Nonlinear magnetic field generation in a collisionless plasma

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We give a theoretical study of the problem of magnetic field generation in a uniform collisionless plasma with high-frequency (Langmuir or electromagnetic) waves. We obtain a nonlinear dispersion equation for the magnetostatic oscillations of such a plasma, taking into account the effect of the anisotropy of the energy spectrum of the high-frequency waves. We study the generation of oscillations caused by this effect. We show that this generation may be due to two concrete mechanisms, one of which is a nonlinear wave-particle type process and is relativistic in nature while the other one is a wave-wave type process which occurs in the nonrelativistic approximation. Both mechanisms lead to the same order of magnitude for the growth rate and the characteristic wavelength of the perturbations, but the first one is less sensitive to an isotropization of the high frequency wave spectrum and is therefore more important.

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1. INTRODUCTION

Let there be a collisionless spatially uniform plasma without a magnetic field in which high-frequency (HF); electromagnetic or Langmuir) waves are excited. We are interested in the non-linear magnetic field generation in such a plasma, meaning by this the spontaneous excitation, due to the development of some instabilities, of magnetic field fluctuations caused by these waves.

Restricting ourselves to the uniform plasma approximation we leave out of consideration the effect of plasma non-uniformity and HF-field nonuniformity on the instabilities studied by us, and also generation mechanisms caused by the non-uniformity. Taking the first effect into account must lead to a lower bound on the characteristic dimensions of a plasma in which the instabilities studied by us can develop. Such a problem can be studied later. As to magnetic field generation mechanisms caused by plasma and HF-field inhomogeneities, they have been studied extensively and continue to be studied in connection with the observation of appreciable magnetic fields in experiments on the interaction of laser radiation with matter.[1–3] Afanas'ev *et al.*[4] have indicated the main directions of the corresponding theoretical studies.

The basic idea of the present paper consists in the assumption that the magnetic field can be generated thanks to the anisotropy of the energy spectrum of the HF field. (The simplest case of an HF field with an anisotropic energy spectrum is the field of a single monochromatic wave.) This assumption has its analogy with the linear magnetic field generation in a plasma with an anisotropic particle momentum distribution (see Refs. 5, 6 and literature quoted there).

Using this analogy and dimensionality considerations one can without calculations predict the characteristic scale of the spatial structure and the order of magnitude of the growth rate of the magnetic field generated in a plasma with anisotropic HF waves. We recall that in the case of an anisotropic plasma (when there are no HF waves) perturbations with a square of the wave number k of the order[5,6]

$$k^2 \approx (\omega_p/c)^2 \Delta T/T, \tag{1.1}$$

turn out to be unstable, where ω_p is the electron plasma frequency, c the light velocity, T the average plasma temperature, and ΔT the temperature anisotropy (we assume that $\Delta T/T \ll 1$). In the case of a plasma with anisotropic HF waves the part of $\Delta T/T$ is played by a quantity proportional to $m\tilde{v}^2/T$, where $\tilde{v}$ is the velocity of the oscillating particles in the HF field and m their mass. Instead of (1.1) we must thus have in this case

$$k^2 \approx (\tilde{v}\omega_p/cv_T)^2 \varphi, \tag{1.2}$$

where $v_T = (T/m)^{1/2}$ is the electron thermal velocity and φ some function of the dimensionless parameter $(v_T/c)^2$. The instability mechanism must also occur in a cold plasma ($v_T \to 0$) and thus we must have $\varphi \sim (v_T/c)^2$. In that case

$$k \approx \omega_p \tilde{v}/c^2, \tag{1.3}$$

which is confirmed by the calculations.

In the case of an anisotropic plasma the growth rate δ is connected with the wave number through the following estimate

$$\delta \approx k^3 c^2 v_T/\omega_p^2. \tag{1.4}$$

Using (1.3) and the indicated analogy we may assume that in the case of a plasma with HF waves and $\tilde{v} \lesssim v_T$

$$\delta \approx \omega_p v_T \tilde{v}^3/c^4. \tag{1.5}$$

This is also confirmed by calculations.

By virtue of what we have just said we call the instabilities studied here "wave anisotropy instabilities" (WAI).

The presence of the relativistic parameter $\varphi \sim (v_T/c)^2$ indicates that the WAI theory must, in general, be developed taking relativistic effects of the plasma into account. In the following considerations we shall take that fact into consideration.

Our analysis indicates the presence of two actual WAI mechanisms. One of them is relativistic in nature. The corresponding instability we call the "relativistic WAI." It is not connected with the occurrence of secondary HF waves (beats) characteristic for the so-called magneto-modulational instabilities (MMI).[7,8] (In that case, however, there occurs a modulation of the particle distribution function; for

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details see below.) The second mechanism observed by us for the WAI excitation is connected with the appearance of secondary HF waves. For that reason we call it a modulated mechanism. It can be studied neglecting relativistic effects. Our analysis of the modulational magnetic field generation differs from Refs. 7, 8 in that we consider a plasma with electromagnetic waves whereas the authors of Refs. 7, 8 consider a plasma with Langmuir waves. Modulational effects in the magnetic field generation problem have also been discussed in Ref. 9, but there this effect was used not as a generation mechanism but as a mechanism to amplify a field generated through other causes.

2. BASIC EQUATIONS

2.1 *General relations.* We write the electromagnetic field in the form

$$(\mathbf{E},\ \mathbf{B})=(\mathbf{E}^h,\ \mathbf{B}^h)+(\mathbf{E}^s,\ \mathbf{B}^s)+(\mathbf{E}^{hs},\ \mathbf{B}^{hs}). \qquad (2.1)$$

Here $\mathbf{E}^h$, $\mathbf{B}^h$ are the electric and magnetic fields of the primary (high-frequency) waves; $\mathbf{E}^s$, $\mathbf{B}^s$ the fields of the low-frequency waves generated by the primary waves; $\mathbf{E}^{hs}$, $\mathbf{B}^{hs}$ the fields of the secondary waves, the so-called virtual waves of beats produced by the interaction of the primary high-frequency waves with the low-frequency ones. Similarly we write the distribution function of each kind of particle f:

$$f=f_0+f^h+f^s+f^{hs}+\bar{f}^{hh}. \qquad (2.2)$$

Here f_0 is the distribution function when there is no electromagnetic field present (equilibrium distribution function), while f^h, f^s, f^{hs} have a similar meaning as the fields with the same upper indexes, $\bar{f}^{hh}$ is a correction to the equilibrium distribution function caused by the primary waves, averaged over space and time. We get equations for f^h, f^s, f^{hs}, starting from the Vlasov equation

$$\hat{L}f+e\mathbf{F}\partial f/\partial\mathbf{p}=0. \qquad (2.3)$$

Here $\hat{L}=\partial/\partial t+\mathbf{v}\nabla$; $\mathbf{v}$, $\mathbf{p}$ are the particle velocity and momentum, e the particle charge,

$$\mathbf{F}=\mathbf{E}+[\mathbf{v}\times\mathbf{B}]/c, \qquad (2.4)$$

c the light velocity. Apart from $\mathbf{F}$ we use in what follows also the notation $\mathbf{F}^h$, $\mathbf{F}^s$, $\mathbf{F}^{hs}$, the meaning of which is clear from (2.1), (2.4).

