Transformation of the spatial statistics of a partially coherent light beam in a nonlinear medium

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The self-consistent problem of the self-action of a multimode light beam in a nonlinear cubic medium is treated by the Monte Carlo method. The transformation of the statistics of the intensity and phase distributions of the radiation is investigated, and the behavior of the mean intensity and the dispersion of its fluctuations, as well as the spatial coherence range both along and across the beam, are examined. It is shown that the field distribution of a broad-band beam retains its normal form under self-focusing, whereas self-focusing of a narrow-band beam results in the beam breaking up into filaments, and consequently, in a strong change in the intensity distribution. The phase distribution remains uniform, however, even in this case.

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I. INTRODUCTION

By now the basic laws of the self-action of beams of coherent radiation have been fairly well investigated (see, e.g., Refs. 1 and 2). Our understanding of the processes that take place in the self-action of incoherent radiation has not yet reached such a high level.

Thus, a perturbation method has been successfully used to trace the initial stage of the transformation of the spatial statistics of a plane wave[2] and a bounded beam,[3] both modulated by weak noise. It was shown that in a nonlinear cubic medium these perturbations are unstable against small amplitude and phase fluctuations in a certain range of spatial frequencies, and as a result, the perturbations increase. This has been used to explain the phenomenon of small-scale self-focusing—the breakup of a beam into filaments under its self-action—which has been observed in a number of experiments.

The transformation of the radii of the envelope and the spatial coherence under self-action of a beam with a broad spatial-temporal radiation spectrum has been investigated in Ref. 4 in the aberration-free approximation, and in Ref. 5 by numerical methods. In those papers the problem was solved using a closed equation for the second-order spatial correlation function. As is well known, however, such an equation is obtained by expressing the higher moments of the field as products of the second moments. Although it is clear from general physical considerations that such a procedure can be justified if the coherence time is so short that the nonlinear medium cannot "keep up" with the rapid fluctuations of the wave field and the induced refractive index will be determined by the mean intensity of the beam, nevertheless, not even considering the necessity of estimating the errors of such an approximation, an analysis of the closed equation does not afford the possibility of tracing the transformation of the statistics of the amplitude and phase distribution, nor of investigating the fluctuations of the light-wave field, etc. And finally, the above technique becomes entirely inapplicable in the case of the self-action of radiation having a narrow frequency spectrum.

Perhaps the only way to carry through a comprehensive analysis of the self-action of a multimode light beam is to solve the nonlinear parabolic equation exactly, using random simulations of the wave field with given statistical properties as initial conditions, and then to average these solutions. Our work is based on this method of statistical trials (the Monte-Carlo method).

We have investigated the self-action of spatially incoherent two-dimensional light beams with an arbitrarily broad frequency spectrum. In that investigation the problem was solved in three stages.

In the first stage, random complex fields having Gaussian statistics with specified mean values of the intensity profile, the beam radii, and the spatial correlation were simulated.

Next, each simulated random field was used as the initial condition for the nonlinear parabolic equation, which was solved numerically for a local cubic nonlinearity with allowance for the time lag of the latter. All the simulated fields with the same duration τ were introduced steadily, one after another, so in our model of multimode radiation the duration τ represents the coherence time of the radiation.

In the last stage we averaged the solutions thus obtained. We used up to 1400 solutions for a single set of initial conditions in order to achieve good accuracy and to be able to construct reliable histograms.

2. MATHEMATICAL FORMULATION OF THE PROBLEM. SIMULATION OF THE RADIATION OF A MULTIMODE LASER

Let us assume that a partially coherent two-dimensional light beam is propagating along the z axis in a nonlinear cubic medium. The equation for the self-action of a light wave with the complex amplitude A has the form

$$\left(\frac{\partial}{\partial z}+\frac{i}{2k}\frac{\partial^2}{\partial x^2}\right)A(x,z,\eta)=\frac{ik}{2\varepsilon_0}\varepsilon_{nl}A(x,z,\eta). \qquad (1)$$

Here $\eta = t - z/u_0$ is the local time, u_0 is the group velocity, k is the wave number, ε_{nl} is the nonlinear addition of the dielectric constant ε_0, which for a relaxing cubic medium is

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determined by the equation

$$\left(\frac{\partial}{\partial \eta}+\frac{1}{\tau_{nl}}\right)\varepsilon_{nl}=\frac{\varepsilon_2}{\tau_{nl}}|A|^2, \quad (2)$$

in which τ_{nl} is the characteristic relaxation time for the nonlinearity. Since the complex amplitude $A(x,z,\eta)$ is a random function of the coordinates and time, at the entrance to the medium ($z=0$) any of its realizations (say the l-th) will have the form

$$A^{(l)}(x,0,\eta)=u^{(l)}(x)+iv^{(l)}(x), \quad (3)$$

where the random functions u and v for a specific realization do not depend on η.

