstone boson correspond to the symmetry realized with the aid of the generator F can be constructed in the following manner (we leave out the normalization):

$$G = \sum_\varphi [\langle\varphi^+\rangle F\varphi - \varphi^+ F\langle\varphi\rangle], \tag{27}$$

where the sum is taken over all the scalar fields. If $\langle\varphi\rangle$ is real and the operator $F = Y$ is a diagonal operator, then (27) reduces to

$$G = \sum_\varphi Y(\varphi)\langle\varphi\rangle \operatorname{Im}\varphi. \tag{28}$$

For our case, obviously,

$$G = v\zeta + r_1\omega_1 + r_2\omega_2 + r_3\omega_3, \tag{29}$$

where

$$\zeta = \operatorname{Im}\zeta^3, \qquad \omega_\alpha = \operatorname{Im}\omega_{\alpha\alpha}.$$

Formula (29) would be perfectly correct if there were in the theory no other Goldstone bosons that can be mixed in with G. In our case such bosons are Goldstone particles connected with the spontaneous breaking of the symmetry with respect to the transformations λ_3 and λ_8 of the $SU(3)_h$ group. (These bosons, owing to the Higgs mechanism, are absorbed by the corresponding gauge bosons.) We write down explicitly these two Goldstone bosons, the first corresponding to the genreator $\lambda_3 = [1, -1, 0]_{\text{diag}}$ and the second to the U-spin, i.e., to the generator

$$(\sqrt{3}/2)\lambda_8 - {}^1/_2\lambda_3 == [0, 1, -1]_{diag}:$$

$$T = 2(r_1\omega_1 - r_2\omega_2), \qquad U = 2(-r_2\omega_2 + r_3\omega_3) - v\zeta. \tag{30}$$

It is necessary now to add to expression (29) for G a linear combination of T and U bosons such that the obtained state be orthogonal to T and U. We have utlimately for the sought Goldstone boson α

$$\begin{aligned}\alpha = \frac{1}{N}\{&[(T\cdot T)(U\cdot U) - (T\cdot U)^2]G \\ &- [(G\cdot T)(U\cdot U) - (T\cdot U)(G\cdot U)]T \\ &- [(G\cdot U)(T\cdot T) - (T\cdot U)(G\cdot T)]U\}\end{aligned} \tag{31}$$

or, substituting (30) for T and U:

$$\begin{aligned}\alpha = \frac{1}{N}\{&2vr_3(r_1^2 + r_2^2)\zeta + 4r_1r_2^2r_3\omega_1 + 4r_1^2r_2r_3\omega_2 \\ &+ [4r_1^2r_2^2 + v^2(r_1^2 + r_2^2)]\omega_3\},\end{aligned} \tag{32}$$

where N is a normalization factor.

We write down now the mass matrix M_{RL}, which connects the left and right fermions

$$-\mathscr{L}_M = \overline{l_R{}^\alpha}(M_{RL})_{\alpha\beta} l_L{}^\beta + \text{H.c.}. \tag{33}$$

As can be seen from (24), M_{RL} is given by

$$M_{RL} = \begin{pmatrix} fr_1, & hv, & 0 \\ -hv, & fr_2, & 0 \\ 0, & 0, & fr_3 \end{pmatrix}. \tag{34}$$

Thus, only the first two states, e and μ, require in fact diagonalization.

Although we are dealing only with a 2×2 matrix, we are faced with a rather cumbersome algebraic problem, since the diagonalization is carried out with the aid of independent rotations of left-hand and right-hand fermions

$$l_L \to V_L l_L, \qquad l_R \to V_R l_R,$$

for which

$$M_{RL}\rightarrow V_R^+M_{RL}V_L.$$

It is simplest to diagonalize the two Hermitian matrices

$$m_1=M_{RL}^+M_{RL}\quad (m_1\rightarrow V_L^+m_1V_L)$$

and

$$m_2=M_{RL}M_{RL}^+\quad (m_2\rightarrow V_R^+m_2V_R)$$

and obtain thus their eigenvalues and matrices V_L and V_R, but the latter only accurate to multiplication of one column by a phase factor. The uncertainty in the multiplication of one and the same column V_L and V_R by an identical phase factor is of no significance, since it corresponds to redefinition of the phases of the physical fields e and μ. However, the relative phases V_L and V_R are substantial for a correct construction of physical states. It is possible therefore to proceed in the following manner; after determining arbitrary V_L and V_R with the aid of diagonalization of m_1 and m_2, we calculate next the matrix $V_R^+ M_{RL} V_L$ and, stipulating that its eigenvalues (fermion masses) be real and positive, obtain the necessary phase factors that must be introduced into the columns of any of the matrices V_R or V_L. After completing this rather cumbersome program, we arrive at the following answer