We take the equation for f^h in the linear approximation in the electromagnetic field so that

$$\hat{L}f^h+e\mathbf{F}^h\partial f_0/\partial\mathbf{p}=0. \qquad (2.5)$$

To describe f^{hs} we use the equation

$$\hat{L}f^{hs}+e\mathbf{F}^{hs}\partial f_0/\partial\mathbf{p}+e\mathbf{F}^s\partial f^h/\partial\mathbf{p}=0. \qquad (2.6)$$

Here we neglected the term with $\mathbf{F}^h f^s$ which is unimportant for problems of modulational instabilities (cf. Ref. 10). Using (2.3) we find the following equation for f^s:

$$\hat{L}f^s+e\mathbf{F}^s\partial(f_0+\bar{f}^{hh})/\partial\mathbf{p}+e\partial\langle\mathbf{F}^{hs}f^h+\mathbf{F}^h f^{hs}\rangle/\partial\mathbf{p}=0. \qquad (2.7)$$

The angle brackets denote the low-frequency part of the corresponding quantity. Finally

$$\bar{f}^{hh}=\frac{1}{2}\langle\delta p_i\delta p_j\rangle\frac{\partial^2 f_0}{\partial p_i\partial p_j}, \qquad (2.8)$$

where $\delta\mathbf{p}$ is the correction to the particle momentum caused by the field of the primary waves. The quantity $\delta\mathbf{p}$ is determined by the equation of motion of the particle

$$\hat{L}\delta\mathbf{p}=e\mathbf{F}^h. \qquad (2.9)$$

We add to the equations given here the Maxwell equations in their standard form and the expressions for the electric current density

$$\mathbf{j}=\sum e\int\mathbf{v}f d\mathbf{p}. \qquad (2.10)$$

The summation is over the different kinds of particles. For the sake of simplicity we dropped in (2.10) the superscripts h, s, hs for $\mathbf{j}$ and f.

2.2. *Description of the primary waves.* We write the electromagnetic field of the primary waves as a set of Fourier harmonics:

$$\mathbf{F}^h=\int\mathbf{F}_{\mathbf{k}\omega}{}^h\exp(-i\omega t+i\mathbf{k}\mathbf{r})\,d\mathbf{k}\,d\omega. \qquad (2.11)$$

We assume that the equilibrium distribution function of each kind of particle f_0 is isotropic in momentum space: $f_0=f_0(p)$. In that case we can split the primary waves into two kinds: longitudinal (the so-called electrostatic or Langmuir) waves and transverse (electromagnetic) waves. In the case of longitudinal waves

$$\mathbf{F}^h=\mathbf{E}^h\equiv\mathbf{E}^l,\quad \mathbf{E}_{\mathbf{k}\omega}{}^l\|\mathbf{k},$$

and in the case of transverse waves

$$\mathbf{F}^h=\mathbf{E}^t+[\mathbf{v}\times\mathbf{B}]/c,\ \mathbf{E}_{\mathbf{k}\omega}{}^t\perp\mathbf{k},\ \mathbf{B}_{\mathbf{k}\omega}{}^t=c[\mathbf{k}\times\mathbf{E}_{\mathbf{k}\omega}{}^t]/\omega.$$

For both types of waves it follows from (2.5) that

$$f_{\mathbf{k}\omega}{}^h=-\frac{ie}{\omega-\mathbf{k}\mathbf{v}}\mathbf{v}\mathbf{E}_{\mathbf{k}\omega}{}^h\frac{\partial f_0}{\partial\mathscr{E}}. \qquad (2.12)$$

Here $\mathscr{E}=mc^2\gamma$ is the relativistic particle energy, m its rest mass, $\gamma=(1+w^2)^{1/2}$ the Lorentz factor, and $w=p/mc$ the dimensionless particle momentum. Substituting (2.12) into (2.10) and using the Maxwell equations we get the well known dispersion equations of the linear approximation for primary longitudinal and transverse waves:

$$\varepsilon^l(\mathbf{k},\ \omega)=0, \qquad (2.13)$$

$$\varepsilon^t(\mathbf{k},\ \omega)-(ck/\omega)^2=0. \qquad (2.14)$$

Here ε^l, ε^t are the longitudinal and transverse permittivities of the linear approximation given by the equations

$$\varepsilon^\nu(\mathbf{k},\omega)=1+\sum\frac{4\pi e^2}{\omega}\int\frac{(\mathbf{e}^\nu\mathbf{v})^2}{\omega-\mathbf{k}\mathbf{v}}\frac{\partial f_0}{\partial\mathscr{E}}d\mathbf{p}, \qquad (2.15)$$

where $\nu=l,\ t$, $\mathbf{e}^\nu=\mathbf{E}^\nu_{\mathbf{k}\omega}/E^\nu_{\mathbf{k}\omega}$ is the unit polarization vector of the wave of kind ν and $E^\nu_{\mathbf{k}\omega}\equiv|\mathbf{E}^\nu_{\mathbf{k}\omega}|$.

We assume the function f_0 to be Maxwellian:

$$f_0=\frac{n}{4\pi(mc)^3}\frac{\alpha}{K_2(\alpha)}\exp(-\alpha\gamma). \qquad (2.16)$$

Here $\alpha=mc^2/T$ is a parameter characterizing the degree of relativity for the corresponding plasma component, T is the temperature of that component, and K_2 a Macdonald function. A nonrelativistic plasma corresponds to $\alpha\gg1$ and an ultra-relativistic one to $\alpha\ll1$.