We shall assume a given initial intercoherence function

$$\Gamma(x_1,x_2,0)=\langle A(x_1,0)A^*(x_2,0)\rangle$$
$$=I_0\exp\left[-\frac{x_1^2}{2a_0^2}-\frac{x_2^2}{2a_0^2}-\frac{(x_1-x_2)^2}{4r_{c0}^2}\right], \quad (4)$$

in which a_0 and r_{c0} are the initial values of the beam radius and the correlation length. Taking into account what has been said above and using a computer programed to follow the algorithm described in Ref. 6, we generated two statistically independent pseudorandom vectors $(u_1,u_2,\ldots,u_m,\ldots,u_n)$ and $(v_1,v_2,\ldots,v_m,\ldots,v_n)$ having multidimensional normal distributions with zero means and specified covariation matrices $\langle u_m u_{m'}\rangle$ and $\langle v_m v_{m'}\rangle$ whose forms are determined by the initial condition.[4] Each l-th realization of the vectors $(u_1^{(l)},\ldots,u_n^{(l)})$ and $(v_1^{(l)},\ldots,v_n^{(l)})$ corresponded to distributions of the real and imaginary parts of the complex amplitude among n points on the x axis. The complex random fields thus simulated have Gaussian statistics and adequately simulate the radiation of a multimode laser.[7]

The initial mean intensity profile

$$\langle I(x,0)\rangle=\langle A(x,0)A^*(x,0)\rangle$$

and the degree of coherence

$$\gamma(x_1,x_2,0)=\Gamma(x_1,x_2,0)\left[\langle I(x_1,0)\rangle\langle I(x_2,0)\rangle\right]^{-1/2}$$

agree very accurately with Eq. (4), while for the relative dispersion of the intensity fluctuations we have

$$\sigma_I(x,0)=\left[\langle I^2(x,0)\rangle-\langle I(x,0)\rangle^2\right]^{1/2}\langle I(x,0)\rangle^{-1}=1$$

within the initial cross section of the beam.

Since u and v are distributed normally, the intensity $I=u^2+v^2$ should have the exponential distribution

$$\omega(I)=\langle I\rangle^{-1}\exp\left[-I/\langle I\rangle\right],$$

while the phase should be uniformly distributed. Figure 1 shows the computer-calculated histogram for the intensity distribution at the center of the beam at $z=0$ while the full curve, which is given for comparison, is a graph of the above exponential. The size of the sample used in the numerical calculations ($L=1400$) ensures the reliability of the results presented below.

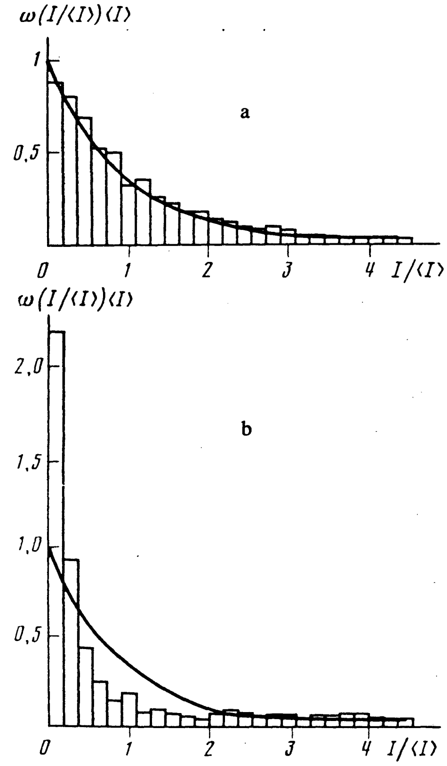

FIG. 1. Probability density distribution for the intensity on the axis of a narrow-band beam ($\delta=0$) for $p=40$ and $N=3$ at the points $z=0$ (a) and $z=0.15ka_0^2$ (b).

3. NUMERICAL CALCULATIONS

The computer simulations of the random field serve as initial conditions for the parabolic equation (1). The following quantities were varied in the numerical solution of that equation: the parameter $p=P_0/P_{cr}$, where

$$P_0=c\varepsilon_0^{1/2}I_0a_0/8\pi^{1/2},$$

is the input power flux and

$$P_{cr}=c\varepsilon_0^{1/2}/8\pi^{1/2}a_0k^2\varepsilon_2$$

is the critical power for self-focusing of the two-dimensional coherent beam; the number $N=a_0/r_{c0}$ of inhomogeneities in the initial cross section of the beam, which characterizes the transverse mode composition of the radiation; and the parameter $\delta=\tau_{nl}/\tau$ (τ is the coherence time), which is proportional to the width of the frequency spectrum.

