

Tunnel transparency of disordered systems in a magnetic field

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It is shown that multiple nonresonant scattering in a magnetic field alters substantially the character of the decay (the argument of the exponential) of the wave function of a tunneling electron. For example, the wave function in a strong magnetic field is proportional to $\exp(-x^2/2\lambda^2)$ without allowance for scattering, but when scattering is taken into account it takes the form $\exp(-|x|/b)$, where x is the coordinate in the direction perpendicular to the magnetic field, λ is the magnetic length, $b = \lambda / |\ln B|$, and B is a parameter that describes the scattering. The mean square modulus of the Green's function with negative energy in a magnetic field is calculated for scattering by a random Gaussian potential. It is shown that in semiconductor solid solutions this quantity can be used to describe the tunnel transparency of films in a magnetic field parallel to the surface, as well as the magnetoresistance of bulk samples in the region of hopping conduction.

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1. STATEMENT OF PROBLEM

The problem of electron tunneling with allowance for below-barrier scattering is frequently encountered in the physics of disordered systems. The simplest example is the calculation of the tunnel transparency of a dielectric film with impurities, dealt with by Chaplik and Entin¹ and by Lifshitz and Kirpichenkov.² In Ref. 2 were introduced the concepts of resonant and nonresonant tunneling. In resonant tunneling an important role is played by impurity centers with energy levels very close to the energy of the tunneling electron. The existence of such resonant centers increases sharply the tunnel transparency of the barrier.¹ When nonresonant tunneling is considered it is assumed that there are no resonant impurities at all or that they are very sparse, and can therefore be disregarded. The present paper is devoted entirely to nonresonant tunneling.

In an impurity-free film of thickness L the transparency is known to be given by $D = D_0 \exp(-2L/a)$, where a is a characteristic length that depends on the tunneling-electron energy ϵ reckoned downward from the bottom of the conduction band. In the effective mass approximation, for example, $a = \hbar/(2m\epsilon)^{1/2}$. As shown in Ref. 2, in nonresonant tunneling the below-barrier scattering by impurities alters little the argument $2L/a$ of the exponential, introducing in it a correction proportional to the impurity density N .

We show in this paper that in a magnetic field H parallel to the film surface the nonresonant below-barrier scattering leads to much more substantial effects. We consider a film bounded by the planes $x = 0$ and $x = L$. A magnetic field oriented along the z axis produces for the electron an additional potential

$$U(x) = (e^2 H^2 / 2mc^2) (x - x_0)^2,$$

where x_0 is the center of the Landau oscillator. To calculate the argument of the tunnel-transparency exponential it suffices to describe the tail of the electron wave function with $x_0 = 0$ below the barrier. The presence of a parabolic "magnetic" increment to the tunnel barrier (Fig. 1) causes the electron wave function to fall off in a direction perpendicular

to the film surface (the x axis) not in accord with the usual formula $\exp(-x/a)$, but following the laws

$$\psi(x) \propto \exp\left(-\frac{x}{a} - \frac{x^3 a}{6\lambda^4}\right), \quad x \ll \frac{\lambda^2}{a}, \quad (1a)$$

$$\psi(x) \propto \exp\left(-\frac{x^2}{2\lambda^2}\right), \quad x \gg \frac{\lambda^2}{a}, \quad (1b)$$

where $\lambda = (c\hbar/eH)^{1/2}$ is the magnetic length. If $\lambda < a$, only Eq. (1b) has meaning. The transparency has a similar dependence on H , namely, $D \propto |\psi(L)|^2$. It is shown in the present paper that scattering by impurities changes strongly the fall-off of the wave function (1), so that the argument of exponential of the transparency depends linearly on L :

$$D(L) = D_0 e^{-2L/b}, \quad (2)$$

where b is a quantity that depends on H and N . Thus, at large L the scattering by impurities increases the tunnel transparency exponentially. This phenomenon can be compared with the influence of impurity scattering on the transverse electric conductivity of free electrons in a magnetic field.

The role of the below-barrier scattering can be qualitatively explained by recognizing that each scattering act changes the quantum number x_0 jumpwise and brings it

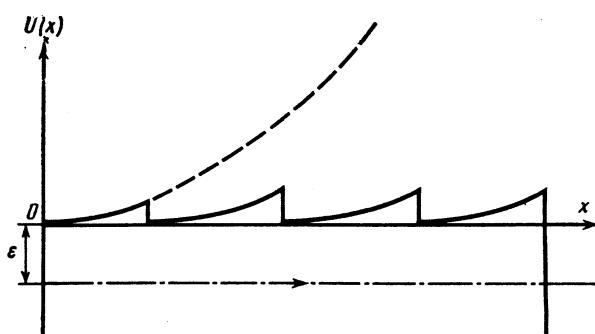


FIG. 1. Effective tunnel barrier in the presence of a magnetic field in a pure film (dashed line) and in a film with impurities (solid line that terminates at $x = L$). The energy level of the tunneling electron is shown by the horizontal dash-dot line.

closer to the coordinate of the scattering center. The magnetic potential then "drops" to zero, and in the state with the new value x_0 it is reckoned already from the coordinate of the scattering center. As a result, the effective potential $U(x)$ that acts on the electron is not monotonic but has a sawtooth shape (Fig. 1), so that the tunneling is greatly facilitated. Such a potential leads only to an upward shift of the bottom of the conduction band. Disregarding fluctuations, it turns out therefore that $|\psi(x)|^2$ decreases like $\exp(-2x/b)$, and not according to Eqs. (1).