$$m_{e,\mu}^2={}^1/_2[|f|^2(r_1^2+r_2^2)+2|h|^2v^2]\mp{}^1/_2[|f|^4(r_1^2-r_2^2)^2+4|h|^2|f|^2v^2(r_1^2+r_2^2-2r_1r_2\cos\theta)]^{1/2},\quad \theta=\arg(f^*h),\tag{35}$$

where

$$\begin{gathered}
V_L=\begin{pmatrix} pe^{i\chi}, & -q\\ q, & pe^{-i\chi}\end{pmatrix},\\
V_R=e^{i\varphi}\begin{pmatrix} pe^{-i\chi+i\delta_1}, & qe^{i\delta_2}\\ -qe^{i\delta_1}, & pe^{i\chi+i\delta_2}\end{pmatrix}\\
p=\cos\theta_c=\frac{1}{\sqrt{2}}\left(1+\frac{r_2^2-r_1^2}{\Delta^{1/2}}\right)^{1/2},\\
q=\sin\theta_c=\frac{1}{\sqrt{2}}\left(1-\frac{r_2^2-r_1^2}{\Delta^{1/2}}\right)^{1/2}(r_2>r_1),\\
\chi=\arg(r_2e^{i\theta}-r_1e^{-i\theta}),\quad \varphi=\arg f,\quad \varphi+\theta=\arg h,\\
\operatorname{ctg}\delta_1=\operatorname{ctg}2\theta-\frac{r_2}{r_1\sin 2\theta}\left[1-\frac{2(r_1^2+r_2^2-2r_1r_2\cos 2\theta)}{r_2^2-r_1^2+\Delta^{1/2}}\right],\\
\operatorname{ctg}\delta_2=\operatorname{ctg}2\theta-\frac{r_1}{r_2\sin 2\theta}\left[1+\frac{2(r_1^2+r_2^2-2r_1r_2\cos 2\theta)}{r_2^2-r_1^2+\Delta^{1/2}}\right]\\
\Delta=(r_1^2-r_2^2)^2+(4|h|^2/|f|^2)v^2(r_1^2+r_2^2-2r_1r_2\cos 2\theta).
\end{gathered}\tag{36}$$

The foregoing unattractive expressions can be used in the following manner. Separating in the Lagrangian of the interaction of the Higgs fields with the fermions the interaction with ζ, ω_1, ..., we can easily find the interaction of the fermions l^α with the Goldstone field a (to this end it is necessary to recognize that according to (32) the field ζ contains a with weight $2vr_3(r_1^2+r_2^2)/N$, etc.). It is then necessary to change over to physical fermions $l_{\rm phys}=V_Ll_L+V_Rl_R$, using the expressions (36) for V_L and V_R. Having done this, we can verify explicitly that the scalar diagonal couplings $(\bar{e}e)a$ and $(\bar{\mu}\mu)a$ (e and μ are physical states) vanish. This confirms the general theorem proved in Sec. 2.

To prove the vanishing of the constant of the scalar interaction that is diagonal in the flavors it is natural to use the total expression (36) for V_L and V_R, for if the constants h and f are real, i.e., the initial Lagrangian is CP-invariant, this property follows in trivial fashion from the fact that the interction is Hermitian. The expression for the interaction that is not diagonal in the flavors, which is of interest to us, will be presented only for the case of a *CP*-invariant Lagrangian, i.e., for real constants h and f ($\theta=\varphi=0$). In this case expressions (35) and (36) become much simpler ($\chi=\delta_1=\delta_2=0$), and for the constant of the sought interaction we can obtain the following formula:

$$-\mathscr{L}_{int}=g(\bar{\mu}e+\bar{e}\mu),$$

$$g=h\frac{(m_\mu-m_e)^2\cos 2\theta_c}{(m_\mu-m_e)^2+(m_\mu+m_e)^2\cos^2 2\theta_c}\left[1+\frac{4r_1^2r_2^2}{v^2(r_1^2+r_2^2)}\right]^{-1/2}\times\left[1+\frac{1}{4}\frac{v^2}{r_3^2}+\frac{r_1^2r_2^2}{r_3^2(r_1^2+r_2^2)}\right]^{-1/2}.\tag{37}$$