We assume that the primary waves are long: $\omega \gg kv$. In that approximation, and with f_0 of the form (2.16), Eq. (2.15) means[11]

$$\varepsilon^v(\mathbf{k}, \omega) = 1 - \sum \frac{\omega_p^2}{\omega^2} [g_1^v(\alpha) + (c^2k^2/\omega^2) g_2^v(\alpha)], \quad (2.17)$$

where $\omega_p^2 = 4\pi e^2 n/m$ is the square of the nonrelativistic plasma frequency of the corresponding kind of particle,

$$g_1^l(\alpha) = g_1^t(\alpha) \equiv g_1(\alpha) = \frac{\alpha^2}{K_2(\alpha)} \int\limits_\alpha^\infty \frac{K_2(x)}{x^2} dx, \quad (2.18)$$

$$g_2^l(\alpha) = 3g_2^t(\alpha) = \frac{3\alpha^2}{2K_2(\alpha)} \int\limits_\alpha^\infty \frac{K_1(x)}{x^5} (3\alpha^2 - x^2) dx. \quad (2.19)$$

It follows from (2.13), (2.14), (2.17) that the eigenfrequencies of the primary waves $\omega = \omega_{\mathbf{k}}^v$ are given by the relations[11]

$$\omega_{\mathbf{k}}^l = \omega_0 + {}^1/_2 (g_2^l/g_1)(c^2k^2/\omega_0), \quad (2.20)$$

$$\omega_{\mathbf{k}}^t = \omega_0 + {}^1/_2 (1 + g_2^t/g_1)(c^2k^2/\omega_0), \quad (2.21)$$

where

$$\omega_0 = \omega_p g_1^{1/2}. \quad (2.22)$$

We assume here that in the case of an electron-positron plasma ω_p is calculated using the total particle density while in the case of an electron-ion plasma we use the electron density. We note also that, due to our assumption that we are dealing with long-wavelength waves, terms with k^2 in (2.20), (2.21) are corrections compared to ω_0, so that approximately $\omega_k^l \approx \omega_k^t \approx \omega_0$.

2.3. *Linear theory of magnetostatic oscillations*. The low-frequency waves studied by us are magnetostatic oscillations. Like the high-frequency (normal) electromagnetic waves, magnetostatic oscillations are described by the dispersion equation (2.14). The formal difference between these two kinds of waves consists in that in the case of the normal electromagnetic waves $\omega > kv$ while in the case of magnetostatic oscillations $\omega \ll kv$.

We take the electromagnetic field of the magnetostatic oscillations in the form of a single plane wave, i.e., in the form (2.11) without an integral on the right-hand side of the equation. The distribution function $f_{\mathbf{k}\omega}^s$ has a form analogous to (2.12) (with the substitution $\mathbf{E}_{\mathbf{k}\omega}^h \to \mathbf{E}_{\mathbf{k}\omega}^s$). The transverse permittivity of the plasma in the linear approximation corresponding to the magnetostatic oscillations and denoted by ε_0^s, has the form[11]

$$\varepsilon_0^s(\mathbf{k}, \omega) = i \sum \frac{\omega_p^2}{c^2 |k| \omega} g(\alpha), \quad (2.23)$$

where

$$g(\alpha) = \frac{\pi}{2} \frac{(1+\alpha)\exp(-\alpha)}{\alpha K_2(\alpha)}. \quad (2.24)$$

Equation (2.23) follows from (2.15) when $\omega \ll kv$. The subscript zero of ε^s indicates the linear approximation.

We get from (2.14), (2.23) the following formula in the linear approximation for the frequency of the eigenoscillations of the magnetostatic waves:

$$\omega_0^s = -i(|k|^3 c^3/\omega_p^2) g(\alpha). \quad (2.25)$$

As in (2.20) to (2.22) ω_p is evaluated here using either the total particle density or the electron density depending on the composition of the plasma.

The wave number k in (2.25) is assumed to be sufficiently small, $kc/\omega_p \ll 1$ so that $\omega/kc \ll 1$. As $\omega/|k|c$ is small the magnetic field of waves such as (2.25)

$$\mathbf{B}_{\mathbf{k}\omega}^s = c[\mathbf{k}\mathbf{E}_{\mathbf{k}\omega}^s]/\omega$$

is large compared to the electric field, $|\mathbf{B}_{\mathbf{k}\omega}^s| \gg |\mathbf{E}_{\mathbf{k}\omega}^s|$. Moreover, it is clear from (2.25) that the real part of the frequency of the waves considered vanishes, $\mathrm{Re}\,\omega_0^s = 0$. This justifies the name "magnetostatic oscillations." It is also clear that $\mathrm{Im}\,\omega_0^s < 0$, i.e., in the linear approximation the magnetostatic oscillations are aperiodically damped plasma perturbations. The following analysis reveals some nonlinear excitation mechanisms whereby magnetostatic oscillations become growing waves.

3. DERIVATION OF THE NON-LINEAR DISPERSION EQUATION FOR MAGNETOSTATIC OSCILLATIONS

3.1. *General consequences of the set of nonlinear equations of Sec.* 2. We write the solution of Eq. (2.3) in the form [cf. (2.12)]

$$f^h = -e\hat{L}^{-1}(\mathbf{F}^h \partial f_0/\partial \mathbf{p}), \quad (3.1)$$

where $\hat{L}^{-1}$ is the operator which is the inverse of $\hat{L}$. Similarly, using (3.1) we get from (2.6)

$$f^{hs} = -e\hat{L}^{-1}\left(\mathbf{F}^{hs}\frac{\partial f_0}{\partial \mathbf{p}}\right) + e^2\hat{L}^{-1}\left[\left(\mathbf{F}^s\frac{\partial}{\partial \mathbf{p}}\right)\hat{L}^{-1}\left(\mathbf{F}^h\frac{\partial f_0}{\partial \mathbf{p}}\right)\right]. \quad (3.2)$$

Using (2.8), (2.9) we find

$$\bar{f}^{hh} = -\frac{1}{2}\langle F_i^h \hat{L}^{-2} F_j^h \rangle \frac{\partial^2 f_0}{\partial p_i \partial p_j}. \quad (3.3)$$