Other examples where the below-barrier scattering in a magnetic field is significant are the Franz-Keldysh and the Zener effects in crossed electric and magnetic fields.³ Scattering by impurities should increase strongly the probability of tunneling in the electric-field direction and change its dependence on the magnetic field. It is known⁴ that in a pure material, in the two-band approximation, the Zener effect vanishes completely in a sufficiently strong magnetic field. At the same time, when account is taken of scattering by impurities, the effect has no threshold magnetic field, i.e., Zener tunneling is possible in an arbitrarily strong magnetic field.

The law that governs the fall-off of the wave function at close distances plays an important role also in the theory of hopping conduction. This law determines the probability of hopping from impurity to impurity and the concentration dependence of the resistance. In a magnetic field, the law that governs the fall-off of the wave function determines the hopping magnetoresistivity.⁵ At low temperatures the length of the electron hop can exceed considerably the average distance between impurities, so that the electron can undergo multiple scattering in the course of tunneling. The existing theory does not take this phenomenon into account. Without a magnetic field, the scattering has little influence on the fall-off of the wave function but, as already mentioned, in a magnetic field the fall-off is substantially altered by the scattering, and this changes strongly the exponential dependence of the resistivity on the magnetic field.

The idea that the law governing the wave-function fall-off is altered by below-barrier scattering was advanced in a paper by one of us,⁶ where the temperature dependence of the hopping conductivity with variable hopping range in a strong magnetic field was obtained with allowance for below-barrier scattering.

In the present paper we calculate the value of b in Eq. (2) for tunnel transparency in the case when the impurity potential can be regarded as Gaussian with a δ -function correlator (white noise). Such a potential is produced, for example, by fluctuations in the composition in a substitutional solid solution. As a rule, in isoelectronic substitution the difference between the potentials of the atoms is found to be so weak, that the substituting atom produces no impurity level. Therefore the main contribution to the scattering is made by composition fluctuations that involve a large number of substituting atoms. Such fluctuations can be regarded with good accuracy as Gaussian.

The object of the investigation in this paper is the quantity $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$, where $G_\epsilon(\mathbf{r}, \mathbf{r}')$ is the Green's function of an

electron in a magnetic field and in a random impurity field. The energy ϵ of the tunneling electron lies lower than the renormalized bottom of the band. The symbol $\langle \dots \rangle$ denotes averaging over the impurity configurations.

The quantity $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$ can be used to investigate the transparency of a film. Let the film be bounded by the planes $x = 0$ and $x = L$, and let the magnetic field be directed along the z axis. It turns out that in an impurity-containing film the argument of the exponential of the tunnel transparency of the film coincides with the argument $\Phi_\epsilon(L)$ of the exponential in the quantity $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$ at $\mathbf{r} = (0, 0, 0)$ and $\mathbf{r}' = (L, 0, 0)$, i.e.,

$$\ln D/D_0 = \Phi_\epsilon(L). \quad (3)$$

In the absence of a magnetic field the property (3) is quite obvious. In a magnetic field, however, it appears only because of the scattering by the random impurities. In Sec. 3 we present expressions for the Green's function of a free electron with negative energy in a magnetic field [Eqs. (7) and (8)]. Comparing these equations with (1a) and (2b) we see that the numerical coefficients in the arguments of the exponentials are different. Consequently, it would be incorrect to calculate the transparency of a pure film by using Eq. (3). The reason is the interference of the waves that emerge from different points of the plane $x = 0$. In a magnetic field, two waves that follow different paths acquire a phase difference proportional to the magnetic flux linked with the contour enveloping these paths and bounded by the plane $x = 0$. As a result, waves from closely located points suppress one another.

In a film with impurities this interference is inessential. The point is that waves that follow different paths are scattered by different sections of the Gaussian potential. Since the correlation radius of the potential is assumed to be very small, the phases of these waves are not correlated at all and their intensities are additive. To calculate the argument of the exponential of the tunnel transparency it suffices therefore to find the probability of tunneling along the shortest path, i.e., along a path perpendicular to the film, and average the probability over the impurity configurations. The result is Eq. (3).

To calculate the pre-exponential factor D_0 we must formulate correctly the boundary condition, as was done in Ref. 2.

When $|G_\epsilon(\mathbf{r}, \mathbf{r}')|^2$ is averaged over the impurity configurations, the main contribution is made by the configurations corresponding to exponentially large values of $|G_\epsilon(\mathbf{r}, \mathbf{r}')|^2$, although their probability can be exponentially small. This rule reflects the physics of the film problem, since the film transparency D is determined by the most transparent sections with which an exponentially small fraction of the surface can be connected. To calculate the influence of the scattering on the Franz-Keldysh and Zener effects in crossed electric and magnetic fields we also need the quantity $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$, but calculated with a uniform electric field taken into account.

