Stimulated scattering of spatially incoherent optical radiation

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The process of backward scattering of spatially incoherent optical radiation is investigated theoretically. It is shown that the spatial orthogonality condition for the frequency components of the initiating radiation leads to a significant simplification of the dynamical equations, which can then be analyzed for arbitrary relations between the characteristic lifetimes T_2 of the respective oscillations and the reciprocal width $\Delta\nu_p^{-1}$ of the pump spectrum. The space-time structure of the scattered Stokes radiation is computed for different numbers of the spectral components of the pump. It is found that the physical cause of the sharp decrease occurring, when $\Delta\nu_p T_2 \gtrsim 1$, in the gain increments of the Stokes waves that are correlated with the pump is the inversion of the spectra of these waves with respect to the spectrum of the initiating radiation.

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I. INTRODUCTION

The effect of the nonmonochromaticity of the exciting radiation on the processes of stimulated scattering has been considered time and again in the literature, both the characteristics of the scattering of plane waves and the characteristics of the scattering of spatially inhomogeneous waves (from the point of view of the reconstruction or inversion of the wave front (WFI) of the corresponding fields) having been investigated. But all these investigations are limited to the case of the broad-band pump, when the exciting radiation line width $\Delta\nu_{p\lambda}$ is much larger than the spontaneous scattering line width $\Delta\nu_{sp}$. The analyses in the majority of cases are carried out with the use of the method developed in Refs. 1 and 2, and based on the derivation and investigation of the dynamical equations describing the interaction between the initiating and Stokes waves, which consist of a discrete set of independent spectral components. For $\Delta\nu_p \gg \Delta\nu_{sp}$ the actual spectra can be correctly approximated by a set of N spectral components whose spacing Ω satisfies the condition $\Omega T_2 > 1$, where $T_2 = 1/\Delta\nu_{sp}$ is the characteristic lifetime of the vibrations of the medium and $\Delta\nu_p = (N-1)\Omega$. The use of the last condition $\Omega T_2 > 1$ allows us to treat the vibrational wave of the active medium as a single-frequency wave. As a result, it is possible to write out a closed system of dynamical equations and solve it. Thus, this method, in contrast to the other methods, allowed the determination of not only the gain increments of the Stokes waves, but also the structure of the corresponding fields.

When the condition for a broad-band pump is violated, i.e., when $\Delta\nu_p \lesssim \Delta\nu_{sp}$, we can no longer use the condition for the orthogonality of the spectral components over the time interval T_2, i.e., the condition $\Omega T_2 > 1$. In this case two or more spectral components of the signal lie within the limits of the spontaneous scattering line width $\Delta\nu_{sp}$. Therefore, the oscillations of the active medium cease to be monochromatic; there begins the generation of side frequencies, i.e., the broadening of the spectrum of the scattered radiation in comparison with the pump spectrum.[3] As a result, it is not possible in the general case to derive a closed system of dynamical equations. It is precisely for this reason that the case $\Delta\nu_p \lesssim \Delta\nu_{sp}$ has thus far not been analyzed.

As noted above, success was achieved in the solution of the problem of stimulated scattering of a broad-band pump owing to the use of the condition for the orthogonality of the spectral components over the time interval T_2. Other orthogonality conditions for an arbitrary number of spectral components do not exist for the scattering of a plane exciting-radiation wave. But such conditions appear in the case of the scattering of a spatially inhomogeneous pump. Indeed, there appears here a second independent variable: a transverse space coordinate. Therefore, we can use the orthogonality of the spatial configurations of the light fields.

We shall carry out a consistent analysis with the use of this idea. For the exciting radiation we choose the following model. We shall represent the spectral composition of the radiation by a set of discrete components, such that $\Delta\nu_p = (N-1)\Omega$, where Ω can vary from zero to infinity, but each spectral component has a spatial configuration that is orthogonal to the configurations of all the other components. It turns out that the use of the orthogonality condition for the spatial configurations of the spectral components of the pump also allows us to derive a system of uncoupled dynamical equations, find the gain growth rates, determine the structure of the fields, etc. Let us note that such a model of the pump adequately describes the spatially incoherent radiation of a laser in which transverse mode selection does not occur.