The dashed curve on Fig. 2,a depicts an individual random simulation of the initial intensity profile, while those in Figs. 2,b and 2,c show that profile as modified by passage through the nonlinear medium to the point $z=0.15\,ka_0^2$ for the cases of narrow-band ($\delta=0$) and broad-band ($\delta=100$) radiation, respectively. The full curves show the mean intensity profiles obtained by averaging over 1400 simulations. The mean power of the beam as it entered the medium was $P_0=20P_{cr}$. In each of these figures, the bar near the top shows the spatial coherency range near the beam axis and is

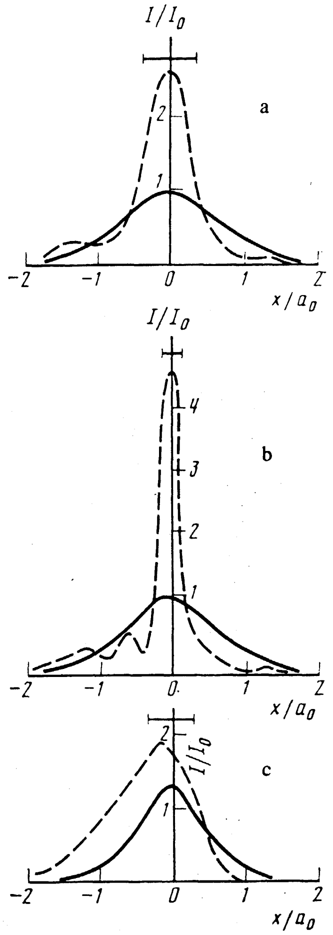

FIG. 2. Average (solid curves) and instantaneous (dashed curves) intensity profiles for a beam with $p = 20$ and $N = 3$ at the entrance to the medium (a) and at the point $z + 0.15ka_0^2$ for $\delta = 0$ (b) and $\delta = 100$ (c).

evidently roughly equal to the characteristic scale of the transverse inhomogeneity of the beam.

It is very important that a broad-band beam focuses as a whole, whereas a narrow-band beam in a nonlinear medium exhibits an additional spatial modulation, which leads to its stratification (in the case of a three-dimensional beam, to its breakup into filaments). The appearance of filaments (we shall use this term even for two-dimensional beams) is further confirmed by an analysis of the intensity distribution histrogram (Fig. 1,b) for the paraxial part of a beam with $P_0 = 40P_{cr}$ and $N = 3$ at the distance $z = 0.15ka_0^2$. The smooth curve shows the initial distribution of the beam. Since a filament is an indeterminate inhomogeneity of the radiation field in which the maximum intensity is considerably higher than average while the intensity at its boundary is close to zero, it follows that, from a statistical point of view, the presence of filaments will increase the probability for the occurence of intensity extrema. Such an increase in the number of intensity extrema is clearly to be seen in the corresponding transformation of the intensity distribution function depicted in Fig. 1,b.

An analysis of the phase distribution of the field permits us to conclude with considerable confidence that the distribution is invariant in the nonlinear medium:

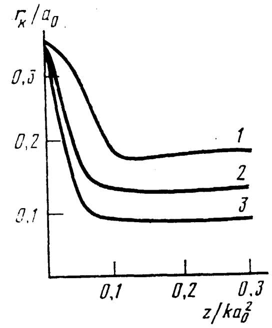

FIG. 3. Correlation range in the propagation direction near the beam axis vs z/ka_0^2 for $N = 3$, $\delta = 0$, and the following values of p: 1–10, 2–20, 3–40.

$$\omega(\varphi) = 1/2\pi, \quad \varphi = \operatorname{arctg}(v/u).$$

The breakup of a narrow-band multimode beam into filaments progresses rapidly at first and continues until the power in each filament is equal to the critical power; then the self-channeling of the radiation within an individual filament that is characteristic for a nonlinear cubic medium sets in. The characteristic transverse dimension of a filament is essentially determined by the spatial coherence range, whose variation, calculated near the beam axis in the propagation direction, is presented in Fig. 3. The rapid breakup into filaments then ceases, regardless of the initial spatial structure of the beam. Of course the size of a filament decreases as its input power increases.

The small-scale self-focusing is accompanied by a rapid increase of the relative intensity fluctuations, with subsequent saturation.