It must be borne in mind that in other physical problems the averaging of the quantity $|G_\epsilon(\mathbf{r}, \mathbf{r}')|^2$ may turn out to be an utterly inadequate procedure. Consider, for example the problem of hopping conduction with a large hopping range in a semiconductor doped, say, with donors. We choose in the sample all the donor pairs joined by the vector $\mathbf{r}' - \mathbf{r}$. The values of $|G_\epsilon(\mathbf{r}, \mathbf{r}')|^2$ for different pairs can differ greatly from one another. The mean value $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$ is determined by the pairs for which the Green's function has an anomalously large modulus. The donors that form such a "bound" pair are connected by a chain of other donors that are anomalously close to one another and are located on a straight line drawn from \mathbf{r} to \mathbf{r}' . The bound pairs can be exceedingly sparse and be very far from one another in space. It is clear that such pairs cannot determine the hopping conductivity, which requires that the donors be connected by paths that penetrate through the entire sample. The use of the quantity $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$ to calculate the donor overlap integral in the theory of electric hopping conduction can greatly underestimate the resistivity.

To calculate the hopping conductivity by percolation theory we must know the distribution of the arguments of the exponentials of $|G_\epsilon(\mathbf{r}, \mathbf{r}')|^2$ for pairs with specified values of $\mathbf{r}' - \mathbf{r}$. If many other donors are located between the points \mathbf{r}' and \mathbf{r} , the variance of the argument of the exponential of $|G_\epsilon(\mathbf{r}, \mathbf{r}')|^2$ is relatively small. We can therefore confine ourselves to calculation of the mean value of the argument of the exponential, i.e., of the quantity $\langle \ln |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$. This quantity, in contrast to $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$, is determined by the typical zigzag chains that join the points \mathbf{r} and \mathbf{r}' and in which the donors are separated from one another by distances of the order of the mean distance between them.⁶ As a result the quantities $\langle \ln |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$ and $\ln \langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$ differ greatly from each other. The possibility of such a difference was already noted in the monograph of Lifshitz, Gredeskul, and Pastur.⁷

The model studies in this paper, that of a Gaussian random potential, has, in contrast to the just-considered case, of an impurity band made up of states localized on donors, a unique property that follows from the assumption that the weakly scattering centers have a high density:

$$|\ln \langle |G_\epsilon|^2 \rangle - \langle \ln |G_\epsilon|^2 \rangle| \ll |\ln \langle |G_\epsilon|^2 \rangle| \quad (4)$$

(the mean logarithm is close to the logarithm of the mean). This property reflects the relative weakness of the fluctuations of $|G_\epsilon(\mathbf{r}, \mathbf{r}')|^2$ and will be discussed below. Here we note only that it allows us to use the quantity $\ln \langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$ to construct a theory of hopping conduction of solid solutions of semiconductors. We have in mind here that a solid solution doped by non-isoelectronic impurities that produce local states (e.g., donors). The conduction is due to hops over these donors. In the space between the donors, the electron is scattered by the fluctuations of the composition of the solid solution, and this scattering forms the wave function of the electron. According to the foregoing, its asymptotic form can be described with the aid of the quantity $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$ calculated in the present paper.

2. QUALITATIVE DERIVATION OF THE ARGUMENT OF THE EXPONENTIAL OF $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$

The potential $V(\mathbf{r})$ connected with the fluctuations of the composition in a solid solution $A_y B_{1-y}$ is a Gaussian random function with a δ -function correlator (white noise);

$$\langle V(\mathbf{r}) V(\mathbf{r}') \rangle = \gamma \delta(\mathbf{r} - \mathbf{r}'), \quad (5)$$

where

$$\gamma = (\alpha^2/N) y(1-y), \quad \alpha = dE_c/dy,$$

and $E_c(y)$ is the energy of the bottom of the conduction band of the solution $A_y B_{1-y}$, and N is the density of the lattice sites at which the atoms A and B can be located. Equation (5) is valid under the condition that the lengths that are significant in the problem greatly exceed the mean distance between the atoms of the component with the minimum density. The Green's function $G_\epsilon(\mathbf{r}, \mathbf{r}')$ can be represented by a series

$$G_\epsilon(\mathbf{r}, \mathbf{r}') = G_\epsilon^0(\mathbf{r}, \mathbf{r}') + \int d\mathbf{r}_1 G_\epsilon^0(\mathbf{r}, \mathbf{r}_1) V(\mathbf{r}_1) G_\epsilon^0(\mathbf{r}_1, \mathbf{r}') + \int d\mathbf{r}_1 d\mathbf{r}_2 G_\epsilon^0(\mathbf{r}, \mathbf{r}_1) V(\mathbf{r}_1) G_\epsilon^0(\mathbf{r}_1, \mathbf{r}_2) V(\mathbf{r}_2) G_\epsilon^0(\mathbf{r}_2, \mathbf{r}'), + \dots, \quad (6)$$