II. BASIC EQUATIONS

Let us represent the pumping radiation in the form of a set of monochromatic spectral components, each of which is characterized by its own spatial structure:

$$E_p(\mathbf{r},t) = \sum_{m,n} A_{m,n} \exp\{-i[(\omega_0 + m\Omega)t - \mathbf{k}_n\mathbf{r}]\}, \tag{1}$$

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where ω_0 is the mean frequency, Ω is the spectral line spacing, and m and n are whole numbers: $m(n) = 1, \dots, M(N)$. Then the scattered field wave propagating in the direction opposite to the direction of propagation of the pump can be similarly represented:

$$E_s(\mathbf{r}, t) = \sum_{-\infty}^{\infty} \sum_{q} a_{m,q} \exp\{-i[(\omega_s + m\Omega)t + \mathbf{k}_q \mathbf{r}]\},$$

$$q = 1, \dots, N. \qquad (2)$$

Assuming that the scattering process is a steady-state process, that the conditions for the realization of the inversion effect are fulfilled,[4] and that the length of the scattering region is sufficiently short, so that the effects to the group retardation can be neglected, we obtain for the amplitudes $a_{m,q}$ in the prescribed pump field approximation the system of equations

$$\frac{da_{m,q}}{dz} = \frac{g}{2} \sum_{k} \sum_{\alpha} \sum_{t} \frac{A_{t,k} A^{*}_{\alpha,k} a_{\alpha - t + m, q}}{1 + i(t - m)\Omega T_2} + \frac{g}{2} \sum_{n} \sum_{p \neq q} \sum_{t} \frac{A^{*}_{n,q} A_{t,p} a_{n - t + m, p}}{1 + i(t - m)\Omega T_2}, \qquad (3)$$

where g is the gain of the active medium, z is the longitudinal coordinate for the propagation of the Stokes waves, and T_2 is the lifetime of the medium oscillations.

The equations (3) describe the amplification of a Stokes signal of the form (2) in the pump field (1), and goes over, as $\Omega T_2 \to \infty$, into the system of equations obtained for a broadband pump in Ref. 5. The spatial-orthogonality condition for the various frequency components can be written as follows:

$$\sum_{k} A_{t,k} A^{*}_{\alpha,k} = I_\alpha \delta_{t,\alpha}, \qquad (4)$$

where I_α is the intensity of the pump's spectral component with frequency $\omega_0 + \alpha\Omega$ and $\delta_{t,\alpha}$ is the Kronecker symbol.

The spatial-orthogonality condition (4) can be realized, in particular, when active medium is pumped by N frequency components whose directivity patterns do not coincide (N spatially inhomogeneous beams with different frequencies). Another case in which the condition (4) is realized is the case of the multimode radiation of a laser in which transverse mode selection does not occur; it can then be assumed that the spatial configurations of the fields corresponding to the various eigenfrequencies of the laser are not related with each other, and, hence, the condition (4) is fulfilled.

Let us introduce a sum of the form

$$S_m^{\alpha}(z) = \sum_{q} A_{\alpha,q} a_{m,q}(z). \qquad (5)$$

Then, using (4), we obtain from the system (3) for $S_m^{\alpha}(z)$ the system of equations

$$\frac{dS_{m+k}^{n-k}}{dz} = \frac{g}{2} S_{m+k}^{n-k} \sum_{\alpha=0}^{n} \frac{I_\alpha}{1 - (m + k - \alpha) i\Omega T_2} + \frac{g}{2} I_{n-k} \sum_{\alpha=0}^{n} \frac{S^{\alpha}_{m+n-\alpha}}{1 - (m + k - \alpha) i\Omega T_2}, \qquad (6)$$

where $k = 0, 1, \dots, n$ and $n + 1 = N$ is the number of frequency components of the pump. It can be seen that the system (6) comprises a set of independent subsystems ($m = 0, \pm 1, \dots$), the solution for each of which can be sought separately. In view of this, the general solution to the system (3) is a superposition of the solutions to the subsystems (6). For $m = 0$, and for the solutions that are correlated with the pump, the system (6) describes the case of the inversion of the reflected wave spectra with respect to the pump spectrum; for $m = 1$, the inversion accompanied by a shift by the intermode distance Ω, etc.

Thus, in the presence of N pump frequency components, and under conditions when the spatial orthogonality condition (4) is fulfilled, the spectral structure of the reflected Stokes signal is a superposition of the pump spectra inverted and shifted through different distances $m\Omega$.