Substituting (3.1) to (3.3) into (2.7) and acting on the result with the operator $\hat{L}^{-1}$ we find

$$f^s = f_0^s + f_1^s + f_2^s, \quad (3.4)$$

where

$$f_0^s = -e\hat{L}^{-1}(\mathbf{F}^s \partial f_0/\partial \mathbf{p}), \quad (3.5)$$

$$f_1^s = -e^3\hat{L}^{-1}\left\langle\left(\mathbf{F}^h\frac{\partial}{\partial \mathbf{p}}\right)\hat{L}^{-1}\left[\left(\mathbf{F}^s\frac{\partial}{\partial \mathbf{p}}\right)\hat{L}^{-1}\left(\mathbf{F}^h\frac{\partial f_0}{\partial \mathbf{p}}\right)\right]\right\rangle + \frac{1}{2}e^3\hat{L}^{-1}\left(\mathbf{F}^s\frac{\partial}{\partial \mathbf{p}}\right)\langle F_i^h\hat{L}^{-2}F_j^h\rangle\frac{\partial^2 f_0}{\partial p_i \partial p_j}, \quad (3.6)$$

$$f_2^s = e^2\hat{L}^{-1}\left\langle\left(\mathbf{F}^{hs}\frac{\partial}{\partial \mathbf{p}}\right)\hat{L}^{-1}\left(\mathbf{F}^h\frac{\partial f_0}{\partial \mathbf{p}}\right) + \left(\mathbf{F}^h\frac{\partial}{\partial \mathbf{p}}\right)\hat{L}^{-1}\left(\mathbf{F}^{hs}\frac{\partial f_0}{\partial \mathbf{p}}\right)\right\rangle. \quad (3.7)$$

It is clear that the term f_0^s corresponds to the linear approximation of the theory of magnetostatic oscillations [cf. (3.5) with (2.12)]. The term f_1^s describes the anisotropic nonlinear effect studied in Ref. 12. Finally, it is clear from (3.7) that the term f_2^s is caused by the virtual wave effect. This effect was studied in Ref. 7.

Using (2.1), (3.2) to evaluate the electric current $\mathbf{j}^{hs}$ and substituting this current into the Maxwell equations we can express $\mathbf{E}^{hs}$ in terms of $\mathbf{E}^h$ and $\mathbf{E}^s$ so that

$$\mathbf{E}^{hs}=\mathbf{E}^{hs}(\mathbf{E}^h,\ \mathbf{E}^s). \tag{3.8}$$

From (3.8) it is clear that all terms on the right-hand side of (3.4) are proportional to $\mathbf{E}^s$. We then conclude, taking (3.4), (2.10), and the Maxwell equations into account, that the non-linear dispersion equation for the magnetostatic oscillations must have the form

$$|\varepsilon_{ij}^{(0)}+\varepsilon_{ij}^{(1)}+\varepsilon_{ij}^{(2)}-(k^2\delta_{ij}-k_ik_j/k^2)c^2/\omega^2|=0. \tag{3.9}$$

Here $\varepsilon_{ij}^{(0)}$ is the linear permittivity tensor which is related to (2.23) through the equation

$$\varepsilon_{ij}^{(0)}=(\delta_{ij}-k_ik_j/k^2)\varepsilon_0{}^s+(k_ik_j/k^2)\varepsilon_0{}^l, \tag{3.10}$$

$\varepsilon_{ij}^{(1)}$, $\varepsilon_{ij}^{(2)}$ are the parts of the nonlinear permittivity tensor produced by the non-linear anisotropy and virtual waves effects. We have added in (3.10) also a term with the linear longitudinal permittivity ε_0^l, the form of which is unimportant for what follows.

Using (2.10), (3.2) and the Maxwell equations we can check that in the case of an electron-positron plasma with identical electron and positron distribution functions there is no virtual wave effect:

$$\mathbf{E}^{hs}=0. \tag{3.11}$$

This was pointed out in Ref. 12. In that case we have the dispersion Eq. (3.9) with $\varepsilon_{ij}^{(2)}=0$.

3.2. *The evaluation of* $\varepsilon_{ij}^{(1)}$. We simplify the expression for f_1^s by using the fact that for waves such as (2.20), (2.21) we have the approximate relation

$$\hat{L}^{-2}\mathbf{F}^h\approx-\mathbf{F}^h/\omega_0{}^2. \tag{3.12}$$

Moreover, using the fact that the magnetostatic oscillations are low-frequency waves we put approximately

$$\mathbf{F}^s=[\mathbf{vB}^s]/c. \tag{3.13}$$

In that case (3.6) reduces to the form

$$f_1{}^s=-\hat{L}^{-1}\Delta, \tag{3.14}$$

where

$$\Delta=\frac{e^3}{m^2\omega_0{}^2c^2\gamma^2}\langle E_i^hE_j^h\rangle v_i\left[\frac{\mathbf{v}}{c}\,\mathbf{B}^s\right]_j\frac{\partial f_0}{\partial\mathscr{E}}. \tag{3.15}$$

When Fourier transforming (3.14) the operator $\hat{L}^{-1}$ is replaced by $i(\omega-\mathbf{k}\cdot\mathbf{v})^{-1}$ and Δ by $\Delta_{\mathbf{k}\omega}$ where $\Delta_{\mathbf{k}\omega}$ differs from Δ through the formal substitution

$$\mathbf{B}^s\to\mathbf{B}_{\mathbf{k}\omega}{}^s=c[\mathbf{kE}_{\mathbf{k}\omega}{}^s]/\omega.$$

Therefore

$$f_{1,\mathbf{k}\omega}^s=-\frac{ie^3\langle(\mathbf{E}^h)^2\rangle}{m^2\omega_0{}^2\omega(\omega-\mathbf{kv})c^2\gamma^2}(\mathbf{e}^h\mathbf{v})[\mathbf{e}^h\mathbf{v}][\mathbf{kE}_{\mathbf{k}\omega}{}^s]\partial f_0/\partial\mathscr{E}. \tag{3.16}$$

Here $\mathbf{e}^h=\mathbf{E}^h/E^h$ is the unit vector along $\mathbf{E}^h$ [cf. (2.15)]. In the case of primary waves with different directions of $\mathbf{e}^h$ we must in the right-hand side of (3.16) also sum (or integrate) over the appropriate directions.