where $G_\epsilon^0(\mathbf{r}, \mathbf{r}')$ is the free Green's function in a magnetic field with negative energy. It is of the form

$$G_\epsilon^0(\mathbf{r}, \mathbf{r}') = \left(\frac{m}{2\pi\hbar^2} \right) \frac{1}{R} \exp \left\{ -\frac{R}{a} - \frac{R^3 a}{24\lambda^4} \sin^2 \vartheta \right\} e^{-i\Phi(\mathbf{r}, \mathbf{r}')}, \quad (7)$$

at $\lambda \gg a$, $R^2 \sin^2 \vartheta \ll \lambda^4/a^2$, and

$$G_\epsilon^0(\mathbf{r}, \mathbf{r}') = \left(\frac{m}{2\pi\hbar^2} \right) \frac{a}{\lambda^2} \exp \left(-\frac{R|\cos \vartheta|}{a} - \frac{R^2 \sin^2 \vartheta}{4\lambda^2} \right) e^{-i\Phi(\mathbf{r}, \mathbf{r}')}, \quad (8)$$

at $\lambda \ll a$. In (7) and (8), $R \equiv |\mathbf{r}' - \mathbf{r}|$, ϑ is the angle between the vectors $\mathbf{r}' - \mathbf{r}$ and \mathbf{H} , and

$$\Phi(\mathbf{r}, \mathbf{r}') = (ie/2\hbar c) \mathbf{H}[\mathbf{r}, \times \mathbf{r}']$$

is the phase determined by the choice of the vector potential.

Let us investigate the asymptotic form of the Green's function $G_\epsilon(0, x)$ along the x axis perpendicular to the magnetic field. We consider that term of the series (6) which describes n scattering acts, and define as the scattering region (SR) that region of space in which is concentrated the integration of this term of the series with respect to one of the n variables. Each scattering act takes place in one of the points of the corresponding SR. Assume that the SR are equidistant along the x axis and that their dimensions are small compared with the distance between them (see Fig. 2). Then the argument of the exponential of the product of $n+1$ free Green's functions in this term of the series turns out to be smaller than the argument of the exponential of the function $G_\epsilon^0(0, x)$. For example, at $\lambda \gg a$ the argument of the exponential of the product is

$$-\left[\frac{x}{a} + (n+1) \left(\frac{x}{n+1} \right)^{\frac{1}{n}} \frac{a}{24\lambda^4} \right], \quad (9)$$

whereas the argument of the exponential of $G_\epsilon^0(0, x)$ is equal to $-(x/a + x^3 a/24\lambda^4)$. Thus, the scattering decreases

strongly the probability of the below-barrier tunneling connected with the magnetic field, and the effect is stronger the larger the number of scattering acts. (It is easily shown that at a fixed n and at a nonequidistant spacing of the SR along the x axis the gain due to the scattering is decreased.) On the other hand, the probability of each scattering is low. This means that if each integral with respect to \mathbf{r}_i is set in correspondence with a dimensionless factor B it turns out that $B \ll 1$. The point is that the volume of each SR is relatively small. The transverse dimension ρ of the SR cannot increase strongly because in this case the tunneling paths become more and more zigzag-like and their length increases, and with increasing longitudinal dimension h of the SR the scattering points become nonequidistant, thereby decreasing the gain described by Eq. (9). Thus, the $(n+1)$ -st term of the series (6) is proportional to

$$B^n \exp\left(-\frac{x}{a} - (n+1)\left(\frac{x}{n+1}\right)^3 \frac{a}{24\lambda^4}\right). \quad (10)$$

Expression (10) has a sharp extremum with respect to n at

$$n = n_{\max} = \left(\frac{x^3 a}{12\lambda^4}\right)^{1/3} |\ln B|^{-1/3} - 1. \quad (11)$$

Substituting (11) in (10) we find that at $\lambda \gg a$

$$G_s(0, x) \approx \exp\left[-\frac{x}{a} - \frac{3^{2/3}}{2^{4/3}} \frac{x}{a} \left(\frac{a}{\lambda}\right)^{4/3} |\ln B|^{1/3}\right] \approx e^{-x/b}, \quad (12)$$

where

$$b = a \left[1 - \frac{3^{2/3}}{2^{4/3}} \left(\frac{a}{\lambda}\right)^{4/3} |\ln B|^{1/3}\right]. \quad (13)$$

In the derivation of (12) and (13) we used for the Green's function Eq. (7), which is valid if $R \ll \lambda^2/a$. This is correct if the distance between neighboring SR

$$R_0 = \frac{x}{n_{\max} + 1} \approx \left(\frac{12\lambda^4}{a}\right)^{1/3} |\ln B|^{1/3} \quad (14)$$

satisfies the inequality $R_0 \ll \lambda^2/a$, which follows in fact from the condition $\lambda \gg a$.