Let us point out here that, recently, Sidorovich[6,7] attempted to investigate the stimulated scattering of a pump of this type for $N = 4$. But he limited himself to the consideration of the case of spectrum inversion (i.e., of the $m = 0$ case) and the artificially introduced case of pump spectrum reconstruction, which, as will be shown below, can be realized only in the limiting case $\Omega \gg 1/T_2$ and in the case of special initial conditions. These cases are a result of the use by Sidorovich of the procedure for seeking the gain increments of Stokes fields with a form prescribed beforehand. But to obtain sufficient results, we must, firstly, go over in a consistent manner from the Maxwell equations to equations for slow field amplitudes and, secondly, solve them correctly, which is done in the present paper.

III. TWO-MODE SPATIALLY INCOHERENT PUMP

For practical applications, and for the elucidation of the physical meaning of the various components of the solutions, it is of interest to investigate the system (6) for $N = 2$. It then admits of a simple analytic solution for the eigenvectors (modes) and eigenvalues (growth rates) for arbitrary m. On the other hand, it is possible in the $N = 2$ case to analyze the effect of the phase modulation of the reference waves in the regime of nonthreshold reflection, as well as in a system of the WFI generator + WFI amplifier type.[8,9] For $N = 2$ we have from (6) the equations

$$\frac{dS^{0}_{m+1}}{dz} = \frac{g}{2} \left[\frac{2I_0}{1 - i(m+1)\Omega T_2} + \frac{I_1}{1 - im\Omega T_2} \right] \times S^{0}_{m+1} + \frac{g}{2} \frac{I_0}{1 - im\Omega T_2} S_m^{1},$$

$$\frac{dS_m^{1}}{dz} = \frac{g}{2} \left[\frac{I_0}{1 - im\Omega T_2} + \frac{2I_1}{1 - i(m-1)\Omega T_2} \right] \times S_m^{1} + \frac{g}{2} \frac{I_1}{1 - im\Omega T_2} S_{m+1}. \qquad (7)$$

Let us first investigate this system, assuming that the intensities of the frequency components of the pump are equal: $I_\beta = I$; $\beta = 0, 1$. Going over to the equations for the eigenvalues $\Gamma = \frac{1}{2} g I \lambda$ of the system (7), we obtain

$$\lambda S^0_{m+1} = \left[\frac{2}{1-i(m+1)\Omega T_2}+\frac{1}{1-im\Omega T_2}\right] S^0_{m+1}+\frac{1}{1-im\Omega T_2}S_m^1,$$

$$\lambda S_m^1 = \frac{1}{1-im\Omega T_2}S^0_{m+1}+\left[\frac{1}{1-im\Omega T_2}+\frac{2}{1-i(m-1)\Omega T_2}\right]S_m^1,$$

(8)

where the λ's are the eigenvalues of the system (8); whence we find for the quantity $S^0_{m+1}/S^1_m=\alpha$ the equation

$$\alpha^2-\frac{4i\Omega T_2(1-im\Omega T_2)}{[1-i(m+1)\Omega T_2][1-i(m-1)\Omega T_2]}\alpha-1=0 \qquad (9)$$

and

$$\alpha_1=-\frac{1-i(m+1)\Omega T_2}{1-i(m-1)\Omega T_2},\qquad \alpha_2=\frac{1-i(m-1)\Omega T_2}{1-i(m+1)\Omega T_2}. \qquad (10)$$

Substituting these values $\alpha_{1,2}$ into (8), we obtain for the real parts of the growth rates Γ (they determine the amplification of the Stokes waves) the expressions

(11)

$$\mathrm{Re}\,\Gamma_{1,2} = \left[\frac{2}{1+(m-1)^2\Omega^2T_2^2}+\frac{1+\mathrm{Re}\,\alpha_{1,2}-m\Omega T_2\,\mathrm{Im}\,\alpha_{1,2}}{1+m^2\Omega^2T_2^2}\right]\frac{gI}{2},$$

$$\mathrm{Re}\,\Gamma_1=\frac{gI}{2}\frac{2}{1+m^2\Omega^2T_2^2},$$

$$\mathrm{Re}\,\Gamma_2=\frac{\frac{gI}{2}4[1+(m^2+1)\Omega^2T_2^2]}{[1+(m-1)^2\Omega^2T_2^2][1+(m+1)^2\Omega^2T_2^2]}. \qquad (12)$$

Figure 1 shows the dependences of $\mathrm{Re}\Gamma_{1,2}$, obtained in accordance with (12) and normalized to the gain growth rate in the mean pump field for $\Omega=0$, on the frequency shift Ω between the pump components for different m.