It can be seen that the constant g vanishes at $m_\mu=m_e$, as stated in Sec. 2. At $\cos 2\theta_c\sim 1$ we have $g\sim(m_\mu-m_e)^2$, but at $\cos_2\theta_c\sim(m_\mu-m_e)$, the linear smallness $g\sim(m_\mu-m_e)$ is also possible.

To help a reader who would like to verify the calculations we present the following useful relations:

$$m_\mu-m_e=f(r_2-r_1),\qquad m_\mu+m_e=[f^2(r_1+r_2)^2+4h^2v^2]^{1/2},$$
$$\sin 2\theta_c=2hv[(r_1+r_2)^2f^2+4h^2v^2]^{-1/2}.\tag{38}$$

4. THE SU(5)×SU(3)$_h$ MODEL

In this section we consider the model proposed in Ref. 5, in which the standard SU(5) symmetry is supplemented by a horizontal gauge SU(3)$_h$ symmetry. In the model of Ref. 5 is assumed a rather rich content of the Higgs sector, which makes it possible to describe quite successfully the mass matrix (the masses and the mixing angles) of the fermions. Referring to the reader for details to the original article,[5] we recall only one achievement of the model, which is of rather general character: it becomes possible in this model to reconcile in natural fashion the necessarily small values (∼hundreds of GeV) of the vacuum mean values of the Higgs bosons, which give the masses of fermions of different populations, to the very large mass of these bosons. The latter is needed to surpess the neutral currents with the flavor violation that occur in the model on account of exchange of the corresponding Higgs bosons. This is accomplished by introducing "projective couplings"—Higgs-Lagrangian terms in which the bosons mentioned above enter in linear fashion together with the other Higgs fields, including a pentaplet that develops a small vacuum mean value. (The small vacuum mean value of the pentaplet is obtained, of course, at the expense of (one!) unnatural hierarchy condition that must be artificially imposed, just as in other versions of the SU(5) model.) The projective couplings lead to the appearance of an effective linear term in the Higgs field, which gives mass to the fermions when all other fields are replaced by their mean vacuum values. As a result, even at a mass, say, of the order of 10^{15} GeV the mean value of the discussed Higgs field connected with the fermions may turn out to be shifted from zero only by an amount of the order of a hundred GeV.

Specifically, the $SU(5)\times SU(3)_h$ model contains the following Higgs multiplets[5]:

$$\begin{gathered}\Phi_i^{\ j}\sim(24.1)\qquad H^i\sim(5.1),\\ \xi^\alpha\sim(1.3),\quad \eta^\alpha\sim(1.3),\quad \chi^{(\alpha,\beta)}\sim(1.6),\\ \omega^{i(\alpha,\beta)}\sim(5.6),\quad \rho_i^{\{\alpha,\beta\}}\sim(\bar{5}.6),\\ \zeta_{i\alpha}\sim(\bar{5}.\bar{3}),\quad \sigma^{\ k}_{[ij]\alpha}\sim(\overline{45}.\bar{3}).\end{gathered}\tag{39}$$

The first figure in the parentheses denotes the dimensionality of the SU(5) multiplet, and the second the dimensionality of the $SU(3)_h$ group. Small vacuum mean values are possessed by the fields H^i ($\langle H^5\rangle\neq0$) and by all the fields listed in the last two lines which give mass to fermions on account of the Yukawa couplings:

$$\begin{gathered}h_\omega(\psi_L)_\alpha^{\ [ij]}\,c(\psi_L)_\beta^{\ [kl]}\,\omega^{m(\alpha,\beta)}\varepsilon_{ijklm}+h_\rho(\psi_L)_{i\alpha}c(\psi_L)_\beta^{\ [ik]}\,\rho_k^{\{\alpha,\beta\}}\\ +h_\zeta(\psi_L)_{i\alpha}c(\psi_L)_\beta^{\ [ik]}\,\zeta_{k\gamma}\varepsilon^{\alpha\beta\gamma}+h_\sigma(\psi_L)_{i\alpha}c(\psi_L)_\beta^{\ [jk]}\,\sigma^{\ i}_{[jk]\gamma}\varepsilon^{\alpha\beta\gamma}+\text{H.c.},\end{gathered}\tag{40}$$

where the left-hand fermions are

$$(\psi_L)_\alpha^{\ [ij]}\sim(10.\bar{3}),\quad (\psi_L)_{i\alpha}\sim(\bar{5}.\bar{3}).$$

Without dwelling in detail on the description of the Higgs coupling,[5] we formulate a fact that is fundamental in what follows. The model can include the global $U(1)$ symmetry, i.e., the phase transformations $\exp(iY\theta)$, if the fields are assigned the following values of the hypercharge Y [the values of Y are given below for the components of the same covariance as those written out in (39)]

$$\begin{gathered}Y_\Phi=Y_H=0,\qquad Y_\xi=Y_\eta=-Y_\chi=1,\\ Y_\omega=Y_\rho=Y_\zeta=Y_\sigma=-1,\qquad Y_{\psi_L}={}^1/_2.\end{gathered}\tag{41}$$

It is easily seen that $U(1)$ invariance holds for the Yukawa couplings presented above, as well as by the Higgs couplings[5] that are significant for the model:

$$\begin{gathered}\xi^\alpha\xi^{\alpha'}\chi^{(\beta,\beta')}\chi^{(\gamma,\gamma')}\varepsilon_{\alpha\beta\gamma}\varepsilon_{\alpha'\beta'\gamma'},\qquad \eta^\alpha\eta^{\alpha'}\chi^{(\beta,\beta')}\chi^{(\gamma,\gamma')}\varepsilon_{\alpha\beta\gamma}\varepsilon_{\alpha'\beta'\gamma'},\\ \omega^{i(\alpha,\beta)}\chi_{(\alpha,\beta)}\Phi_i^{\ j}H_j,\quad \rho_i^{\ \{\alpha,\beta\}}\chi_{(\alpha,\beta)}\Phi_j^{\ i}H^j,\\ \zeta_{i\alpha}\Phi_j^{\ i}H^j\eta^\alpha,\quad \sigma^{\ k}_{[ij]\alpha}H^i\Phi_k^{\ j}\xi^\alpha\end{gathered}$$

(and also the couplings that include only the fileds Φ and H).

At the same time, the postulated symmetry forbids a number of Higgs couplings, e.g.,

$$\begin{gathered}\xi_\alpha\xi_\beta\chi^{(\alpha,\beta)},\quad \zeta_{i\alpha}\xi_\beta\omega^{i(\alpha,\beta)},\\ \xi_\alpha\xi_\beta H_i\omega^{i(\alpha,\beta)},\quad \xi_\alpha\xi_\beta H^i\rho_i^{\{\alpha,\beta\}},\\ \xi^\alpha\eta^\beta H_i\zeta^{i\gamma}\varepsilon_{\alpha\beta\gamma},\quad \xi_\alpha\zeta_{i\beta}\omega^{j(\alpha,\beta)}\Phi_j^{\ i},\quad \operatorname{Det}\chi\end{gathered}$$

(and the same couplings with $\xi\to\eta$). None of these interactions are needed in the model of Ref. 5 to obtain for the fermions a mass matrix that agrees with experiment, and some of the forbidden couplings would actually destroy the constructed mass matrix. Such, e.g., are the couplings

$$\xi_\alpha\xi_\beta H_i\omega^{i(\alpha,\beta)},\quad \xi_\alpha\xi_\beta H^i\rho_i^{\{\alpha,\beta\}},$$

which would lead to inadmissably large values

$$\langle\omega^{5(1,1)}\rangle,\ \langle\rho_5^{\{1,1\}}\rangle.$$

Thus, assuming the presnece of $U(1)$ symmetry $\exp(iY\theta)$, we construct a Goldstone boson G in accordance with the prescription (28) explained in the preceding section. For the non-normalized state of G we have