In a strong magnetic field, when $\lambda \ll a$, we obtain for the $(n+1)$ -st term of the series (6)

$$B^n \exp\left[-(n+1)\left(\frac{x}{n+1}\right)^2 \frac{1}{4\lambda^2}\right]. \quad (15)$$

This expression has a sharp maximum at

$$n = n_{\max} = \frac{x}{2\lambda} |\ln B|^{-1/2} - 1. \quad (16)$$

Substitution of (16) and (15) gives at $\lambda \ll a$

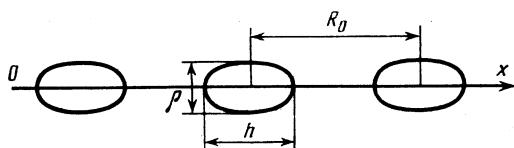


FIG. 2. Chain of scattering regions along the electron tunneling path for $n = 3$.

$$G_s(0, x) \propto \exp\left(-\frac{x}{\lambda} |\ln B|^{1/2}\right). \quad (17)$$

Comparing (17) and (12), (13) with (1) or (7), (8) we see that the scattering does indeed alter radically the character of the falloff of the wave function across the field. Expressions (12) and (17) are valid under the condition that the length x is large enough, so that $n_{\max} \gg 1$.

In the foregoing derivation we have neglected the phase factors in (7) and (8). The possibility of this simplification will be confirmed in the next section. It is physically connected with the fact that the phase of the wave function varies randomly because of the scattering, so that there is little interference between the scattered waves.

We discuss now the parameter B in greater detail. We shall show below that at $\lambda \gg a$

$$B = \frac{m}{2\pi\hbar^2} \frac{\alpha}{N} \frac{[N\Omega y(1-y)]^{1/2}}{R_0}, \quad (18)$$

where

$$\Omega = 3^{1/2} \cdot 2^{4/3} \pi^{3/2} \lambda^{3/2} a^{1/2} |\ln B|^{1/2}, \quad (19)$$

is the volume of the SR. It will be shown below that the SR dimensions are approximately

$$\lambda \approx \lambda^2 / (R_0 a)^{1/2} \approx R_0 |\ln B|^{-1/2}, \quad (20)$$

$$\rho \approx (R_0 a)^{1/2} \approx \lambda^{3/2} a^{1/2} |\ln B|^{1/4}. \quad (21)$$

In the preceding reasoning we used the fact that $B \ll 1$. It can be seen from (20) that the longitudinal dimension h of the SR is smaller than the distance between the neighboring SR only if this condition is satisfied. To clarify the physical meaning of the condition $B \ll 1$, we use expressions (19) and (14) and transform (18) into

$$B |\ln B|^{1/2} = (178E_0/48\pi\epsilon)^{1/2}, \quad (22)$$

where

$$E_0 = \alpha^4 y^2 (1-y)^2 m^3 / 178\hbar^3 N^2$$

is the characteristic energy of the tail of the state density $g(\epsilon)$ in the forbidden band; this tail is connected with the fluctuations of the composition of the solid solution⁸:

$$g(\epsilon) \propto e^{-(\epsilon/E_0)^{1/2}}. \quad (23)$$

At $\epsilon \gg E_0$ the density of states $g(\epsilon)$ is very small and the states are localized in individual potential wells. At a distance of the order of E_0 from the renormalized bottom of the band, the states become delocalized. We see thus from (22) that the condition $B \ll 1$ coincides with the condition that the states with energy ϵ are localized and are far from the mobility threshold.

With decreasing ϵ/E_0 , the parameter B increases and

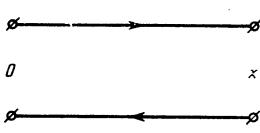


FIG. 3.

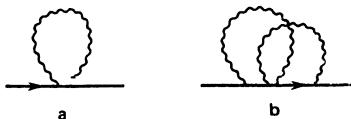


FIG. 4.

the scattering becomes more favorable. According to (20) and (14), at $|\ln B| \gtrsim 1$ the length h becomes comparable with R_0 , i.e., the gaps between the neighboring SR vanish. Moreover, as seen from (11) and (16), as $B \rightarrow 1$ the quantity n_{\max} becomes infinite, attesting to a strong bending of the quantum trajectories and to a tendency to delocalization of the electron states. Of course, the theory expounded above is not intended for a description of the Anderson transition.