Physically, the frequency inversion in the reflected Stokes signal is caused by the fact that, when the frequencies of the pump beams are different, the interference-pattern antinodes produced by these beams move in the direction of the largest wave vector **k** (see Fig. 2). Therefore, for the Stokes signal moving in the opposite direction, and correlated with the pump, and, hence, having the maximum gain growth rate, the frequencies should be connected by the inverse relation, in order for the interference pattern antinodes produced by the Stokes waves to follow the corresponding antinodes for the pump. In the language of the discrete frequency model this means that the greatest growth rates are possessed by those configurations of the Stokes signal which are inverted in frequency with respect to the pumping waves and those that are inverted and shifted by integral multiples of the distance Ω between the frequencies of two waves of the initiating radiation.

Let us further proceed to determine the eigenvectors (modes) of the system (3). We find directly from (9), (10), and (6) that

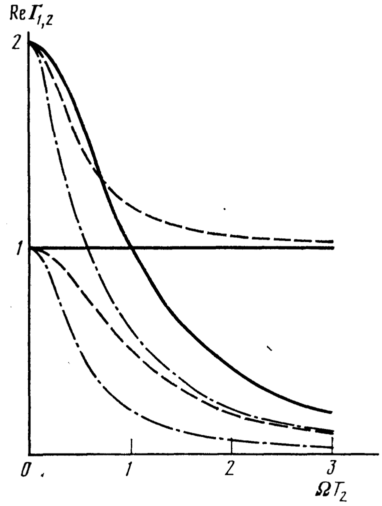

FIG. 1. Dependence of $\mathrm{Re}\,\Gamma_{1,2}$, normalized to the gain increment in the mean field of the monochromatic pump, on the reduced intermode separation in the case of a two-mode pump: the continuous curve is for $m=0$; the dashed curve, for $m=\pm 1$; and the dot-dash curve, for $m=\pm 2$. $I_0=I_1=I$.

$$\begin{pmatrix} a_{m+1,q} \\ a_{m,q}\end{pmatrix}_1 \propto \begin{pmatrix} -[1-i(m+1)\Omega T_2]A^*_{0,q} \\ [1-i(m-1)\Omega T_2]A^*_{1,q}\end{pmatrix}$$

$$\begin{pmatrix} a_{m+1,q} \\ a_{m,q}\end{pmatrix}_2 \propto \begin{pmatrix} [1-i(m-1)\Omega T_2]A^*_{0,q} \\ [1-i(m+1)\Omega T_2]A^*_{1,q}\end{pmatrix}. \qquad (13)$$

Let us estimate the quality of the inversion of the wave front of the pump by the Stokes signal, using the similarity parameter[10]

$$K=\left|\sum_i A_i a_i\right|^2 \Big/ I_p I_s. \qquad (14)$$

Since the solution with the maximum growth rate is usually realized in scattering from spontaneous noise, for $\Omega T_2 \ll 1$ and $m\Omega T_2 \ll 1$ we find for each m in accordance with (13) and (14) that

$$K\approx 1,\qquad \Gamma_2\approx\frac{2gI}{1+m^2\Omega^2T_2^2}\sim 2\Gamma_1. \qquad (15)$$

From this it follows that the wave front inversion effect is realized in the case of small distances between the frequencies of the pump components, i.e., when $\Omega T_2 \ll 1$, and the relations between the increments Γ_2 and Γ_1 and the width of

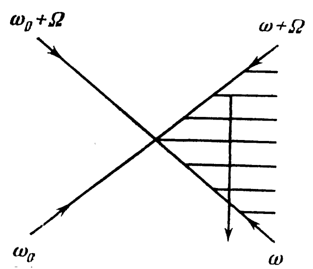

FIG. 2. Principle underlying the formation of a Stokes signal correlated with the pump: $|\mathbf{k}(\omega_0+\Omega)| > |\mathbf{k}(\omega_0)|$. The arrow indicates the direction of motion of the interference pattern.