When evaluating the electric current $\mathbf{j}_{1,\mathbf{k}\omega}^s$, using (3.16), it is convenient to write the velocity $\mathbf{v}$ as a sum of longitudinal and transverse parts:

$$\mathbf{v}=\mathbf{v}^l+\mathbf{v}^t,\quad \mathbf{v}^l=\mathbf{k}(\mathbf{kv})/k^2,\quad \mathbf{v}^t=[\mathbf{k}[\mathbf{vk}]]/k^2. \tag{3.17}$$

Substituting (3.17) into (2.10) we evaluate the integral over the momenta assuming that $\omega<kv$ (see Sec. 2.3). We then consider only the principle part of the integral. The current $\mathbf{j}_1^s$ found in this way we write in the form

$$j_{1,i}^s=(\omega/4\pi i)\varepsilon_{ij}^{(1)}E_j^s. \tag{3.18}$$

We then find

$$\varepsilon_{ij}^{(1)}=a\lambda_{ij}/\omega^2, \tag{3.19}$$

where

$$a=w\omega_p{}^4G_1(\alpha)/\omega_0{}^2,\quad w=\langle(E^h)^2\rangle/4\pi nT,$$
$$G_1(\alpha)=\frac{\alpha}{2K_2(\alpha)}\int_\alpha^\infty\frac{K_2(x)}{x^2}(x-\alpha)^2dx, \tag{3.20}$$

$$\lambda_{ij}=(\delta_{ij}-k_ik_j/k^2)(\mathbf{e}^k\mathbf{e}^h)^2-[\mathbf{e}^k[\mathbf{e}^k\mathbf{e}^h]]_i[\mathbf{e}^k[\mathbf{e}^k\mathbf{e}^h]]_j. \tag{3.21}$$

Here w is the dimensionless energy density of the high-frequency waves, $\mathbf{e}^k=\mathbf{k}/|k|$. The particle density and plasma frequency are taken here in the sense indicated in Sec. 2.3.

We show that $\varepsilon_{ij}^{(1)}$ is non-zero only when the energy spectrum of the primary HF waves is anisotropic. Indeed, in the opposite case, i.e., when the spectrum is isotropic,

$$\langle\langle E_i^hE_j^h\rangle\rangle=(\mathbf{E}^h)^2\delta_{ij}/3. \tag{3.22}$$

The double brackets indicate here an average over time and over the phases of the waves. Substituting (3.22) into (3.15), (3.16) we verify that $f_{1,\mathbf{k}\omega}=0$, and hence that $\varepsilon_{ij}^{(1)}=0$.

3.3. *Evaluation of* $\varepsilon_{ij}^{(2)}$. When evaluating $\varepsilon_{ij}^{(2)}$ we shall as primary waves consider a single monochromatic wave with wave vector $\mathbf{q}$ and frequency $\omega_\mathbf{q}^h$, i.e., put

$$\mathbf{E}^h=\mathbf{E}_+{}^h\exp(-i\omega_\mathbf{q}{}^ht+i\mathbf{qr})+\mathbf{E}_-{}^h\exp(i\omega_\mathbf{q}{}^ht-i\mathbf{qr}), \tag{3.23}$$

where $\mathbf{E}_-^h=\mathbf{E}_+^{h*}$, the asterisk being the symbol of taking the complex conjugate. In terms of Fourier harmonics such a choice of field means [see (2.11)]

$$\mathbf{E}_{\mathbf{k}\omega}{}^h=\mathbf{E}_+{}^h\delta(\mathbf{k}-\mathbf{q})\delta(\omega-\omega_\mathbf{q}{}^h)+\mathbf{E}_-{}^h\delta(\mathbf{k}+\mathbf{q})\delta(\omega+\omega_\mathbf{q}{}^h). \tag{3.24}$$

The results obtained for the case of a single monochromatic wave can be generalized to the case of many waves by summation over the wavenumbers q. We must then use the fact that $\mathbf{E}_\pm^h=\mathbf{E}_\pm^h(\mathbf{q})$.

In accordance with (3.23) we write the electric field $\mathbf{E}^{hs}$ and the electric current $\mathbf{j}^{hs}$ of the virtual waves in the form

$$(\mathbf{E}^{hs},\mathbf{j}^{hs})=\sum_{+,-}(\mathbf{E}_{k\pm q}^{hs},\mathbf{j}_{k\pm q}^{hs})\exp[-i(\omega\pm\omega_\mathbf{q}{}^h)t+i(\mathbf{k}\pm\mathbf{q})\mathbf{r}]. \tag{3.25}$$

We write out explicitly Eq. (3.7) for the function f_2^s and afterwards evaluate the current $\mathbf{j}_2^s$ connected with it. As a result we get

$$\mathbf{j}_2^s=\frac{e\omega_p^2G_2(\alpha)}{4\pi m\omega_.^3}\{[\mathbf{k}\times[\mathbf{E}_{k-q}^{hs}\times\mathbf{E}_+^h]]-[\mathbf{k}\times[\mathbf{E}_{k+q}^{hs}\times\mathbf{E}_-^h]]\}. \quad (3.26)$$

The index $(k\pm q)$ is understood to be four-dimensional, $k\pm q=(\mathbf{k}\pm\mathbf{q},\,\omega\pm\omega_q^h)$;

$$G_2(\alpha)=\frac{\alpha^2}{K_2(\alpha)}\int_\alpha^\infty\frac{K_2(x)}{x^2}(x-\alpha)\,dx. \quad (3.27)$$

Using (3.2) and (2.10) we evaluate the current $\mathbf{j}_{k\pm q}^{hs}$, writing it in the form

$$\mathbf{j}_{k\pm q}^{hs}=\mathbf{j}^L+\mathbf{j}^{NL}, \quad (3.28)$$

where $\mathbf{j}^L$ is the linear part of $\mathbf{j}^{hs}$, induced by the field $\mathbf{E}^{hs}$, while $\mathbf{j}^{NL}$ is the non-linear part produced by the fields $\mathbf{E}^h$, $\mathbf{E}^s$. The expression for $\mathbf{j}^L$ has the standard form

$$j_i^L=\frac{\omega\pm\omega_q^h}{4\pi i}\{[\varepsilon^l(\varkappa)-1]\varkappa_i\varkappa_j/\varkappa^2+[\varepsilon^t(\varkappa)-1]\times(\delta_{ij}-\varkappa_i\varkappa_j/\varkappa^2)\}E_j^{hs}, \quad (3.29)$$

where ε^l and ε^t are given by Eqs. (2.17). We use here the notation $\varkappa=k\pm q$. The expression for $\mathbf{j}^{NL}$ is given by the equation

$$\mathbf{j}^{NL}=-\frac{e\omega_p^2G_2(\alpha)}{4\pi m\omega\omega_0^2}[\mathbf{E}_\pm^h\times[\mathbf{k}\times\mathbf{E}_k^s]]. \quad (3.30)$$

Here the index $k\equiv(\mathbf{k},\omega)$.