3. CALCULATION OF $\langle |G_\epsilon(\mathbf{r}, \mathbf{r}')|^2 \rangle$

We shall use a diagram technique in coordinate space. The free Green's function $G_\epsilon^0(\mathbf{r}_1, \mathbf{r}_2)$ defined by Eqs. (7) and (8) will be represented by a solid line, and the interaction line will be wavy. Since the Green's function in a magnetic field is complex, it is necessary to take into account the direction, marked by an arrow, of the solid lines. Since the random-potential correlator (5) is of the form of a δ -function, each wavy line must start and end at one coordinate point. To each wavy line corresponds a factor γ . Integration is carried out with respect to the coordinates of all the points to which wavy lines are secured. In each diagram for $\langle |G(0, x)|^2 \rangle$ the points 0 with coordinates $(0, 0, 0)$ and $(x, 0, 0)$ are "terminals": one line enters and one line leaves each of them. The points on the diagrams are assumed ordered along the x axis. Figure 3 shows a diagram without interaction. Figure 4 shows two diagrams for the self-energy part of the Green's function. The diagram of Fig. 4a contains the Green's function at $\mathbf{r}_1 - \mathbf{r}_2 = 0$. Such a diagram diverges, thus indicating that it was incorrect to use a δ -function correlator. As shown, for example in Ref. 9, it must be assumed that the δ -function is "smeared" over a small length, and this length determines only the shift of the edge of the band. It can be assumed that the energy is reckoned from the renormalized edge and the diagram of Fig. 4a can be disregarded. The diagrams of the type shown in Fig. 4b (and more complicated ones) for the self-energy part describe the appearance of levels in the forbidden band. They can be disregarded in the study of nonresonant tunneling. We consider now diagrams on which are connected lines going in different directions. Diagrams "with backtracking," one of which is shown in Fig. 5a, need not be taken into account. Indeed the diagram in Fig. 5a is clearly much smaller than that of Fig. 5b, since on the section (x_1, x_2) the diagram 5a contains not two but four Green's functions, each of which has a small tunnel exponential. It turns out as a result that it is necessary to sum only the "ladder" diagrams shown in Figs. 5b and 6.

The contribution of a diagram containing n wavy lines is

$$\langle |G_n|^2 \rangle = \gamma^n \int |G^0(0, \mathbf{r}_1)|^2 |G^0(\mathbf{r}_1, \mathbf{r}_2)|^2 \dots |G^0(\mathbf{r}_n, \mathbf{r}_{n+1})|^2 d\mathbf{r}_1 \dots d\mathbf{r}_n. \quad (24)$$

As seen from (24), in this approximation the phase factors of the Green's function are negligible. The derivation that follows is for the case of a weak magnetic field, when expression (7) can be used for the Green's function. The argument of the exponential in the integrand of (24) is of the form

$$\sum_{i=0}^n \left(-\frac{2|\mathbf{r}_{i,i+1}|}{a} - \frac{|\mathbf{r}_{i,i+1}|^3 \beta}{a^3} \right), \quad (25)$$

where $\mathbf{r}_{i,i+1} = \mathbf{r}_{i+1} - \mathbf{r}_i$, $\beta = a^4 \sin^2 \vartheta / 12\lambda^4$, and ϑ is the angle between the magnetic field and the x axis. It is assumed that $\mathbf{r}_0 = 0$ and $\mathbf{r}_{n+1} = (x, 0, 0)$. As already mentioned, the transverse dimensions of the SR are much smaller than the distances between the SR. Assuming this beforehand, we expand the first term of (25) in terms of the ratios $y_{i,i+1}/x_{i,i+1}$ and $z_{i,i+1}/x_{i,i+1}$. In the second term, which is small compared with the first, it suffices to put $r_{i,i+1} = x_{i,i+1}$. Using the ratio

$$\frac{2x}{a} = \sum_{i=0}^n \frac{2x_{i,i+1}}{a},$$

we can write the argument of the exponential in (24) in the form $-2x/a - F$, where

$$F = \sum_{i=0}^n \frac{y_{i,i+1}^2 + z_{i,i+1}^2}{x_{i,i+1}a} + \frac{x_{i,i+1}\beta}{a^3}. \quad (26)$$

The integration in (24) is by the saddle-point method in $3n$ -dimensional coordinate space. As seen from (26). At the saddle point $y_i = z_i = 0$ for all i from 0 to $n+1$. Since $x_{n+1} = x$, the values of x_i at the saddle point are not equal to zero. Differentiating F at $y_i = z_i = 0$ we obtain

$$\partial F / \partial x_i = 3(x_i - x_{i-1})^2 - 3(x_{i+1} - x_i)^2. \quad (27)$$

Equating to zero the right-hand sides of (27), we find that at the extremum the points x_i are equidistant, so that $x_{i,i+1} = x/(n+1)$. The function F at the saddle point is equal to

$$F_0 = (n+1) [x/(n+1)]^3 \beta/a^3. \quad (28)$$

To calculate the integrals we must find the second derivative of the function F at the saddle point:

$$\begin{aligned} \frac{\partial^2 F}{\partial x_i^2} &= \frac{12\beta x}{a^3(n+1)}, & \frac{\partial^2 F}{\partial x_i \partial x_{i+1}} &= -\frac{6\beta x}{a^3(n+1)}, \\ \frac{\partial^2 F}{\partial y_i^2} &= \frac{\partial^2 F}{\partial z_i^2} = \frac{4(n+1)}{xa}, & & \\ \frac{\partial^2 F}{\partial y_i \partial y_{i+1}} &= \frac{\partial^2 F}{\partial z_i \partial z_{i+1}} = -\frac{2(n+1)}{xa}. & & \end{aligned} \quad (29)$$

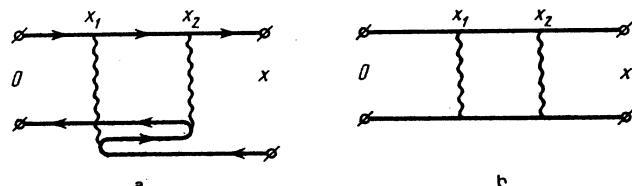


FIG. 5.