the scattered radiation spectrum are the same in this case as in the case of a monochromatic pump. When $0.2 \lesssim \Omega T_2 \lesssim 1$, because of the discrimination of the gain growth rates corresponding to different values of m, we can take into account only the growth rates with $m = 0$ and $m = \pm 1$. From (13) and (14) we obtain

$$K(m=0) = 1/(1+\Omega^2 T_2^2),$$

$$K(m=\pm 1) = (1+\Omega^2 T_2^2)/(1+2\Omega^2 T_2^2).$$

Assuming that the quality of the wave front inversion is no longer satisfactory when $K \lesssim 0.8$, we find that, for the wave front inversion effect to be realized, the following condition should be fulfilled:

$$\Omega T_2 < 0.5. \qquad (16)$$

As can easily be seen, the scattered radiation spectrum in the present approximation consists of four equidistant components; it is three times broader than the pump spectrum. As the frequency shift between the pump beams is increased further, i.e., as $\Omega T_2 \to \infty$, the nonresonance components of the Stokes signal attenuate, and the spectrum of the radiation in each of the reflected beams begin to duplicate the spectrum of the initiating radiation, which is in accord with previously obtained results.[3,11]

Notice that the expressions (12) describe the gain increments for the Stokes waves that are correlated in spatial structure with the pump. The gain increments for the uncorrelated configurations are the coefficients attached to the S^0_{m+1} and S^1_m in the system (7). They correspond, as in the case of a monochromatic pump, to amplification in the mean field, and are significantly smaller than the maximum increments of the Stokes waves that are correlated with the pump.

As has already been noted above, the system of equations (7) allows us to carry out the analysis in two practically important cases: a) the nonthreshold reflection of weak signals[9] and b) systems of the WFI generator + WFI amplifier type.[8] In both of these cases the reference Stokes signal is usually fed at the frequency of the resonance with the reference pump wave, e.g., with $A_{1,q}$, and then we must set $m = 1$ in (7). The Stokes signal that is inverted with respect to the signal pump wave will then be generated at the frequency 2Ω (see Fig. 2). In the case a) we can assume that $I_1 \gg I_0$, $S^1_1(0) = S_0$, and $S^0_2 = 0$. Then from (7) we obtain the system

$$\frac{dS_1^1}{dz} = gI_1 S_1^1, \qquad (17)$$

$$\frac{dS_2^0}{dz} = \frac{g}{2}\frac{I_1}{1-i\Omega T_2}S_0^2 + \frac{g}{2}\frac{I_0}{1-i\Omega T_2}S_1^1.$$

It follows directly from this that, as Ω increases, the efficiency of the nonthreshold reflection decreases in proportion to the coefficient $(1 - 2i\Omega T_2)^{-1}$ (see also Ref. 12) in the pre-exponential factor.

In the case b) the amplitudes of the pump components are connected by the inverse relation: $I_0 \gg I_1$. Using (7), we obtain

$$\frac{dS_1^1}{dz} = \frac{g}{2}\frac{I_0}{1-i\Omega T_2}S_1^1 + \frac{g}{2}\frac{I_1}{1-i\Omega T_2}S_2^0,$$

$$\frac{dS_2^0}{dz} = \frac{g}{2}\frac{I_0}{1-2i\Omega T_2}S_2^0 + \frac{g}{2}\frac{I_1}{1-i\Omega T_2}S_1^1. \qquad (18)$$

On the basis of (18), we can derive for the quantity $\mathrm{Re}\,\Gamma$, which determines the gain, the relation $\mathrm{Re}\,\Gamma \propto (1 + 4\Omega^2 T_2^2)^{-1}$. This circumstance can significantly lower the efficiency of the WFI generator + WFI amplifier system when $\Omega T_2 \gtrsim 1$, and decrease the signal-to-noise ratio (see Ref. 8) because the noise is generated mainly at the frequency of the resonance with the signal wave.