$$G=p\xi+q\eta-\sum_{\alpha=1}^{3}r_\alpha\chi_\alpha-\lambda\sum_{\alpha=1}^{3}r_\alpha\omega_\alpha-\mu\sum_{\alpha=1}^{3}r_\alpha\rho_\alpha-\nu q\zeta-\varkappa p\sigma,\tag{42}$$

where

$$\begin{gathered}\xi=\frac{1}{i\sqrt2}(\xi^1-\xi^{1*}),\quad \eta=\frac{1}{i\sqrt2}(\eta^3-\eta^{3*}),\quad H=\frac{1}{i\sqrt2}(H^5-H^{5*}),\\ \chi_\alpha=\frac{1}{i\sqrt2}(\chi^{(\alpha,\alpha)}-\chi^{(\alpha,\alpha)*}),\\ \omega_\alpha=\frac{1}{i\sqrt2}(\omega^{5(\alpha,\alpha)}-\omega^{5(\alpha,\alpha)*}),\\ \rho_\alpha=\frac{1}{i\sqrt2}(\rho_5^{\{\alpha,\alpha\}}-\rho_5^{\{\alpha,\alpha\}*}),\quad \zeta=\frac{1}{i\sqrt2}(\zeta_{53}-\zeta_{53}^*),\\ \sigma=\frac{1}{i\sqrt{12}}(\sigma_1^{[15]1}+\sigma_2^{[25]1}+\sigma_3^{[35]1}-3\sigma_4^{[45]1}-\text{H.c.}),\end{gathered}\tag{43}$$

and the real vacuum mean values $p, q, \dots$ are[5]

$$\begin{gathered}p=\langle\xi^1\rangle,\quad q=\langle\eta^3\rangle,\quad r_\alpha=\langle\chi^{(\alpha,\alpha)}\rangle,\\ \lambda r_\alpha=\langle\omega^{5(\alpha,\alpha)}\rangle,\quad \mu r_\alpha=\langle\rho_5^{\{\alpha,\alpha\}}\rangle,\\ \nu q=\langle\zeta_{53}\rangle,\quad \varkappa p=\langle\sigma_1^{[15]1}\rangle=\langle\sigma_2^{[25]1}\rangle=\langle\sigma_3^{[35]1}\rangle=-{}^1/_3\langle\sigma_4^{[45]1}\rangle.\end{gathered}\tag{44}$$

The quantities p, q, and r_α characterize the spontaneous breaking of the horizontal symmetry and are therefore large. The rather neutral possibility here is that $r_3\sim10^{15}$ GeV, i.e., of the same order as the scale of the breaking of the SU(5) symmetry, whereas q, r_2, and r_1 are subject to the conditions[5]

$$r_3\gg p\gtrsim r_2\gg q\gg r_1.\tag{45}$$

The remaining vacuum mean values λr_α, μr_α, νq, $\varkappa p$ are only of the order of hundreds of GeV, i.e., $\lambda,\mu,\nu,\varkappa\ll1$ [we emphasize that $\ll$ in (45) means smallness of the order of the ratio of the masses of the fermions of the different generations, say $p/r_3\sim m_\mu/m_\tau\sim\frac{1}{10}$, and does not mean in any way smallness of $\lambda,\mu,\nu,\varkappa\sim10^{-13}$].

Just as in the case of the simplified model considered in the preceding section, account must be taken of the mixing of G with Goldstone bosos corresponding to destroyed generators of the $SU(5)\times SU(3)_h$ group. Two such Goldstone states correspond to the generators λ_3 and λ_8 (or, more conveniently, $(\sqrt3/2)\lambda_8-\frac12\lambda_3$) of the group $SU(3)_h$, just as in the preceding section. The third state is the result of disturbance of the generator $T_3-\sin^2\theta_W Q$ of the group $SU(2)_L\times U(1)$ [of the subgroup SU(5)], with which the Z boson interacts. All three states can be easily expressed:

$$T=p\xi+2r_1\chi_1-2r_2\chi_2+2\lambda(r_1\omega_1-r_2\omega_2)+2\mu(r_1\rho_1-r_2\rho_2)-\varkappa p\sigma,$$
$$U=-q\eta+2r_2\chi_2-2r_3\chi_3+2\lambda(r_2\omega_2-r_3\omega_3)+2\mu(r_2\rho_2-r_3\rho_3)+\nu q\zeta,$$
$$z=-vH-\lambda\sum_{\alpha=1}^{3}r_\alpha\omega_\alpha+\mu\sum_{\alpha=1}^{3}r_\alpha\rho_\alpha+\nu q\zeta+\varkappa p\sigma. \qquad (46)$$