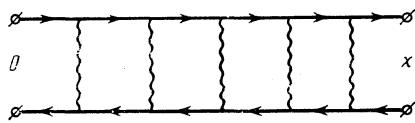


FIG. 6.

Here i ranges from 1 to n . The other second derivatives are equal to zero.

When calculating the integral (24) it is necessary to replace R in the pre-exponential factors of the Green's functions by $x/(n+1)$. Then

$$\langle |G_n|^2 \rangle = \gamma^n \left[\left(\frac{m}{2\pi\hbar^2} \right)^2 \left(\frac{n+1}{x} \right)^2 \right]^{n+1} \times e^{-F_0} \int \dots \int e^{-(r-r_0)} dr_1 \dots dr_n. \quad (30)$$

We expand $F - F_0$ near the saddle point, using the second derivatives of (29), and introduce the new variables

$$x_i' = \left[\frac{6\beta x}{a^3(n+1)} \right]^{\frac{1}{2}} x_i, \quad y_i' = \left[\frac{2(n+1)}{xa} \right]^{\frac{1}{2}} y_i, \\ z_i' = \left[\frac{2(n+1)}{xa} \right]^{\frac{1}{2}} z_i.$$

We obtain

$$\langle |G_n|^2 \rangle = \gamma^n \left[\left(\frac{m}{2\pi\hbar^2} \right)^2 \left(\frac{n+1}{x} \right)^2 \right]^{n+1} \times e^{-F_0} \left[\left(\frac{a^3(n+1)}{6\beta x} \right)^{\frac{1}{2}} \frac{xa}{2(n+1)} \right]^n Q_n^3, \quad (31)$$

$$Q_n = \int_{-\infty}^{\infty} dx_1 \dots \int_{-\infty}^{\infty} dx_n \exp \left\{ -\frac{1}{2} \Gamma_{ik}^{(n)} x_i x_k \right\} = \frac{(2\pi)^{n/2}}{(\det \Gamma^{(n)})^{\frac{1}{2}}}, \quad (32)$$

$$\Gamma^{(n)} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \end{pmatrix}. \quad (33)$$

It is known that

$$\det \Gamma^{(n)} = (-1)^n C_n^1(-1) = n+1,$$

where $C_n^1(-1)$ is a Gegenbauer polynomial equal to $(-1)^n(n+1)$. As a result we get

$$\langle |G_n|^2 \rangle = \left(\frac{m}{2\pi\hbar^2} \right)^2 \left(\frac{n+1}{x} \right)^2 \frac{1}{(n+1)^{\frac{1}{2}}} \exp \{-F_0(n) + n \ln B_n^2\}, \quad (34)$$

$$B_n^2 = \frac{\pi^{\frac{n}{2}}}{\sqrt{3}} \gamma \left(\frac{m}{2\pi\hbar^2} \right)^2 \left(\frac{n+1}{x} \right)^{\frac{n}{2}} \frac{a^{\frac{n}{2}}}{\beta^{\frac{n}{2}}}. \quad (35)$$

The quantity $\langle |G_n|^2 \rangle$ as a function of n has a sharp maximum at $n = n_{\max} \gg 1$. Therefore in the calculation of

$$\langle |G|^2 \rangle = \sum_n \langle |G_n|^2 \rangle$$

we can replace the sum over n by an integral, which can be calculated by the saddle-point method. The expression in the

argument of the exponential of (34) is a maximum at

$$n = n_{\max} = 2^{\frac{1}{2}} x \beta^{\frac{1}{2}} / a |\ln B^2|^{-\frac{1}{2}} - 1, \quad (36)$$

where B^2 satisfies the transcendental equation

$$B^2 |\ln B^2|^{-\frac{1}{2}} = \gamma \left(\frac{m}{2\pi\hbar^2} \right)^2 \left(\frac{2}{3} \right)^{\frac{1}{2}} \pi^{\frac{n}{2}} a = 2^{\frac{1}{2}} \left(\frac{178E_0}{48\pi e} \right)^{\frac{1}{2}}. \quad (37)$$

Equations (11) and (22) coincide with (36) and (37). Calculation by the saddle-point method yield

$$\langle |G_e|^2 \rangle = \left(\frac{2\pi}{3} \right)^{\frac{1}{2}} \left(\frac{m}{2\pi\hbar^2} \right)^2 \left(\frac{a^4 \sin^2 \vartheta}{6\lambda^4} \right)^{\frac{1}{2}} \frac{1}{B^2 |\ln B^2|^{\frac{1}{2}}} \\ \times \exp \left\{ -\frac{2x}{a} - \left(\frac{3}{2} \right)^{\frac{1}{2}} \frac{x}{a} \left(\frac{a^4 \sin^2 \vartheta}{\lambda^4} \right)^{\frac{1}{2}} |\ln B|^{-\frac{1}{2}} \right\}. \quad (38)$$

As expected, at $\sin \vartheta = 1$ the argument of the exponential in (38) is larger than the one in (12).