IV. MULTIMODE SPATIALLY INCOHERENT PUMP

When the number of pump components $N > 2$, the analytic expressions for the growth rates and the eigenvectors of the system (6) have an extremely unwieldy form, but the system (6) for the case in which the spectra are inverted (i.e., for the $m = 0$ case) is amenable to a relatively simple numerical solution. As an example, we show in Fig. 3 the dependence of the gain growth rates on the intermode separation for a three-mode pump with $I_\beta = I$, where $\beta = 0,1,2$. Notice that the asymptotic forms of the solutions obtained for $\Omega = 0$ and $\Omega \to \infty$ coincide with the results obtained earlier for these limiting cases.[4,5,12]

Experimentally, the solutions corresponding to the growth rates with the maximum real parts are usually realized in investigations of the generation of Stokes radiation from spontaneous noise. Therefore, for the purpose of comparing the results obtained with experiment, we show the dependence of max $\mathrm{Re}\,\Gamma$ on the total spectral width $\Delta\nu_p$ for $N = 2$, 3, and 4 in Fig. 4 and the corresponding dependences of the inversion parameter in Fig. 5. It is worth noting that the growth rates and the inversion parameters for $N = 3$ and 4 behave virtually identically in the region of variation of $\Delta\nu_p$ from 0 to $\approx 2/T_2$. This circumstance indicates that we can use the three-mode approximation in the case $N > 4$ and in the limit of a continuous spectrum with width $\Delta\nu_p$. Assuming, as was done in Sec. III, that the quality of the inversion is no longer satisfactory when $K < 0.8$, we obtain on the basis of Fig. 5 the condition for the realization of the wave front inversion effect:

$$\Delta\nu_p T_2 < 0.7, \quad \text{or} \quad \Delta\nu_p < 0.35\Delta\nu_{sp}. \qquad (19)$$

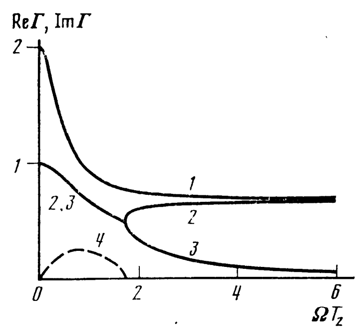

FIG. 3. Dependence of the normalized gain growth rates $\Gamma_{1,2,3}$ on the reduced mode spacing for a three-mode pump: 1) Γ_1; 2) Re Γ_2; 3) Re Γ_3; 4) Im $\Gamma_{2,3}$; Im $\Gamma_1 = 0$. $I_0 = I_1 = I_2 = I$.

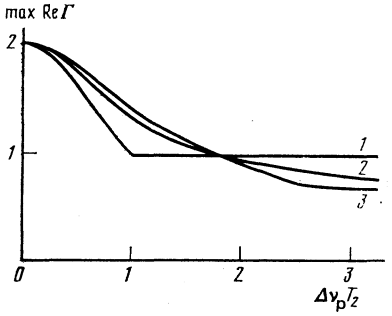

FIG. 4. Dependence of the maximum real parts max Re Γ of the normalized gain growth rates on the total spectral width for equal amplitudes of the frequency components of: 1) a two-mode pump; 2) a three-mode pump; and 3) a four-mode pump.

For $m = 0, \pm 1,...$ we can find directly from (6) that, as $\Omega \to 0$,

$$\Gamma_0 \to g \sum_{i=0} I_i, \quad \Gamma_{k\neq 0} \to \frac{g}{2} \sum_{i=0} I_i;$$

qualitatively, the behavior in this case of the frequency spectrum of the Stokes radiation as Ω varies from 0 to ∞ is in full accord with what obtains in the two-mode case considered in Sec. III, except that the scattered radiation spectrum consists largely of $3N-2$ components when $\Delta\nu_p T_2 \sim 1$ and N components as $\Delta\nu_p T_2 \to \infty$.

Let us consider the last case. Here we can consider only the resonance terms in the system (6):

$$\frac{dS_{m+k}^{n-k}}{dz} = \frac{g}{2} S_{m+k}^{n-k} I_{m+k} + \frac{g}{2} I_{n-k} S_{n-k}^{m+k}, \tag{20}$$

where $m + k = 0, 1,...,n$ and $k = 0, 1,...,n$. Introducing the notation $p = m + k$, $t = n - k$, we obtain

$$\frac{dS_p^t}{dz} = \frac{g}{2} S_p^t I_p + \frac{g}{2} I_t S_t^p. \tag{21}$$

For $I_p \approx I/N$, where I is the total intensity of the spectral components of the pump, we find from (21) that

$$\Gamma_{\rm corr} \approx g\frac{I}{N}, \quad S_p^t \propto \exp\left(g\frac{I}{N}z\right). \tag{22}$$