We write the field and the current of the virtual waves as a sum of a longitudinal and a transverse part:

$$(\mathbf{E}_{k\pm q}^{hs},\mathbf{j}_{k\pm q}^{hs})=(\mathbf{E}_{k\pm q}^{l},\mathbf{j}_{k\pm q}^{l})+(\mathbf{E}_{k\pm q}^{t},\mathbf{j}_{k\pm q}^{t}), \quad (3.31)$$

where the quantities with the superscripts l and t are understood in the sense (3.17) with the substitution $\mathbf{k}\to\mathbf{k}\pm\mathbf{q}$. Using (3.27) to (3.29) we then find from the Maxwell equations

$$\mathbf{E}_\varkappa^l=\pm\frac{ie\omega_p^2G_2}{m\omega\omega_0^3D^l(\varkappa)}\boldsymbol{\varkappa}([\boldsymbol{\varkappa}\times\mathbf{E}_\pm^h][\mathbf{k}\times\mathbf{E}_k^s]), \quad (3.32)$$

$$\mathbf{E}_\varkappa^t=\pm\frac{ie\omega_p^2G_2}{m\omega\omega_0^3D^t(\varkappa)}\{[\boldsymbol{\varkappa}\times[\mathbf{k}\times\mathbf{E}_k^s]](\boldsymbol{\varkappa}\mathbf{E}_\pm^h)-[\boldsymbol{\varkappa}\times\mathbf{E}_\pm^h](\boldsymbol{\varkappa}[\mathbf{k}\times\mathbf{E}_k^s])\}, \quad (3.33)$$

where

$$D^l(\varkappa)=\varkappa^2\varepsilon^l(\varkappa),\ D^t(\varkappa)=\varkappa^2[\varepsilon^t(\varkappa)-c^2\varkappa^2/\omega_\varkappa^2],\quad \omega_\varkappa=\omega\pm\omega_q^h. \quad (3.34)$$

Substituting (3.31) to (3.33) into (3.26) we evaluate the current $\mathbf{j}_2^s$. We write the result of our calculations in the form

$$\mathbf{j}_2^s=\mathbf{j}_2^{s,l}+\mathbf{j}_2^{s,t}, \quad (3.35)$$

where $\mathbf{j}_2^{s,l}$, $\mathbf{j}_2^{s,t}$ are the contributions to $\mathbf{j}_2^s$ from $\mathbf{E}_\varkappa^l$, $\mathbf{E}_\varkappa^t$, respectively. Similarly we write

$$\varepsilon_{ij}^{(2)}=\varepsilon_{ij}^{(2)l}+\varepsilon_{ij}^{(2)t}. \quad (3.36)$$

We then get for $\varepsilon_{ij}^{(2)\nu}$

$$\varepsilon_{ij}^{(2)\nu}=-\frac{e^2\omega_p^4G_2^2E_+^hE_-^h}{m^2\omega^2\omega_0^6}\mu_{ij}^\nu, \quad (3.37)$$

where

$$\mu_{ij}^l=b_i^-b_j^+/D_-^l+b_i^+b_j^-/D_+^l, \quad (3.38)$$

$$\mu_{ij}^t=[\mathbf{kq}]_i[\mathbf{kq}]_jX_0+([\mathbf{kq}]_i[\mathbf{ke}^h]_j+[\mathbf{kq}]_j[\mathbf{ke}^h]_i)X_1+(\delta_{ij}k^2-k_ik_j)X_2. \quad (3.39)$$

We have used here the notation:

$$\mathbf{b}^\pm=[\mathbf{k}[\mathbf{k}\pm\mathbf{q},\ \mathbf{e}^h]], \quad (3.40)$$

$$X_n=\frac{(-1)^n(\mathbf{k}-\mathbf{q},\mathbf{e}^h)^n}{D_-^t}+\frac{(\mathbf{k}+\mathbf{q},\mathbf{e}^h)^n}{D_+^t},\qquad n=0,1,2, \quad (3.41)$$

$$D_\pm^l=D^l(k\pm q),\ D_\pm^t=D^t(k\pm q). \quad (3.42)$$

Equations (3.37) take into account the contribution to $\varepsilon_{ij}^{(2)}$ from a single Langmuir wave. In the case of a large number of primary waves we must sum the contributions from all these waves in the right-hand side of (3.37).

3.4. *Canonical form of the nonlinear dispersion equation.* We find from (3.19) to (3.21), (3.26) to (3.41)

$$k_i\varepsilon_{ij}^{(1)}=k_i\varepsilon_{ij}^{(2)}=0. \quad (3.43)$$

From this it follows, in particular, that there is no non-linear contribution from the low-frequency perturbations to the longitudinal permittivity:

$$k_i\varepsilon_{ij}^{(1)}k_j=k_i\varepsilon_{ij}^{(2)}k_j=0. \quad (3.44)$$

We assume that the wave vector $\mathbf{k}$ lies in the x,z-plane, i.e., $\mathbf{k}=(k_x,0,k_z)$. Using (3.43) and the symmetry of the tensors $\varepsilon_{ij}^{(1)}$, $\varepsilon_{ij}^{(2)}$ in their indexes we conclude that these tensors can be characterized by three independent quantities ε_{yy}^γ, $\varepsilon_{\perp\perp}^\gamma$, $\varepsilon_{\perp y}^\gamma=\varepsilon_{y\perp}^\gamma$ $(\gamma=1,2)$ where

$$\varepsilon_{\perp\perp}^\gamma=\varepsilon_{xx}^\gamma k^2/k_z^2=-\varepsilon_{xz}^\gamma k^2/k_xk_z=\varepsilon_{zz}^\gamma k^2/k_x^2,$$
$$\varepsilon_{\perp y}^\gamma=\varepsilon_{yz}^\gamma k/k_x=-\varepsilon_{yx}^\gamma k/k_z. \quad (3.45)$$

We then get from (3.9) the dispersion equation

$$\begin{vmatrix}\varepsilon_{yy}-c^2k^2/\omega^2, & \varepsilon_{y\perp}\\ \varepsilon_{\perp y}, & \varepsilon_{\perp\perp}-c^2k^2/\omega^2\end{vmatrix}=0, \quad (3.46)$$

where

$$\varepsilon_{\alpha\beta}=\delta_{\alpha\beta}\varepsilon_0^s+\varepsilon_{\alpha\beta}^{(1)}+\varepsilon_{\alpha\beta}^{(2)l}+\varepsilon_{\alpha\beta}^{(2)t},\qquad(\alpha,\beta)=(\perp,y). \quad (3.47)$$

The quantities $\varepsilon_{\alpha\beta}^{(1)}$, $\varepsilon_{\alpha\beta}^{(2)\nu}$ are given by Eqs. (3.19), (3.37) with the substitution $(i,j)\to(\alpha,\beta)$, and the expressions for the coefficients $\lambda_{\alpha\beta}$, $\mu_{\alpha\beta}^\nu$ by Eqs. (3.21), (3.38), (3.39) with the change in notation following from (3.45).