4. FLUCTUATIONS OF THE GREEN'S FUNCTION

We discuss now the differences that should arise when averaging $\ln |G_e(r, r')|^2$ and $|G_e(r, r')|^2$. This calls for identification of the character of the Green's-function fluctuations connected with various configurations of the impurity in the space between the points r and r' . We shall do this very approximately, assuming that each term of the series (6) breaks up into a product of integrals over the volumes of the individual SR. In the arguments of the Green's functions we substitute then the differences of the coordinates of the centers of neighboring SR. We denote the dimensionless factor that results from integration with respect to i th SR by B_i . The quantity B_i is proportional to $\int V(r) d^3r$, where the integration is over the volume of the i th SR, and $V(r)$ is a random Gaussian potential. The quantities B_i are random and cause the fluctuations of the Green's function. The fluctuations of B_i are due to the fact that the potential $V(r)$, averaged over the given SR, has random sign and magnitude. The mean value of B_i is zero, and the mean square is $\langle B_i^2 \rangle^{1/2} = B$ and is determined by (18). It is natural for this equation to contain the quantity $[N\Omega y(1-y)]^{1/2}$ which, roughly speaking, is the mean squared fluctuation of the number of substitutional atoms in the SR. Clearly, the condition for the applicability of Gaussian statistics to the fluctuations of the number of substitutional atoms in the SR and of the quantities B_i is of the form

$$N\Omega y(1-y) \gg 1. \quad (39)$$

If this inequality is satisfied, the probability density of B_i takes the form

$$\frac{1}{(2\pi)^{\frac{1}{2}} B} \exp \left(-\frac{B_i^2}{2B^2} \right).$$

The square of the Green's function includes the product

$$\prod_{i=1}^{n_{\max}} B_i^2.$$

Therefore

$$\langle \ln|G|^2 \rangle = \left\langle \sum_{i=1}^{n_{\max}} \ln B_i^2 \right\rangle = n_{\max} \langle \ln B_i^2 \rangle, \quad (40)$$

whereas

$$\ln \langle |G|^2 \rangle = \ln \prod_{i=1}^{n_{\max}} \langle B_i^2 \rangle = n_{\max} \ln B^2. \quad (41)$$

It follows therefore that

$$\begin{aligned} \langle \ln|G|^2 \rangle - \ln \langle |G|^2 \rangle &= n_{\max} \left\langle \ln \frac{B_i^2}{B^2} \right\rangle \\ &= n_{\max} \int_{-\infty}^{\infty} \ln x^2 \exp(-x^2/2) dx = -1.27 n_{\max}. \end{aligned} \quad (42)$$

Comparing (42) with (40) and (41) we see that the inequality (4) is satisfied. The derivation presented makes no claims with respect to the numerical coefficient in (42), but it seems to us that the correct order of magnitude of the difference $\langle \ln|G|^2 \rangle - \ln \langle |G|^2 \rangle$ was obtained.

5. CONCLUSION

We have investigated in this paper the influence of the magnetic field on nonresonant tunneling in a solid solution. We examine now when the resonant tunneling can be neglected. According to (23), at the energies $\varepsilon \gg E_0$ of interest to us, the density of the localized states connected with the composition fluctuations of the solid solution is small but finite. If a state with energy ε is encountered approximately half-way between the points r and r' , the exponentially small factor in $G_\varepsilon(r, r')$ vanishes because of the resonance.^{1,2}

For hopping conduction over donors with level energy $\varepsilon \gg E_0$, the presence of exponentially sparse donor pairs, between which the fluctuation state turns out to be accidentally resonant, is negligible. Therefore the resonant scattering does not demarcate the regions of applicability of our results in the theory of hopping conduction. Things are different in the case of tunneling through the film or in the case of the Franz-Keldysh and the Zener effects. In these cases even exponentially sparse resonant configurations can determine the transparency. For example, in the case of a film the transparency turns out to be proportional to $g(\varepsilon)\Delta\varepsilon$, where

$g(\varepsilon)$ is the density of states (23) and $\Delta\varepsilon$ is the level width of a local state with energy ε , located midway in the film. (The width is connected with the tunneling in vacuum.) For our theory to be valid in a weak magnetic field it is necessary that the nonresonant transparency in a zero field prevail over the resonant. Since $\Delta\varepsilon \propto \exp(-L/a)$ at $H = 0$, the condition under which the nonresonant transparency prevails is of the form

$$e^{-2L/a} \gg e^{-(\varepsilon/E_0)^{1/2}} e^{-L/a}, \quad (43)$$

i.e.,

$$L < a(\varepsilon/E_0)^{1/4} = \hbar/(2mE_0)^{1/4} \equiv L_0, \quad (44)$$

where L_0 is the characteristic radius of the composition fluctuations that produce the localized states.⁸

It must be noted, however, that in the investigation of the influence of the magnetic field on the resonant tunneling it is also necessary to take into account the multiple nonresonant scattering by impurities, which changes the value of $\Delta\varepsilon$. This question is outside the scope of the present paper.

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