For the Stokes signals that are uncorrelated in spatial structure we obtain directly from (3) the estimate

$$\Gamma_{\rm uncorr} \approx {}^1/_2 gI/N, \tag{23}$$

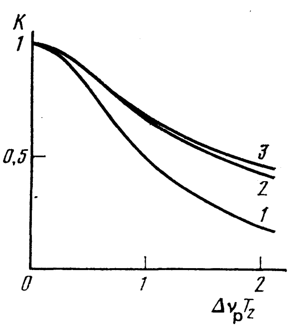

FIG. 5. Dependence of the inversion parameter on the total spectral width for equal amplitudes of the frequency components of: 1) a two-mode pump; 2) a three-mode pump; and 3) a four-mode pump.

which is two times smaller than the increment for the signals that are correlated with the pump [see (22)]. It can also be seen that, by specially choosing the initial conditions, namely, by requiring that $S_p^t(0) = S(0)\delta_{t,p}$, we can realize the case in which the Stokes signal reproduces the pump spectrum, though the reproduction in question will clearly occur only when $\Delta\nu_p \gg 1/T_2$.

Notice that the system (21) fully coincides with the system obtained in Ref. 12, where it is investigated in greater detail.

V. CONCLUSION

Thus, we have derived and investigated a system of equations, (3), describing the stimulated scattering of a pump of the form (1) in the case when the Stokes signal and the pump propagate in opposite directions and the relation between the spontaneous scattering line width and the mode spacing Ω is arbitrary. It is shown that the spatial orthogonality condition allows us to substantially simplify the analysis and investigate the system for a number of practically interesting cases. Thus, for example, we have obtained the condition, (19), for the realization of the wave front inversion effect, investigated the nonthreshold-reflection and WFI generator + WFI amplifier schemes, and shown on the basis of the exact solutions of the dynamical equations for a two-mode pump and approximate solutions for an N-mode pump with $\Omega \gg 1/T_2$ how the scattered radiation spectrum varies as ΩT_2 varies from 0 to ∞. It is established that the physical cause of the sharp decrease occurring even in the absence of group retardation effects in the gain increments ($\Gamma \propto 1/N$) of waves that are correlated with the pump when these waves and the pump propagate in opposite directions and $\Delta\nu_p \gg 1/T_2$ is the inversion of the spectrum of the Stokes signal. As a result, only pairs of the spectral components of the Stokes waves and the pump can effectively interact; the contribution of the remaining spectral components is a nonresonant one.

Let us note that the results obtained also allow us to account for the hitherto unexplained observation, reported in Ref. 13, that amplification does not occur in the case of counterpropagation of the pump and the external Stokes signal. Indeed, this observation was made in experiments[13] in which a laser operating in the regime of active Q-switching without transverse mode selection was used as a pump source. The lasing line width was $\Delta\nu_p \approx 0.3$ cm^{-1}, while the spontaneous scattering line width $\Delta\nu_{sp} = 0.067$ cm^{-1}, i.e., the condition for a broad band was fulfilled. From this it follows that, even without allowance for the effect of the group retardation, the amplitude growth rate of the gain is, according to (23), equal to $\Gamma \approx gI/2N$, where N can be estimated at $N \sim 2\Delta\nu_p/\Delta\nu_{sp} \sim 10$. The experiments were performed under conditions when $gIl \lesssim 3$, where l is the length of the nonlinear medium. From this it follows that the amplification of the Stokes signal was not higher than $e^{0.3}$, and could have been missed, considering the measurement accuracy that could be achieved at the time. Similarly, we can explain the unsuccessful attempts of a number of researchers to obtain stimulated Mandel'shtam-Brillouin scattering of

CO_2-laser radiation, since the spontaneous scattering line width decreases as the frequency is varied from the visible to the CO_2-laser band ($\lambda_{CO_2} = 10.6\ \mu m$): $\Delta\nu_{sp} \propto 1/\nu_p^2$; hence the requirement that the pump radiation be spatially coherent is much stricter.

Let us also point out that the results obtained can be regarded as the results of an investigation of the influence of the phase modulation of the pumping waves on the processes of stimulated scattering with the use of the linear dependence $\varphi(t) = \varphi_0 + \Omega t$ as a model of the temporal behavior of the phase. It is, however, clear that, qualitatively, they are applicable to other models for the function $\varphi(t)$ with Ω replaced by $1/\tau_{ph}$, where τ_{ph} is the characteristic variation time of the phase φ.

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