4. RELATIVISTIC INSTABILITY CONNECTED WITH WAVE ANISOTROPY

In the present section we consider WAI caused by the non-linear dielectric permittivity $\varepsilon_{\alpha\beta}^{(1)}$. Such a kind of instability is (within the framework of the assumptions made) the only possible mechanism for magnetic field generation in the case of an electron-positron plasma as, according to what we have said above, in such a plasma $\varepsilon_{\alpha\beta}^{(2)}=0$.

It follows from Sec. 3 (for details *vide infra*) that $\varepsilon_{\alpha\beta}^{(1)} \neq 0$ only when we take relativistic effects in the plasma into account. In this connection we call the instability discussed in the present section the relativistic WAI.

For the sake of simplicity we restrict ourselves in the main to the case where the electric field of the HF waves has the same direction as the wave vector of the perturbations $\mathbf{E}^h \| \mathbf{k}$. We take the vectors $\mathbf{E}^h$ and $\mathbf{k}$ to be directed along the z-axis. At the end of the section we discuss briefly perturbations with an arbitrary angle between $\mathbf{E}^h$ and $\mathbf{k}$.

4.1 Dispersion equation

We obtain under the stated assumptions from (3.46) two identical dispersion equations for perturbations with different polarizations:

$$\varepsilon_0^{(s)}+\varepsilon^{(1)}-c^2k^2/\omega^2=0, \tag{4.1}$$

where

$$\varepsilon^{(1)}=w(\omega_p/\omega)^2T/mc^2. \tag{4.2}$$

We assume here that the plasma is non-relativistic, $T \ll mc^2$.

It follows from (4.1), (4.2) that

$$\omega=ic^3k(k_*^2-k^2)/\omega_p^2, \tag{4.3}$$

where

$$k_*^2=w(\omega_p/c)^2T/mc^2. \tag{4.4}$$

It is clear that when

$$k<k_* \tag{4.5}$$

there occurs an aperiodic instability, $\delta \equiv \mathrm{Im}\,\omega > 0$, $\mathrm{Re}\,\omega = 0$; this is the relativistic WAI. The wave vector k_* plays the role of the boundary of the instability. The growth rate δ reaches a maximum when $k_0 = k_*/3^{1/2}$. In that case

$$\delta_{max}=2\pi^{-1/2}c^3k_*^3/3^{3/2}\omega_p^2. \tag{4.6}$$

From (4.4), (4.6) follow the estimates (1.3), (1.5) with $\tilde{v} \approx eE^h/m\omega_0$.

4.2. Role of relativistic effects, plasma heating when there is non-resonant wave-particle interaction, and rigidity of the system

The approach developed in Secs. 2, 3 while rather general, does not allow us with the necessary clarity to elucidate several aspects of the relativistic WAI. Therefore we give in the present subsection a certain interpretation of it.

To fix the ideas we assume the magnetic field of the perturbations to be oriented along the y-axis, i.e., $\mathbf{B}^s = (0, B_y^s, 0)$. We assume that the system is at the boundary of the instability. Taking into account the aperiodic nature of the instability we may then assume that the low-frequency perturbations are time-independent, $\partial f^s/\partial t = \partial \mathbf{B}^s/\partial t = \mathbf{E}^s = 0$. Under the stated assumptions the kinetic equation (2.7) for the low-frequency part of the distribution function takes the form

$$v_z\frac{\partial f^s}{\partial z}=-\frac{e}{c}B_y^s\left(v_x\frac{\partial}{\partial p_z}-v_z\frac{\partial}{\partial p_x}\right)\bar{f}^{hh}-e\left\langle E_z^h\frac{\partial f^{hs}}{\partial p_z}\right\rangle. \tag{4.7}$$

The first term on the right-hand side of (4.7) is connected with plasma heating when the wave-particle interaction is non-resonant.[13] Such a heating is characterized by the function $\bar{f}^{hh}$ which according to (2.8), (2.9) is in the case considered given by the relation

$$\bar{f}^{hh}={}^1/_2\langle\delta p_z^2\rangle\partial^2 f_0/\partial p_z^2, \quad \partial\delta p_z/\partial t=eE_z^h. \tag{4.8}$$

The second term on the right-hand side of (4.7) we treat as a manifestation of the rigidity of the system caused by the oscillatory particle motion in the high-frequency field with respect to the low-frequency perturbations. The basis for such a treatment is the analogy with the perturbed motion of a particle in a plasma in a static magnetic field. The particle then performs Larmor oscillations which are analogous to oscillations in a high-frequency field. It is well known that the presence of a static field makes the response of particle to the excitation of an electromagnetic field (magnetization effect) more difficult and this produces the system rigidity. Using the above-mentioned analogy we are led to the concept of the rigidity of a system in a high-frequency field. Of course, such an analogy is not complete but it gives us a correct way to orientate ourselves.

When we use the above-mentioned interpretation of the terms on the right-hand side of (4.7) it becomes clear that the effects described by us are competing. The calculations given in Sec. 3 show that if we neglect relativistic effects in the plasma such a competition leads to a complete cancelling of these effects, i.e., the right-hand side of (4.7) vanishes. When relativistic effects are taken into account the rigidity effect dominates over the heating effect and this is the physical reason for the relativistic WAI.

4.3. Excitation of oblique perturbations ($\mathbf{k}\cdot\mathbf{E}^s \neq 0$, $\mathbf{k}\times\mathbf{E}^s \neq 0$) and stabilization of the instability when the anisotropy of the HF waves decreases

When $\mathbf{k}\times\mathbf{E}^s \neq 0$ we have instead of (4.1) the following two dispersion equations

$$\varepsilon_0^s+\varepsilon_{yy}^{(1)}-c^2k^2/\omega^2=0, \tag{4.9}$$

$$\varepsilon_0^s+\varepsilon_{\perp\perp}^{(1)}-c^2k^2/\omega^2=0, \tag{4.10}$$

where

$$\varepsilon_{yy}^{(1)}=\varepsilon^{(1)}\cos^2\theta, \quad \varepsilon_{\perp\perp}^{(1)}=\varepsilon^{(1)}(\cos^2\theta-\sin^2\theta), \quad \theta=\mathrm{arctg}(k_x/k_z), \tag{4.11}$$

while the expression for $\varepsilon^{(1)}$ is given by Eq. (4.2). Equation (4.9) describes perturbations with $\mathbf{E}^s = (0, E_y^s, 0)$ and (4.10) those with $\mathbf{E}^s = (E_x^s, 0, E_z^s)$.