The sought Goldstone boson is

$$\alpha=N^{-1}(G-A\cdot T-B\cdot U-Cz), \qquad (47)$$

where the coefficients A, B, and C are determined from the condition that α be orthogonal to T, U, and Z. The states G, T, and U contain both large $(-\xi,\eta,\chi)$ and small terms, while z contains only small ones. An expression for α can be written, accurate to the ratio of the small vacuum mean values to the large ones, in the form

$$\alpha=N^{-1}(G-A_0T-B_0U-Cz), \qquad (48)$$

where A_0 and B_0 are calculated without allowance for the fields that have small vacuum mean values, i.e.,

$$A_0=\frac{(G_0\cdot T_0)(U_0\cdot U_0)-(T_0\cdot U_0)(G_0\cdot U_0)}{(T_0\cdot T_0)(U_0\cdot U_0)-(T_0\cdot U_0)^2},$$
$$B_0=\frac{(G_0\cdot U_0)(T_0\cdot T_0)-(T_0\cdot U_0)(G_0\cdot T_0)}{(T_0\cdot T_0)(U_0\cdot U_0)-(T_0\cdot U_0)^2}. \qquad (49)$$

Since the terms $\Delta A\cdot T_0$, $\Delta B\cdot U_0$ $(A=A_0+\Delta A, T=T_0+\Delta T,\ldots)$ make a small contribution to the normalization, while T_0 and U_0 do not interact with fermions.

The coeffiient C is determined from the condition $(\alpha\cdot z)=0$, which yields

$$C=\frac{1}{(z\cdot z)}((G-A_0\cdot T-B_0\cdot U)\cdot z)$$
$$=\frac{1}{(z\cdot z)}((\Delta G-A_0\Delta T-B_0\Delta U)\cdot z). \qquad (50)$$

We use now the hierarchy of the parameters (45). The cumbersome expressions for the scalar products become then much simpler, and direct calculation yields the values

$$A_0\approx 1, \quad B_0\approx {}^1/_2, \quad C\approx 0, \qquad (51)$$

the expression for α becoming extremely simple:

$$\alpha\approx N^{-1}[G-T-{}^1/_2U]$$
$$\approx N^{-1}[{}^3/_2q\eta-3r_1\chi_1-3\lambda r_1\omega_1-3\mu r_1\rho_1-{}^3/_2\nu q\zeta]$$
$$\approx N^{-1}({}^3/_2q\eta-{}^3/_2q\nu\zeta)=\eta-\nu\zeta. \qquad (52)$$

Separating in (40) the interaction of the fermions with ζ_{53}, we easily obtain the interaction of α with fermions. We neglect here the insignficant d-s mixing:

$$\mathscr{L}=i\sqrt{2}(h_\zeta\nu)\alpha[(\bar{s}d)-(\bar{d}s)+(\bar{\mu}e)-(\bar{e}\mu)]. \qquad (53)$$

As shown in Ref. 5, the quantity $h_\zeta\nu q$ $(q=\langle\eta\rangle)$ is connected with the masses of the fermions of the first two generations:

$$h_\zeta\nu q=(m_dm_s)^{1/2}=(m_\mu m_e)^{1/2}. \qquad (54)$$

Accordingly, we rewrite the expressions for $\mathscr{L}$ in the following form (redefining $s\to is$ and $\mu\to i\mu$):

$$\mathscr{L}=\frac{(2m_dm_s)^{1/2}}{\langle\eta\rangle}\alpha(\bar{s}d+\bar{d}s)+\frac{(2m_em_\mu)^{1/2}}{\langle\eta\rangle}\alpha(\bar{\mu}e+\bar{e}\mu). \qquad (55)$$

This notation reflects the effects of the renormalization of the Yukawa constants on going from the grand-unification mass to contemporary energies (i.e., according to Ref. 5, that the equality $m_dm_s=m_em_\mu$ takes place at energies $\sim 10^{15}$ GeV). Equation (55) is our final result. It is interesting that the nondiagonal $\bar{d}s$ and $\bar{e}\mu$ interaction of the Goldstone α (55) is the principal interaction of α with fermions. Diagonal interactions, just as an interaction with a τ lepton (b quark) turn out to be small by virtue of the conditions (45).