As the beam power increases the above-mentioned fluctuations decrease because of the increase in the averge intensity associated with self-focusing. In the case of beams having a frequency spectrum of finite width, the small-scale structure of the self-focusing is not so strongly expressed since the lag of the nonlinear response of the medium begins to make itself felt. For a broad band beam with $\delta \gg 1$ the nonlinear refraction will be determined by the average intensity profile, and the spatial modulation disappears entirely. Figs 4,a and 4,b show the change in the nature of the self-focusing when the width δ of the frequency spectrum is increased. It is quite evident that the broadening of the fre-

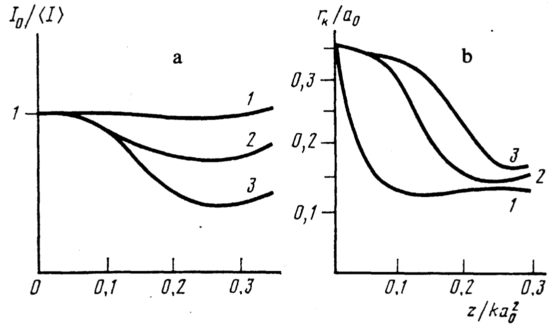

FIG. 4. Intensity (a) and correlation range (b) on the beam axis vs z/ka_0^2 for $N = 3$, $p = 20$, and the following values for the width δ of the frequency spectrum: 1–0, 2–10, 3–100.

TABLE I. Comparison of the normalized values of the second, third, and fourth central moments $\Delta = M_2/\sigma^2$, $S = M_3/\sigma^3$, and $\mathscr{E} = M_4/\sigma^4$ of the intensity on the beam axis at $z = 0.15ka_0^2$ for $P_0 = 40P_{cr}$, obtained for various values of δ in the Monte-Carlo calculations (superscript MC) with the corresponding theoretical values for the exponential law (superscript T).

δ	Δ^T	Δ^{MC}	S^T	S^{MC}	$\mathscr{E}^T$	$\mathscr{E}^{MC}$
100	1	1.00	2	2.00	4	4.00
10	1	1.02	2	2.32	4	4.24
0	1	2.19	2	8.79	4	24.98

quency spectrum results in a decrease of the mean intensity at the beam axis (Fig. 4,a). At the same time, the spatial coherence range (b) and the relative dispersion of the intensity fluctuations vary smoothly on passing to broad-band beams with $\delta \gg 1$; this indicates the absence of small-scale self-focusing. On comparing the ways in which the effective beam width $a \sim \langle I \rangle^{-1/2}$ and the coherence range r_c decrease (curves 3 on Figs. 4,a and 4,b) we see that the ratio r_c/a does not decrease, but in a number of cases may even increase on account of the aberration noise.[5]

As an analysis of the intensity and phase distribution histograms shows, the statistics of broad-band radiation approaches the Gaussian form as the width of the frequency spectrum increases.

CONCLUSION

1. The numerical treatment of the self-action problem for multimode radiation shows that the self-focusing of a narrow-band beam is a small-scale phenomenon, while the statistics of the intensity distribution changes strongly, although the phase distribution remains uniform. The transverse spatial structure is determined by the thickness of the filaments whose power is of the order of the critical power.

2. A quantitative analysis of the intensity-distribution histograms for various values of δ was carried through, using Kolmogorov's criterion for the fit; it showed that the probability $W(\delta)$ for not rejecting the exponential law increases with increasing δ and has the values

$$W(0)=0.23;\quad W(10)=0.87;\quad W(100)=0.94.$$

3. An additional test of the exponential-law hypothesis for the intensity distribution was carried through by comparing the third and fourth central moments M_3^e and M_4^e for the intensity with the corresponding theoretical values M_3^t and M_4^t for various values of δ. The results of this comparison are presented in Table I.

4. An analysis of the phase-distribution histrograms using an analogous technique gave grounds for asserting that the phase distribution very probably remains uniform.

5. The self interaction of a broad-band light beam can also be investigated by solving the equation for the mutual-coherence function,[4,5] but this only makes it possible to trace the behavior of the beam intensity and of the spatial coherence range. Analysis shows that the criterion for the applicability of this approach reduces to the requirement that the fluctuations of the nonlinear addition to the dielectric constant be small compared with its average value:

$$\sigma^2_{\varepsilon_{nl}}/[\varepsilon_2\langle I\rangle]^2=\pi G_I(0)/\tau_{nl}\langle I\rangle^2\ll 1,$$

where $G_I(0)$ is the spectrum of the intensity fluctuations near zero frequency.

6. In some problems of nonlinear optics, the small-scale self-focusing that arises is a negative factor that limits the construction of high-quality amplifiers and systems of adaptive optics. The use of broad-band laser light for such purposes makes it possible substantially to improve the spatial structure of the light beam in a nonlinear medium.

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