It is clear that the problem of oblique perturbations reduces to the one considered in Sec. 4.1 with the substitution

$$k_*^2=\bar{k}_*^2\begin{cases}\cos^2\theta, & \mathbf{E}^s\|\mathbf{y},\\ \cos^2\theta-\sin^2\theta, & \mathbf{E}^s\perp\mathbf{y}'\end{cases} \tag{4.12}$$

where $\bar{k}_*^2$ denotes the right-hand side of (4.4). Perturbations with $\mathbf{E}^s\|\mathbf{y}$ are unstable up to $\theta = \pi/2$ (their growth rate decreases with increasing θ) whereas perturbations with $\mathbf{E}^s\perp\mathbf{y}$ are stabilized for $\theta \geqslant \pi/4$.

The situation considered here corresponds to extremely strong anisotropy of the HF wave spectrum. When the degree of anisotropy decreases Eqs. (4.9), (4.10) must be mixed up with one another so that instead of two dispersion equations we shall have a single one—Eq. (3.46). Moreover, the quantities $\varepsilon_{\alpha\beta}^{(1)}$ occurring in (3.46) will be integrals of angular functions such as (4.11), where now θ has the meaning of the angle between the wave vectors of the perturbations and the direction of the electric field of the various HF waves. The characteristic value $\theta = \theta_0$ then means the same as the characteristic solid angle of the wave numbers of the HF waves (or the characteristic angle of the direction of the polarization in the case of electromagnetic HF waves). It is thus clear that the instabilities considered above must occur up to $\theta_0 \approx 1$.

5. ROLE OF MODULATIONAL EFFECTS IN THE WAI OF A PLASMA WITH ELECTROMAGNETIC WAVES

We supplement the analysis of Sec. 4 with taking into account the effects connected with $\varepsilon_{\alpha\beta}^{(2)}$. As in Sec. 4 we start with considering the case $\mathbf{k}\|\mathbf{E}^h\|\mathbf{z}$. The primary HF waves are assumed to be electromagnetic, $\mathbf{q}\perp\mathbf{E}^h$. To fix the ideas we take $\mathbf{q}\|\mathbf{x}$. We assume the plasma to be non-relativistic.

Using the formulae in Sec. 3 we conclude that $\varepsilon_{1y}^{(2)l} = \varepsilon_{1y}^{(2)t} = 0$, i.e., the tensor $\varepsilon_{\alpha\beta}^{(2)}$ is diagonal. Moreover, $\varepsilon_{yy}^{(2)l} = 0$. Two dispersion equations then follow from (3.46):

$$\varepsilon_0^{(s)} + \varepsilon^{(1)} + \varepsilon_{22}^{(2)t} - c^2k^2/\omega^2 = 0, \quad (5.1)$$

$$\varepsilon_0^{s} + \varepsilon^{(1)} + \varepsilon_{\perp\perp}^{(2)t} - c^2k^2/\omega^2 = 0. \quad (5.2)$$

The first one describes perturbations with $\mathbf{E}^s\|\mathbf{y}$, $\mathbf{B}^s\|\mathbf{x}$, and the second one those with $\mathbf{E}^s\|\mathbf{x}$, $\mathbf{B}^s\|\mathbf{y}$.

We assume perturbations which are long-wavelength ones relative to the primary HF waves $k \ll q$. The quantities $\varepsilon_{11}^{(2)l}$, $\varepsilon_{11}^{(2)t}$ are then small like k^2/q^2 so that (5.2) reduces to (5.1). Hence, modulational effects are contained solely in (5.1). The quantity $\varepsilon_{yy}^{(2)t}$ characterizing these effects turns out to be exactly the same as the quantity $\varepsilon^{(1)}$ given by Eq. (4.2). In other words, the role of the modulational effects in the case considered turns out to be the same as the role of the relativistic effect discussed in Sec. 4. The concrete results following from (5.1) differ from the ones given in Sec. 4.1 by a numerical factor of order unity so that the estimates (1.3), (1.5) remain in force.

In this connection there arises a paradox: a relativistic effect turns out to be of the same order as a nonrelativistic one. This paradox is cleared up by the fact that the secondary HF waves, with which the modulational effect is connected, are electromagnetic and due to this their electric field is small as $1/c^2$. Indeed, according to (3.33) as $\omega \to 0$ the field $\mathbf{E}^{hs}$ is given by the equation

$$[\omega_{\pm q}^2\varepsilon^t(\omega_{\pm q}) - c^2(\mathbf{k}\pm\mathbf{q})^2]E_y^{hs} + ekE_z^h\omega_p E_y^s/m\omega = 0. \quad (5.3)$$

We have here for the sake of simplicity omitted the index $\mathbf{k}\pm\mathbf{q}$, $\pm\omega_{\mathbf{q}}^h$ of E_y^{hs} and the subscript $\pm$ of E_z^h. In the case considered

$$\omega_{\pm q}^2\varepsilon^t(\omega_{\pm q}) - c^2(\mathbf{k}\pm\mathbf{q})^2 = -c^2k^2, \quad (5.4)$$

so that $E^{hs} \propto 1/c^2$.

We show that the modulational effect discussed here is important only when the spectrum of the primary HF waves is appreciably anisotropic. To see this we turn again to (5.3) and note that, if we use (5.4)

$$E^{hs} \propto 1/k. \quad (5.5)$$

Such a result is obtained only in the case considered by us, $\mathbf{k}\perp\mathbf{q}$. When there is a spread in the directions of the wave vectors of the primary waves we must assume that $\mathbf{kq} \neq 0$ and instead of (5.5) it follows then from (5.3) that

$$E^{hs} \propto k/(k^2 - 2\mathbf{qk}). \quad (5.6)$$

The expressions for $\varepsilon_{\alpha\beta}^{(2)t}$ obtained using (5.6) have a characteristic structure

$$\varepsilon_{\alpha\beta}^{(2)t} \propto \int \frac{k\,d\mathbf{q}}{k^2 - 2\mathbf{qk}}. \quad (5.7)$$

It is clear that when $q \gg k$ the field of the secondary waves E^{hs} must decrease appreciably even for a small angular spread $\Delta\theta$ of the wavevectors of the primary waves, $\Delta\theta \gtrsim k/q$. The modulational mechanism for generation will then be appreciably weakened whereas the relativistic one is not weakened up to $\Delta\theta \approx 1$ (see Sec. 4.3).

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