Equation (55) enables us to calculate the probabilities of the decays $\mu\to e+\alpha$ and $K^+\to\pi^++\alpha$; to calculate the amplitudes of the second process we can use the relations

$$\langle\pi^+|\bar{d}s|K^+\rangle=i(m_s-m_d)^{-1}\langle\pi^+|\partial_\mu(\bar{d}\gamma_\mu s)|K^+\rangle$$
$$=(p-p')_\mu[f_+(p+p')_\mu+f_-(p-p')_\mu](m_s-m_d)^{-1}$$
$$=f_+(m_K^2-m_\pi^2)(m_s-m_d)^{-1}\approx m_K^2/m_s,$$

where p and p' are the momenta of the K and π mesons, and $(p-p')^2=0$. We have

$$\Gamma(\mu\to e+\alpha)=\frac{1}{8\pi}\frac{m_em_\mu^2}{\langle\eta\rangle^2},$$
$$\Gamma(K^+\to\pi^++\alpha)\approx\frac{1}{8\pi}\frac{m_dm_K^3}{\langle\eta\rangle^2m_s}. \qquad (56)$$

This is the cause of the restriction on the vacuum mean value $\langle\eta\rangle$. A more stringent limitation is imposed by the absence of experimental decay $K^+\to\pi^+\alpha$ [$\Gamma(K^+\to\pi^+\alpha)/\Gamma(K^+\to all)<3.7\cdot10^{18}$ (Ref. 9)], from which we can obtain the limit

$$\langle\eta\rangle>1.1\cdot10^{10}\ \text{GeV}.$$

We note that in attempts to observe the $\mu\to e+\gamma$ decay an electron with momentum $m_\mu/2$ is always sought in correlation with the γ quantum; to observe the decay $\mu\to e=\alpha$ it is necessary simply to seek an electron with $p_e=m_\mu/2$.

It is difficult to state definitely which value of $\langle\eta\rangle=q$ is typical of the considered model. If we assume that $r_3\sim10^{15}$ GeV, it is possible that[5] $q=\langle\eta\rangle\sim[(m_dm_s)^{1/2}/m_b]\,r_3\sim10^{12}-10^{13}$ GeV. To observe the $K^+\to\pi^+\alpha$ decay it is then necessary to improve the experimental accuracy by several orders of magnitude. It is possible, however, that the characteristic scale of the vacuum mean value responsible for the breaking of the horizontal symmetry is still less than the grand-unification scale (strictly speaking, this would be desirable in the model of Ref. 5 from purely empirical considerations, to obtain the correct form of the mass matrix of the fermions). It is not excluded then that to observe $K^+\to\pi^+\alpha$ it is necessary only to improve somewhat the existing experimental accuracy. At any rate, as already mentioned in the introduction, the $K\to\pi\alpha$ and $\mu\to e\alpha$ decays can be observed at incomparably larger values of $\langle\eta\rangle$ than other effects connected with the existence of flavor-changing neutral currents.

In conclusion we note the following. Another type of a Goldstone boson with interactions nondiagonal in flavor could be majoron.[2] Although the parameters that characterize the majoron are highly indeterminate, it can be assumed that the width of $\mu \to e + \chi$ (the χ majoron) is smaller by many orders of magnitude than (56) even, e.g., at $\langle \eta \rangle \sim 10^{13}$ GeV. (If the constant of such a nondiagonal interaction is estimated in analogy with the constant of the diagonal interaction as being $\sim 10^{-20}$ (Ref. 2, the width of $\mu \to e + \alpha$ is less than the width of $\mu \to e + \alpha$ at $\langle \eta \rangle \sim 10^{13}$ GeV by ten orders of magnitude.) We note that in models with the majoron the non-diagonal transitions take place only in the lepton spectrum.

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